Cosmological multi-black-hole solutions

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We present simple, analytic solutions to the Einstein-Maxwell equation, which describe an arbitrary number of charged black holes in a spacetime with a positive cosmological constant Λ. In the limit $\Lambda=0$, these solutions reduce to the well-known Majumdar-Papapetrou (MP) solutions. Like the MP solutions, each black hole in a $\Lambda>0$ solution has charge Q equal to its mass M, up to a possible overall sign. Unlike the $\Lambda=0$ limit, however, solutions with $\Lambda>0$ are highly dynamical. The black holes move with respect to one another, following natural trajectories in the background de Sitter spacetime. Black holes moving apart eventually go out of causal contact. Black holes on approaching trajectories ultimately merge. To our knowledge, these solutions give the first analytic description of coalescing black holes. Likewise, the thermodynamics of the $\Lambda>0$ solutions is quite interesting. Taken individually, a $|Q|=M$ black hole is in thermal equilibrium with the background de Sitter Hawking radiation. With more than one black hole, because the solutions are not static, no global equilibrium temperature can be defined. In appropriate limits, however, when the black holes are either close together or far apart, approximate equilibrium states are established.

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I. INTRODUCTION

In this paper we give simple, analytic solutions to the Einstein-Maxwell equation, which describe collections of charged black holes in a spacetime with a positive cosmological constant Λ. In the limit of a vanishing cosmological constant, our solutions reduce to the well-known Majumdar-Papapetrou (MP) solutions [1]. For all values of $\Lambda\geq 0$, individual black holes in the solutions have charge Q equal to mass M, up to a possible overall sign. For $\Lambda>0$, however, this is no longer the condition of extremality [2]. We study the mechanical and thermodynamic properties of the $\Lambda>0$ solutions, which turn out to generalize the properties of the MP solutions in interesting ways.

For $\Lambda=0$, the black holes are static. In contrast, the black holes in a $\Lambda>0$ solution are highly dynamical. The black holes ignore one another and follow natural trajectories in the background de Sitter spacetime. The black holes eventually either merge or move out of causal contact. As far as we known, these solutions give the first analytic description of coalescing black holes. However, there are many questions about the causal structure and dynamics of these solutions which remain to be answered.

The thermodynamics of solutions with $\Lambda>0$ is also quite interesting. The black holes in the MP solution each have vanishing Hawking temperature, and so are in thermal equilibrium with each other and also with the background flat spacetime. With $\Lambda>0$, the background de Sitter spacetime has an ambient Hawking temperature [3]. A single $|Q|=M$ black hole actually has a temperature equal to the de Sitter temperature, and hence would be in a state of thermal equilibrium with the background spacetime. The temperature, however, depends on both the cosmological constant and the mass of the black hole. Adding more black holes, with differing masses, then, does not produce a thermal equilibrium state. Indeed, with more than one black hole, because the solutions are not static, no global equilibrium temperature can be defined. Approximate temperatures, however, can be defined in the limits where the black holes are either widely separated or coalesced. A given solution, then, describes a transition between one state of thermal equilibrium in the far past and another equilibrium state in the far future. However, an understanding of the nonequilibrium thermodynamics of the transition between these states will require further investigation.

II. MAJUMDAR-PAPAPETROU SOLUTIONS

In Newtonian mechanics, a collection of charged point particles, each having charge equal to its mass,[1] can stay at rest in a state of mechanical equilibrium. The electrostatic repulsions of the particles exactly balance the gravitational attractions. Remarkably, the same balance holds in general relativity. The MP solutions to the source-free Einstein-Maxwell equation correspond to this Newtonian situation. The MP solutions, themselves, are geodesically incomplete. Hartle and Hawking [4] showed how the MP solutions could be analytically extended and interpreted as a system of charged black holes. The metric and gauge field for the MP solutions are given by

$$ds^2=-\Omega^{-2}dt^2+\Omega^2(dx^2+dy^2+dz^2)\ ,$$
$$A_t=\Omega^{-1}\ ,\quad \Omega=1+\sum_i \frac{m_i}{r_i}\ , \tag{2.1}$$
$$r_i=\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}\ ,$$

