Electromagnetic vacuum fluctuations and electron coherence

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The coupling of the quantized electromagnetic field to coherent electrons is investigated. The effects both of photon emission and of the electromagnetic vacuum fluctuations upon electron interference are analyzed. The modifications of the vacuum fluctuations due to a conducting plate lead to a decrease in the amplitude of the interference oscillations. The possibility of observing this effect is discussed. It is also shown that there is an analogue of the Aharonov-Bohm effect in which electron interference is sensitive to vacuum fluctuations in regions from which the electrons are excluded.

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I. INTRODUCTION

Two of the more fundamental aspects of quantum theory are quantum coherence and vacuum fluctuations. Quantum coherence and the associated interference phenomena are the essence of quantum mechanics. An especially subtle manifestation of quantum coherence occurs in the Aharonov-Bohm effect [1]. Here the nonlocal aspects of quantum mechanics are revealed in the ability of coherent electrons to respond to a classical magnetic field in a region from which the electrons are excluded. Similarly, vacuum fluctuations are the essence of quantum field theory, the quantum theory of a system with an infinite number of degrees of freedom. Again a simple but subtle manifestation of the vacuum fluctuations of the electromagnetic field is the Casimir effect [2], the force of attraction between a pair of parallel, uncharged perfectly conducting plates. Although it has apparently been observed [3], the experiments are not as precise as one would like for such a fundamental effect.

In this paper, we will discuss effects which arise when coherent electrons are coupled to the quantized electromagnetic field. (By the phrase "coherent electrons," we mean any single- or multiple-electron quantum state with definite phase information, as opposed to a mixed or thermal state.) An obvious effect is the emission of photons, which will in turn alter the interference pattern. This will take the form of both a phase shift and of a distortion whereby the amplitude of the interference oscillations is changed. The latter can be understood as due to decoherence, the electromagnetic field coupling causing a loss of quantum coherence. It can also be understood as an aspect of quantum measurement theory: Photon emission may make it possible to determine which path an electron takes in a double slit experiment. A less obvious effect is that of the electromagnetic vacuum fluctuations upon the interference pattern. These are best understood in a context such as that of the Casimir effect, when there are conducting boundaries present which modify the vacuum fluctuations. It will be shown that this modification can also change the amplitude of the interference oscillations (leading to "vacuum decoherence") [4]. It will also be shown that the combined effects of photon emission and vacuum fluctuations can be expressed as a double surface integral [Eq. (33)] over a world sheet bounded by two electron world lines. Consequently, coherent electrons are sensitive to vacuum fluctuations in regions from which they are excluded, an effect which combines the features of the Aharonov-Bohm and Casimir effects.

In Sec. II the essential formalism is developed. The effects of vacuum fluctuations and of photon emission are considered separately, and expressions for the phase shift and amplitude distortion due to each are given. In Sec. III, the effects of conducting boundaries are discussed. The phase shift and amplitude distortion are calculated explicitly for the case of electrons traveling parallel to a single conducting plate. It is shown that the phase shift is due to the electrostatic Aharonov-Bohm effect. The change in the amplitude of the interference pattern is the true effect of the vacuum fluctuations. In Sec. IV the results are summarized and discussed. The magnitude of the change in amplitude is estimated and the possibility of its experimental detection is discussed. Unless otherwise noted, Lorentz-Heaviside units with $\hbar = c = 1$ will be used.

II. BASIC FORMALISM

A. Vacuum persistence amplitude

The effects of vacuum fluctuations upon an electron interference pattern may be most simply derived using the formula for the vacuum persistence amplitude first given by Schwinger [5]. If j^μ is a classical current which is coupled to the quantized photon field, there will in general be photon creation. We consider a situation in which the current is switched on at a finite time in the past and off at a finite time in the future. If $|\text{in}\rangle$ is the photon in vacuum, and $|\text{out}\rangle$ is the out vacuum, then $\langle\text{out}|\text{in}\rangle$ is the vacuum persistence amplitude. This is the probability amplitude for the vacuum to remain the vacuum, that is, the amplitude for no photons to be emitted. This amplitude has a simple expression in terms of the photon Feynman propagator $D_F^{\mu\nu}$:

$$\langle\text{out}|\text{in}\rangle = \exp\left[-\frac{i}{2}\int j_\mu(x)j_\nu(x')D_F^{\mu\nu}(x,x')d^4x d^4x'\right]. \tag{1}$$

If the current is due to a point particle of charge e, then it has the form

$$j^\mu(x) = e\int d\tau\, u^\mu(\tau)\delta^{(4)}(x - x(\tau)), \tag{2}$$

where u^μ is the particle's four-velocity, τ is its proper time, and its world line is given by $x^\alpha = x^\alpha(\tau)$. In this case, the vacuum persistence amplitude may be expressed as

$$\langle \text{out}|\text{in}\rangle = \exp\left[-\frac{ie^2}{2}\int D_F^{\mu\nu}(x,x')dx_\mu dx'_\nu\right], \tag{3}$$

where the line integrations are taken over the world line of the charge. In the classical limit, one expects the argument of the above exponential to have a large, negative real part. This expresses the improbability that no photons will be emitted by a large, time-dependent current.

Now consider an electron interference experiment in which coherent electrons may travel from x_i to x_f along either of two classical paths C_1 or C_2. First let us recall the analysis of this experiment when the effects of the electromagnetic field are ignored. Let ψ_1 and ψ_2 be the amplitudes for an electron to travel along C_1 and C_2, respectively. Then the superposed amplitude is $\psi = \psi_1 + \psi_2$, and the number density of electrons detected at x_f is

$$n_0(x_f) = |\psi|^2 = |\psi_1|^2 + |\psi_2|^2 + 2\,\text{Re}(\psi_1\psi_2^*), \tag{4}$$

the last term being responsible for the interference pattern. Next we incorporate the coupling to the quantized electromagnetic field. Suppose that no photons are emitted. This is the interference pattern that would be detected by a veto experiment. In such an experiment, we would set up detectors for both electrons and photons. The interference pattern will be observed even if the flux of electrons is sufficiently small that there is only one electron in the apparatus at any one time. In the veto experiment, we would use such a low flux and arrange that whenever a photon is detected, the electron detectors are shut off for a time at least as long as the electron travel time through the system. This ensures that we are observing the interference pattern produced by those electrons which have not emitted a photon. The net amplitude for an electron to pass through either slit without emitting a photon is

$$\Psi = \langle \text{out}|\text{in}\rangle_1\psi_1 + \langle \text{out}|\text{in}\rangle_2\psi_2, \tag{5}$$

where, e.g., $\langle \text{out}|\text{in}\rangle_1$ is the amplitude that no photon is emitted when an electron travels along C_1. Now the number density of electrons detected at x_f is

$$n_V(x_f) = |\Psi|^2 = |\langle \text{out}|\text{in}\rangle_1\psi_1|^2 + |\langle \text{out}|\text{in}\rangle_2\psi_2|^2 + 2\,\text{Re}(\langle \text{out}|\text{in}\rangle_1\langle \text{out}|\text{in}\rangle_2^*\psi_1\psi_2^*). \tag{6}$$

