Classical and quantum production of cornucopions at energies below 10^{18} GeV

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We argue that the paradoxes associated with infinitely degenerate states, which plague relic particle scenarios for the end point of black hole evaporation, may be absent when the relics are horned particles. Most of our arguments are based on simple observations about the classical geometry of extremal dilaton black holes, but at a crucial point we are forced to speculate about classical solutions to string theory in which the infinite coupling singularity of the extremal dilaton solution is shielded by a condensate of massless modes propagating in its infinite horn. We use the nonsingular $c=1$ solution of $(1+1)$-dimensional string theory as a crude model for the properties of the condensate. We also present a brief discussion of more general relic scenarios based on large relics of low mass.

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In a previous paper in collaboration with two of our colleagues [1], we proposed a novel solution to the puzzles of Hawking evaporation of black holes. Our work was based on the seminal paper of Callan, Giddings, Harvey, and Strominger (CGHS) [2], which was in turn inspired by a number of papers on charged black holes in dilaton gravity and string theory [3]. The essential new conceptual idea in all of these papers was the observation that many of the charged black hole solutions in these theories had a geometric structure quite different from that of the Schwarzschild black hole of general relativity.

In a theory involving both a metric and one or more scalar fields, one is at liberty to make Brans-Dicke transformations [4] in which the metric is Weyl transformed by some positive function of the scalars. From the point of view of Lagrangian mechanics this is a point transformation and no physics can depend upon it[1] but the geometry of spacetime can change radically under such transformations. Consequently, the physics may be more transparent in one Brans-Dicke (BD) frame rather than another. In particular, when dealing with effective low-energy Lagrangians derived from string theory, a natural BD frame is picked out by choosing the metric along whose geodesics strings propagate. This is the *σ-model metric* used by Garfinkle, Horowitz, and Strominger [3]. With this choice, the spatial geometry outside the horizon of a charged black hole is shown in Fig. 1. Typically, there will be a large region (which we call the horn of the black hole) with the geometry of $I\times S^2$, where I is a real interval and S^2 is the round two sphere. Most of the degrees of freedom of string theory propagate as massive particles in this region, with only a few massless two-dimensional fields. In particular, the extremal charged black hole has a completely static metric, with no horizon, no singularity, and an infinite horn.

It was suggested in the paper of CGHS and explained in our previous paper [1] that the novel geometry of dilaton black holes could provide an intuitive resolution of most of the puzzles of Hawking evaporation. Although the black hole appears to an outside observer to be a rather small object, confined to a bounded region in space, in reality its horn contains a potentially infinite volume, which can serve as a repository of information and conserved quantum numbers. In particular, Bekenstein's upper bound [6] on the amount of information that can be hidden behind the horizon of a black hole of mass M is evaded by black holes with horny geometry. Bekenstein's argument relied on the notion that only particles with Compton wavelength smaller than the radius of the horizon could "fit in" to the black hole. But this is untrue for the infinite volume *cornucopion* [1]. Thus we are no longer faced with the dilemma of having to explain how all the information contained in

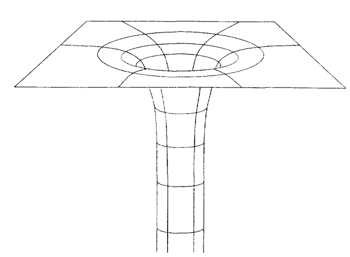

FIG. 1. The static cornucopion geometry at fixed polar angle.

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[1]However, as we pointed out in [5], one must be careful to insist that the transformation be single valued and preserve positivity of the conformal factor. Some recent work in $(1+1)$-dimensional gravity ignores these restrictions. We believe that such transformations change the physics of the original model in an essential way if they are used as more than a mathematical trick to obtain exact solutions to the classical equations of motion.

a large mass black hole is emitted in the process of Hawking evaporation down to a smaller mass hole.[2]

In the present paper, we wish to present more details of the arguments which led us to believe that cornucopions resolve the coherence loss problem of black holes. We begin with a short discussion of the cornucopion solution in string theory, arguing that the Gibbons-Maeda-Garfinkle-Horowitz-Strominger (GMGHS) solution is singular in this context. We then suggest that results on $c=1$ string theory imply that the GMGHS solution is part of a one-parameter family of solutions, all the other members of which have a nonzero condensate of massless fermion pair modes. Since the effective string coupling for scattering within the horn of the cornucopion is inversely proportional to the strength of this condensate, quantum corrections to the bare dilaton black hole are large, but the solutions with large condensate can be described semiclassically.[3] We take this dilaton black hole with a large condensate as a paradigm for the cornucopion.

In Sec. III we describe the classical formation of near extremal charged black holes in dilaton gravity coupled to electromagnetism. We argue that the cornucopion begins life as a finite volume dimple on flat space time, whose tip grows without bound. The static cornucopion solution is the asymptotic limit of this growing solution.[4] This leads us to the concept of finite volume cornucopions, which represent the instantaneous configurations of the near extremal black hole a finite time after its formation. These are approximately static solutions of the equations of motion whose only time dependence is in the rapidly receding tip. In Sec. IV we review various scenarios for the end point of black hole evaporation, concentrating on the difficulties of scenarios which invoke the existence of relic particles.

The problems with relic particle scenarios are all a consequence of the infinite number of approximately degenerate states that must be present if the relics are to possess the information content of the large black holes from which they were formed. In the case of cornucopions this infinite number of states is associated with the infinite volume of the horn. We begin Sec. V by expanding on the arguments made in [1] about the difficulty of bringing cornucopions into thermal equilibrium with an external heat bath. We argue that the combination of these arguments with those given below, which suggest small amplitudes for cornucopion pair production, remove the problems posed for thermodynamics by an infinite set of degenerate states. We then study the contribution of finite volume cornucopion configurations to virtual loops by heuristic semiclassical techniques, and argue that the amplitude for this virtual process is of order e^{-V/g_0^2} where V is the cornucopion volume and g_0 is related to the value of the dilaton field at infinity in the usual manner. We argue that the sum over the large but finite number of states in the cornucopion corrects this by a factor e^{cV} where c is a positive constant independent of g_0. Thus, if the coupling is small enough, the sum over cornucopion volumes and internal states converges and cornucopions give only small finite corrections to low-energy scattering processes. We show that this argument is consistent with crossing symmetry by drawing an analogy with the scattering and production of solitons in weakly coupled field theories. Order-one scattering amplitudes for elementary particles scattering off solitons are consistent with exponentially small production cross sections because the production cross section is related by crossing to a large momentum transfer scattering amplitude. The particle-soliton scattering amplitude can be large for small momentum transfers but falls exponentially with the momentum transfer because the soliton form factor is the Fourier transform of a smooth classical field. By analogy, we argue that virtual production of pairs of large volume cornucopia is related by crossing to processes in which the cornucopion volume is changed by a large amount due to scattering from an elementary particle. These processes are extremely improbable, and there is no inconsistency with our estimate for the contribution of cornucopions to virtual loops.

Unfortunately, these semiclassical arguments are not valid for the model of cornucopions given by the extremal black hole of four-dimensional Einstein-Maxwell-dilaton gravity. For this solution, the effective coupling grows as one proceeds down the horn of the cornucopion, and one finds a finite production probability by naive semiclassical estimation. Of course, the semiclassical approximation breaks down in this case and one must understand strong coupling physics. We cannot do this at present for real cornucopions, but some progress can be made by studying an analogous problem in $(1+1)$-dimensional string theory where the strong-coupling singularity is shielded by a condensate of massless modes. In that model we argue that the classical Euclidean action is indeed of order the volume. However, we must then inquire whether the effective volume of the system is truly infinite. This turns out to depend on how one resums the semiclassical perturbation expansion, and at the present time there do not exist reliable criteria for deciding which resummation is correct. Thus, although the case against cornucopia as remnants of black hole evaporation is unproven, the resolution of the argument may depend on strong-coupling physics. We will show that if one takes the pessimistic point of view that strong-coupling effects cut off the horn of the cornucopion, then its residual entropy is of order its mass, in parametric

[2]It is crucial to this argument that our theory have modes which propagate as free particles in the two-dimensional horn of the cornucopion. Information stored in other modes of the field can be transferred into these waves by interactions near the horn of the cornucopion.