[1] We use geometrical units, $G=c=\hbar=1$.

where m_i and (x_i, y_i, z_i) are the masses and locations of the black holes. It can be shown [4] that the points $r_i = 0$ actually represent event horizons of area $4\pi m_i^2$. For the case of one black hole, the metric (2.1) is just the extreme Reissner-Nordström metric in isotropic coordinates.

A single Reissner-Nordström black hole having charge equal to its mass is the simplest example of an extremal black hole. If the charge of the hole were increased (or the mass decreased) further, then the curvature singularity would no longer be hidden behind an event horizon. If such a naked singularity arose in a "physical" solution, it would violate the cosmic censorship conjectures and lead to a breakdown of predictability. That the Hawking temperature of an extremal Reissner-Nordström black hole vanishes is impressive evidence for cosmic censorship. The evaporation of a charged black hole terminates when it reaches the extremal state, leaving the singularity hidden.[2] The MP solutions then describe charged black holes in thermal, as well as mechanical, equilibrium with temperature equal to zero. A related property is that the MP solutions are exact ground states of $N=2$ supergravity, since all quantum corrections to the effective action expanded around the MP solutions vanish [6]. Analogues of the MP solutions have also been written down for dilaton black holes [7,8]. The individual black holes in these solutions are also extremal ones.

III. REISSNER–NORDSTRÖM–de SITTER SOLUTIONS

There is an analogue of the Reissner-Nordström solution for a spacetime with a cosmological constant. The Reissner–Nordström–de Sitter (RNdS) metric and gauge field in Schwarzschild coordinates are given by

$$\begin{aligned} ds^2 &= -V(R)dT^2 + V^{-1}(R)dR^2 + R^2 d\Omega^2 , \\ A_T &= -\frac{Q}{R} , \quad V(R) = 1 - \frac{2M}{R} + \frac{Q^2}{R^2} - \frac{1}{3}\Lambda R^2 , \end{aligned} \tag{3.1}$$

where M and Q are the mass and charge of the hole and Λ is the cosmological constant. The metric has a curvature singularity at the origin. There are event horizons at the radii where (R) vanishes. Taking $M = Q = 0$ in (3.1) gives the static form of the de Sitter metric.

Romans [2] has recently studied the thermodynamics of these solutions. This is more complicated, because for $\Lambda > 0$ the de Sitter horizon also radiates at its own temperature [3]. Indeed, Gibbons and Hawking [3] have extended the laws of black hole thermodynamics to include cosmological event horizons. The Hawking temperature for a horizon at radius ρ is given by

$$T = \frac{|\kappa|}{2\pi} = \frac{1}{4\pi}|V'(\rho)| , \tag{3.2}$$

where κ is the surface gravity at the horizon. As in the $\Lambda = 0$ case, Romans finds extreme RNdS black holes with zero temperature, in which the inner and outer black hole horizons coincide. For $\Lambda > 0$, the extremal black holes have $M < |Q|$. In Appendix A, we show that, as in the $\Lambda = 0$ case [9], it is impossible to destroy the horizon of an RNdS black hole by throwing in charged particles to charge it past the extremal limit

Another interesting class of RNdS solutions has the temperatures of the black hole and de Sitter horizons equal. Remarkably, these solutions have $|Q| = M$.[3] The metric function then has the form

$$V(R) = \left[1 - \frac{M}{R}\right]^2 - \frac{1}{3}\Lambda R^2 . \tag{3.3}$$

The common temperature is given by [2]

$$T = \frac{1}{2\pi}\left[\frac{\Lambda}{3}(1 - 4M\sqrt{\Lambda/3})\right]^{1/2} . \tag{3.4}$$

In the naive picture of black hole evaporation, the $|Q| = M$ solutions are thermodynamically stable states and are the end points of the evaporation process. If $M > |Q|$, then the black hole is hotter than the de Sitter horizon and it will evaporate until it reaches $M = |Q|$. If $M < |Q|$, then the de Sitter horizon is hotter and the black hole will accrete radiation until it reaches $M = |Q|$. It is interesting that for $\Lambda > 0$ the conditions of extremality and thermal equilibrium no longer coincide.