The interference term now contains the factor

$$\langle \text{out}|\text{in}\rangle_1\langle \text{out}|\text{in}\rangle_2^* = \exp\left\{-\frac{ie^2}{2}\left[\int_{C_1} dx_\mu\int_{C_1} dx'_\nu D_F^{\mu\nu}(x,x') - \int_{C_2} dx_\mu\int_{C_2} dx'_\nu D_F^{*\mu\nu}(x,x')\right]\right\}. \tag{7}$$

It is useful to separate the Feynman propagator into its real and imaginary parts:

$$D_F^{\mu\nu}(x,x') = \frac{1}{2}\left[D_r^{\mu\nu}(x,x') + D_r^{\nu\mu}(x',x)\right] - iD^{\mu\nu}(x,x'), \tag{8}$$

where $D_r^{\mu\nu}(x,x')$ is the retarded Green's function, which is nonzero only if x lies to the future of x', and

$$D^{\mu\nu}(x,x') = \tfrac{1}{2}\langle 0|\{A^\mu(x), A^\nu(x')\}|0\rangle \tag{9}$$

is the Hadamard function. Note that both $D^{\mu\nu}$ and $D_r^{\mu\nu}$ are real functions.

The amplitude for an electron to traverse path C_i without emitting a photon is

$$\langle \text{out}|\text{in}\rangle_i = A_i e^{i\phi_i}, \tag{10}$$

where

$$A_i = \exp\left\{-\frac{1}{2}e^2\left[\int_{C_i} dx_\mu\int_{C_i} dx'_\nu D^{\mu\nu}(x,x')\right]\right\} \tag{11}$$

is the magnitude of the amplitude and

$$\phi_i = -\frac{1}{2}e^2\int_{C_i} dx_\mu\int_{C_i} dx'_\nu D_r^{\mu\nu}(x,x') = -\frac{1}{2}e^2\int_{C_i} dx_\mu\int_{C_i} dx'_\nu D_r^{\nu\mu}(x',x) \tag{12}$$

is the phase shift. Now we can express the electron number density as

$$n_V(x_f) = A_1^2|\psi_1|^2 + A_2^2|\psi_2|^2 + 2A_1A_2\text{Re}[e^{i(\phi_1-\phi_2)}\psi_1\psi_2^*]. \tag{13}$$

The factors A_1^2 and A_2^2 are probabilities for not emitting photons. If, for example, $\psi_2 = 0$, then $n_V(x_f) = A_1^2|\psi_1|^2$, the factor A_1^2 representing the fraction of the electrons which are counted in a veto experiment. In empty space, the retarded Green's function is nonzero only for lightlike separated points. However, the electrons move along timelike world lines, for which no two distinct points may be separated by light rays. Thus, points at which $x \neq x'$ do not contribute to ϕ_1 and ϕ_2. These phase shifts are formally infinite, but they do not depend upon any of the characteristics of the paths and are hence unobservable. When boundaries are present, this is no longer true, and finite, observable phase shifts arise.

B. Approximation of classical trajectories

An issue which was not addressed in the previous discussion is the justification for assuming that the electrons move along classical trajectories, when in fact they are quantum particles subject to uncertainties in both position and momentum. In this subsection we will examine the accuracy of this approximation. In order to do this, let us outline a formalism which is in principle capable of going beyond the approximation of classical trajectories.

Describe the electron by a Dirac spinor ψ. (It is sufficient to consider only a first-quantized Dirac theory, because we are dealing with effects at energies far below the electron's rest-mass energy.) The current is then $j^\mu = e\bar{\psi}\gamma^\mu\psi$. Let us adopt the interaction picture to describe the quantum dynamics. In this case, the time evolution is described by the U matrix:

$$U(t,t_0) = T\big(e^{-i\int_{t_0}^{t} H_I dt}\big) = T\big(e^{-i\int_{t_0}^{t} j_\mu A^\mu d^4x}\big). \quad (14)$$

Here $H_I = \int j_\mu A^\mu\, d^3x$ is the interaction Hamiltonian, A^μ is the second-quantized photon operator in an appropriate choice of gauge, and T denotes the time-ordered product. We now wish to make two simplifying assumptions. The first is that the electron is moving nonrelativistically in a wave packet state. The second is that we may ignore magnetic moment effects. The effect of the magnetic moment will be discussed in Sec. III C, where it will be argued to be very small. The Gordon decomposition of the Dirac current, j^μ, allows it to be written as a sum of a convection current and a spin current. The latter carries the information about the magnetic moment effects and is neglected. To the extent that the wave packet is sharply peaked in both position and momentum, the former takes the form of the classical current of a point charge given in Eq. (2). In this case, the U matrix takes the form

$$U(t,t_0) = T\big(e^{-ie\int_{t_0}^{t} A_\mu dx^\mu}\big). \quad (15)$$

We may assess the accuracy of this approximation by assuming that the wave packet has the minimum uncertainties in position and momentum, Δx and Δp, respectively, allowed by the uncertainty principle, so that

$$\Delta p \Delta x \approx \hbar. \quad (16)$$

Suppose that the wave packet moves with a mean speed v and that L is the characteristic length of the apparatus. We wish to require that when the electrons traverse a distance L, the momentum uncertainty introduces a spreading of the wave packet which is of the order of the original position uncertainty, Δx. This implies

$$\frac{\Delta p}{m}\frac{L}{v} \approx \Delta x, \quad (17)$$

which, when combined with Eq. (16), leads to

$$\frac{\Delta x}{L} \approx \sqrt{\frac{\lambda_C}{Lv}} = \sqrt{\frac{\lambda_{\rm dB}}{L}}, \quad (18)$$

where m is the electron's rest mass, λ_C is its Compton wavelength, and $\lambda_{\rm dB}$ is the de Broglie wavelength. The ratio $\frac{\Delta x}{L}$ is a dimensionless measure of the accuracy of the approximation which treats the electron as traveling on a classical trajectory. This is the usual criterion for a quantum particle to exhibit classical behavior. As long as L is of macroscopic dimensions and v is not exceedingly small, this ratio is very small and the approximation is excellent. For example, if $v = 0.01$ and $L = 10$ cm, then $\frac{\Delta x}{L} \approx 10^{-5}$.