[3]Most resummations of the perturbation series in the $c=1$ model give a finite limit for the S matrix as the tachyon condensate goes to zero [7]. Thus, in the quantum theory the solution is nonsingular, but it is not amenable to semiclassical investigation.

[4]It is likely that extremal charged matter does not collapse classically, because of magnetostatic repulsion. Our discussion can be taken as a description of near extremal collapse. The cornucopion is hypothesized to be the remnant of Hawking evaporation of this near extremal object.

agreement with the Bekenstein-Hawking formula.

We end Sec. V with a discussion of the real pair production of cornucopia in weak static magnetic fields. It has been argued, that in the weak field limit, the calculations of Affleck and Manton for solitons [8] and/or Garfinkle and Strominger for Wheeler wormholes [9] indicate that cornucopions will be produced at the same rate as elementary particles of the same mass in such background fields. We give a critical discussion of these arguments, and explain an alternative semiclassical picture of cornucopion pair production, which makes the Affleck-Manton estimate of the amplitude for production of the cornucopion geometry consistent with finite total cross sections. We argue that the tunneling process which gives rise to cornucopion pairs produces cornucopions with a size of the order of the characteristic Schwinger length for production of elementary particles of the same mass as the cornucopion. These relatively small cornucopions then grow classically to arbitrarily large size. In this method of pair production, the external field cannot create most of the states of the infinite cornucopion geometry. It creates only those that can arise from initial conditions which "fit in" to the original small volume. As in the inflationary universe, this is a small subset of the total number of cornucopion states.

In the conclusions we note that cornucopions may be only one of a number of classes of objects that look like small black holes from the outside, but conceal a large interior world. We stress that black holes whose singularity is replaced by an interior de Sitter space (which appear in the work of Farhi and Guth [10], Frolov, Markov, and Mukhanov [11], Morgan [12], and Strominger [13]) may represent a particularly attractive end point for black hole evaporation.

I. IS THE EXTERNAL DILATION BLACK HOLE SINGULAR?

Garfinkle, Horowitz, and Strominger argued that the extremal dilaton black hole was nonsingular. The basis of their argument was that the apparent singularity was an infinite distance away in the "stringy metric":

$$ds^2 = -dt^2 + e^{4\phi} d\mathbf{x}^2, \tag{1.1}$$

$$e^{2\phi} = g^2 = e^{2\phi_0} + \frac{2Me^{\phi_0}}{|x|}. \tag{1.2}$$

Taking geodesic distance as a predictor of physical evolution is dangerous in theories which involve scalar fields. One can always perform Brans-Dicke transformations in which the metric is Weyl transformed by a positive function of the scalars. This is a point transformation on the configuration space of fields, and cannot change the physics. However it does change geodesic distance, in a way that can be singular if the scalar fields develop singularities. The physics of the model depends on how all the fields in the theory are coupled not only to the metric, but also to the dilaton and other scalars. GHS argued that, since the world sheet Lagrangian of string theory describes geodesic motion in the stringy metric, this was the appropriate physical measure of distance in string theory. However, once we begin to calculate scattering amplitudes, the dilaton field begins to play an important role. In string theory, each vertex operator carries a factor of the string coupling constant, which becomes infinite at the end of the infinite horn of the GHS solution.

There is as yet no known exact conformal field theory representation of the extremal dilaton black hole solution of heterotic string theory. However, if we concentrate on physics in the horn of the cornucopion we can make a plausible guess at some of the features of this conformal field theory. The world sheet Lagrangian describing scattering within the horn should be the world sheet supersymmetric completion of a Lagrangian of the form

$$\mathcal{L}_{ws} = \partial y \bar{\partial} y - \partial \tau \bar{\partial} \tau - QR^{(2)} y + \mathcal{L}_{\text{compact}}. \tag{1.3}$$

Here $y = \ln x$, τ is the time coordinate, and $\mathcal{L}_{\text{compact}}$ describes a unitarity conformal field theory with a discrete positive spectrum of conformal dimensions. It represents the angular degrees of freedom of three-dimensional space, as well as the six compactified dimensions. We do not know the right and left central charges of the Lagrangian $\mathcal{L}_{\text{compact}}$, but they cannot be equal to zero. The spectrum of the theory then contains potential tachyons, which (it is to be hoped) are eliminated by the physical state conditions.

As a consequence of the discrete spectrum of dimensions of $\mathcal{L}_{\text{compact}}$ most of the particles in this theory propagate with large effective masses in the horn. The low-energy field theoretic description of the system implies that there are exceptions to this rule, which are related to charged fermion zero modes around the monopole. These states were discussed in [14] and [1]. They propagate as massless two-dimensional fermions. The vertex operators for these states have the form $e^{(\alpha+ik)y+iEt}\mathcal{O}$, where $\mathcal{O}$ is constructed from the degrees of freedom of the $\mathcal{L}_{\text{compact}}$ theory. On-shell vertex operators satisfy $E^2 = -(\alpha + ik - Q/2)^2 + (Q^2/4) + \Delta - 2$, where Δ is the dimension of $\mathcal{O}$. In order that the states have real energies we must have $\alpha = Q/2$, and in order that they be massless, $\Delta = -Q^2/4 + 2$.

Inserted into the y path integral, these operators generically cause divergences in the integral over the zero mode of y. This is analogous to the divergences of tachyon amplitudes in the $c=1$ matrix model when the cosmological constant is equal to zero. In that case, the cure for the disease is well known. The zero energy tachyon vertex operator can be added to the Lagrangian without destroying conformal invariance. This tachyon condensate is a new classical solution of $c=1$ string theory and the perturbation expansion around it is computable and finite. Study of that expansion shows that the effective expansion parameter is g_{st}^2/μ where μ is the coefficient of the tachyon condensate in the world sheet Lagrangian (the two-dimensional cosmological constant). The singularity of the $\mu = 0$ solution is seen as a failure of the semiclassical expansion. Various nonperturbative resummations of the expansion give a perfectly sensible S matrix at $\mu = 0$.

It seems rather hopeless to try to find an analogous nonperturbative solution of the string theory associated with the horn of the cornucopion. We do not even have an exact conformal field theory to start from. Rather we should look for the analog of the solution with finite μ. It does not seem to make sense to simply put the vertex operators of the massless modes into the world sheet Lagrangian, for they are spacetime fermions. From a spacetime point of view, we would expect a condensate of these zero modes to be described by the bosonized fermion current. We have no idea how to represent these scalar fields in terms of vertex operators in string theory. Although the bosons are two fermion states, it is not entirely outlandish to expect to find them in the one string Hilbert space. The two to two fermion scattering matrix should have a pole corresponding to exchange of these bosons. Since they are derivatively coupled, it should show up as a 0/0 contribution to the zero energy scattering amplitudes. Such structures in spacetime often have signatures in the behavior of vertex operator correlation functions near the boundary of moduli space which can be associated with other vertex operators.[5]

We hope to return to these fascinating issues at some future time, for they are crucial to a semiclassical understanding of the GMGHS solution. For the purposes of the present paper we will make the optimistic assumption that an analog of the nonsingular $\mu \neq 0$ solutions exists, and that its properties are similar to those of the $c=1$ model. It is this hypothetical nonsingular solution that we want to use as a model for the cornucopion. The careful reader will note that there is only one point below in which we make use of the nonzero value of the condensate. Most of the properties of the cornucopion follow from its geometrical structure, and would be valid for other sorts of black hole remnants which conceal a large internal space behind their apparent horizons.

Before closing this section we should draw attention to a possible problem with the idea of using the $c=1$ string theory as a model for a cornucopion. Shenker [16] has pointed out to us that if, as one expects in the cornucopion, the string coupling goes to a finite constant in the throat region, the effective volume of the $c=1$ world appears in perturbation theory to be finite and of order the logarithm of the string coupling. In the extremal dilaton black hole, the volume over which the two-dimensional world is weakly coupled is also proportional to the mass of the black hole. Thus if we make the pessimistic assumption that the strong-coupling region is really inaccessible, the cornucopion will be capable of storing an amount of information that is bounded by a constant of order its mass. This bound is parametrically the same as that given by the Bekenstein-Hawking formula. Note however that in many nonperturbative resummations of the $c=1$ perturbation series, the barrier that prevents tachyons from penetrating the strong-coupling region is finite, and there is another weakly coupled region on the other side of it which contains an infinite number of states.[6] Thus, the question of whether a nonsingular cornucopion can have an infinite number of states is bound up with nonperturbative physics.