We note that the total gravitational entropy of an RNdS black hole is maximized, for fixed charge and cosmological constant, in this equilibrium state. The total gravitational entropy is given by the sum of the areas of the black hole and cosmological event horizons:

$$S_{\text{grav}} = \tfrac{1}{4}A_{\text{BH}} + \tfrac{1}{4}A_{\text{dS}} . \tag{3.5}$$

The result then follows from the generalized first law of thermodynamics given in [3]. For an infinitesimal perturbation between RNdS solutions of fixed charge, the generalized first law states that

$$\kappa_{\text{dS}}\delta A_{\text{dS}} - \kappa_{\text{BH}}\delta A_{\text{BH}} = 0 , \tag{3.6}$$

where κ_{dS} and κ_{BH} are the surface gravities of the de Sitter and black hole event horizons. The change in entropy of such a variation can then be written as

$$\delta S_{\text{grav}} = \frac{1}{4}\left[1 + \frac{\kappa_{\text{BH}}}{\kappa_{\text{dS}}}\right]\delta A_{\text{BH}} . \tag{3.7}$$

Clearly, the entropy is extremized for $\kappa_{\text{dS}} = -\kappa_{\text{BH}}$, which coincides with the condition that the black hole and de Sitter temperatures be equal, and happens when $|Q| = M$. From the discussion in the previous paragraph, we see that the extremum is a maximum. This result is in contrast with an evaporating Schwarzschild black hole, where the gravitational entropy decreases. Here, the de Sitter horizon acts like the walls of a box. The entropy of the de Sitter horizon measures, in some sense, the entro-

[2]Holzhey and Wilczek [5] have studied how the evaporation process for charged dilaton black holes can terminate at an extremal state, even though the temperature of this state may be nonzero.

[3]This had been noted previously by Mellor and Moss [10].

py of the Hawking radiation carried away from, or absorbed by the black hole.

IV. MOTION OF TEST PARTICLES

Are cosmological analogues of the MP solutions built out of extremal or $|Q|=M$ black holes? The black holes in the MP solution "ignore" one another. We would like to find some similar phenomenon for charged black holes in a de Sitter background. To find the right criterion, we look at the motion of charged test particles in the RNdS metric (3.1). We will see that the motion of $q=m$ test particles in a $Q=M$ RNdS background is particularly simple.

The conserved energy E of a test particle of charge q, mass m, and four-velocity u^a on a radial path in an RNdS background is

$$\frac{E}{M}=-\xi^a\left[u_a+\frac{q}{m}A_a\right]=V(R)\frac{dT}{d\tau}-\frac{q}{m}A_T(R)\ , \qquad (4.1)$$

where ξ^a is the static Killing vector and τ is the proper time. Together with the normalization condition $u^a u_a=-1$, this gives the equation of motion

$$\left[\frac{dR}{d\tau}\right]^2=-V(R)+\left[\frac{E}{m}+\frac{q}{m}A_T(R)\right]^2 . \qquad (4.2)$$

Substituting in the gauge field and metric function for a $Q=M$ black hole, this becomes

$$\left[\frac{dR}{d\tau}\right]^2=-\left[1-\frac{M}{R}\right]^2+\frac{1}{3}\Lambda R^2+\left[1-\frac{qM}{mR}\right]^2 . \qquad (4.3)$$

If the test particle has $q=m$ and has energy to equal its rest mass (i.e., $E=m$), then this further reduces to

$$\frac{dR}{d\tau}=\pm\left[\frac{\Lambda}{3}\right]^{1/2}R\ . \qquad (4.4)$$

This in turn is the same as the equation of motion for a minimum energy test particle in a background de Sitter spacetime (i.e., $Q=M=0$). This looks like what we want. The $q=m$ test particle is, in some sense, not affected by the presence of the $Q=M$ black hole. This hint will turn out to be what we need to guess an exact solution.