If one wished to go beyond the classical trajectory approximation, it would be necessary to take into account the finite spread of the wave packet in both position and momentum. In this case, the spacetime integration in Eq. (14) will not reduce to a line integral, but rather will receive a contribution from all points inside the world tube of the wave packet. If one were to adopt a path-integral formulation rather than the operator formulation of quantum theory used here, this contribution corresponds to summing over paths in addition to the classical trajectory.

We can use Eq. (14) to derive the vacuum persistence amplitude, Eq. (1), which is the vacuum expectation value of the S matrix: $\langle 0|U(\infty,-\infty)|0\rangle$. If we expand the right-hand side of Eq. (14), apply Wick's theorem, and use the relation

$$D_F^{\mu\nu}(x,x') = -i\langle 0|T(A^\mu(x)A^\nu(x'))|0\rangle, \quad (19)$$

the result is Eq. (1).

C. Effects of photon emission

We are now in a position to consider the situation in which we do not attempt to detect the emitted photons. Thus we have to allow for the possibility that the final state of the electromagnetic field is a multiphoton state. The quantum state at late times is $U(\infty,-\infty)|0\rangle$, which is a superposition of all possible photon number eigenstates. Let $|\nu\rangle = |\{n_i\}\rangle$ denote an arbitrary photon number eigenstate with the set of occupation numbers $\nu = \{n_i\}$. Also let U_1 and U_2 be as given by Eq. (15), where the line integration is taken along the paths C_1 and C_2, respectively. Then, for example, $b_1(\nu) = \langle\nu|U_1|0\rangle\psi_1$ is the amplitude for an electron to travel along path C_1 and emit photons into state $|\nu\rangle$, and $b_2(\nu)$ is the corresponding amplitude for path C_2. The photon state which results when an electron travels along path C_i is

$$|\varphi_i\rangle = \sum_\nu b_i(\nu)|\nu\rangle. \quad (20)$$

Thus the quantum state of the combined photon-electron system, when both paths are available, is

$$|\Psi\rangle = |\varphi_1\rangle\psi_1 + |\varphi_2\rangle\psi_2. \quad (21)$$

In our previous discussion, we obtained the electron number density as the absolute square of an amplitude. However, it could also be interpreted as the expectation value of the operator

$$\hat{n}(x) = \delta(\mathbf{x}-\mathbf{x}'). \quad (22)$$

That is, if ψ is the Schrödinger wave function, then the corresponding number density is

$$n(x) = \langle\hat{n}(x)\rangle = \int \psi^*\hat{n}\psi d^3\mathbf{x}' = |\psi(x)|^2. \quad (23)$$

This operator acts only in the subspace of electron states, and so if the state vector is as given by Eq. (21), then the number density is

$$n(x) = \langle\Psi|\hat{n}(x)|\Psi\rangle = \langle\varphi_1|\varphi_1\rangle|\psi_1|^2 + \langle\varphi_2|\varphi_2\rangle|\psi_2|^2 + 2\,\mathrm{Re}(\langle\varphi_2|\varphi_1\rangle\psi_1\psi_2^*)$$
$$= |\psi_1|^2 + |\psi_2|^2 + 2\,\mathrm{Re}(\langle\varphi_2|\varphi_1\rangle\psi_1\psi_2^*). \quad (24)$$

In contrast with Eq. (13), if, for example, $\psi_2 = 0$ (path 2 is blocked), we have that $n(x) = |\psi_1|^2$. This is due to the fact that here we are including all electrons, regardless of whether or not they emit photons. We are primarily interested in the interference term, and so we need to calculate the factor

$$\langle\varphi_2|\varphi_1\rangle = \sum_\nu b_1(\nu)b_2^*(\nu). \quad (25)$$

Note that the above expression would be equivalent to Eq. (7) if only the $\nu = 0$ term, the contribution of the vacuum state, were to be included. Now we are summing over the contributions of all possible photon states. The above quantity is evaluated explicitly in the Appendix, with the result that

$$\langle\varphi_2|\varphi_1\rangle = \exp\left[e^2\int_{C_1} dx_\mu \int_{C_2} dx'_\nu \langle 0|A^\nu(x')A^\mu(x)|0\rangle\right]$$
$$\times\exp\left\{-\frac{ie^2}{2}\left[\int_{C_1} dx_\mu \int_{C_1} dx'_\nu D_F^{\mu\nu}(x,x') - \int_{C_2} dx_\mu \int_{C_2} dx'_\nu D_F^{*\mu\nu}(x,x')\right]\right\}. \quad (26)$$

The first factor on the right-hand side contains the effects of photon emission, that is, all of the $\nu \neq 0$ terms. This factor is complex, and so it is useful to write it in terms of its amplitude and phase. This is done by use of the identity

$$\langle 0|A^\nu(x')A^\mu(x)|0\rangle = D^{\mu\nu}(x,x') + \frac{i}{2}\left[D_r^{\nu\mu}(x',x) - D_r^{\mu\nu}(x,x')\right]. \quad (27)$$

The result is

$$\exp\left[e^2\int_{C_1} dx_\mu \int_{C_2} dx'_\nu \langle 0|A^\nu(x')A^\mu(x)|0\rangle\right] = \exp\left[e^2\int_{C_1} dx_\mu \int_{C_2} dx'_\nu D^{\mu\nu}(x,x')\right]e^{i\phi_\gamma}. \quad (28)$$

The phase shift due to the effects of photon emission is

$$\phi_\gamma = \frac{1}{2}e^2\int_{C_1} dx_\mu \int_{C_2} dx'_\nu \left[D_r^{\nu\mu}(x',x) - D_r^{\mu\nu}(x,x')\right]. \quad (29)$$

The two terms in this expression can be given a simple physical interpretation: The first is the phase shift introduced when a photon emitted at point x on C_1 later arrives at point x' on C_2, and the second is the corresponding effect of a photon emitted on C_2. It would be tempting to imagine that an electron on one path emits a photon, which later scatters off of another electron on the other path, thereby shifting the interference pattern. However, one must be wary of such a picture, as this phase shift is present even when there is only one electron in the system at any one time. What are actually interfering here are different classical histories for a single electron.