II. COLLAPSING CORNUCOPIONS

In this section we describe the classical collapse processes that could lead to near extremal black holes. Consider a collapsing shell of magnetically charged matter, in dilaton-Einstein-Maxwell theory. One can attempt to construct a solution representing collapse by gluing the four-dimensional exterior solution of GHS onto a smooth interior vacuum solution of the equations with the topology and symmetry of a three-hemisphere. The spherically symmetric geometry is a two sphere with time and radial coordinate dependent radius cross a two-dimensional spacetime geometry for the r-t submanifold. We call the radius of the two sphere $e^{2\sigma(r,t)}$ and use synchronous coordinates. The Lagrangian for the most general solution of this form is

$$S=\int \sqrt{-g}\, e^{-2\phi}\{e^{2\sigma}[-R-2(\partial\sigma)^2-4(\partial\phi)^2+8\partial\phi\partial\sigma] -2+Q^2e^{-2\sigma}\} . \tag{2.1}$$

We have chosen coordinates in which the exterior metric is

$$ds^2=-dt^2+\frac{1}{h(t,r)^2}dr^2+e^{2\sigma(t,r)}d\Omega^2, \tag{2.2}$$

where $h(t,r)=(1-Q/r)$ and $\sigma=\ln(r)$, for the static GMGHS solution.

Now suppose that this is the solution outside a collapsing shell of magnetically charged matter with two-dimensional world line $(t,r)=[T(\tau),R(\tau)]$, where τ is the proper distance along the world line. To obtain the solution inside the collapsing shell we use the dilaton gravity action with zero magnetic field. The latter condition is a consequence of our assumption of spherical symmetry. Inside the shell, there are no magnetic sources and the field equation for the magnetic field is

$$\nabla_\mu(e^{-2\phi}F_{\mu\nu})=0. \tag{2.3}$$

Since ϕ does not depend on the angular variables, and only the angular components of the field are nonzero, the field must be constant inside the shell. Continuity of the solution at the origin restricts this constant to be zero.

Introduce coordinates using the proper time of the shell, so that $ds^2=-d\tau^2+dn^2$, along the world line of the collapsing matter. We will use these coordinates to expand the solution for ϕ and σ in powers of n toward the interior of the shell, with coefficients of the power series being functions of τ. We will further simplify the equations by making a change of variables:

[5]We are thinking of the Dine-Seiberg vertex operators [15] for the auxiliary components of superfields. These are also composite operators from the spacetime point of view.

[6]The structure is reminiscent of the other world in the Kruskal extension of the Schwarzschild solution. The major difference is that in the $c=1$ model there is no singularity or horizon dividing the two worlds, and they can communicate with each other.

$$u(\tau,n)=e^{\sigma-\phi}. \quad (2.4)$$

In terms of these fields the Lagrangian is

$$S=\int\sqrt{-g}\,\{-u^2R+2u^2(\partial\sigma)^2-4(\partial u)^2-2u^2e^{-2\sigma}+Q^2u^2e^{-4\sigma}\}. \quad (2.5)$$

The equations of motion for this Lagrangian are given in Appendix A. Of course, to find interior solutions with zero magnetic field to match onto the exterior extremal dilaton solution, we set $Q=0$ in the above equation.

The power series for the fields σ and ϕ, expanding from the shell towards the interior, is

$$u(\tau,n)=R(\tau)\left[1-\frac{Q}{R(\tau)}\right]^2[1+f_1(\tau)n+f_2(\tau)n^2+f_3(\tau)n^3+\cdots], \quad (2.6)$$

$$\sigma(\tau,n)=[\ln(R(\tau))+d_1(\tau)n+d_2(\tau)n^2+d_3(\tau)n^3+\cdots], \quad (2.7)$$

where the coefficients of the leading terms are determined by continuity of ϕ and σ across the shell. The metric is $g_{\mu\nu}=\text{diag}[-h(\tau,n)^2,g(\tau,n)^2]$, and the coefficients have the expansion;

$$h(\tau,n)=1+h_1(\tau)n+h_2(\tau)n^2+h3(\tau)n^3+\cdots, \quad (2.8)$$

$$g(\tau,n)=1+g_1(\tau)n+g_2(\tau)n^2+g3(\tau)n^3+\cdots. \quad (2.9)$$

The equations that we have to describe this system now consist of the equations for u and σ, and the stress tensor equation. At the boundary of the collapsing shell there is a nontrivial matching equation for the stress tensor component T_{00}.[7]

We will assume that the classical Lagrangian for the matter that constitutes the shell is of the form

$$S=\int\sqrt{-g}\,u^2[-(\partial A)^2-m^2A^2+\cdots]. \quad (2.10)$$

That is, the matter in the collapsing shell couples to the dilaton like some massive mode of the string. In the rest frame of the collapsing shell, the matching equation reads[8]

$$Mu(\tau,0)^2=\int_{-\epsilon}^{\epsilon}T_{00}dn \quad (2.11)$$

which becomes

$$Mu(\tau,0)^2=R\left[1-\frac{Q}{2R}\right]\left[\dot{R}^2+\left[1-\frac{Q}{R}\right]^2\right]^{1/2}-u\partial_n u(\tau,0). \quad (2.12)$$

At this point we must be more specific about the fields on the interior of the shell. In Einstein's theory, there is a unique spherically symmetric nonsingular vacuum solution, but here the dilaton dynamics gives rise to an infinite set of spherically symmetric solutions of the source free field equations in a finite region. We have tried to restrict the solution by assuming a cosmological form for the metric $ds^2=-d\tau^2+a(\tau)^2(dr^2+r^2d\Omega^2)$ inside the shell, but this is inconsistent with the field equations. Similarly, an attempt to keep the three-dimensionally conformally flat form of the metric, with conformal factor tied to the dilaton, is inconsistent. We have not been able to come up with a natural ansatz. Nonetheless, we believe that smooth solutions exist. There are many smooth solutions of the vacuum field equations restricted to a manifold with the topology of a hemi-three-sphere cross time. Our matching conditions fix only the values of the metric functions and dilaton along the timelike world line of the collapsing shell, leaving their normal derivatives undetermined. Thus there seems to be plenty of room for patching in a nonsingular vacuum solution.

To obtain some feeling for the motion of the collapsing shell we have made the fairly arbitrary assumption that

$$f_1(\tau)=\frac{a}{R(\tau)}. \quad (2.13)$$

This gives us a single first-order ordinary differential equation for $R(\tau)$.[9] The solution so obtained behaves like $R(\tau)\approx Q+e^{-\gamma\tau}$, as $\tau\rightarrow\infty$. We can then use this solution to check that the other coefficient functions, to leading order, are well behaved for all finite values of τ. We can continue this procedure perturbatively, to verify that the coefficients in the expansion in powers of n are smooth functions of τ. Of course, this demonstration of a smooth perturbation expansion around the shell, does not guarantee the existence of an everywhere smooth solution. We continue to search for a sensible ansatz that will enable us to demonstrate explicitly the existence of a smooth collapsing solution, but we feel confident that such a solution exists.

The collapsing solution that we have described, begins as a dimple on flat space. At any finite time after its formation, it will have the geometry shown in Fig. 2. We will refer to such an object as a finite volume cornucopion. It is a solution of the field equations that is static over most of space. The time dependence occurs only in the tip of the horn.

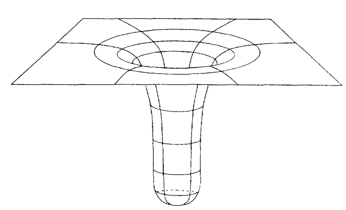

FIG. 2. Instantaneous snapshot of a collapsing cornucopion.

[7]See Appendix B for details.

[8]The full details of the derivation are in Appendix B.