Note that there are two choices for the path of the $q=m$ test particle in (4.4). It can be either "ingoing" or "outgoing." This is in contrast with the $\Lambda=0$ case, where a minimum energy $q=m$ test particle stays at rest in the field of a $Q=M$ black hole. Choosing one path or another breaks the time reversal invariance of the system. We will continue to use the names "ingoing" and "outgoing" to denote these paths below, even though in different coordinates they may not look ingoing or outgoing.

V. COSMOLOGICAL COORDINATES

We seek to promote our $q=m$ test particles to black holes, expecting that they will follow "ingoing" or "outgoing" paths as in (4.4). Such black hole solutions should be quite complicated in static coordinates. For example, moving charged black holes will generate magnetic as well as electric fields. However, in cosmological coordinates the motion of such minimum energy particles is quite simple.

The RNdS solutions can be rewritten in cosmological coordinates. For $Q=M$ this has the form

$$ds^2=-\Omega^{-2}dt^2+a^2(t)\Omega^2(dr^2+r^2d\Omega^2)\ ,$$
$$A_t=\Omega^{-1}\ ,\quad \Omega=\left[1+\frac{m}{ar}\right]\ ,\quad a(t)=e^{Ht}\ , \qquad (5.1)$$
$$H=\pm\left[\frac{\Lambda}{3}\right]^{1/2} .$$

For $M=0$, this is just the standard cosmological form of the de Sitter metric. We will call the coordinate system with $H>0$ expanding and that with $H<0$ contracting. The static Killing vector is given by

$$\xi^a=\left[\frac{\partial}{\partial t}\right]^a-Hr\left[\frac{\partial}{\partial r}\right]^a . \qquad (5.2)$$

The norm of the Killing vector vanishes at the horizons, which implies that the de Sitter horizon r_+ and the outer black hole horizon r_- are located at $Har_\pm\Omega^2=1$, or

$$r_\pm=\frac{1}{2a(t)|H|}(1-2M|H|\pm\sqrt{1-4M|H|})\ . \qquad (5.3)$$

There is also an inner black hole horizon at negative r in these coordinates. Note that the products $a(t)r_\pm$ are constants, and also that the metric (5.1) is nonsingular at $r_\pm$. One can check that the surface gravity κ at the two horizons, defined by

$$\tfrac{1}{2}\nabla_a(\xi^b\xi_b)=-\kappa\xi_a\ , \qquad (5.4)$$

yields the correct value (3.4) for their common temperature.

The transformation between the static and cosmological coordinates is given by

$$a(t)r=R-M\ ,\quad t=T+h(R)\ ,$$
$$h'(R)=-\frac{HR^2}{(R-M)V(R)}\ . \qquad (5.5)$$

Integrating this transformation near the black hole and de Sitter horizons, one finds that the expanding coordinates cover the past black hole horizon and the future de Sitter horizon. Likewise, the contracting coordinates cover the past de Sitter horizon and the future black hole horizon. Thus, the metric (5.1) with $H>0$ actually describes the white hole portion of the extended spacetime, while the metric with $H<0$ describes the black hole part. This can be confirmed by looking at null geodesics. The two-sphere $r=r_-$ is an outer trapped surface for $H<0$. Null rays cannot get out. For $H>0$ it is an inner trapped surface. Null rays cannot get in.

