Can e^+e^- peaks be explained as resonances in Bhabha scattering?

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We study the behavior of two-body continuum states in QED, with special emphasis on the possible appearance of resonances. The starting point is an approximate two-body equation obtained through use of a single-time truncation in the one-photon approximation. A derivation of the relevant equation for scalar QED (the Wick-Cutkosky model) is given to illustrate our method, which has previously been applied to the case of the full QED theory. We then investigate scattering resonances in the $J = 0$ channels for both scalar QED and the standard QED, and show that within the current approximation no resonances are found that can explain the long-lived states observed in heavy-ion scattering at the GSI. We comment on other calculations that have reported positive results for the same problem.

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I. INTRODUCTION

In recent years the problem of the theoretical description of the e^+e^- system in quantum electrodynamics (QED) has received renewed attention [1–3]. This is mainly due to the surprising discovery at GSI of very narrow positron peaks in the scattering of heavy ions [4], that seem to be (almost) independent of the target and projectile mass. Some seem to decay by back-to-back emission of an electron-positron pair [5, 6]. Since the nuclear properties do not seem to play a role, these narrow resonances at center-of-mass energies in the range of 3–4 times the electron rest mass have thus led to numerous hypotheses (that we do not review, see Müller's review [7] and his contribution in [8]) concerning the existence of new states that decay into an e^+e^- pair. Among the suggestions, none of which has been substantiated by experiment, the most conservative is the occurrence of a resonance in e^+e^- scattering. An interesting alternative suggestion for the mechanism that causes these peaks has recently been given by Balantekin and Fricke [9]. They show that for a special form of the time-dependent electromagnetic field, they can get sharp peaks on a scale very different from the natural scale of the em field. It is not at all clear how fields of the required form might play a role in heavy-ion reactions.

Finally, before addressing the theory, it should be added that some doubt remains about the interpretation of the experimental results. The detailed analysis of the most recent results from the EPOS group at GSI [6] does not seem to agree with the back-to-back decay suggested by earlier research. For some of the peaks it appears to be necessary to make unphysical cuts in the data to make them stand out from the background. We also note that there is only limited agreement on the position of the resonances among the two groups (ORANGE and EPOS) that have performed the experiments, even though they were performed at the same accelerator. In Table I we have given a short summary of the experimental data, where we have lined up peaks about which the experiments might agree.

In direct experimental searches using Bhabha (e^+, e^-) scattering these resonances have not been observed (see Ref. [10] for recent measurements by one of the GSI groups; a more general discussion is presented in Ref. [8]). This puts a bound on the width of some of these resonances of about 10^{-2} eV. We are thus looking for states about 1 MeV in the continuum, with a width of at most 10^{-8} times that energy. It seems rather unlikely that "ordinary" QED physics is responsible for such unusual behavior, but the small size associated which such a resonance may mean that the short-distance properties of QED play an important role. For this reason a number of authors have applied approximate two-body equations. Surprisingly enough there appear claims in

TABLE I. A tabulation of the observed peaks in heavy-ion scattering. We compare the energies seen by the ORANGE group to those of the EPOS group.

E (keV)	
ORANGE	EPOS
518 ± 20	
	608 ± 8
642 ± 12	625 ± 8
	748 ± 8
792 ± 10	760 ± 20
811 ± 10	807 ± 8

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the literature that resonances really exist in calculations based on a two-body truncation, either in full QED [1, 3] or in the simplified Wick-Cutkosky model of spinless QED [2]. These results seem to contradict the "proof" in Ref. [11] that the appearance of high-energy narrow resonances is not easily reconciled with the scaling behavior of QED. We have shown earlier that there do not appear to be resonances in scalar QED [12], contradicting Ref. [2]. The equation used by Wong and Becker [1] to describe resonances is known to be ill conditioned, and thus makes that result untrustworthy. Finally the positive results by Spence and Vary [3] have been disputed by Horbatsch [13] using a similar numerical method (a spline technique), with a different choice of collocation points that includes the k value corresponding to the scattering energy. Unfortunately, one still needs a finite photon mass in these calculations, and it is not completely clear whether one can extrapolate to zero photon mass. Anyway we feel that the study of a related, but different, equation using a completely different numerical technique might shed some new light on these questions.

A serious problem with studies based on an approximate relativistic two-body equation is that there are many routes to such an equation (see, e.g., Ref. [14]), each giving a different, but related, result. An example of some importance to the current paper is a procedure, introduced by Gross [15], where the Bethe-Salpeter equation in the one-photon approximation is reduced to a single-time equation by putting one of the particles on the mass shell. This produces an equation with a Hermitian, energy-independent, nonlocal interaction, in contrast to some of the recent work that uses an energy-dependent non-Hermitean interaction, the non-Hermiticity arising from the possibility of real photon production.

Because of the interest in the results obtained in the procedures sketched above, it is clear that it is necessary both to check the published results and to try one or more additional equations to see whether similar results are obtained. The equation we chose to study emerges directly from the field theory, after making some physical approximations. The derivation was recently given by two of us [17] (A.K. and R.M.D.) for the case of QED. In the derivation we put one of the particles on the mass shell from the beginning. It is therefore not surprising that the result agrees with the corresponding equation derived by Gross.

