Renormalization of Yang-Mills theories in light-cone gauge: Recent results

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Recent results concerning the light-cone gauge are reported. In particular we revisit the nonsinglet Altarelli-Parisi kernel in the leading-log approximation. Ultraviolet and infrared singularities are neatly disentangled thanks to the Mandelstam-Leibbrandt prescription in the vector propagator. A new interpretation emerges of "real" and "virtual" diagram contributions. Then we exhibit the one-loop renormalization of the composite operator $A_\mu^a(0)A_\nu^b(0)$ as a first step towards full control of gauge-dependent insertions.

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I. INTRODUCTION

Perturbative calculations in non-Abelian gauge theories started being performed several years ago in the light-cone gauge, owing to its simple partonic interpretation. As a matter of fact, in the deep-inelastic limit planar Feynman diagrams give the dominant contribution to structure functions, as is well known.

Beyond the tree level, however, some difficulties were met in handling the extra singularities coming from the vector propagator [1,2]. The usually adopted Cauchy principle value (CPV) prescription led indeed to inconsistencies already at the one-loop level. If taken literally, it entails renormalization "constants" which depend on longitudinal momentum fractions [1] and a violation of the relevant Ward identities. Although the final results turned out to be correct owing to ingenious physical insights [1], there was a clear feeling that some theoretical developments were missing. We quote from Ref. [1]: "...hopefully, it will be a challenge for field theory experts to provide a more formal support for our "phenomenological rules"."

After the proposal of the Mandelstam-Leibbrandt (ML) causal prescription [3,4] for the spurious singularity and the independent work in which it has been derived in a canonical scheme [5] and renormalization at any order in the loop expansion [6] has been proven, we think that all necessary ideas as well as technical tools [7] are now at hand to put such results on a firmer basis. As a first step in this direction we will discuss a new derivation of the nonsinglet Altarelli-Parisi (AP) kernel in the leading-log approximation and show how ultraviolet (UV) and infrared (IR) singularities can be neatly disentangled in a formulation where renormalization "constants" are truly constant. In so doing the role of real and virtual contributions to the AP kernel is elucidated and a new physical interpretation is obtained.

The second issue we discuss is the renormalization of composite operators [8,9], a topic not touched in Ref. [7]. The main point here is to understand whether nonlocal counterterms are needed as happens for the one-particle-irreducible (1PI) vertices [6,7]; were it the case, an abnormal mixing would occur in gauge-dependent quantities, jeopardizing dimensional considerations. A direct one-loop calculation for $A_\mu^\gamma(0)A_\nu^\delta(0)$ shows that nonlocal singular terms, although present in single diagrams, do indeed cancel in their sum, giving rise to normal mixing. The generalization of this property to any order in the loop expansion will be reported elsewhere [10]. The behavior of more general composite operators is at present under investigation.

II. THE AP KERNEL

In the sequel we use dimensional regularization to control UV divergences whereas IR singularities do not occur as we stay suitably "off shell." We closely follow the concepts and notation of Ref. [1], with which the reader is invited to consult.

The nonsinglet AP kernel is defined as the coefficient in the leading-log approximation when $p^2 \to 0$ of the quantity

$$\Gamma\left[x,\alpha,\frac{Q^2}{-p^2}\right] = \mathcal{V}\left\{1+ixg^2C_F\int\frac{d^4k}{(2\pi)^4}\delta\left[x-\frac{nk}{np}\right]\frac{1}{(k^2)^2}\times\mathrm{Tr}\left[\frac{\not n}{4nk}\not k\gamma^\mu\not p\gamma^\nu\not k\right]\mathrm{disc}[D_{\mu\nu}(p-k)]\right\}, \quad (1)$$