We can look at the paths of $q=m$ test particles in (5.1). The paths (4.4) that were "outgoing" in static coordinates stay at constant spatial comoving coordinate in the expanding coordinates, whereas they move out "doubly" fast in the contracting coordinates (and vice versa for the "ingoing" test particles).

It is useful in understanding the new solutions below to look at the motion of these particles in a little more detail. Consider an "outgoing" $q=m$ test particle in expanding coordinates ($H>0$). It stays at a constant coordinate radius. The de Sitter and white hole horizons $r_{\pm}$, however, redshift like $1/a$. At early times the horizons are both at large radii and the test particle is inside both. At some later time the white hole horizon sweeps past it, and then at some still later time the de Sitter horizon sweeps past it. In static coordinates, we could follow the portion of the particle's path between the two horizons. We would see it leave the white hole horizon at $T=-\infty$ and move out to the de Sitter horizon at $T=+\infty$. Likewise, an "incoming" test particle, which stays at a fixed radial coordinate in contracting coordinates ($H<0$), starts outside the past de Sitter horizon, moves in through it and then through the black hole horizon. In static coordinates, this looks like a particle leaving the de Sitter horizon at $T=-\infty$ and getting to the black hole horizon at $T=+\infty$.

VI. COSMOLOGICAL MP SOLUTIONS

The metric (51.) for one $|Q|=M$ RNdS black hole in cosmological coordinates closely resembles the MP metric (2.1) for the case of one black hole. This suggests a simple form for the cosmological MP solutions:

$$ds^2=-\Omega^{-2}dt^2+a^2(t)\Omega^2(dx^2+dy^2+dz^2)\ ,$$

$$A_t^{-2}=\Omega^{-1},\quad a(t)=e^{Ht}\ ,\quad \Omega=1+\sum_i \frac{m_i}{ar_i}\ , \qquad (6.1)$$

$$r_i=\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}\ ,\quad H=\pm\sqrt{\Lambda/3}\ .$$

Indeed, we will show that more generally a metric and gauge field of the form (6.1) solve the Einstein-Maxwell equations with cosmological constant Λ, if $\partial_t(a(t)\Omega)=\dot a$ and $\overline{\nabla}^2\Omega=0$, where $\overline{\nabla}^2$ is the flat space Laplacian.

First consider the constraint equations. The Hamiltonian, momentum and Gauss law constraints on a spatial slice are given by

$$\begin{aligned} H&=-{}^{(3)}R+\frac{1}{g}(\pi^{ij}\pi_{ij}-\tfrac{1}{2}\pi^2)=-16\pi\rho\ ,\\ H_k&=-\frac{2}{\sqrt{g}}\,{}^{(3)}\nabla_i\pi^i_k=16\pi J_k\ ,\quad {}^{(3)}\nabla_i E^i=0\ , \end{aligned} \qquad (6.2)$$

where g_{ij} and π_{ij} are the spatial metric and momentum, ρ and J_k are the energy and current densities, and E^i is the electric field. For a metric of the form (6.1), if we assume that $\partial_t(a\Omega)=\dot a$ then the momentum is given by

$$\pi_{ij}=-2\frac{\dot a}{a}\sqrt{g}\,g_{ij}\ . \qquad (6.3)$$

Given this simple relation between π_{ij} and the metric, the momentum constraint in (6.2) is satisfied with zero current. The three-dimensional scalar curvature is

$${}^{(3)}R=\frac{1}{a^2\Omega^2}\left\{-\frac{4}{\Omega}\overline{\nabla}^2\Omega+\frac{2}{\Omega^2}\overline{g}^{ij}(\overline{\nabla}_i\Omega)\overline{\nabla}_j\Omega\right\}\ , \qquad (6.4)$$

where $\overline{g}_{ij}$, $\overline{\nabla}_i$ are the flat spatial metric and derivative. The energy density ρ has contributions from the Maxwell field and the cosmological fluid $\rho=\rho_{\max}+\rho_{\cos}$. If $A_t=1/\Omega$, then

$$8\pi\rho_{\max}=\frac{1}{a^2\Omega^4}\overline{g}a^{ij}(\overline{\nabla}_i\Omega)\overline{\nabla}_j\Omega\ . \qquad (6.5)$$