[9]Appendix C.

III. THE PROBLEM OF STABLE RELICS

Since the publication of Hawking's seminal papers on black hole evaporation 18 years ago [17], there have been many attempts to resolve the puzzle of information loss that is apparently implied by the Hawking process. In broad terms, these attempts fall into three classes.[10] In the first, essentially that originally advocated by Hawking, one accepts the information loss at face value and attempts to describe processes in which pure states can transform into mixed states. This idea raises a number of paradoxes [19], and it is not clear[11] that it can lead to a sensible physical theory.

Advocates of the second approach, including Page, 't Hooft, Wilczek, and Susskind and their collaborators [20] insist that the information loss is a consequence of an improper treatment of the quantum mechanics of the gravitational field. They argue that in a more careful analysis, which goes beyond the semiclassical approximation, Hawking radiation will be shown to carry information, encoded in subtle nonlocal correlations, much like those in the "thermal" radiation emitted from any hot body. These authors face the challenge of understanding the entropy produced in Hawking's calculation of the decay of a large classical black hole into one of half the mass. According to the "subtle correlation" viewpoint, and consistent with the Hawking-Bekenstein formula for black hole entropy, the Hawking radiation in this process carries a huge amount of information $\sim(M/M_P)^2$. According to Hawking's calculation, it carries none. Thus there must be a large correction to Hawking's calculation of the density matrix of the emitted radiation, despite the fact that the entire process takes place within what appears to be the domain of validity of the semiclassical approximation. If one takes the semiclassical calculation seriously in regions inside the horizon but away from points of high curvature, one is led to serious problems of causality. The information carried by the infalling matter is still localized behind the horizon on a spacelike surface on which most of the mass of the black hole has been radiated away. In addition to this, the subtle correlation approach can never account for the global quantum numbers that appear to be lost in black hole decay. One is led to claim that black hole physics can only make sense in the context of a theory in which there are no conserved global quantum numbers.

The final approach to the problem of Hawking radiation is to postulate the existence of an infinite number of stable remnant objects, all of whose masses are sufficiently small that the Hawking calculation breaks down near the corresponding Schwarzschild radii. These objects can store any global quantum numbers that have fallen down the black hole, and the infinity of degenerate states is a repository for the information lost in the Hawking process. A recent critical discussion of this scenario can be found in [21].

The problems of the stable relic scenario are all caused by the infinity of degenerate relic states that it requires. All formulas of statistical mechanics are formally infinite (even in the microcanonical ensemble) if we assume that these states have come into equilibrium with the rest of the world. The same can be said for all formulas in quantum field theory, when loops of virtual relic particles are taken into account (at least if we treat the relics as elementary particles). Furthermore, if the probability of pair production of a single relic state in an external environment (say, a weak slowly varying electromagnetic or gravitational field) is bounded from below by a positive number ϵ, no matter how small, then the total production probability is infinite. Typical pair production cross sections for elementary magnetically charged particles, or monopole solitons, in static magnetic fields go like $e^{-M^2/gB}$ where M is the mass of the state being produced. If the infinite set of black hole relics can all be produced at rates of this order of magnitude, then everything in the world will decay into black hole relics in a microscopic time. Given these striking conclusions, it is not surprising that many authors prefer the scenario in which information is emitted with the Hawking radiation to the relic particle scenario.

IV. AND ITS RESOLUTION?

We have just outlined the obstacles faced by any attempt to identify stable relics as the end point of Hawking evaporation. In this section we would like to show how cornucopia overcome these obstacles. A brief discussion of the thermodynamic equilibration of cornucopia has already appeared in [1], so let us begin by expanding on it. Imagine that we have created a single cornucopion in one of its many degenerate quantum states, and that we have filled the universe outside it with a gas of particles at temperature T. Let us ask how long it takes for some subset of states of the cornucopion to come into thermal equilibrium with the external gas. To begin with, let us imagine that interactions of the gas near the mouth of the horn can excite states inside the cornucopion with probability $\sim e^{-E_{\rm ADM}/kT}$ where $E_{\rm ADM}$ is the Arnowitt-Deser-Misner energy measured by an observer in the asymptotically flat region of spacetime. From causality alone we have a restriction that in time t we can at most excite states of the horn within a distance t of its mouth. The number of such states is of order e^{tM_P} and their contribution to the partition function is at least $e^{tM_P-M_P/T}$ if we assume that typical splitting in ADM energy between these states is of order the Planck mass.[12] Since the time t over which one can imagine thermalization to take place in an expanding universe is always bounded from above by the inverse of the expansion rate H^{-1} we never have to think about a strictly infinite number of states.

[10] S. Giddings [18] has recently presented a discussion of scenarios for the end point of black hole evaporation which overlaps with this section.

[11] At least to the present authors.

[12] If most of the states have zero ADM energy, the temperature-dependent factor is absent.

In fact it is highly unlikely that all of the states within a distance M_P/H of the cornucopion throat will be thermally distributed. There are two basic reasons for this. First, there are large repulsive potential barriers located near the cornucopion throat for most modes of the fields in the external universe.[13] In string theory this can be understood from the statement that the effective world sheet theory which describes the horn of the cornucopion is the tensor product of the Liouville theory and a compact conformal field theory. Thus, apart from the small number of massless modes described above, all states have effective masses in the cornucopion horn that are of order the string scale. The probability for an external particle to penetrate the throat and produce an ingoing wave of these massless modes is very small; at least proportional to inverse powers of the Planck scale.[14] Thus the thermalization of the cornucopion proceeds at a very slow rate, likely to be slower than the expansion rate of the universe at most epochs of interest.

Our second reason for believing that cornucopion states will not come into conventional thermal equilibrium is more difficult to explain because we do not fully understand it. Imagine that some process in the external world has succeeded in injecting a pulse of tachyons into the horn of the cornucopion. Our analogy between the horn of the cornucopion and the $c=1$ model tells us that (at least to all orders in the string loop expansion) the dynamics of this pulse is describable by a two-dimensional field theory of interacting massless particles. The question of which states of this field theory are occupied seems to have little to do with the ADM energy measured by an observer in the asymptotic four-dimensional region of space time. Its dynamics are governed by an effective Hamiltonian $\mathcal{H}$ whose connection with ADM energy is far from clear. One might expect that after enough time, the state of the system interior to the horn would be well described by a density matrix of the form $e^{-\mathcal{H}/T_h}$ where T_h is an internal temperature whose relation to the temperature of the external universe is less than obvious. However, if a finite pulse of massless particles (i.e., the analog of a distribution of tachyons that can be described by a finite perturbation of the Fermi surface in Polchinski's [22] description of the tachyon field theory of the $c=1$ model) was injected, this temperature will be finite. This is not a thermal distribution in ADM energy.

We do not pretend to fully understand these arguments (particularly the latter) so it is fortunate that even if the states of the cornucopion had come into thermal equilibrium they would have little effect on the thermodynamics of the external world. The effect of the extra states of the cornucopion is to endow *each* of the states of the world external to the black hole with an enormous degeneracy. Let us use the letter α to label this degeneracy. Now consider any operator $\mathcal{O}$, localized in a region R of spacetime external to the cornucopion. We claim that $\mathcal{O}$ is essentially the unit operator as far as the α label is concerned. This argument can be stated more precisely. In [1] we argued that the degeneracy in ADM energy of states of the cornucopion was a consequence of the existence of states concentrated in regions far down the horn. The energy difference between two states labeled α and α' goes to zero exponentially as the difference between the two states is taken to be a state localized further and further down the horn. But in the same limit, the difference between the expectation values of $\mathcal{O}$ in these two states vanishes, as does its off diagonal matrix element between them. Thus

$$\sum_{N,\alpha} \langle N,\alpha|\mathcal{O}|\alpha,N\rangle = \sum_{\alpha}\sum_{N} \langle N|\mathcal{O}|N\rangle . \tag{4.1}$$

The sum over α factors out when we compute expectation values. Thus for local quantities located far enough from the cornucopion, thermal averages do not probe the presence of its large number of degenerate states. We emphasize that there is nothing exotic about this argument, and that it depends principally on the fact that the interior of the cornucopion is *far away* from the external observer. If someone increases the local density of states on the moon, it effects thermodynamics on the moon, but not on Earth.