C_F being the color factor and g the coupling constant. In this equation $\mathcal{V}$ represents the virtual Feynman diagram contribution due to the *UV renormalized* quark propagator in the limit when $p^2 \to 0$ [1], p is the spacelike incoming quark momentum, k the (spacelike) four-momentum of the outgoing quark to be integrated up to $-Q^2, Q$ being the momentum of the incoming off-shell photon, and $D_{\mu\nu}$ the gluon propagator. No UV singularities occur as we are considering an absorptive part. The gauge is specified by the vector $n_\mu(n_\nu A^\mu=0)$ and kinematics can be usefully parametrized as

$$p=\left[P+\frac{p^2}{4P},0,P-\frac{p^2}{4P}\right],$$
$$n=\left[\frac{np}{2P},0,-\frac{np}{2P}\right], \tag{2}$$
$$k=\left[\xi P+\frac{k^2+k_\perp^2}{4\xi P},\mathbf{k}_\perp,\xi P-\frac{k^2+k_\perp^2}{4\xi P}\right],$$

the quantity ξ being interpreted as the longitudinal momentum fraction P of p carried by k.

The crucial point in our treatment concerns the discontinuity of the vector propagator. In the ML prescription, the "spurious" pole possesses a causal nature and thereby **contributes** to the absorptive part of $D_{\mu\nu}$, at variance with the CPV prescription. We have indeed [5,7]

$$\begin{aligned}\mathrm{disc}[D_{\mu\nu}(q)]&=-2\pi i\delta(q^2)\theta(q_0)\left[-g_{\mu\nu}+\frac{2\hat{n}q}{\hat{n}n}\frac{n_\mu q_\nu+n_\nu q_\mu}{q_\perp^2}\right]\\&\quad+2\pi i\delta(q^2+q_\perp^2)\theta(q_0)\frac{2\hat{n}q}{\hat{n}n}\frac{n_\mu q_\nu+n_\nu q_\mu}{q_\perp^2}\ .\end{aligned} \tag{3}$$

The vector $\hat{n}$, which is conjugate to n in the ML prescription $[np]^{-1}\equiv\hat{n}p/(\hat{n}pnp+i\epsilon)$ is here parametrized as $(P/np,0,P/np)$.

The second addendum was not considered in Ref. [1]. Introducing Eq. (3) in Eq. (1), an easy calculation gives

$$\Gamma\left[x,\alpha,\frac{Q^2}{-p^2}\right]=\mathcal{V}\left\{1+\frac{\alpha}{2\pi}C_F\ln\left[\frac{Q^2}{-p^2}\right]\times\left[-1-x+\frac{2}{(1-x)_+}\right]\right\}, \tag{4}$$

where $\alpha=g^2/4\pi$ and the distribution $(1-x)_+^{-1}$ is defined for any suitable test function ϕ as $\int_0^1\phi(x)(1-x)_+^{-1}dx\equiv\int_0^1[\phi(x)-\phi(1)](1-x)^{-1}dx$.

We stress that the singularity at $x=1$ is here regularized, not by the virtual contribution (which occurs in $\mathcal{V}$), but by the "spurious" pole, which behaves as a ghost and softens the wild IR behavior of the gluon propagator [11].

The one-loop correction to the quark propagator in light-cone gauge with the ML prescription, has been thoroughly discussed in Ref. [7]. Using dimensional regularization and minimal subtraction scheme [12], we get, for the *UV renormalized* self-energy,

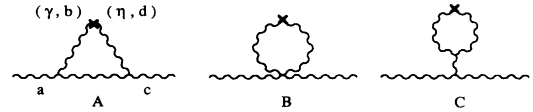

FIG. 1. One-loop diagrams with composite operator insertion.

$$\Sigma=-i\frac{\alpha}{4\pi}C_F\ln\left[\frac{-p^2}{4\pi\mu^2}\right]\left[\not{p}-4\not{n}\frac{\hat{n}p}{n\hat{n}}\right]-\Sigma_f\ , \tag{5}$$

the quantity Σ_f being finite in the limit $p^2\to 0$. From Eq. (5), following the rules given in Ref. [1], one easily derives the expression

$$\mathcal{V}=\left[1-3\frac{\alpha}{4\pi}C_F\ln\left[\frac{-p^2}{4\pi\mu^2}\right]\delta(1-x)+\text{finite terms}\right] \tag{6}$$

which, when inserted in Eq. (4), leads eventually, at order α, to the factor multiplying $\ln(-p^2)$,

$$\frac{\alpha}{2\pi}C_F\left[-1-x+\frac{2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right]\equiv\frac{\alpha}{2\pi}C_F\left[\frac{1+x^2}{1-x}\right]_+\ , \tag{7}$$

namely to the well-known AP result. We remark in passing that, were we interested in "counting" gluons, the ghost contribution should not be considered and then the transverse gluon discontinuity would fully expose the expected IR singular factor $(1-x)^{-1}$ [13].