From the expressions (6.3), (6.4), and (6.5), one sees that the Hamiltonian constraint can be satisfied by having the "black hole parts" and the "cosmological parts" vanish separately. That is, one has a solution if

$$\overline{\nabla}^2\Omega=0\quad\text{and}\quad\left[\frac{\dot a}{a}\right]^2=\frac{8\pi}{3}\rho_{\cos}\ . \qquad (6.6)$$

Further, the Gauss law constraint is satisfied if (6.6) holds. Note that (6.6) can be satisfied by any time dependent, but spatially constant, $\rho_{\cos}$.

Some insight into the nature of the solutions is gained from the matter equations of motion: $\nabla_a(T^{ab}_{\max}+T^{ab}_{\cos})=0$. The time component of this equation is

$$\frac{1}{\Omega}\frac{d}{dt}\rho_{\cos}+3\frac{\dot a}{a}(p_{\cos}+\rho_{\cos})+\frac{1}{2\Omega}\frac{d}{dt}\rho_{\max}+2\frac{\dot a}{a}\rho_{\max}=0\ . \qquad (6.7)$$

From (6.5) it follows that the Maxwell terms themselves sum to zero. Therefore, the part involving the cosmological fluid must also vanish independently. At this point there are two choices for a solution. From (6.6), $\rho_{\cos}$ cannot depend on the spatial coordinates. If $\rho_{\cos}$ is allowed to be time dependent, then, since Ω has spatial dependence, the pressure $p_{\cos}$ must be spatially dependent. This makes sense physically; ordinary matter would tend to accrete around the black hole. The inhomogeneous pressure is needed to keep the density constant. In the one black hole case, such solutions have been studied by McVittie [11] and others (see, e.g., [12]). The other choice, which we shall make, is to take $\rho_{\cos}$ to be independent of time. We then have $p_{\cos}=-\rho_{\cos}=-\Lambda$, the form for a cosmological constant. Again, this makes sense physically, because a cosmological constant cannot accrete.[4] Finally, we note that the full set of evolution equations for π_{ij} are straightforward to check and yield no more constraints.

VII. GEOMETRIC AND THERMODYNAMIC PROPERTIES

The solutions (6.1) with $H<0$ appear to describe a system of "incoming" charged black holes. The solutions

[4]An idealized model of a cosmic string, spacetime minus a wedge, can be embedded in any flat Robertson-Walker background since the string does not accrete [13].

with $H>0$ would describe the time-reversed situation; a system of "outgoing" charged white hole. The first thing to establish is that the solutions really do contain black hole (or white hole) horizons. One expects this to be the case. However, it is not straightforward to locate the horizons. The solutions, with more than one black hole, do not appear to have a stationary Killing vector. Thus, one cannot simply look for the surfaces on which the Killing vector becomes null. One can look for apparent horizons (boundaries of regions of trapped surfaces) in a given spatial slice, but this, too, is complicated by the lack of symmetry.

The situation does simplify for early and late times, when the holes are either far apart or close together. For concreteness, consider two "incoming" black holes in a background with $H<0$. At early times the holes are far apart, and near each one the metric approaches that for a single hole. In this limit, one can verify that there are regions of outer trapped surfaces surrounding each of the holes. These regions extend out to radii $r_{-,i}$ given by (5.3). As time goes on, the universe contracts, and the coordinate size of each of these apparent horizons grows. Their shapes will distort due to the presence of the other hole. At some point in time the two apparent horizons will collide with one another and presumably merge. Indeed, in the late time limit one can verify that outer trapped surfaces surround both of the holes.