This paper presents, mainly for didactic purposes, a more detailed discussion of the Wick-Cutkosky model, thus extending the discussion in our letter [12]. After the derivation of the basic two-body wave equation in Sec. II, we give the partial-wave decomposition of the equation (Sec. III), and study the numerical solution of this equation in Sec. IV and Appendix A. We examine, in some detail, our definition of resonances, as well as the technique we employ to locate resonances. This technique, complex coordinate rotation, is commonly used in nonrelativistic quantum mechanics, but has to our knowledge never been used in the current context. We then come to the main problem and study the calculation of resonances in full QED. We shall give only an outline of these calculations, as many steps parallel those in scalar QED. The detailed partial-wave decomposition of the integral kernel is given in Appendix C. In Sec. VI we discuss the relation of our work to other approaches found in the literature.

II. A TWO-BODY EQUATION FOR THE WICK-CUTKOSKY MODEL

To show the techniques underlying the derivation of a two-body equation by Klein and Dreizler [17], we apply similar techniques to the study of the Wick-Cutkosky model [16]. This is a model of QED without spin, which is specified by the Lagrange density

$$\mathcal{L} = \psi^\dagger(\Box - m^2)\psi + \tfrac{1}{2}\chi\Box\chi + g\psi^\dagger\psi\chi, \tag{1}$$

where the coupling constant g is identified with $2em$, twice the (dimensionless) unit of charge times the "electron" mass. The equations of motion derived from this equation are (note that χ and ψ commute)

$$\Box\chi(x) = g\psi^\dagger(x)\psi(x), \tag{2}$$

$$(\Box + m^2)\psi(x) = g\psi(x)\chi(x), \tag{3}$$

$$(\Box + m^2)\psi^\dagger(x) = g\psi^\dagger(x)\chi(x). \tag{4}$$

Following the lead of Klein and Dreizler [17] we consider the asymmetric matrix element of a field operator between a single-particle state with positive charge and momentum $-\mathbf{k}$, $|-\mathbf{k}\rangle$, and a two-particle state with momentum $\mathbf{P}$ and intrinsic quantum number Λ (Λ includes the rest mass of the state), $|\mathbf{P}\Lambda\rangle$. We have thus chosen the arbitrary convention that ψ annihilates a negative charge. We now find, using translational invariance,

$$\begin{aligned}\langle -\mathbf{k}|\psi(x)|\mathbf{P}\Lambda\rangle &= \psi_{\mathbf{P}\Lambda}(\mathbf{k}|\mathbf{r}t) \\ &= \exp\{-i[P_0 - E_k]t \\ &\quad + i(\mathbf{P}+\mathbf{k})\cdot\mathbf{r}\}\psi_{\mathbf{P}\Lambda}(\mathbf{k}),\end{aligned} \tag{5}$$

$$\psi_{\mathbf{P}\Lambda}(\mathbf{k}) \equiv \langle -\mathbf{k}|\psi(0)|\mathbf{P}\Lambda\rangle. \tag{6}$$

The matrix element of the equation of motion (3) between the same states is, in the center-of-mass system where $\mathbf{P} = 0$,

$$\left\{-[P_0 - E_k]^2 + E_k^2\right\}\psi_\Lambda(\mathbf{k}) = g\langle -\mathbf{k}|\chi(0)\psi(0)|\Lambda\rangle, \tag{7}$$

or, slightly simplified,

$$-P_0(P_0 - 2E_k)\psi_\Lambda(\mathbf{k}) = g\langle -\mathbf{k}|\chi(0)\psi(0)|\Lambda\rangle. \tag{8}$$

The right-hand side of this equation is now evaluated in the on-shell no-pair-creation approximation, i.e., we assume that insertion of only one-body intermediate states between χ and ψ in (8) is sufficient. In consequence, we now need the matrix elements of χ between one-body states. From the "Maxwell equation" (2) we derive

$$(\partial_0^2 - \nabla^2)\langle -\mathbf{k}|\chi(x)|-\mathbf{k}'\rangle = g\langle -\mathbf{k}|\psi^\dagger(x)\psi(x)|-\mathbf{k}'\rangle. \tag{9}$$

Again, using translational invariance, the x dependence of the matrix elements can be expressed by the exponential factor

$$\exp\{-i(E_k - E_k')t + i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}\} \tag{10}$$

multiplying the matrix element at $x = 0$. Thus we have

$$\left[(\mathbf{k}-\mathbf{k}')^2-(E_k-E_{k'})^2\right]\langle-\mathbf{k}|\chi(0)|-\mathbf{k}'\rangle = g\langle-\mathbf{k}|\psi^\dagger(0)\psi(0)|-\mathbf{k}'\rangle. \tag{11}$$

With the usual definition

$$Q^2 = q^2-(E_k-E_{k'})^2 = (\mathbf{k}-\mathbf{k}')^2-(E_k-E_{k'})^2, \tag{12}$$

and explicit evaluation of the matrix element on the right-hand side, again using the same no-pair approximation,

$$\langle-\mathbf{k}|\psi^\dagger(0)\psi(0)|-\mathbf{k}'\rangle \approx \langle-\mathbf{k}|\psi^\dagger(0)|0\rangle\langle 0|\psi(0)|-\mathbf{k}'\rangle = \frac{1}{(2\pi)^3 2}\frac{1}{\sqrt{E_k E_{k'}}} \tag{13}$$

[here we use the usual normalization, e.g., Eq. (12.7) in Ref. [18], of the Fock-space states for the Klein-Gordon equation] we conclude that

$$\langle-\mathbf{k}|\chi(0)|-\mathbf{k}'\rangle = g\frac{1}{(2\pi)^3 2}\frac{1}{\sqrt{E_k E_{k'}}}\frac{1}{Q^2}. \tag{14}$$