The effect of cornucopia on thermodynamics in regions far from the throat of the black hole is thus seen to be rather innocuous. We believe that the widespread belief that infinite numbers of black hole remnants, degenerate in ADM energy, contradict thermodynamics is based on the misconception that these remnants could be described as particles, and that one should be able to produce them all in the laboratory. The fundamental problem of thermodynamics in the stable relic scenario is thus intimately related to the estimate that we made in the previous section of production of a stable relic, thought of as an elementary particle.

We are thus led to investigate the fundamental problem of any relic scenario, the pair production of cornucopia, and their contribution to virtual loops. The most serious sounding argument against the existence of stable relics of the black holes is based on crossing symmetry. The scattering of low-energy photons and gravitons from a cornucopion is completely determined by the object's mass and charge, and the scattering amplitudes are not small. Crossing symmetry, it is argued, should relate these amplitudes to cornucopion production amplitudes, and (by unitarity) to their contributions to virtual loops. If each state of the cornucopion is produced with finite amplitude, and/or gives a finite contribution in loops, the sum over the infinite number of virtual relic states will give rise to infinite production cross sections and infinite renormalizations of all low-energy amplitudes.

We will argue in a moment that this argument does not take sufficient account of the infinitely extended nature of the cornucopion geometry. However, it is worth pointing out that such arguments from crossing symmetry can be

[13]This is shown, for example, in the paper of Holzhey and Wilczek cited in [20], and was also apparently known to GHS.

[14]One should avoid being confused by the strong Callan-Rubakov interaction of external particles with a magnetically charged cornucopion. This describes processes going on very far (in Planck units) from the cornucopion throat.

misleading even for extended objects of finite extent, namely, solitons in ordinary weakly coupled local field theories. In particular, naive application of crossing symmetry would lead us from the finite Thompson cross section for low-energy photon scattering off a 't Hooft Polyakov monopole, to the conclusion that the production cross section for such monopoles above threshold in electron positron annihilation was of order one. From unitarity we would then conclude that monopoles contributed to photon vacuum polarization at some finite order in perturbation theory (some inverse power of the monopole mass). In fact, as argued long ago by Drukier and Nussinov [23] the monopole production cross section is of order $e^{-c/\alpha}$. The essence of their argument was that in a weakly coupled theory, it costs a power of α in probability to create a quantum of the bare fields from the vacuum. The coherent monopole state contains of order $1/\alpha$ quanta.

To understand where the crossing symmetry argument went wrong, we note that the Thompson crossing section is related to the form factor of the electromagnetic current (we suppress irrelevant Lorentz indices and kinematic factor)

$$\langle p+q|J|p\rangle = F(q^2) \tag{4.2}$$

at small spacelike momentum transfers q^2. The monopole production cross section, on the other hand, is, by crossing symmetry, related to the analytic continuation of the same function F to large timelike q^2 of order the monopole mass $\sim m_W/\alpha$. A better crossing symmetry estimate of the production cross section would be to relate it to the behavior of the monopole form factor at large spacelike q^2 where it falls exponentially (it is essentially the Fourier transform of the smooth monopole field configuration).[15]

We have introduced this example not only because it illustrates how a naive application of crossing symmetry can grossly overestimate the production rate of an extended semiclassical object, but because we believe that it provides a good analogy to cornucopion production, with the cornucopion volume playing the role that momentum plays in the monopole form factor. The conventional pictorial argument for crossing symmetry is shown in Fig. 3. In the left half of this figure, a particle is produced by some classical apparatus, scatters off a photon, and is absorbed by another classical machine. The amplitude is nonzero even when the emission, absorption, and scattering events are in spacelike relation. Thus, some Lorentz observers can see the scattering occur before the emission or absorption, and interpret the amplitude as a production process.

[15]Even this estimate is suspect, though it gives a more plausible answer. One could encounter Stokes lines in the analytic continuation from large spacelike to large timelike q^2. However there is an intuitive connection between this estimate and the Drukier-Nussinov argument. The smoothness of the high-energy monopole configuration is a consequence of the fact that it contains a large number of soft quanta.

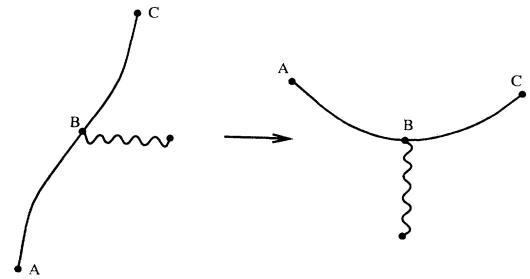

FIG. 3. A pictorial argument for crossing symmetry.

In [1] we associated a particlelike coordinate variable $x^{\mu}(\tau)$ to a single cornucopion. The dynamics of the system involved a conventional functional integral over this variable, and so it is still presumably true that the photon cornucopion scattering amplitude is nonvanishing when the events A, B, and C are in a spacelike relation. However, the meaning of this picture is quite different for cornucopia and for particles. Even A is a classical cornucopion production event. We can imagine creating it by letting some matter collapse into a black hole, possibly followed by Hawking radiation to produce a cornucopion with Planck sized throat. It is thus a violent classical process, not a microscopic event. Event B is an interaction of the photon with the throat of the cornucopion, it is presumably much more complicated than the corresponding interaction with a point particle, but involves no new issues of principle. However, event C has a drastically different interpretation for cornucopia and elementary particles. Event C is pictured as occurring at a point in spacetime. However, it symbolizes the destruction of the cornucopion, which involves the shrinking of its horn back to flat space. The tip of the horn is however a finite and presumably very large spacelike distance away from event B. Given the initial conditions in which the cornucopion was formed by gravitational collapse, we find it unlikely that its classical motion will ever shrink the horn back to zero size to agree with the configuration assumed in event C. Even if this could happen, it must take a long time. Thus, in order for the cornucopion to classically disappear at event C, C must be far in the future of B, and there is no confusion about the time ordering. The alternative is, that between points B and C, a quantum tunneling event occurred in which the volume of the cornucopion shrank to zero.

Thus crossing symmetry relates photon pair production of cornucopia not to simple scattering processes in which the cornucopion's horn is unaffected, but to processes in which the volume of the horn changes drastically. In the analogy to monopole production discussed above, we would say that ordinary scattering measures the cornucopion "form factor at small volume change," while the production process is obviously related to scattering with large volume change. To estimate the probability of such large changes in the geometry we will first attempt to use semiclassical ideas.

We will first discuss the contribution of cornucopia to virtual loops. The positive energy theorem assures us of the semiclassical stability of flat space in quantum gravity. There should be no exact instantons describing the decay of flat space into another geometry. What we must do is try to estimate the probability for a virtual process, rather than a tunneling process which terminates in motion through an allowed classical region. Thus we want to calculate the Euclidean action for a virtual transition from flat space to a specified configuration of the geometry, which then subsides into flat space. We will take the intermediate configuration to be the finite volume cornucopion discussed in Sec. III, and treat the volume of the horn as a collective coordinate. The full amplitude for this virtual process will then take the form

$$A \sim \int dV\, e^{-S(V)/g_0^2} Z(V), \tag{4.3}$$

where $S(V)$ is the Euclidean action for the process, $g_0^2 = e^{+2D_0}$ is the coupling constant written in terms of the value of the dilaton in the asymptotically flat region of spacetime, and $Z(V)$ is the partition function for small fluctuations around the instanton. To all orders in the loop expansion, $Z(V)$ will behave as e^{NM^3V}, for large V, where M is the cutoff scale (the string tension scale in string theory) and N is the number that goes to a constant as $g_0 \to 0$. We are assuming that the theory is weakly coupled at all length scales, and in particular that (as in string theory) the cutoff is smaller than the Planck Mass by a factor of g_0. In the weak-coupling limit, $Z(V)$ counts the number of states of the cornucopion of volume V. Its logarithm is extensive in the volume.