By use of Eqs. (26) and (27), we can write

$$\langle\varphi_2|\varphi_1\rangle = e^W\, e^{i(\phi_\gamma+\phi_1-\phi_2)}. \quad (30)$$

The phase factor is the combined phase shift due to vacuum effects and photon emission effects. The amplitude factor e^W also contains the factors A_1 and A_2, representing vacuum effects, and the factor

$$\exp\left[e^2\int_{C_1} dx_\mu \int_{C_2} dx'_\nu D^{\mu\nu}(x,x')\right] \quad (31)$$

representing photon emission. It may be written as

$$W = -\frac{1}{2}e^2\oint_C dx_\mu \oint_C dx'_\nu D^{\mu\nu}(x,x'), \quad (32)$$

where $C = C_1 - C_2$ is the closed path obtained by traversing C_1 in the forward direction and C_2 in the backward direction. By means of the four-dimensional Stokes theorem, we may write

$$W = -\frac{1}{2}e^2\int da_{\mu\nu}\int da'_{\rho\sigma} D^{\mu\nu;\rho\sigma}(x,x'), \quad (33)$$

where $da_{\mu\nu}$ is the area element of the timelike two-surface enclosed by C, and

$$D^{\mu\nu;\rho\sigma}(x,x') = \tfrac{1}{2}\langle 0|\{F^{\mu\nu}(x), F^{\rho\sigma}(x')\}|0\rangle \quad (34)$$

is the Hadamard (anticommutator) function for the field strengths. The electron interference pattern is now described by the number density

$$n(x) = |\psi_1|^2 + |\psi_2|^2 + 2e^W\mathrm{Re}[e^{i(\phi_\gamma+\phi_1-\phi_2)}\psi_1\psi_2^*]. \quad (35)$$

In the present case, all electrons are counted, regardless of whether photon emission occurs, and so no factor of A_1^2 arises. The combined effects of vacuum fluctuations and of photon emission have some similarities to the Aharonov-Bohm effect with classical electromagnetic fields. The quantity W may be expressed as either a double line integral of the symmetrized vacuum expectation value of the vector potential or as a double surface integral of the corresponding expectation value of the field strength. The electron interference pattern is thus sensitive to the vacuum fluctuations in regions from which the electrons are excluded. It is possible to construct explicit examples of this using conducting boundaries. If the electrons move in a region outside of a closed, perfectly conducting surface, the field strength Hadamard function $D^{\mu\nu;\rho\sigma}(x,x')$ in the exterior region is independent of the details of what is inside the surface. However, according to Eq. (33), we could move around conductors within this surface, change $D^{\mu\nu;\rho\sigma}(x,x')$ on the interior, and hence alter the electron interference pattern. More generally, conducting boundaries provide a means for altering and hence probing the effects of electromagnetic vacuum fluctuations upon the interference pattern and will be the topic of the remainder of this paper.

III. EFFECT OF CONDUCTING BOUNDARIES

We are interested in a situation where there is a conducting boundary present. In this case, the propagator (Feynman Green's function) may be expressed as

$$D_F^{\mu\nu}(x,x') = D_{F0}^{\mu\nu}(x-x') + D_{FR}^{\mu\nu}(x,x'), \tag{36}$$

where $D_{F0}^{\mu\nu}(x-x')$ is the photon propagator in the absence of the boundary, and the renormalized propagator $D_{FR}^{\mu\nu}(x,x')$ is the correction introduced by the presence of the boundary. Similarly, the Hadamard and retarded Green's functions may be expressed as sums of empty space and of renormalized contributions. If we are interested in comparing the interference pattern in the presence of the boundary with that in the absence of the boundary, then the relevant quantities are those formed from the renormalized Green's functions. The electron number density may now be written as

$$n(x) = |\psi_1|^2 + |\psi_2|^2 + 2e^{W_0}e^{W_R}\mathrm{Re}[e^{i(\phi_0+\phi_R)}\psi_1\psi_2^*]. \tag{37}$$

Here ϕ_0 and W_0 are the phase shift and distortion, respectively, in the absence of a boundary, and ϕ_R and W_R are the changes in the phase shift and distortion introduced by the presence of the conducting boundary.

The particular geometry which we will consider in this paper is that of a single, perfectly conducting plate. A convenient choice of gauge is the Feynman gauge, in which the empty-space Feynman propagator takes the form

$$D_{0F}^{\mu\nu}(x-x') = \eta^{\mu\nu}\Delta_F(x-x') = -\frac{i\eta^{\mu\nu}}{4\pi^2[(x-x')^2 - i\epsilon]}. \tag{38}$$

Here $\eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ is the metric tensor. The corresponding Green's function in the presence of a single perfectly conducting plate in the $z=0$ plane is of the form of Eq. (38), where the renormalized Green's function has the "image" form,

$$D_{FR}^{\mu\nu}(x,x') = -(\eta^{\mu\nu} + 2n^\mu n^\nu)\Delta_F(x-\tilde{x}'). \tag{39}$$

Here $\tilde{x}' = (t',x',y',-z')$ is the reflection of the point x' in the $z=0$ plane, and $n^\mu = (0,0,0,1)$ is the unit vector normal to the plate. If one takes the four-dimensional curl of $D_F^{\mu\nu}$ in both x and x', the result is the field strength Green's function [6] $D_F^{\alpha\beta;\rho\sigma}(x,x')$. This latter function must satisfy the boundary conditions that its transverse electric and normal magnetic components in each variable must vanish at the plate. For example, $D_F^{01;\rho\sigma}(z=0,x') = D_F^{\alpha\beta;12}(x,z'=0) = 0$, etc. One may verify that these conditions are indeed satisfied. Of course, $D_F^{\alpha\beta;\rho\sigma}(x,x')$ also satisfies the Maxwell equations in both variables.