Setting $M = P_0$, since the zeroth component of momentum in the c.m. frame is just the invariant mass of the e^+e^- system, we find the integral equation

$$-M(M-2E_k)\psi_\Lambda(\mathbf{k}) = g\int d\mathbf{k}'\langle-\mathbf{k}|\chi(0)|-\mathbf{k}'\rangle\langle-\mathbf{k}'|\psi(0)|\Lambda\rangle = \frac{g^2}{2(2\pi)^3}\int d\mathbf{k}'\frac{1}{\sqrt{E_k E_{k'}}}\frac{1}{Q^2}\psi_\Lambda(\mathbf{k}'). \tag{15}$$

The orthogonality relation between states of different invariant mass can be proven to take the form

$$(M_i-M_j)\int d^3k\,\psi_i(\mathbf{k})(M_i+M_j-2E_k)\psi_j(\mathbf{k}) = 0. \tag{16}$$

This suggests that we should normalize the bound state solutions by

$$\int d^3k\,\psi_i(\mathbf{k})(M_i+M_j-2E_k)\psi_j(\mathbf{k}) = \delta_{ij}. \tag{17}$$

In the following, we use scaled momenta and energies that are related to the old ones by multiplication with m. We also introduce $\alpha = e^2/(4\pi) = g^2/(m^2 16\pi)$, and redefine the wave function $\tilde{\psi}(\mathbf{k}) = \psi(\mathbf{k})\sqrt{E_k}$. (Of course we delete all reference to this tilde.) We thus find the following integral equation, that we shall solve below:

$$M(M-2E_k)\psi_\Lambda(\mathbf{k}) = -\frac{\alpha}{\pi^2}\int d\mathbf{k}'\frac{1}{E_{k'}}\frac{1}{Q^2}\psi_\Lambda(\mathbf{k}'). \tag{18}$$

III. PARTIAL-WAVE DECOMPOSITION

Let us now study the partial-wave decomposition of Eq. (18), taking into account the rotational invariance of the original Lagrangian. To that purpose we need to decompose the kernel $1/Q^2$, where

$$\begin{aligned} Q^2 &= (\mathbf{k}-\mathbf{k}')^2-(E_k-E_{k'})^2 \\ &= \mathbf{k}^2+\mathbf{k}'^2-2kk'\cos\theta-(1+k^2) \\ &\quad -(1+k'^2)+2E_kE_{k'} \\ &= -2+2E_kE_{k'}-2kk'\cos\theta, \end{aligned} \tag{19}$$

and, as usual, θ is the angle between $\mathbf{k}$ and $\mathbf{k}'$. We define

$$z = \frac{E_kE_{k'}-1}{kk'}, \tag{20}$$

so that

$$\frac{1}{Q^2} = \frac{1}{2kk'}\frac{1}{z-\cos\theta} = \frac{1}{2kk'}\sum_L x_L(z)P_L(\cos\theta), \tag{21}$$

where P_L are the standard Legendre polynomials. Using the orthogonality of these polynomials $[\int dx\,P_L(x)P_{L'}(x) = \delta_{LL'}2/(2L+1)]$, we find

$$\begin{aligned} x_L(z) &= \int_{-1}^{1}\frac{2L+1}{2}\frac{1}{z-\cos\theta}P_L(\cos\theta)\,d\theta \\ &= (2L+1)Q_L(z), \qquad |z|>1. \end{aligned} \tag{22}$$

Here we have used Eq. (7.124) in Ref. [19] to evaluate the integral in terms of the Legendre functions of the second kind, Q_L. Finally, then,

$$\frac{1}{Q^2} = \frac{1}{kk'}\sum_L\frac{2L+1}{2}Q_L(z)P_L(\cos\theta). \tag{23}$$

The expansion is valid for all points outside the segment of the real axis between -1 and 1. The limiting branch points at $z = 1$ occur for $k = k'$. Since we shall use a numerical method that is not sensitive to this branch point singularity, we shall not use a finite photon mass.

If we now restrict our attention to the S wave ($L = 0$), using $Q_0(z) = -\frac{1}{2}\ln(\frac{z-1}{z+1})$, we find the integral equation (we drop the subscript Λ from now on)

$$M(M-2E_k)\psi(k) = \frac{\alpha 4\pi}{4\pi^2}\int_0^\infty\frac{k'dk'}{k\sqrt{1+k'^2}}\ln\left[\frac{E_kE_{k'}-1-kk'}{E_kE_{k'}-1+kk'}\right]\psi(k'). \tag{24}$$

Once again, the equation simplifies slightly if we introduce $\tilde{\psi} = k\psi$,

$$M(M-2E_k)\psi(k) = \frac{\alpha}{\pi}\int_0^\infty\frac{dk'}{\sqrt{1+k'^2}}\ln\left[\frac{E_kE_{k'}-1-kk'}{E_kE_{k'}-1+kk'}\right]\psi(k'). \tag{25}$$

In order to deal with the nonlinear eigenvalue problem (25) we separate it into a linear eigenvalue problem and a self-consistency condition:

$$M\psi(k) = 2E_k\psi(k) + \tilde{\alpha}\int_0^\infty \frac{dk'}{\sqrt{1+k'^2}} \ln\left[\frac{E_kE_{k'} - 1 - kk'}{E_kE_{k'} - 1 + kk'}\right]\psi(k'), \tag{26}$$

$$\tilde{\alpha} = \frac{\alpha}{\pi M}. \tag{27}$$

The QED eigenvalue problem is linear as we shall see below.