To summarize, when $H<0$, one can show that the two objects are first surrounded individually by outer trapped surfaces, and then later there is an outer trapped surface that surrounds both. For white holes with $H>0$, the situation is time reversed. At early times inner trapped surfaces surround both objects together. Later on, the objects are surrounded only separately by regions of inner trapped surfaces. These results agree with what one expects from the area theorem. Black holes merge, and white holes split.

Another question is whether an extension of the solutions with $H>0$, which covers the black hole portions of the "outgoing" holes (and likewise for the white hole portions of the "ingoing" black holes with $H<0$). For the case of one black hole, (5.5) gives an explicit coordinate transformation from expanding to contracting coordinates. It appears that making this transformation locally about one of the "outgoing" holes (as $t\to\infty$), does indeed extend the spacetime to cover a black hole horizon. However, aspects of this transformation are still confusing, e.g., how the different regions fit together. We defer an explicit presentation to future work.

For the purposes of discussion in this paper, we will assume that the "outgoing" white hole spacetimes are extendable to coordinates that cover black hole horizons and past de Sitter horizons. Likewise, the "incoming" black hole solutions are assumed to be fully extendable. A related question is whether the black holes must all be "outgoing" or all "incoming," or whether there can be arbitrary combinations of "incoming" and "outgoing" holes.

Although a full understanding of the thermodynamics of these solutions must await a better understanding of their analytic extensions and horizon structures, the thermodynamics appears to be quite interesting. Given that there is no stationary Killing vector, the usual definition of temperature in terms of surface gravity at a horizon (3.2) does not work. Indeed, these solutions appear to be highly nonequilbrium. Still, as for the horizon structure, we can give a simple description of the thermodynamic behavior at early and late times, as follows.

Recall that for one $Q=M$ RNdS black hole, the de Sitter and black hole horizons are in thermal equilibrium at a common temperature $T_{\rm BH}=T_{\rm dS}=T(M,\Lambda)$ given by (3.4). Now consider two black holes in a de Sitter background, both satisfying $Q=M$, but with different masses M_1 and M_2. If the black holes are widely enough separated to be out of causal contact, then each black hole will have its own distinct de Sitter horizon. Also, in the region near each black hole there will be an approximate static Killing vector that can be used to define an approximate temperature. Each of the black holes will be in approximate thermal equilibrium with its own de Sitter horizon at temperatures $T(M_1,\Lambda)$ and $T(M_2,\Lambda)$ respectively. If the two black holes are "ingoing" with respect to one another, then this will be the situation at very early times. Later the black holes come into causal contact and eventually merge into a single black hole with mass $M=M_1+M_2$. At very late times, there will again be an approximate static Killing vector. The black hole horizon will be in equilibrium with the de Sitter horizon at temperature $T(M_1+M_2,\Lambda)$.

VIII. CONCLUSIONS AND FURTHER QUESTIONS

We have presented solutions to the Einstein-Maxwell equations with a positive cosmological constant, which plausibly represent a collection of charged black holes moving either towards or away from one another. We have also given a description of the horizon structure and thermodynamics of these solutions in the early and late time limits. There is clearly a good deal more work to be done on these solutions; some of which has been noted in the previous section.

Let us start with questions about the classical properties of the solutions. It is important to know whether these solutions can be fully analytically extended, as has been assumed above, and to have a clear picture of their horizon structures. It should be especially interesting to study the regime in which the black holes are merging (the white hole splitting). These solutions might give insight into an approximation which treats astrophysically interesting mergers of black hole binaries. Although, note that here the mergers take place without any gravitational radiation.

The thermodynamics of the intermediate state, where the black holes are distinct, but still in causal contact, should be interesting. Will the masses of the black holes change due to emission and absorption of Hawking radiation during this period? If so, then they should emerge from this nonequilibrium state with charges in general differing from their masses. Eventually then, each black hole should exchange radiation with the background until it again reaches a $Q=M$ state. A first step towards understanding the exchange of energy would be to study what a particle detector sees, if it follows the path of a $q=m$ test particle in a $Q=M$ RNdS background.