A. Quantum phase shift

Now that we have the renormalized Green's function, we may proceed with the calculation of ϕ_R, the quantum phase shift due to the vacuum fluctuations in the presence of a single conducting plate for electrons which travel a fixed distance above the plate. Consider first the double line integral along C_1:

$$F_1 = \int_{C_1} dx_\mu \int_{C_1} dx'_\nu D_{FR}^{\mu\nu}(x,x'). \tag{40}$$

Recall that the phase shift along this path due to the presence of the plate is

$$\phi_{1R} = -\tfrac{1}{2}e^2\mathrm{Re}(F_1). \tag{41}$$

If C_1 lies in a plane parallel to and a height z_0 above the conducting plate, the $n^\mu n^\nu$ term in $D_{FR}^{\mu\nu}$ does not contribute, and

$$F_1 = -\int_{C_1}\int_{C_1} dx^\mu dx'_\mu \Delta_F(x-\tilde{x}'). \tag{42}$$

We wish to assume that the spatial length L_1 of C_1 is large compared to z_0 and that C_1 is sufficiently smooth that we may break it up into a sequence of straight-line segments, each of which is itself long compared to z_0. Let F_i be the result of a double line integration along one of these segments of length L_i. The electron moves with a speed v along this segment, which we may take to extend in the y direction. Thus,

$$dx^\mu dx'_\mu = dtdt' - dydy' = (1-v^2)dtdt' \tag{43}$$

and

$$F_i = \frac{i}{4\pi^2}\int\int \frac{(1-v^2)dtdt'}{(1-v^2)(t-t')^2 - 4z_0^2 - i\epsilon}. \tag{44}$$

Both of these integrations extend over a time interval $T_i = L_i/v$. Because $T_i \gg z_0$, we may approximate the integral by letting one integration run from $-\infty$ to ∞; the other integration then simply produces a factor of T_i. Thus,

$$F_i \approx \frac{i}{4\pi^2}T_i(1-v^2)\int_{-\infty}^{\infty}\frac{dt}{(1-v^2)t^2 - 4z_0^2 - i\epsilon}. \tag{45}$$

The remaining integration may be performed by residues, with the result

$$F_i = -\frac{L_i\sqrt{1-v^2}}{8\pi v z_0}. \tag{46}$$

However, F_1 is just the sum of the contributions of the various segments and $L_1 = \sum L_i$, and so

$$F_1 = -\frac{L_1\sqrt{1-v^2}}{8\pi v z_0}. \quad (47)$$

Because $L_i \gg z_0$, the contributions in Eq. (42) that arise when x and x' lie on different segments are negligible. In the approximation used here, F_1 is real. Finally, we have the result for the net phase shift, $\phi_R = \phi_{1R} - \phi_{2R}$, due to the presence of the plate

$$\phi_R = \frac{e^2\Delta L\sqrt{1-v^2}}{16\pi v z_0} = \frac{\alpha\Delta L\sqrt{1-v^2}}{4 v z_0}, \quad (48)$$

where $\Delta L = L_1 - L_2$ is the difference in the spatial length of the two paths, and $\alpha = e^2/(4\pi)$ is the fine-structure constant.

This phase shift may be interpreted as the Aharonov-Bohm phase shift due to the potential of the image charge. In the electron's rest frame, this is a purely electrostatic potential:

$$\Phi = \frac{e}{16\pi z}. \quad (49)$$

In the laboratory frame, in which the electron is moving in the x direction with velocity v, the vector potential is

$$A^\mu = \frac{1}{\sqrt{1-v^2}}(\Phi, v\Phi, 0, 0). \quad (50)$$

The corresponding Aharonov-Bohm phase shift is

$$\varphi_{AB} = \left(\int_{C_1} - \int_{C_2}\right) A^\mu dx_\mu. \quad (51)$$

Because $dx_\mu = (1, -v, 0, 0)dt$, we find that

$$\varphi_{AB} = \frac{\alpha\Delta L\sqrt{1-v^2}}{4 v z_0}, \quad (52)$$

and hence $\phi_{AB} = \phi_R$. Note that if $v \ll 1$, the phase shift is an electrostatic Aharonov-Bohm effect in that the dominant contribution comes from the A^0 component of the vector potential.

B. Distortion of the interference pattern

The primary effect of the quantized electromagnetic field upon the electron interference pattern is contained in the real amplitude factor e^W defined in Eq. (30). The approximation employed in the previous subsection was not sufficient to detect this contribution, but it can be computed from Eq. (32). The renormalized Hadamard function for a single conducting plate is

$$D_R^{\mu\nu}(x, x') = -\frac{\eta^{\mu\nu} + 2n^\mu n^\nu}{4\pi^2(x - \tilde{x}')^2}. \quad (53)$$

As before, we restrict attention to electron trajectories $\mathbf{x}(t)$, which lie in a plane parallel to and a height z_0 above the conducting plate. In this case,

$$W_R = \frac{\alpha}{2\pi}\wp \oint_C \oint_C \frac{dx^\mu dx'_\mu}{(t-t')^2 - [\mathbf{x}(t) - \mathbf{x}(t')]^2 - 4z_0^2}, \quad (54)$$

where $\wp$ denotes the principal part.

A simple case in which we may evaluate this integral is when z_0 is large compared to the electron's flight time. (See Fig. 1.) Let T_1 and T_2 be the flight times along paths C_1 and C_2, respectively. If $z_0 \gg T_1, T_2$, then the denominator of the above integrand is approximately $-4z_0^2$. If the motion is nonrelativistic, $v \ll 1$, then $\oint_C \oint_C dx^\mu dx'_\mu \approx (T_1 - T_2)^2$, and

$$W_R \approx -\frac{\alpha(T_1 - T_2)^2}{8\pi z_0^2}. \quad (55)$$

Note that $W_R < 0$, which corresponds to a diminution of the intensity of the interference pattern.

Let us now consider an interference arrangement in which one path C_1 passes over a conducting plate in a straight line for a distance L but the other path C_2, is entirely in empty space, as illustrated in Fig. 2. If we ignore any fringe effects at the edges of the plate, we may write

$$W_R = \frac{\alpha}{2\pi}\wp\int_0^T\int_0^T \frac{(1-v^2)dt dt'}{(1-v^2)(t-t')^2 - 4z_0^2}. \quad (56)$$

Here the electrons move across the plate at a constant speed v, and $T = L/v$ is the flight time. This integral may be explicitly evaluated with the result

$$W_R = \frac{\alpha}{2\pi}\left[\frac{T\sqrt{1-v^2}}{2z_0}\ln\left|\frac{T\sqrt{1-v^2} - 2z_0}{T\sqrt{1-v^2} + 2z_0}\right| - \ln\left|\frac{T(1-v^2) - 4z_0^2}{4z_0^2}\right|\right]. \quad (57)$$

The most interesting limit is that of non-relativistic electrons ($v \ll 1$), for which $T \gg z_0$. In this case

$$W_R \approx -\frac{\alpha}{\pi}\left[1 + \ln\left(\frac{L}{2vz_0}\right)\right]. \quad (58)$$

Note that in this case the entire effect is that due to vacuum fluctuations. The photon emission factor does not arise here because the renormalized Green's function is zero along the path C_2, and hence the sole contribution to W_R comes from the double integral along C_1. Again we find that $W_R < 0$, and so the interference pattern is reduced. One might interpret this as being due to "vacuum decoherence," that is, a tendency of the electromagnetic vacuum fluctuations to decohere the electron wave function, even in the absence of photon emission.