IV. NUMERICAL TECHNIQUES

The partial-wave decomposition of the Coulomb potential, or any of its more complex relatives, in momentum space leads to integral kernels containing terms that behave as $\ln|k-k'|$ for k near k'. Usually this problem is solved by the introduction of a finite photon mass, which is taken to zero after solution of the equation. This procedure is different from the one used in coordinate space, where no finite photon mass is necessary for the solution of bound state problems. It seems plausible that we should be able to forgo the use of a finite photon mass in momentum space. This can indeed be shown to be the case for our numerical method. We rely on the fact that any integral of the form

$$\int_{k_-}^{k_+} \ln|k-k'|P(k')dk', \tag{28}$$

with a polynomial $P(k)$, can be evaluated in closed form and is finite. If we thus treat the logarithmic singularities explicitly, and approximate any function multiplying them by a polynomial, we do not have to regularize the kernel.

In practice, we also map the integration interval $(0,\infty)$ on $(0,1)$, by using a conformal mapping of k to $\rho z/(1-z)$. It can easily be seen that the character of the singularities does not change, and that they now behave as $\ln|z-z'|$. We then decompose the integral kernel into a regular part I^r and a part containing the logarithmic singularities,

$$I(z,z') = I^r(z,z') + I^l(z,z')\ln|z-z'|. \tag{29}$$

We use a seventh-order generalized midpoint rule for the integration. The numerical method was checked on the Coulomb problem, where it was found that the analytic form of the $1s$ state is an eigenvector of the numerical problem with a relative accuracy better than 10^{-12}. Further details of the numerical method are discussed extensively in Appendix A.

In our numerical studies we first look at the eigenvalue problem Eq. (26), and implement the self-consistency conditions later. Even without performing a calculation, we know that the spectrum of (26) should have the following general features: For $\tilde{\alpha}=0$ we have a purely continuous spectrum $M = 2E_k$. In general, for $\tilde{\alpha}$ positive we expect bound states with $M<2$, and a continuous spectrum for $M>2$.

In the numerical calculation of bound states we find that $M=2$ is apparently a spectral accumulation point, and we expect the existence of an infinite Rydberg series converging to this threshold. This situation is drawn in Fig. 1, where we plot the solution M as a function of $\tilde{\alpha}$. In that figure we have also indicated, as the dashed line, the self-consistency condition (27). Each intersection of this curve with an eigenvalue defines a solution of the original equation. We have plotted the solutions to the equation for unrealistically large values of $\tilde{\alpha}$. The physical (Rydberg) spectrum occurs in a small range near $\tilde{\alpha} \approx 1/(2\pi 137)$, which we display in a little more detail in the inset.

We have found that the binding energy of the Rydberg levels can easily be calculated with an accuracy of 10^{-7}, if we change the equation slightly so that we calculate the binding energy directly, instead of the rest mass. The accuracy seems to be limited by the matrix diagonalization, and not by the numerical integration.

To calculate the resonances we should take a different approach, however. First of all let us state that the expectation is that the resonances are so narrow that we can take $\tilde{\alpha}$ to be real. We now have to solve an equation that, apart from the energy-dependent interaction and the square-root-type energy functions, looks very much like a nonrelativistic Schrödinger equation in momentum space. It tells us that we should be able to define resonances in a way similar to the one used in nonrelativistic quantum mechanics. As usual we try to identify a resonance with a pole in the second Riemann sheet of the Green's function $G(\omega)$. In nonrelativistic quantum mechanics one may use the complex coordinate rotation method (discussed in detail in Refs. [20, 21]) for this purpose. This method allows us to find this pole as an eigenvalue of an equation related to the original one by a complex rotation of k,

$$k \longrightarrow k\exp(-i\theta/2). \tag{30}$$

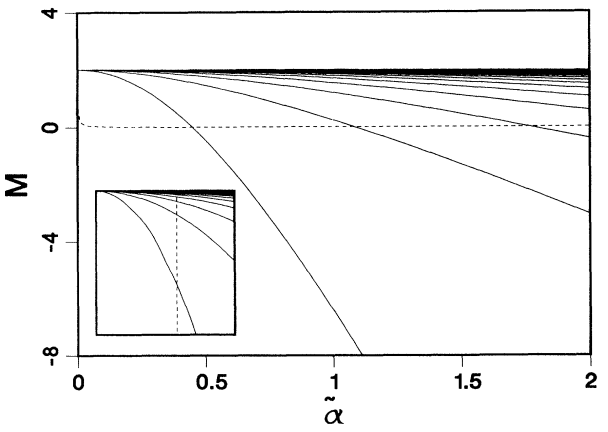

FIG. 1. The solutions of the integral equation (26) as a function of $\tilde{\alpha}$. The dashed line represents the self-consistency condition $M = \alpha/\tilde{\alpha}\pi$. Due to the smallness of α this hyperbola looks almost like a straight line. The inset shows the solutions where $2 - 2\times10^{-5} < M < 2$, $0 < \tilde{\alpha} < 2\times10^{-3}$.

In essence this implies that solutions with complex k which used to blow up at infinity (so-called Siegert boundary conditions),

$$\psi(r) \sim e^{ikr}, \quad \mathrm{Im}(k) < 0, \tag{31}$$

now become square integrable for θ large enough, $\mathrm{Im}[k\exp(-i\theta/2)] > 0$.

The mathematical foundations of the method do not extend to the current problem. Since our equations are only marginally different from the nonrelativistic case, however, we claim that numerical justification of the quality of the method should be sufficient.