In this way of organizing the calculation we see how the infinite number of states of the cornucopion can sum up to give a finite result in virtual loops. The static cornucopion is an idealization. Its infinite number of states come from its infinite volume. Any cornucopion that was created a finite time in the past will only have expanded to a finite but enormously large volume. It will have a finite number of states that grows exponentially with its volume. In virtual loops we sum over these finite volume cornucopions. An infinite contribution will be avoided if the integral of V converges. This will occur if the coupling g_0 is small enough, and if the cost in Euclidean action to create a cornucopion of volume V grows at least as fast as V for large volumes. In an ordinary quantum field theory, it would be essentially obvious that the action for creating a configuration which differs from the vacuum over a volume V will be of order V. In string theory, or the low-energy dilaton gravity theory which it gives rise to, the dilaton field whose exponential multiplies the classical action density, varies linearly along the cornucopion. Regions far from the cornucopion throat give exponentially small contributions to the static solution. Thus, for the external black hole solution of GHS it is not clear that the action for the virtual cornucopion creation process is proportional to V for large volume. However, we have argued above that the GHS black hole is not a good model for the generic cornucopion solution of string theory. In that context it is singular, and we argued that the nonsingular solutions have a condensate which prevents the massless modes in the horn from reaching the strong-coupling region. In order for this to work, the condensate vertex operator (which is the field that appears in the spacetime action) must grow in the strong-coupling region. For example, in the low-energy effective action for two-dimensional string theory, the tachyon appears as

$$\mathcal{L}_T = \tfrac{1}{2} e^{-2D}\sqrt{|g|}\,[-(\Delta T)^2 + T^2]. \tag{4.4}$$

The tachyon condensate increases precisely like e^D as we proceed down the horn of the cornucopion, so its contribution to the action is proportional to the volume of the horn. Although we have no reason to trust the detailed predictions of the low-energy action, they should be valid in the bulk of the cornucopion where tachyon interactions are unimportant. Thus in the $c=1$ model, the spacetime action of the condensate fields is proportional to the volume of the cornucopion. If we assume that the same is true for the hypothetical nonsingular cornucopion solutions described above, we may conclude that for sufficiently small coupling, flat space is stable against decay into cornucopia, and their contribution to virtual loops is finite.

Note that in the above argument we have assumed that the Euclidean action density for the process of cornucopion production is positive. This is of course untrue in the naive definition of Euclidean quantum gravity. It is our belief that a correct treatment of tunneling in quantum gravity requires one to analytically continue the fields in the Euclidean action, perhaps in the manner advocated by Gibbons, Hawking, and Perry, in such a way that the action is positive and tunneling amplitudes always correspond to suppression. Such a continuation would also explain the apparent discrepancy between the fact that the infinite cornucopion has finite mass and our contention that Euclidean processes which create it as an intermediate state have infinite action. The energy density in a theory of gravity is not positive and bulk contributions to the energy cancel.

We would like to emphasize that the result of the foregoing calculation is of great interest in string theory even if our contention that cornucopions resolve the Hawking puzzle is incorrect. There is little doubt that string theory has classical solutions that are highly charged magnetic black holes with the geometry of a cornucopion. The above calculation can be viewed as a weak-coupling estimate of the probability for spontaneous production of a pair of such objects from the vacuum. Our result indicates that flat space is stable against decay into such objects for sufficiently weak coupling. Its failure to show such stability would at least mean the breakdown of the semiclassical argument for the stability of flat space, and might indicate a true instability. Indeed, the calculations suggest a potential instability at larger values of the coupling, but this cannot be verified without more detailed knowledge of the function $N(g_0)$. Known results in the $c=1$ model suggest that there may be a constant contribution to N and that it will not vanish as g_0 gets large. The only degrees of freedom that are massless in the horn of the cornucopion are free throughout most of its volume and give a coupling constant independent contribution to the free energy density. It seems unlikely that this will be precisely cancelled by massive degrees of freedom, so a strong-coupling instability of the flat vacuum seems like a definite possibility. Perhaps the dynamics of virtual cornucopion production contributes to the mechanism by which the string-coupling constant is fixed.

We now come to what is probably the most vexing problem for the cornucopion scenario, real cornucopion production in external magnetic fields. The conventional wisdom on this subject goes back to a paper [8] by Affleck and Manton on the pair production of 't Hooft-Polyakov monopoles. Their argument may be caricatured as follows. The pair production of point magnetic monopoles in a constant magnetic field is described by an instanton in the one particle quantum mechanics which describes single particle motion. If the field points in the x_3 direction, the instanton is a circle in the x_3-x_4 plane in Euclidean space. The radius of the circle is determined by the tunneling condition $RB=2m$, where m is the particle mass and B the strength of the magnetic field.

Affleck and Manton show that there is a field theory instanton which is better and better approximated by this quantum mechanical instanton in the limit that the magnetic radius R is larger than all length scales describing the structure of the monopole soliton. The basic idea is that if $\phi_0(x_1,x_2,x_3)$ represents the static soliton solution, then it is also a solution of the Euclidean equations of motion representing a soliton that exists for some length of Euclidean time. Now consider $\phi(x)=\phi_0(x_1,x_2,\sqrt{x_3^2+x_4^2}-R)$ which represents a soliton moving around an Euclidean circle of radius R. This configuration is not an exact solution of the Euclidean equations, but the leading correction to the equations of motion is canceled in the presence of the background field if R is chosen to take on the Schwinger value $2m/B$ (where m is given by the soliton mass). Thus ϕ can be chosen to be the first term in an expansion of an exact solution in powers of L/R, where L is a characteristic dimension of the soliton. Affleck and Manton show that in the limit $L/R\to 0$, the action for the solution is given by the usual Schwinger formula for point particles, plus a correction coming from the Coulomb interaction which is negligible for weak coupling.

While it is tempting to conclude that the same procedure is immediately applicable to cornucopions, and that they are consequently produced (when we sum over states) at infinite rates, there are several objections to such a conclusion. The first, as we have emphasized, is that purely semiclassical considerations, applied to the GMGHS soliton, are invalid, and the L dependence of the amplitude that we have predicted is only expected to arise semiclassically for a truly nonsingular classical configuration.

Second, the Affleck-Manton calculation gives us the leading behavior in an expansion of the instanton action in powers of the external field multiplied by length scales associated with the structure of the object. In order B^{-1} the structure dependence is all encoded in the soliton mass, and the soliton behaves like an elementary particle. In general we might expect a structure-dependent term of order B^0. Structure dependence would appear as a dependence of this coefficient on the ratio of scalar and gauge couplings which was not a simple function of the monopole mass. This term is absent for 't Hooft-Polyakov monopoles because of the symmetry of the instanton configuration.[16] The Garfinkle-Strominger [9] calculation of Wheeler wormhole production reveals a term of this order proportional to the Euler character. If other curvature squared terms were added to the action, modifying the structure of the instanton without modifying the behavior expected for point monopoles, we would obtain other corrections to the action of the same order. These corrections could be interpreted as dependence on the monopole structure. It seems to us that the term in the action proportional to cornucopion length, which we have suggested would be present, will show up in the coefficient of this order zero term in the Affleck-Manton expansion, if it shows up at all. The Affleck-Manton expansion assumes that all length scales characterizing the soliton are smaller than the magnetic length $2m/B$, which is surely not valid for infinitely long cornucopions. Thus, the question of whether the action has a term proportional to L can be reliably studied in this expansion only for L much smaller than the magnetic length. As we study the production of cornucopions of various sizes, the extensive term should first appear as L dependence of the coefficient of B^0. Then, when this term begins to dominate, the expansion breaks down. Thus we believe that the Affleck-Manton approximation is invalid for estimating the probability of tunneling to a virtual geometry of size larger than the magnetic length. However, we can imagine a process of tunneling to a cornucopion of a size within the domain of validity of the Affleck-Manon calculation, which then expands classically to become infinitely long. We will discuss this process in a moment.

Our final objection to the conclusion that the Affleck-Manton argument is applicable to cornucopia is much less clearly formulated. The Affleck-Manton instanton is topologically trivial (relative to the constant background field), while its analog for cornucopions is not. The corresponding Euclidean space time has an extra boundary. Thus there is no clean argument that such instantons should be included in the path integral.[17] By contrast, the sort of instanton depicted in Fig. 5, is a smooth deformation of flat Euclidean space. We have depicted this in-

[16]There is a term of order B^0 in the monopole production rate, but it comes from Coulomb interactions and is structure independent.