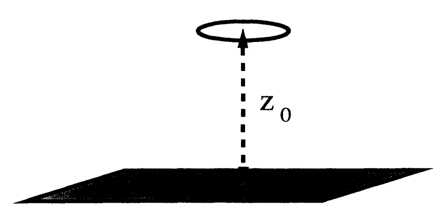

FIG. 1. Closed path for which the flight times are long compared to the electron's distance z_0 from a conducting plate.

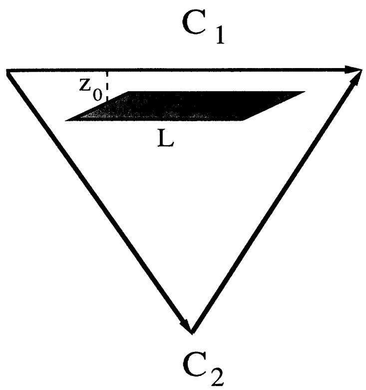

FIG. 2. Path C_1 passes over a conducting plate at a height z_0 for a distance $L \gg z_0$. Path C_2 bypasses the plate.

One might intuitively visualize this as a loss of phase information as the electron is jostled by the fluctuating electromagnetic field. What is not clear is whether this vacuum decoherence can be interpreted as arising from the ability to distinguish the paths, as one can interpret the decoherence due to photon emission.

Let us now consider a geometry in which both vacuum effects and photon emission arise. We now let both paths pass a distance z_0 over a conducting on intersecting straight lines, as illustrated in Fig. 3. The remainder of the paths are assumed to be in empty space, where the renormalized Green's function vanishes. Again fringe effects at the edge of the plate are ignored. The electrons on both paths have a speed v relative to the laboratory frame. Let L_1 and L_2 be the lengths of the trajectories over the plate, respectively, and T_1 and T_2 be the corresponding flight times. The double line integral around the closed path C may be expressed as

$$\oint_C \oint_C = \oint_{C_1} \oint_{C_1} + \oint_{C_2} \oint_{C_2} - 2 \oint_{C_1} \oint_{C_2} . \tag{59}$$

The corresponding contributions to W_R may be written as

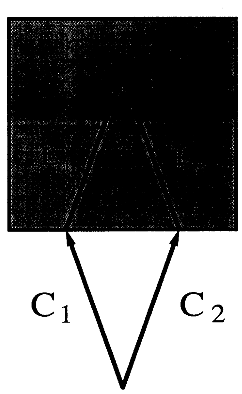

FIG. 3. Paths C_1 and C_2 begin in empty space and finish by traveling distances L_1 and L_2, respectively, above a conducting plate.

$$W_R = \frac{\alpha}{2\pi}(I_{11} + I_{22} - 2I_{12}). \tag{60}$$

The integrals I_{11} and I_{22} are of the form of the integral in Eq. (56). In the limits that $v \ll 1$ and $z_0 \ll \min(T_1, T_2)$, they are given by

$$I_{11} \approx -2\left[1 + \ln\left(\frac{L_1}{2vz_0}\right)\right], \qquad I_{22} \approx -2\left[1 + \ln\left(\frac{L_2}{2vz_0}\right)\right]. \tag{61}$$

The cross term involving both paths is

$$I_{12} = \wp \oint_{C_1} \oint_{C_2} \frac{dx^\mu dx'_\mu}{(t-t')^2 - [\mathbf{x}(t) - \mathbf{x}(t')]^2 - 4z_0^2}. \tag{62}$$

This integral is a Lorentz invariant which may be evaluated in the rest frame of the first electron, in which case the trajectory of the second electron, path C_2, is given by

$$|\mathbf{x}(t')| = v_{\rm rel}(T_1 - t), \tag{63}$$

where $v_{\rm rel}$ is the relative velocity of the two electrons, and

$$I_{12} = \wp \int_0^{T_1} dt \int_{T_1 - T_2}^{T_1} dt' \frac{1}{(t-t')^2 - v_{\rm rel}^2 (T_1 - t')^2 - 4z_0^2}. \tag{64}$$

In the non-relativistic limit, we may ignore the $v_{\rm rel}^2$ term in the demoninator. The resulting integral may be explicitly evaluated; however, the resulting expression is rather complicated. In any case, we are primarily interested in its limiting form for small z_0. For $z_0 \ll T$, where T is the smallest of T_1, T_2 and $|T_1 - T_2|$, this form is

$$I_{12} \approx -\frac{1}{2}\left\{\ln\left(\frac{L_1}{2vz_0}\right) + \ln\left(\frac{L_2}{2vz_0}\right) - \ln\left[\frac{(T_1 - T_2)^2}{T_1 T_2}\right]\right\} - 1 + O\left(\frac{z_0^2}{T^2}\right). \tag{65}$$

Finally, if we combine Eqs. (60), (61), and (65), we find

$$W_R \approx -\frac{\alpha}{2\pi}\left\{ \ln\left(\frac{L_1}{2vz_0}\right) + \ln\left(\frac{L_2}{2vz_0}\right) + \ln\left[\frac{(T_1 - T_2)^2}{T_1 T_2}\right] + 2\right\} + O\left(\frac{z_0^2}{T^2}\right). \tag{66}$$

Note that the argument of the third logarithm in the above expression cannot be too small because of the restriction that $|T_1 - T_2| \gg z_0$. As in the previous example, $W_R < 0$, and so the interference pattern is diminished by the presence of the plate. The photon emission contribution I_{12} is of the same form as the vacuum terms, but yields a positive contribution to W_R. This reflects the fact that the presence of the plate tends to reduce photon emission (the radiation field of the image charge interferes destructively with the electron's radiation field).

Note that although we have considered straight trajectories, those portions of the trajectories which are above the plate are finite, and so in general there can still be photon emission occurring.