The integral equation under study is

$$(M - 2E_k)\psi(k) = \frac{\tilde{\alpha}}{\pi}\int_0^\infty \frac{e^{-i\theta/2}dk'}{E_{k'}} \ln\left[\frac{E_k E_{k'} - 1 - kk'e^{-i\theta}}{E_k E_{k'} - 1 + kk'e^{-i\theta}}\right]\psi(k'), \tag{32}$$

where $E_k = \sqrt{1 + e^{-i\theta}k^2}$. From the nonrelativistic limit we know that the ln term generalizes the Fourier transform of a local potential. As a test of the approach we replace the logarithm in Eq. (32) by the S wave of the Fourier transform of the "Gaussian-shell" potential

$$V(\mathbf{r}) = A\int \sin\theta\, d\theta\, d\phi \exp\{-\beta[\mathbf{r} - \mathbf{r}_0(\theta,\phi)]^2\} = 2\pi A\{\exp[-\beta(r - r_0)^2] - \exp[-\beta(r + r_0)^2]\}. \tag{33}$$

(The Fourier transform and S wave decomposition are given in Appendix D.) For positive A this potential function has as a spherically symmetric barrier through which a wave function localized on the inside can leak to the outside, and thus becomes a resonance with a finite width.

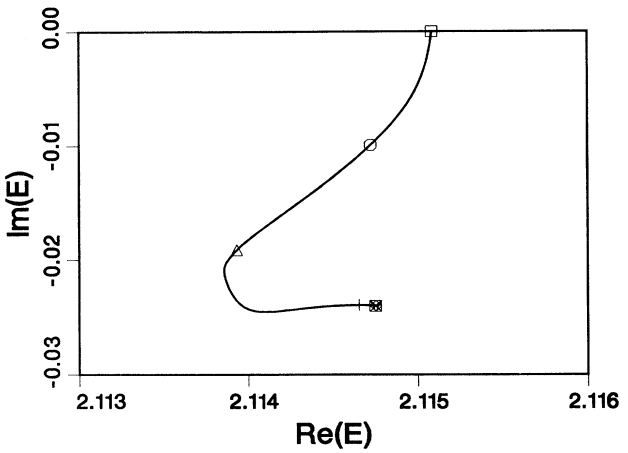

FIG. 3. Trajectory $E(\theta)$ of the lowest resonance in Fig. 2. θ runs from 0 to 0.7. Markers are drawn at each tenth radian.

We have chosen $r_0 = 4$, $\beta = 1$, and two different positive values for A, one intermediate value, $A = 0.065$, where the resonances have an appreciable width, and a large value, $A = 1.032$, where the resonances are very narrow, simulating the behavior we are trying to find for the QED problem. We have diagonalized the complex-scaled problem (using the finite difference method of Appendix A) for several values of θ. The results for intermediate A are given in Fig. 2, where one can see the appearance of two resonances as θ increases above a certain threshold. This is a very general feature: As we change θ so that the continuous spectrum rotates into the complex plane, at some point the continuous spectrum crosses a resonance (on the second Riemann sheet) which now becomes a complex eigenvalue, and remains so as we further increase θ. This uncovering can be seen

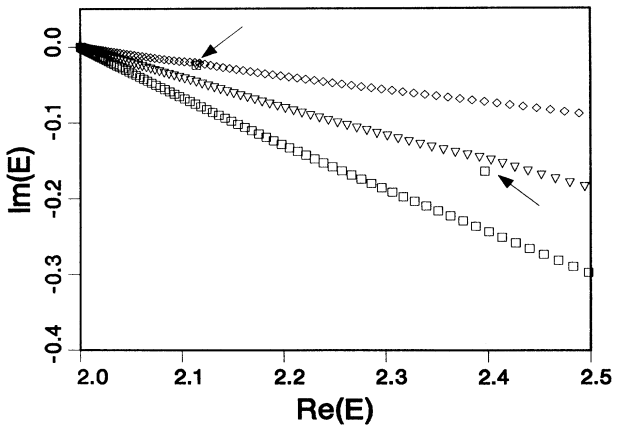

FIG. 2. Numerical complex energy solutions for the scaled eigenvalue problem with the Gaussian-shell potential for intermediate coupling A. We have used $\theta = 0.1$ (lozenges), 0.2 (triangles), and 0.3 (squares) rad. Note the two resonances indicated by arrows.

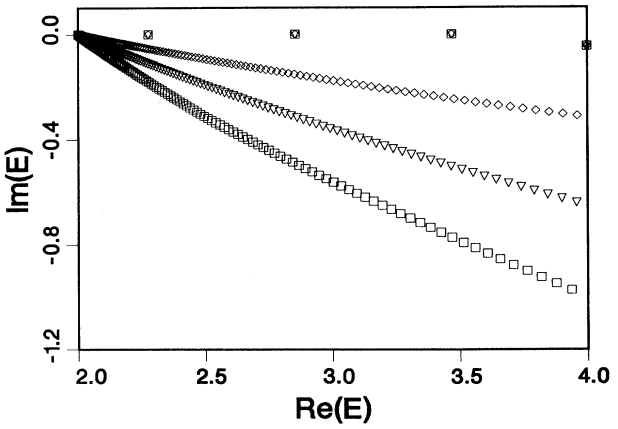

FIG. 4. Complex energy solutions for the scaled eigenvalue problem with the Gaussian-shell potential for large coupling A. We have used $\theta = 0.1$ (lozenges), 0.2 (triangles), and 0.3 (squares) rad. Note the very narrow resonances.

very nicely in Fig. 2. We show the trajectory $E(\theta)$ for the first of these two resonances in Fig. 3, which demonstrates the stabilization properties of the energy: If we had an infinite basis we would not have to worry about the choice of rotation angle, since after uncovering the resonance it would remain in the same spot. If we use a finite basis, however, we should choose an optimal rotation angle, where the energy is stationary with respect to θ. The pole in our example is almost stationary for θ from 0.3 to 0.7 rad, so we have good stabilization. Finally, we show in Fig. 4 that we can also obtain narrow resonances by increasing A. Note that due to the relativistic nature of our energy term the spectrum only rotates by θ for large energies (2θ for the usual nonrelativistic case).