[17]And of course, no clean argument that they should not. Note that while the Garfinkle-Strominger instanton is also topologically nontrivial, it can be made to look more and more like a Melvin universe by increasing the external field. The topologically nontrivial part of the manifold (the maximal size of the wormhole neck), becomes metrically smaller and smaller. With a little bit of coarse graining we can make it disappear. Thus there is a sense in which this configuration is nearly continuously connected to the Melvin background. There is no corresponding coarse grained sense in which the circular deformation of the static cornucopion is continuously connected to the vacuum.

stanton as a sequence of time slices in which virtual cornucopion production proceeds by dimple formation and growth much as real formation by gravitational collapse does. Note that, in keeping with our prejudices about the dependence of Euclidean action on cornucopion length, we have imagined that the cornucopion configuration produced by the external field will have a finite length L at the point when the virtual state "pops into existence." We believe that L will be of order the magnetic length $2m/B$.

The cornucopions produced by the above process have finite volume but can now evolve classically into infinite volume objects. Classical evolution involves no further suppression of the amplitude, so we must inquire whether we have demonstrated a method of generating an infinite number of states with a fixed amplitude per state. We believe that we have not. The initial classical configuration of the cornucopion produced by an instanton like that shown in Fig. 5 is of finite volume. Even if we assume equal amplitudes for producing all of the excited states in this volume, the number of available initial states is finite and of order e^L.[18] This is much smaller than the essentially infinite number of states of the final cornucopion. Thus, we agree with the claim that cornucopia will be pair produced in an external field with about the amplitude suggested by the Affleck-Manton calculation. However, the method of production will be to tunnel to a cornucopion whose length is of order the magnetic length, and then inflate classically. The set of initial data for this classical evolution is too small to populate the large and ever growing set of states of the final geometry.

Our argument here is reminiscent of the argument that inflation solves the homogeneity problem of the Big Bang. In an inflationary universe, an initial microscopic domain becomes much bigger than the currently observable universe. Any state of the initial universe becomes essentially homogeneous in the final geometry. Thus the full space of possible initial states of the domain are in one to one correspondence with the very small subspace of the possible states of the inflated universe, namely, those which are homogeneous. This sort of expansion of the Hilbert space would seem to be an essential ingredient of a quantum theory of geometry.

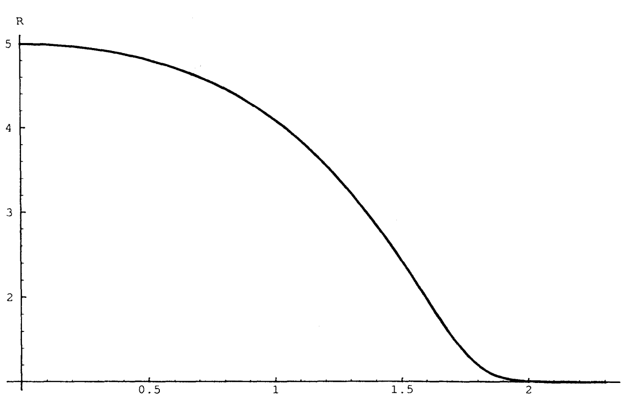

FIG. 4. Trajectory of the collapsing shell.

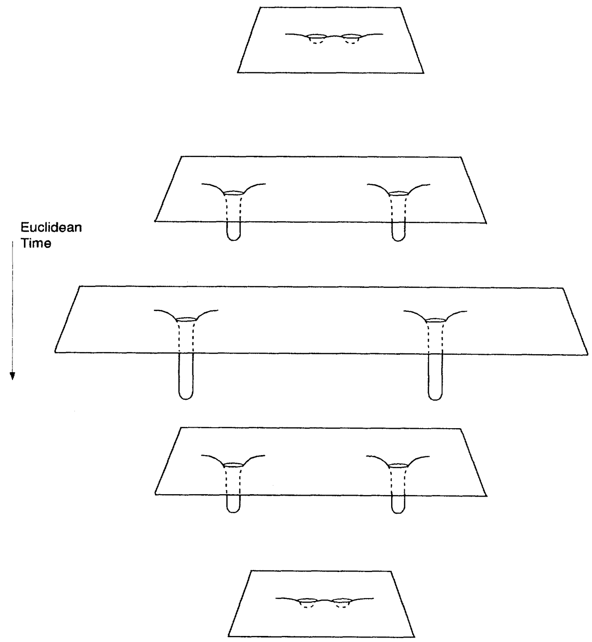

FIG. 5. Time slices of a Euclidean trajectory for pair-producing cornucopions.

We make no pretense that the above arguments are conclusive or rigorous. We believe however that the knee jerk reaction that cornucopions behave in every way like particles because they look like particles to an external observer are on no more a solid footing. This is a true statement for solitons in an ordinary Lorentz-invariant quantum field theory, but we believe that it is much less than obviously true in a theory in which geometry is dynamical. Unfortunately, the GMGHS solution does not lend itself to semiclassical analysis that would enable us to settle this question by a simple computation.

V. CONCLUSIONS

There are a number of pressing issues which must be addressed in order to satisfy ourselves that cornucopions are really a solution of the Hawking problem. First, it is imperative to find a way to establish the existence of the nonsingular solution with condensate upon which our considerations were based. This is not so much a problem of finding an exact conformal field theory for the GMGHS solution, as one of understanding spacetime bosonization in a string theoretic language. Are there vertex operators which represent the bosonic particle states formed by two parallel massless fermions? Is there a classical solution of string theory which corresponds to a nontrivial static background of these bose fields, and are its physical properties similar to those of the $c = 1$ model? Or is the formation of the fermion condensate not

[18]Here, as always, we assume an ultraviolet cutoff on the number of states in finite volume.

describable in classical language? Classical or not, what are the properties of the condensate? Most particularly, does the resulting state of the full theory have infinite volume like the classical solution, or finite effective volume like the linear dilaton electrodynamics of [24]? The latter scenario would solve the problem of infinite numbers of degenerate states in string theory, but would eliminate cornucopia as candidates for black hole remnants. The resulting object would have finite volume, of order its Schwarzchild radius and information content consistent with the Bekenstein-Hawking bound. Cornucopia will resolve the black hole information paradox only if, like the "symmetric" solution of the $c=1$ matrix model, they are truly infinite spaces.

We should note that the extremal dilaton black hole is not the only example of a possible remnant object with infinite volume hidden behind an apparently particulate facade. In their discussion of creation of a universe in the laboratory, Farhi and Guth [10] noted that a small patch of inflating universe looks to an outside observer like a black hole. If the neck that connects the inflating bubble to external flat space remains of finite extent, this would be a remnant object with an infinite number of states.[19] Strominger's "decoupled ghost" model [13], of semiclassical two-dimensional gravity seems to contain remnants of this type. Perhaps if one could find a four-dimensional model with remnants of this type, reliable semiclassical computations of production amplitude could be performed. It seems plausible that the relevant instanton would be a piece of Euclidean four sphere of finite radius (determined perhaps by the total ADM mass) attached to flat space. This would correspond to the nucleation of a finite size universe which would then undergo classical inflation. As in our discussion of the previous section the nucleation process could create only a small number of the states of the final de Sitter universe. However, if the singularity of a black hole were replaced with such an internal de Sitter universe, it could serve as a repository for the information that is apparently lost in the Hawking process. This is perhaps a more general and attractive scenario for the end point of black hole evaporation than that employing extremal dilaton black holes. It is based on the same general principle: in a theory of dynamical geometry, things can be larger on the inside than they are on the outside. We remain convinced that the most plausible scenario for pair production of such large extended objects is via nucleation of a small geometry which then expands classically. As in the inflationary universe, such a process is not able to access most of the states that can be accommodated in the eventual large geometry. Correspondingly, virtual processes will all involve the temporary appearance of finite volume geometries, which then subside into the vacuum. The intermediate set of states will always be finite (given an ultraviolet cutoff). These observations reconcile the existence of essentially infinite repositories of information, with the absence of observable production amplitudes for these exotic states in ordinary processes.

