Gravitational form factors of the neutrino

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The gravitational properties of the neutrino are studied in the weak-field approximation. By imposing the Hermiticity condition, *CPT*, and *CP* invariance on the energy-momentum tensor matrix element, we show that the allowed gravitational form factors for Dirac and Majorana neutrinos are very different. In a *CPT*- and *CP*-invariant theory, the energy-momentum tensor for a Dirac neutrino of the same species is characterized by four gravitational form factors. As a result of *CPT* invariance, the energy-momentum tensor for a Majorana neutrino of the same species has five form factors. In a *CP*-invariant theory, if the initial and final Majorana neutrinos have the same (opposite) *CP* parity, then only tensor- (pseudotensor-)type transitions are allowed.

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I. INTRODUCTION

If a neutrino has mass, then the question of whether the neutrino is a Dirac- or Majorana-type particle arises naturally. This is because the neutrino may be its own antiparticle (Majorana particle). The difference between a Dirac and Majorana neutrino is clearly exhibited in the neutral current interaction process [1], observation of neutrinoless double β decay and in their electromagnetic properties [2,3]. For example, a spin-$\frac{1}{2}$ Majorana neutrino can only have the anapole moment form factor if *CPT* invariance holds. This result was generalized to an arbitrary half integral spin Majorana fermion in Ref. [4], and an arbitrary spin Majorana fermion in Ref. [5].

However, there are relatively few discussions on the gravitational properties of a spin-$\frac{1}{2}$ fermion [6,7]. In this paper, we extend their work by performing a complete study of the gravitational properties of the neutrino. In Sec. II, we present a general analysis of the energy-momentum tensor $\theta_{\alpha\beta}$ matrix element between two spin-$\frac{1}{2}$ neutrinos. Using the Dirac equation, the symmetric properties of $\theta_{\alpha\beta}$ and the energy-momentum conservation condition, we arrive at the most general expression for the gravitational form factors of the neutrino. By imposing the Hermiticity condition, *CPT*, and *CP* invariance on the $\theta_{\alpha\beta}$ matrix element, we obtained certain conditions on the gravitational form factors for the neutrino. We summarize the results in Sec. III.

II. GRAVITATIONAL FORM FACTORS OF THE NEUTRINO

In this section we study the allowed form of the couplings for the energy-momentum tensor $\theta_{\alpha\beta}$ matrix element between two neutrino states. We carry out the analysis in the weak field approximation,

$$g_{\alpha\beta}=\eta_{\alpha\beta}+\kappa h_{\alpha\beta} , \tag{2.1}$$

where $\eta_{\alpha\beta}$ is the flat space-time metric, $h_{\alpha\beta}$ is the graviton field, and $\kappa=32\pi G$. In our paper we closely follow the notation used in Ref. [3].

A. General analysis

Consider the invariant amplitude for the process $\nu_i \rightarrow \nu_f + g$, where ν_i and ν_j are two Dirac neutrinos with masses m_i and m_j ($m_i > m_f$) and g is the graviton (virtual or real). The transition amplitude for this process is given by

$$\langle \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)\rangle=\bar{u}(p_f)(\Gamma_{\alpha\beta})_{fi}u(p_i) , \tag{2.2}$$

where $|\nu_i\rangle$ and $\langle \nu_f|$ are the initial and final neutrino states respectively, and $(\Gamma_{\alpha\beta})_{fi}$ is the dressed vertex function that characterizes the above invariant amplitude.