Having satisfied ourselves about the viability of our method we have tried extensively to find resonances in the scalar QED problem, for $\tilde{\alpha} < 10^{-3}$. We have varied (ρ) and the scaling angle, but have been unable to discover any resonance. This, and its consequences, are discussed in our previous paper [12].

V. THE REALISTIC QED PROBLEM

Even though the result discussed in the previous section is interesting in itself, it does not allow us to draw any conclusions about the situation encountered in QED. Here one can use a formalism similar to the one used in Sec. II to derive an integral equation. As discussed in detail by Klein and Dreizler [17], there is the additional problem that the electron and positron are distinguishable, so that we need an explicit symmetrization to reflect the underlying charge-conjugation symmetry of QED. After a considerable amount of algebra, one finds the following integral equation when the small components of all Dirac spinors are eliminated:

$$[M - 2E(k)]\Psi(\mathbf{k}) = -\frac{\alpha}{4\pi}\int d\mathbf{k}' I(\mathbf{k},\mathbf{k}'|\boldsymbol{\sigma}^{(e)},\boldsymbol{\sigma}^{(p)})\Psi(\mathbf{k}'), \tag{34}$$

where the kernel I is defined as

$$\begin{aligned}\frac{4EE'}{(E+m)(E'+m)} I = {} & (2q^{-2} - Q^{-2})[1 + \boldsymbol{\kappa}\cdot\boldsymbol{\kappa}' + i\boldsymbol{\sigma}^{(e)}\cdot(\boldsymbol{\kappa}\times\boldsymbol{\kappa}')] \\ & \times[1 + \boldsymbol{\kappa}\cdot\boldsymbol{\kappa}' + i\boldsymbol{\sigma}^{(p)}\cdot(\boldsymbol{\kappa}\times\boldsymbol{\kappa}')] \\ & + Q^{-2}\boldsymbol{\sigma}^{(e)}\cdot\boldsymbol{\sigma}^{(p)}[\boldsymbol{\sigma}^{(e)}\cdot(\boldsymbol{\kappa}-\boldsymbol{\kappa}')][\boldsymbol{\sigma}^{(p)}\cdot(\boldsymbol{\kappa}-\boldsymbol{\kappa}')] \\ & + Q^{-2}[4\boldsymbol{\kappa}\cdot\boldsymbol{\kappa}' + 4i\mathbf{S}\cdot\boldsymbol{\kappa}\times\boldsymbol{\kappa}'].\end{aligned} \tag{35}$$

Here the following quantities have been introduced: Q was already defined in Eq. (19), $q^2 = (\mathbf{k}-\mathbf{k}')^2$, $\boldsymbol{\kappa} = \mathbf{k}/(E_k+m)$, and $\boldsymbol{\sigma}$ denotes the Pauli matrix for electrons (e) or positrons (p), respectively. Finally $\mathbf{S} = \frac{1}{2}(\boldsymbol{\sigma}^{(e)} + \boldsymbol{\sigma}^{(p)})$ is the total spin.

In contrast to Eq. (25), the mass M appears only linearly. This makes the equation a little easier to solve, since we do not have to impose additional self-consistency.

Equation (34) has been shown to contain the Breit limit, and as such contains much of the same physics as the equation used by Spence and Vary [3]. In particular it contains a magnetic force, claimed to be responsible for the appearance of resonances. Of course the task of making a partial-wave decomposition of this kernel is much more formidable than for the scalar case. The final result, which can be summarized sufficiently concisely, is listed in Appendix C. We have first checked that we can obtain a reasonable description of the bound state properties of the equation. These are given quite well by our numerical method, and agree to fourth order in α with the results give in Bethe and Salpeter [22].

Next we have complex scaled the integral kernel, which has analytic properties paralleling those of the kernel discussed in Sec. III. We have concentrated our attention on the $J = 0^+$ and the $J = 0^-$ channels, since the last is usually considered to be responsible for the resonant structures seen in the experiment. We have again performed extensive numerical calculations, using a variety of values for θ and ρ. A result of a representative calculation is given in Fig. 5. If we compare this to the resonances seen in Fig. 3, we see that there is not even a hint of any nonsmooth behavior in the discretized continuous spectrum.

VI. DISCUSSION AND OUTLOOK

We have thus found no resonances in the current approximation to QED. As argued in our previous paper

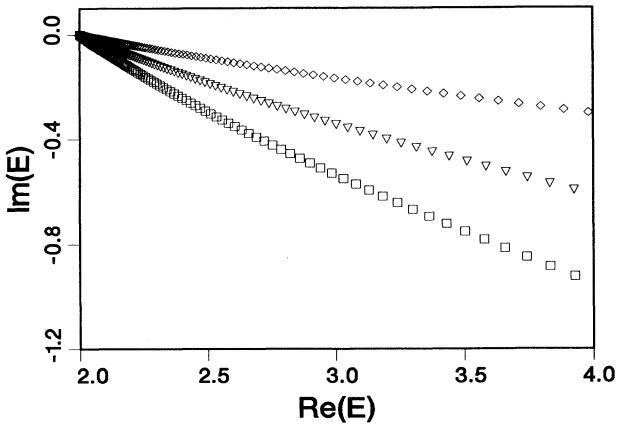

FIG. 5. A representative example calculation for the $J = 0^-$ channel. We have used $\theta = 0.1$ (lozenges), 0.2 (triangles), and 0.3 (squares) rad.