C. Image charge and other effects

In addition to the quantum field-theoretic effects which are the main topic of this paper, an electron moving near a conductor will also be influenced by the classical electromagnetic field of the charges in the conductor. In the case of a single plane boundary, this is the field of a single image charge. The electrostatic attraction of the image will tend to pull the electron toward the plate and possibly make it difficult to construct the situation which we have analyzed, that of a trajectory at a constant height $z = z_0$ above the plate. Let the electron now have a height $z = z_0 - \xi(t)$ above the plate. The equation of motion for the electron in the direction normal to the plate is

$$m\ddot{\xi} = \frac{e^2}{2(z_0 - \xi)^2}. \tag{67}$$

As long as $\xi \ll z_0$, the solution of this equation is approximately

$$\xi(t) \approx \frac{e^2}{4mz_0^2}t^2. \tag{68}$$

Thus the characteristic time during which the electron falls into the plate is

$$t_0 = \frac{2\sqrt{z_0^3 m}}{e}, \tag{69}$$

and the image effect will be negligible so long as this time is long compared to the electron's flight time, $T = L/v$, i.e., so long as

$$L \ll v\frac{\sqrt{z_0^3 m}}{e}. \tag{70}$$

We can rewrite this constraint as

$$L \ll (6cm)v\left(\frac{z_0}{1\ \mu\text{m}}\right)^{\frac{3}{2}}. \tag{71}$$

It is possible to satisfy this constraint over a large range of the parameters. We will discuss the various constraints on an experimental configuration in more detail below.

In this paper, the single-plate geometry is our primary example due to its simplicity. However, other geometries could be of interest in attempts to observe the vacuum fluctuation effects. In particular, one could minimize the image charge effect by using two parallel plates. In this case, an electron traveling exactly midway between the plates will "feel" no net force. Such a trajectory is, however, unstable on a time scale t_0. By arranging the initial trajectory of the electrons sufficiently close to the midpoint, one might be able to improve upon the constraint Eq. (71) by perhaps an order of magnitude.

As noted above, we have ignored the effects of the electron's magnetic moment. This magnetic moment has the magnitude $\mu = e\lambda_C$ and dimensions of length. Quantities such as W_R which describe the distortion of the interference pattern are dimensionless, and the only other length scales are those which characterize the geometry. Hence in the case of electrons traveling a long distance at a height z_0 above a plate, we expect that magnetic moment effects will be suppressed by a factor of the order of λ_C/z_0 and will hence be negligible.

IV. SUMMARY AND DISCUSSION

We have seen that coupling of coherent electrons to the quantized electromagnetic field gives rise to both photon emission and vacuum fluctuation effects. These combined effects are similar to the Aharonov-Bohm effect in a classical magnetic field. The interference pattern can be sensitive to changes in the vacuum fluctuations in a region from which the electrons are excluded. In the single-plate geometry which is the primary example used in this paper, we found that the phase shift, Eq. (48), can be interpreted as an electrostatic Aharonov-Bohm effect due to the potential of the image charge. Although the Aharonov-Bohm effect due to a classical magnetic field has been verified experimentally, the electrostatic effect has not yet been observed. Thus it is of interest to contemplate whether the phase shift, Eq. (48), could be used as the basis for an experimental test of the electrostatic Aharonov-Bohm effect.

The primary effect of the vacuum fluctuations is to distort the interference pattern through the factor of e^W in Eq. (35). The presence of a conducting plate modifies the amplitude of the interference oscillations by a factor of $e^{W_R} \approx 1 + W_R$. An attempt to measure this effect might utilize the geometry of Fig. 2, where one beam passes over a conducting plate for a distance L at a height z_0. Here W_R is given by Eq. (58). The maximum possible value of L for fixed z_0, or equivalently the minimum value of z_0 for fixed L, is given by Eq. (71), the constraint which comes from the classical image charge effect. This constraint may be rewritten as

$$\frac{L}{2vz_0} < 3 \times 10^4 \left(\frac{z_0}{1\ \mu\text{m}}\right)^{\frac{1}{2}}. \tag{72}$$

Thus the maximum magnitude of W_R is bounded:

$$|W_R| < 2.3 \times 10^{-3} \left[11.3 + \frac{1}{2}\ln\left(\frac{z_0}{1\ \mu\text{m}}\right)\right]. \tag{73}$$

Thus it is possible to arrange for $|W_R|$ to be of the order of 2%. Note that for fixed L, $|W_R|$ increases with decreasing z_0. However, if we saturate the constraint Eq. (71), then both L and $|W_R|$ increase with increasing z_0. Because $|W_R|$ depends only logarithmically upon z_0, larger values for z_0 have little effect. (Increasing z_0 from $1\ \mu$m to 10^{28} cm, the size of the observable Universe, only increases $|W_R|$ from 2% to 20%.) If we were to change z_0 by a factor of 2, with L fixed, the change in $|W_R|$ would be

$$|\Delta W_R| \approx 1.6 \times 10^{-3}. \tag{74}$$

Whether such changes in the magnitude of the interference oscillations are in fact observable in a realistic experiment remains to be determined.

The emphasis in the above discussion has been on free electrons moving in the vicinity of a conductor. However, there is an alternative possibility, that of using coherent electrons inside of a material [7]. For example, in a superconducting quantum interference device (SQUID), coherent Cooper pairs produce interference effects. It may be possible to observe the phase shift ϕ_R and distortion factor W_R in a Josephson-junction device in which the Cooper pairs move in a plane parallel to, but a distance z_0 above a conducting plate. There should then be a change in the amplitude of the interference oscillations similar to that for free electrons. A more detailed analysis of this situation is required, however. In particular, it is not clear what will play the role of the electron's velocity in this case.

ACKNOWLEDGMENTS

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APPENDIX

Here Eq. (26), which describes the effects of both vacuum fluctuations and photon emission, will be derived. We may express the photon states which arise when an electron travels along the paths C_1 and C_2 as $|\varphi_1\rangle = U_1|0\rangle$ and $|\varphi_2\rangle = U_2|0\rangle$, respectively, where

$$U_i = T\left(e^{-ie\int_{C_i} A_\mu dx^\mu}\right). \tag{A1}$$

We may break the paths into a set of discrete segments. Let C_1 be the set $\{\Delta x^\mu(j)\}$ and C_2 be the set $\{\Delta x'^\mu(k)\}$. Then if

$$b_j = -ie\Delta x_\mu(j)A^\mu(j) \tag{A2}$$

and

$$c_k = ie\Delta x'_\mu(k)A^\mu(k), \tag{A3}$$

we can write $U_1 = \prod_j e^{b_j}$ and $U_2 = \prod_k e^{-c_k}$. Furthermore, $U_2^\dagger = \prod_k^* e^{c_k}$. Here $\prod$ is a time-ordered product and $\prod^*$ is an anti-time-ordered product.