III. COMPOSITE OPERATORS

As a first example we discuss the operatorial quantity $A^{\gamma,b}(0)A^{\eta,d}(0)$ in the one-loop approximation. As is expected [14], this quantity develops a singularity which is here regularized dimensionally.

The relevant diagrams to be considered are (Bose symmetric ones are understood) shown in Fig. 1, where γ,η are Lorentz indices, a,b,c,d refer to color and the cross denotes the composite operator insertion. All singularities unrelated to the insertion are understood to be already renormalized [14]. A lengthy but straightforward calculation gives [10]

$$\begin{aligned}A^{\gamma,b}(0)A^{\eta,d}(0)=[A^{\gamma,b}(0)A^{\eta,d}(0)]+\frac{1}{2-\omega}\left[\frac{g}{4\pi}\right]^2\Bigg\{&\frac{\eta^\gamma n^\eta}{(n\hat{n})^2}\{T^b,T^d\}_+^{pq}\hat{n}A^p(0)\hat{n}A^q(0)\\&+[T^b,T^d]^{pq}\mathcal{A}^{\gamma\eta}_{\mu\rho}A^{\mu,p}(0)A^{\rho,q}(0)+\frac{1}{g}f^{bde}\Delta^{\gamma\eta}_\tau A^{\tau,e}(0)\Bigg\}\ ,\end{aligned} \tag{8}$$

where T^a are the color algebra generators in the adjoint representation; the antisymmetric tensor $\mathcal{A}^{\gamma\eta}_{\mu\rho}$ is defined as

$$\mathcal{A}^{\gamma\eta}_{\mu\rho}=\tfrac{1}{2}(g^\gamma_\rho\mathcal{P}^\eta_\mu-g^\gamma_\mu\mathcal{P}^\eta_\rho+g^\eta_\mu\mathcal{P}^\gamma_\rho-g^\eta_\rho\mathcal{P}^\gamma_\mu)\ , \tag{9}$$

the projection operator $\mathcal{P}^\eta_\mu$ being

$$\mathcal{P}^\eta_\mu=g^\eta_\mu-\frac{n_\mu\hat{n}^\eta+\hat{n}_\mu n^\eta}{n\hat{n}}\ , \tag{10}$$

and the antisymmetric differential operator $\Delta_\tau^{\gamma\eta}$ is

$$\Delta_\tau^{\gamma\eta}=g_\tau^\gamma\left[2\partial^\eta-2\frac{\hat{n}^\eta n\partial}{n\hat{n}}+\frac{n^\eta\hat{n}\partial}{n\hat{n}}\right]-\frac{n^\gamma\hat{n}\tau}{n\hat{n}}\left[\partial^\eta-\frac{\hat{n}^\eta n\partial}{n\hat{n}}\right]-\{\gamma\leftrightarrow\eta\} . \quad (11)$$

In Eq. (8) the first addendum at the right-hand side is the one-loop renormalized composite operator [14].

One should remark that Eq. (8) exhibits a "normal" mixing; in particular no nonlocal operators of the kind $(n^\gamma n^\delta/n\hat{n})(\hat{n}\partial/n\partial)$, which *do contribute* in graphs A and B, but cancel in their sum, are involved. One can prove that this property holds true to any order in the loop expansion [10].

The final goal is of course to get a full control over gauge-dependent structures which may appear in concrete QCD perturbative calculations [15]. We consider the result we have derived above as a first step in this direction.

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