[12] the positive result obtained by [2] is probably due to the erroneous interpretation of numerical apects of their result. The work by Spence and Vary is also in doubt, since the results by Horbatsch [13] seem to show an extreme sensitivity to the choice of collocation points, so that the numerical apects are once again suspect.

Together with the results of our calculation we draw the tentative conclusion that the mechanism studied in this paper (two-body states with one-photon exchange) cannot be responsible for the appearance of resonances. This is in accordance with the virial argument by Grabiak *et al.* [11]. We do not claim that this argument rules out resonances caused by multiple-photon exchange [23].

We have also not ruled out that the high field effects occurring in the heavy-ion reactions may actually be responsible for the occurrence of these resonances, even though there are no convincing theoretical grounds for such a conclusion. (See however [9] where it is shown that resonant production with a moderately slow—but unrealistic—time variation can give rise to sharp resonances.) It is not clear to us whether the fact that pair production seems to occur near the two-pair-creation threshold can be explained in this scenario.

We are therefore very interested in the results of the ongoing experiments at Argonne. We believe that these will be able to shed some light on this confusing but interesting issue.

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APPENDIX A: FINITE DIFFERENCE SOLUTION OF THE LINEAR EIGENVALUE PROBLEM

In order to facilitate the numerical solution of Eq. (26) we introduce the new integration variable (ρ denotes a scale)

$$k = \rho z/(1-z), \quad z = k/(k+\rho). \tag{A1}$$

An integral equation of the form

$$M\psi(k) = 2E(k)\psi(k) - \alpha \int_0^\infty I(k,k')\psi(k')dk' \tag{A2}$$

becomes

$$M\psi(z) = 2E_\phi\psi(z) - \alpha\rho \int_0^1 dz' \frac{1}{(1-z')^2} I(z,z')\psi(z'). \tag{A3}$$

The kernel I is next decomposed into a singular and a regular part,

$$I(z,z') = I^r(z,z') + I^l(z,z')\ln[zz'/(z-z')^2]. \tag{A4}$$

The integral over z' is then evaluated numerically by approximating $I^{r,l}(z,z')\frac{1}{(1-z')^2}\psi(z')$ for fixed z with a seventh-order interpolating polynomial in z', using function values at the grid points $z_i = (i+1/2)h$. We calculate the integral from ih to $(i+1)h$, however. For given i we use the function values at $(i-5/2)h, \ldots, (i+7/2)h$, except near the end points where we use an asymmetric interpolating polynomial, e.g., $0, 1/2h, 3/2h, \ldots, 11/2h$ for the integration from 0 to $3h$. In this process we use the boundary conditions $\psi(k=0) = \psi(k=\infty) = 0$. This leads to a high-order accurate version of the midpoint rule.

We thus integrate the logarithm exactly, approximating all other parts of the integral by polynomials, and obtain a discretized version of the integral equation, that schematically looks like

$$M\psi_i = 2E_i\psi_i - \alpha I_{ij}\psi_j. \tag{A5}$$

This can then be solved for M by a simple matrix diagonalization. We refrain from giving the complicated form of I_{ij} here, which is caused by treating the logarithmic singularity exactly.

APPENDIX B: FOURIER TRANSFORM OF THE GAUSSIAN-SHELL POTENTIAL

In order to calculate the Fourier transform of (33),

$$V(\mathbf{r}) = A \int \sin\theta\, d\theta\, d\phi \exp\{-\beta[\mathbf{r} - \mathbf{r}_0(\theta,\phi)]^2\}, \tag{B1}$$

we calculate the Fourier transform before the angular integration,

$$\int d^3r e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\beta(r-r_0)^2} = (\pi/\beta)^{3/2} \exp\left(-\frac{k^2}{4\beta}\right) \exp(i\mathbf{k}\cdot\mathbf{r}_0). \tag{B2}$$

We then find

$$\tilde{V}(\mathbf{k}) = A\left(\frac{\pi}{\beta}\right)^2 \exp\left(-\frac{k^2}{4\beta}\right) \times \int \sin\theta\, d\theta\, d\phi \exp[i\mathbf{k}\cdot\mathbf{r}_0(\theta,\phi)]$$

$$= \frac{4\pi^{5/2}}{\beta^{3/2}} A \frac{\sin kr_0}{kr_0} \exp[-k^2/(4\beta)]. \tag{B3}$$

The S-wave partial wave of this equation is given by

$$V_0(k,k') = \tfrac{1}{2}\int_{-1}^1 V(\mathbf{k}-\mathbf{k}')d\cos\theta. \tag{B4}$$

Using

$$s^2 = k^2 + k'^2 - 2kk'\cos\theta, \tag{B5}$$

we find

$$sds = -kk'd\cos\theta. \tag{B6}$$

Thus

$$V_0(k,k') = \frac{2\pi^{5/2}A}{r_0\beta^{3/2}kk'}\int_{|k-k'|}^{k+k'} e^{-\frac{s^2}{4\beta}}\sin sr_0\,ds$$
$$= \frac{2\pi^{5/2}A}{r_0\beta^{3/2}kk'}\frac{1}{2}\int_{|k-k'|}^{k+k'} e^{-\frac{s^2}{4\beta}}(e^{isr_0}-e^{-isr_0})\,ds$$
$$= \frac{2\pi^{5/2}A}{r_0\beta^{3/2}kk'}\sqrt{\pi\beta}e^{-\beta r_0^2}\mathrm{Im}\left[\mathrm{erfc}(ir_0\sqrt{\beta}+|k-k'|)-\mathrm{erfc}(ir_0\sqrt{\beta}+[k+k'])\right]. \tag{B7}$$