In our view, the version of the remnant scenario that we have presented is the most conservative resolution of the paradox of Hawking radiation that has been proposed. The apparently conservative idea that "all the information comes out in subtle correlations" seems to require us to envisage large quantum corrections to classical geometries in regions in which the classical description shows no sign of breakdown, while Hawking's original proposal of nonunitary evolution of density matrices seems to resist incorporation into a local effective theory of low-energy processes [19]. In the horned particle scenario, the physics of dynamical geometry turns out to be stranger than we had imagined, but not stranger than we are capable of imagining.

[19]The idea that such objects could be the end point of black hole evaporation was first presented to us by Michael Douglas.

ACKNOWLEDGMENTS

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APPENDIX A: FIELD EQUATIONS FOR THE LAGRANGIAN OF (2.5)

Given the Lagrangian written in terms of the fields u and σ, we can derive the following Euler-Lagrange equations

$$2uQ^2e^{-4\sigma}-2uR+4u(\partial\sigma)^2+8\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu u)-4ue^{-2\sigma}=0, \quad \text{(A1)}$$

$$u^2e^{-2\sigma}-\partial_\mu(u^2\sqrt{-g}\,g^{\mu\nu}\partial_\nu\sigma)-u^2Q^2e^{-4\sigma}=0, \quad \text{(A2)}$$

$$-(\nabla_\mu\nabla_\nu-g_{\mu\nu}\nabla^2)u^2-2u^2\partial_\mu\sigma\partial_\nu\sigma+4\partial_\mu u\,\partial_\nu u-g_{\mu\nu}[u^2e^{-2\sigma}+2(\partial u)^2-u^2(\partial\sigma)^2-\tfrac{1}{2}Q^2u^2e^{-4\sigma}]=0 \quad \text{(A3)}$$

and in particular

$$T_{00}=g_{00}g^{11}\nabla_1^2u^2-2u^2\dot{\sigma}^2+4\dot{u}^2=g_{00}[u^2e^{-2\sigma}+2(\partial u)^2-u^2(\partial\sigma)^2-\tfrac{1}{2}Q^2u^2e^{-4\sigma}]=0. \quad \text{(A4)}$$

APPENDIX B: THE MATCHING EQUATIONS

To carry out the expansion in powers of n towards the interior of the shell we will express all functions in terms of τ and n. In particular, we will need the derivative in the direction orthogonal to the shell, and so need $\partial/\partial n$ in terms of $\partial/\partial t$ and $\partial/\partial r$.

Consider the general coordinate transformation given by $(\tau,t)\to(t(\tau,n)r(\tau,n))$. Using the known forms of the metric inside and outside the shell we find the following equations:

$$\begin{aligned}-1&=g_{\tau\tau}\\&=g_{tt}\left[\frac{\partial t}{\partial\tau}\right]^2+g_{rr}\left[\frac{\partial r}{\partial\tau}\right]^2\\&=-\left[\frac{\partial t}{\partial\tau}\right]^2+\frac{1}{\left[1-\frac{Q}{r}\right]^2}\left[\frac{\partial r}{\partial\tau}\right]^2\end{aligned}\tag{B1}$$

and

$$1=g_{nn}=-\left[\frac{\partial t}{\partial n}\right]^2+\frac{1}{\left[1-\frac{Q}{r}\right]^2}\left[\frac{\partial r}{\partial n}\right]^2\tag{B2}$$

and

$$0=g_{\tau n}=-\frac{\partial t}{\partial n}\frac{\partial t}{\partial\tau}+\frac{1}{\left[1-\frac{Q}{r}\right]^2}\left[\frac{\partial r}{\partial n}\right]\left[\frac{\partial r}{\partial\tau}\right].\tag{B3}$$

Now put $(\partial r/\partial\tau)|_{n=0}=\dot R(\tau)$.
Then (B1) gives

$$\frac{\partial t}{\partial\tau}=\left[1+\frac{1}{\left[1-\frac{Q}{R}\right]^2}\dot R(\tau)^2\right]^{1/2}\tag{B4}$$

and using (B2) and (B3)

$$\frac{\partial r}{\partial n}=\left[\dot R(\tau)^2+\left[1-\frac{Q}{R}\right]^2\right]^{1/2}.\tag{B5}$$

For a thin shell of matter that acts as a source for the stress tensor of the theory with Lagrangian (2.10), we have

$$\mu(\tau)=\int_{-\epsilon}^{\epsilon}T_{00}dn,\tag{B6}$$

where $\mu(\tau)$ is the energy density of the shell in the (τ,n) coordinates. Only the singular part of T_{00} will contribute to (B6) and by looking at the expression for T_{00}, we see that (as a consequence of the continuity of U and σ across the boundary of the shell)

$$Mu(\tau,0)^2=\int_{-\epsilon}^{\epsilon}g_{00}g^{11}\nabla^2(u^2)dn=2u\partial_n u|_{-\epsilon}^{\epsilon}.\tag{B7}$$

Using the results of Appendix A, we find that at $n=-\epsilon$,

$$u\partial_n u=f_1(\tau)R(\tau)^2e^{-2\phi_0}\left[1-\frac{Q}{R}\right]\tag{B8}$$

and at $n=\epsilon$,

$$u\partial_n u=e^{-2\phi_0}R\left[1-\frac{Q}{2R}\right]\left[\dot R(\tau)^2+\left[1-\frac{Q}{R}\right]^2\right]^{1/2}.\tag{B9}$$

So we have the final form of the matching condition;

$$MR^2\left[1-\frac{Q}{R}\right]=R\left[1-\frac{Q}{2R}\right]\left[\dot R(\tau)^2+\left[1-\frac{Q}{R}\right]^2\right]+R^2f_1\left[1-\frac{Q}{R}\right].\tag{B10}$$

Finally, we note that the parameter M in the above equations is the ADM mass of the extremal dilaton black hole: $M=Qe^{-\phi_0}/2$.

APPENDIX C: AN ANSATZ FOR THE MOTION OF THE SHELL

As in the text we assume that $f_1(\tau)=a/R(\tau)$, which gives an ordinary differential equation for $R(\tau)$. Let us first analyze this equation for $\tau\to\infty$, by assuming that $R(\tau)=Q+\epsilon(\tau)$ with $\epsilon\ll Q$. Then $\dot\epsilon=-\gamma\epsilon$, and thus $R(\tau)$ behaves for $\tau\to\infty$ in the manner asserted in the text.

Let us now plug the ansatz into the matching equation and solve for $\dot R(\tau)$. This gives,

$$\begin{aligned}\dot R(\tau)&=-\frac{(R-Q)}{R(2R-Q)}\sqrt{4(f_1R^2-MR^2)^2-(2R-Q)^2}\\&=-\frac{(R-Q)}{R(2R-Q)}\sqrt{4(aR-MR^2)^2-(2R-Q)^2}.\end{aligned}\tag{C1}$$

This can be integrated numerically to obtain the solution for a choice of $R(0)$ and $\dot R(0)$.

For $\dot R(0)=0$ at $R(0)=5$, and with $a=109.10$, we find the solution displayed in Fig. 4—a smooth collapse from $R=5$ to $R=1$. If we then look at the remaining equations, expanded to the lowest order in n, then we have five equations for the five functions, $d_1(\tau)$, $h_1(\tau)$, $g_1(\tau)$, $d_2(\tau)$, and $f_2(\tau)$, with all coefficients going to zero or a finite number as $\tau\to\infty$. So this provides a regular collapse situation for this ansatz. We could now proceed to solve for higher-order terms in the power series expansion around the shell, finding smooth solutions for all τ. Of course, this procedure does not guarantee a solution which is everywhere smooth.

To analyze the collapse situation completely, one should first find a time-dependent solution to the field equations (ignoring for the moment the matching equations) arising from some smooth initial data on a Cauchy hypersurface, which has a dilaton field that is an increasing function of time which approaches infinity asymptotically. One then uses the matching equations to join this solution onto the GMGHS solution along the collapsing shell. As indicated in the text, we believe that there is a great deal of freedom in this procedure, and we have not yet been able to find a simple ansatz which gives explicit solutions for the interior of the shell.

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