The quantity which we wish to evaluate is the vacuum expectation value

$$\langle\varphi_2|\varphi_1\rangle = \langle 0|U_1U_2^\dagger|0\rangle. \tag{A4}$$

This is done most readily if we separate operators into their positive- and negative-frequency parts. For example,

$$b_j = b_j^+ + b_j^- \tag{A5}$$

and

$$c_k = c_k^+ + c_k^-, \tag{A6}$$

where $b_j^+|0\rangle = c_k^+|0\rangle = 0$ and $\langle 0|b_j^- = \langle 0|c_k^- = 0$. We may write, for example,

$$e^{b_j} = e^{(b_j^+ + b_j^-)} = e^{b_j^-}e^{b_j^+}e^{\frac{1}{2}\langle b_j^2\rangle}. \tag{A7}$$

Here, we have used the Campbell-Baker-Hausdorff formula

$$e^{(A+B)} = e^A e^B e^{-\frac{1}{2}[A,B]} \tag{A8}$$

for any two operators A and B which commute with their commutator, and the fact that $[b_j^+, b_j^-] = \langle b_j^2\rangle$.

The operator product $U_1U_2^\dagger$ may now be expressed as

$$U_1U_2^\dagger = \exp\left[\frac{1}{2}\left(\sum_j\langle b_j^2\rangle + \sum_k\langle c_k^2\rangle\right)\right] \times\prod_k{}^* e^{c_k^-}e^{c_k^+}\prod_j e^{b_j^-}e^{b_j^+}. \tag{A9}$$

In this product, we wish to bring all of the $(-)$ operators to the left and all of the $(+)$ operators to the right. Let us first examine the expression $\prod_j e^{b_j^-}e^{b_j^+}$. Because this product is already time ordered, a given b_j^- sits to the right of only those b_l^+ which are later in time, which will be denoted by $l > j$. As the b_j^- is moved to the left, we pick up a factor of $e^{\langle b_l b_j\rangle}$ from each $l > j$, i.e.,

$$e^{b_l^+}e^{b_j^-} = e^{b_j^-}e^{b_l^+}e^{\langle b_l b_j\rangle}. \tag{A10}$$

Thus,

$$\prod_j e^{b_j^-}e^{b_j^+} = \exp\left(\sum_{l>j}\langle b_l b_j\rangle\right)\prod_j e^{b_j^-}\prod_j e^{b_j^+}. \tag{A11}$$

Similarly,

$$\prod_k{}^* e^{c_k^-}e^{c_k^+} = \exp\left(\sum_{k>m}\langle c_m c_k\rangle\right)\prod_k e^{c_k^-}\prod_k e^{c_k^+}. \tag{A12}$$

The c_k^- need to move past those c_m^+ which are earlier in time. The operators within each of the two products on the right-hand side of Eq. (A12) commute with one another, and so the distinction between time ordering and anti-time-ordering is irrelevant. Next, it is necessary to commute all of the b_j^- to the left of all of the c_m^+. This introduces a factor of $\exp(\sum_{kj}\langle c_k b_j\rangle)$. If we combine this fact with Eqs. (A11) and (A12), the operator product, Eq. (A9), becomes

$$U_1U_2^\dagger = \exp\left[\frac{1}{2}\left(\sum_j\langle b_j^2\rangle + \sum_k\langle c_k^2\rangle\right) + \sum_{k>m}\langle c_m c_k\rangle + \sum_{l>j}\langle b_l b_j\rangle + \sum_{kj}\langle c_k b_j\rangle\right]\prod_k e^{c_k^-}\prod_j e^{b_j^-}\prod_k e^{c_k^+}\prod_j e^{b_j^+}. \tag{A13}$$

The vacuum expectation value of the operator products in this expression is just unity because of the fact that $e^{b_j^\dagger}|0\rangle = |0\rangle$, etc. Thus

$$\langle\varphi_2|\varphi_1\rangle = \langle U_1 U_2^\dagger\rangle = \exp\left[\frac{1}{2}\left(\sum_j \langle b_j^2\rangle + \sum_k \langle c_k^2\rangle\right) + \sum_{k>m}\langle c_m c_k\rangle + \sum_{l>j}\langle b_l b_j\rangle + \sum_{kj}\langle c_k b_j\rangle\right]. \tag{A14}$$

We may express the sums which appear in the above expression back in terms of integrals of two point functions as follows:

$$\sum_j \langle b_j^2\rangle + 2\sum_{l>j}\langle b_l b_j\rangle = \left\langle T\left(\sum_l b_l \sum_j b_j\right)\right\rangle = -e^2 \int_{C_1} dx_\mu \int_{C_1} dx'_\nu \langle 0|T(A^\mu(x)A^\nu(x'))|0\rangle, \tag{A15}$$

$$\sum_k \langle c_k^2\rangle + 2\sum_{m<k}\langle c_m c_k\rangle = \left\langle T^*\left(\sum_m c_m \sum_k c_k\right)\right\rangle = -e^2 \int_{C_2} dx_\mu \int_{C_2} dx'_\nu \langle 0|T^*(A^\mu(x)A^\nu(x'))|0\rangle$$
$$= -e^2 \int_{C_2} dx_\mu \int_{C_2} dx'_\nu \langle 0|T(A^\mu(x)A^\nu(x'))|0\rangle^*, \tag{A16}$$

and

$$\sum_{kj}\langle c_k b_j\rangle = e^2 \int_{C_1} dx_\mu \int_{C_2} dx'_\nu \langle 0|A^\nu(x')A^\mu(x)|0\rangle. \tag{A17}$$

Here T^* denote the anti-time-ordered product. If we substitute these expressions into Eq. (A14) and use the relation, Eq. (19), we obtain the final result of this appendix, Eq. (26). It is also possible to obtain this result by use of functional techniques [8].

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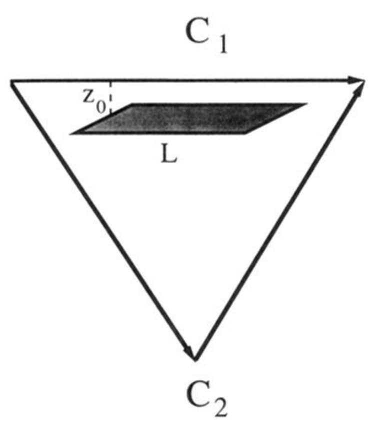

FIG. 2. Path C_1 passes over a conducting plate at a height z_0 for a distance $L \gg z_0$. Path C_2 bypasses the plate.

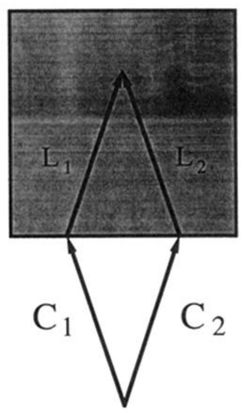

FIG. 3. Paths C_1 and C_2 begin in empty space and finish by traveling distances L_1 and L_2, respectively, above a conducting plate.