Lorentz invariance implies that the vertex function in general can have 24 types of coupling: 12 tensor types and 12 pseudotensor types. The 12 possible tensor types of coupling have the following forms:

$$\eta_{\alpha\beta} ,\quad q_\alpha q_\beta ,\quad \{qP\}_{\alpha\beta} ,\quad \{q\gamma\}_{\alpha\beta} ,\quad P_\alpha P_\beta ,$$
$$\{P\gamma\}_{\alpha\beta} ,\quad \{\sigma_{\alpha\mu}q^\mu q_\beta\}_{\alpha\beta}=\{\sigma qq\}_{\alpha\beta} ,$$
$$\{\sigma qP\}_{\alpha\beta} ,\quad \{\sigma q\gamma\}_{\alpha\beta} ,\quad \{\sigma PP\}_{\alpha\beta} ,\quad \{\sigma Pq\}_{\alpha\beta}$$

and $\{\sigma P\gamma\}_{\alpha\beta}$, where we have suppressed the Lorentz indices, $\{\ \}_{\alpha\beta}$ denote symmetrization over the indices α and β, $q=p_f-p_i$, $P=p_f+p_i$, and $\sigma=\sigma_{\alpha\mu}=i/2[\gamma_\alpha,\gamma_\mu]$. The pseudotensor types of coupling are obtained by the addition of a γ_5 factor.

Using the Dirac equation, $(\gamma_\mu p^\mu - m)u=0$, one obtains identities which relate the various types of coupling (such as the Gordon decomposition relation), and hence reduces the number of independent couplings. We collect these relations in the Appendix. Thus, the energy-momentum matrix element between two Dirac neutrino states may be written as

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$$\langle \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)\rangle = \bar{u}(p_f)(\Gamma_{\alpha\beta})_{fi}u(p_i)$$
$$= \bar{u}(p_f)[T_{1fi}\eta_{\alpha\beta} + T_{2fi}q_\alpha q_\beta + T_{3fi}P_\alpha P_\beta + T_{4fi}\{qP\}_{\alpha\beta} + T_{5fi}\{\sigma qq\}_{\alpha\beta} + T_{6fi}\{\sigma qP\}_{\alpha\beta} + P_{1fi}\gamma_5\eta_{\alpha\beta} + P_{2fi}\gamma_5 q_\alpha q_\beta + P_{3fi}\gamma_5\{q\gamma\}_{\alpha\beta} + P_{4fi}\gamma_5\{P\gamma\}_{\alpha\beta} + P_{5fi}\gamma_5\{\sigma qq\}_{\alpha\beta} + P_{6fi}\gamma_5\{\sigma qP\}_{\alpha\beta}]u(p_i) , \quad (2.3)$$

where $T = T(q^2, m_i, m_f)$ and $P = P(q^2, m_i, m_f)$ are the tensor- and pseudotensor-type form factors respectively.

Conservation of the energy-momentum tensor ($q^\beta\theta_{\alpha\beta} = 0$) implies the following relations among the form factors:

$$T_1 + T_2 q^2 + \delta m_{fi}^2 T_4 = 0 , \quad (2.4)$$

$$\delta m_{fi}^2 T_3 + q^2 T_4 = 0 , \quad (2.5)$$

$$q^2 T_5 + \delta m_{fi}^2 T_6 = 0 , \quad (2.6)$$

$$P_1 + P_2 q^2 - M_{if} P_3 = 0 , \quad (2.7)$$

$$i(q^2 P_5 + \delta m_{fi}^2 P_6) - M_{if} P_4 = 0 , \quad (2.8)$$

$$q^2 P_3 + 2\delta m_{fi}^2 P_4 = 0 , \quad (2.9)$$

where $M_{if} = m_i + m_f$, and $\delta m_{fi}^2 = m_f^2 - m_i^2$. Lorentz invariance and energy-momentum conservation imply that for $q^2 \neq 0$, the general form for the energy-momentum matrix element between two neutrino states is given by