The splitting and merging of holes raises interesting questions about the parameter space of $\Lambda=0$ solutions. Extreme RN black holes are regarded a solitons of general relativity, satisfying a kind of Bogomolnyi bound [14]. But in at least one respect they appear to differ from other solitons. Consider a magnetic monopole in the Bogomolnyi limit with two units of topological charge. The solution will have zero modes corresponding to the possibility of breaking it up into two monopoles, each having unit topological charge. By analogy, we would expect an extreme RN black hole to have zero modes, corresponding to the possibility of breaking it up into smaller extreme black holes. By the area theorem, though, this cannot be the case. On the other hand, a single $Q=M$ RNdS white hole appears to be unstable. It can be split into any number of charge equal mass fragments, which are then carried apart by the expansion of the universe. In the limit of zero cosmological constant, this may correspond to marginal stability of an extreme RN white hole. This picture is supported by analysis at the level of test particles. A $q=m$ test particle can stay at rest in a $Q=M$ RN background, whether it is inside or outside the event horizon. Hence, there should be analogues of the MP solutions describing merged black holes.

Another interesting set of questions involves supersymmetry. Multiobject solutions are usually associated with Bogomolnyi bounds arising from an underlying supersymmetry of the solution. Romans [2] has noted, though, that the relevant supersymmetry (coming from $N=2$ Yang-Mills supergravity) is consistent only with $\Lambda<0$. Our solutions, then, are not supersymmetric, at least in this sense. On the other hand, for $\Lambda<0$, while the $Q=M$ RNdS holes are supersymmetric, they are also naked singularities. It should be interesting to see whether these can also be assembled into multihole solutions and to understand the role played by supersymmetry [15].

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APPENDIX

In Ref. [9], Wald asked whether you can destroy a black hole by overcharging it. One might think that, by throwing in particles with a high charge-to-mass ratio, one could charge the hole past the extremal limit of $Q=M$. It turns out [9] that for a charged particle to get over the Coulomb barrier into the hole, it must have more energy greater than or equal to its charge.

Here we do the analogous calculations for the extreme RNdS black holes [2]. For a metric of the form

$$ds^2=-V(r)dt^2+V(r)^{-1}dr^2+r^2d\Omega^2 \tag{A1}$$

with gauge field $A_t(r)=-Q/r$, the equation of motion for a test particle (of energy E, rest mass m, and charge q) on a radial geodesic is given by (4.3):

$$\left[\frac{dr}{d\tau}\right]^2=-V(r)+\frac{1}{m^2}[E+qA_t(r)]^2 \ . \tag{A2}$$

In order for the particle to get into the hole it must, at least, reach the event horizon. Looking for the minimum energy particle which reaches the horizon, we set $E=m$ and $dr/d\tau=0$ at the horizon. At the horizon radius ρ, we then have $[V(\rho)=0]$

$$\left[1-\frac{qQ}{m\rho}\right]^2\geq 0 \ . \tag{A3}$$

This translates to

$$\frac{m}{q}\geq\frac{Q}{\rho} \ . \tag{A4}$$

For the extreme RNdS black holes we have [2]

$$M=\rho(1-\tfrac{2}{3}\Lambda\rho^2) \ , \quad Q^2=\rho^2(1-\Lambda\rho^2) \ . \tag{A5}$$

From these we can compute how the mass and charge of an extremal hole change with each other. We find

$$\frac{\partial M}{\partial Q}=\sqrt{1-\Lambda\rho^2}=\frac{Q}{\rho} \ , \tag{A6}$$

which coincides with the bound (A4). Hence an extreme RNdS black hole cannot be pushed over the limit. Given the nontrivial relation (A5) between the charge and mass of the extreme black holes, this is a somewhat striking confirmation of cosmic censorship.

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