APPENDIX C: PARTIAL-WAVE DECOMPOSITION OF THE INTEGRAL KERNEL

If we employ the following definitions,

$$\begin{aligned} q^2 &= (\mathbf{k}-\mathbf{k}')^2,\\ Q^2 &= q^2-(E_k-E_{k'})^2,\\ z &= \frac{E_kE_{k'}-m^2}{kk'},\\ y &= \frac{\frac{1}{2}(k^2+k'^2)}{kk'},\\ \tilde{Q}_L(z) &= \frac{LQ_{L-1}(z)+(L+1)Q_{L+1}(z)}{2L+1},\\ \hat{Q}_L(z) &= \frac{L\tilde{Q}_{L-1}(z)+(L+1)\tilde{Q}_{L+1}(z)}{2L+1},\\ \tilde{I} &= \frac{4EE'}{(E+m)(E'+m)}\frac{kk'}{2\pi}I, \end{aligned} \tag{C1}$$

the partial-wave decomposition of the integral kernel can be written as

$$\begin{aligned} \langle LSJM|\tilde{I}|L'S'JM\rangle = {}& \delta_{SS'}\delta_{LL'}\left(2\left[Q_L(y)+2\kappa\kappa'\tilde{Q}_L(y)+(\kappa\kappa')^2\hat{Q}_L(y)\right]-\left[Q_L(z)-2\kappa\kappa'\tilde{Q}_L(z)+(\kappa\kappa')^2\hat{Q}_L(z)\right]\right)\\ &+\delta_{SS'}\delta_{LL'}[Q_L(z)(\kappa^2+\kappa'^2)-\tilde{Q}_L(z)2\kappa\kappa'](-1)^L(\tfrac{1}{4}+S)\\ &+2\sqrt{6}\kappa\kappa'\begin{Bmatrix} L & S & J\\ S & L' & 1\end{Bmatrix}\delta_{SS'}\hat{S}\sqrt{S(S+1)}\sum_{L_a}\left(2[Q_{L_a}(y)+\kappa\kappa'\tilde{Q}_{L_a}(y)]+[Q_{L_a}(z)-\kappa\kappa'\tilde{Q}_{L_a}(z)]\right)\\ &\times\begin{Bmatrix} 1 & 1 & 1\\ L & L' & L_a\end{Bmatrix}[L_a]C^{L_a\ 1\ L}_{0\ 0\ 0}C^{L_a\ 1\ L'}_{0\ 0\ 0}\\ &+36\kappa^2\kappa'^2\hat{S}\hat{S}'\sum_{L_aL_bL_1L_2}[L_a][L_b]\hat{L}_1\hat{L}_2[2Q_{L_b}(y)-Q_{L_b}(z)](-1)^{L_1+L'+S+J}\\ &\times\begin{Bmatrix} 1/2 & 1/2 & S\\ 1/2 & 1/2 & S'\\ 1 & 1 & L_a\end{Bmatrix}\begin{Bmatrix} 1 & 1 & L_1\\ 1 & 1 & L_2\\ 1 & 1 & L_a\end{Bmatrix}\begin{Bmatrix} L & S & J\\ S' & L' & L_a\end{Bmatrix}\begin{Bmatrix} L_1 & L_2 & L_a\\ L' & L & L_b\end{Bmatrix}C^{1\ 1\ L_1}_{0\ 0\ 0}C^{1\ 1\ L_2}_{0\ 0\ 0}C^{L_1\ L_b\ L}_{0\ 0\ 0}C^{L_2\ L_b\ L'}_{0\ 0\ 0}\\ &+6\hat{S}\hat{S}'\sum_{L_aL_1L_2}Q_{L_a}(z)\hat{L}_aC^{L_1\ L\ L'}_{0\ 0\ 0}(-1)^{L+S+J}\\ &\times\sum_{L_1L_2}\hat{L}_1C^{1\ 1\ L_1}_{0\ 0\ 0}\begin{Bmatrix} L & S & J\\ S' & L' & L_1\end{Bmatrix}\begin{Bmatrix} 1/2 & 1/2 & S\\ 1/2 & 1/2 & S'\\ 1 & 1 & L_1\end{Bmatrix}\{\kappa^2\delta_{L_2L}\delta_{L_aL'}+\kappa'^2\delta_{L_2L'}\delta_{L_aL}(-1)^{L_1}\}\\ &+12\kappa\kappa'(-1)^{L'+S+J}\hat{S}\hat{S}'\sum_{L_aL_1}\delta_{L_1,\mathrm{even}}Q_{L_a}(z)\begin{Bmatrix} L & S & J\\ S' & L' & L_1\end{Bmatrix}\begin{Bmatrix} 1/2 & 1/2 & S\\ 1/2 & 1/2 & S'\\ 1 & 1 & L_1\end{Bmatrix}[L_1]\\ &\times\begin{Bmatrix} 1 & 1 & L_1\\ L' & L & L_a\end{Bmatrix}[L_a]C^{1\ L_a\ L}_{0\ 0\ 0}C^{1\ L_a\ L'}_{0\ 0\ 0}. \end{aligned} \tag{C2}$$

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