$$\langle \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)\rangle = \bar{u}(p_f)\Bigg[T_{1fi}\left[\eta_{\alpha\beta} - \frac{1}{\delta m_{fi}^2}\{qP\}_{\alpha\beta} + \frac{q^2}{(\delta m_{fi}^2)^2}P_\alpha P_\beta\right] + T_{2fi}\left[q_\alpha q_\beta - \frac{q^2}{\delta m_{fi}^2}\{qP\}_{\alpha\beta} + \frac{q^4}{(\delta m_{fi}^2)^2}P_\alpha P_\beta\right] + T_{6fi}\left[\{\sigma qP\}_{\alpha\beta} - \frac{\delta m_{fi}^2}{q^2}\{\sigma qq\}_{\alpha\beta}\right] + P_{1fi}\gamma_5\left[\eta_{\alpha\beta} + \frac{1}{M_{if}}\{q\gamma\}_{\alpha\beta} - \frac{q^2}{2\delta m_{fi}^2 M_{if}}\{P\gamma\}_{\alpha\beta} + \frac{i}{2\delta m_{fi}^2}\{\sigma qq\}_{\alpha\beta}\right] + P_{2fi}\gamma_5\left[q_\alpha q_\beta + \frac{q^2}{M_{if}}\{q\gamma\}_{\alpha\beta} - \frac{q^4}{2\delta m_{fi}^2 M_{if}}\{P\gamma\}_{\alpha\beta} + \frac{iq^2}{2\delta m_{fi}^2}\{\sigma qq\}_{\alpha\beta}\right] + P_{6fi}\gamma_5\left[\{\sigma qP\}_{\alpha\beta} - \frac{\delta m_{fi}^2}{q^2}\{\sigma qq\}_{\alpha\beta}\right]\Bigg]u(p_i) . \quad (2.10)$$

For the same neutrino flavor $m_i = m_f$, the solutions for Eqs. (2.4)–(2.9) are $T_4 = T_5 = 0$, $T_2 = -T_1/q^2$, $P_3 = 0$, $P_2 = -(P_1/q^2)$, and $P_5 = -(2miP_4/q^2)$. Thus the energy-momentum matrix element is reduced to

$$\langle \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)\rangle = \bar{u}(p_f)\left[T_{1ii}\left[\eta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right] + T_{3ii}P_\alpha P_\beta + T_{6ii}\{\sigma qP\}_{\alpha\beta} + P_{1ii}\gamma_5\left[\eta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right] + P_{4ii}\gamma_5\left\{\left[\gamma - \frac{\gamma_\mu q^\mu q}{q^2}\right]P\right\}_{\alpha\beta} + P_{6ii}\gamma_5\{\sigma qP\}_{\alpha\beta}\right]u(p_i) . \quad (2.11)$$

This result agrees with Refs. [6,7], except the P_4 term. In analogy with the electromagnetic form factors, T_6 is called the anomalous gravitational magnetic moment form factor, P_4 the gravitational anapole moment form factor, and P_6 the gravitational dipole moment form factor [6].

B. Gravitational form factors of a Dirac neutrino

The energy-momentum tensor $\theta_{\alpha\beta}$ is proportional to $p_\alpha p_\beta$ [8], where $p_\alpha = (ip_0, \mathbf{p})$, whereas the Hermiticity of the energy-momentum tensor operator is given by $\theta_{\alpha\beta}^\dagger = \eta_\alpha\eta_\beta\theta_{\alpha\beta}$. The Hermiticity condition implies

$$\langle \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)\rangle^\dagger = \eta_\alpha\eta_\beta\langle \nu_i(p_i)|\theta_{\alpha\beta}|\nu_f(p_f)\rangle , \quad (2.12)$$

where $\eta_\alpha = (-1,1,1,1)$. As a result of Hermiticity, we have

$$\gamma_0(\Gamma_{\alpha\beta})_{fi}^\dagger\gamma_0 = \eta_\alpha\eta_\beta(\Gamma_{\alpha\beta})_{if} . \quad (2.13)$$

This implies the following relations among the gravitational form factors:

$$(T_1, T_2, T_6, P_1, P_2, P_6)_{fi}^* = (T_1, T_2, -T_6, -P_1, -P_2, P_6)_{if} \quad (2.14)$$

and

$$(T_1,T_3,T_6,P_1,P_4,P_6)^*_{ii}$$
$$=(T_1,T_3,-T_6,-P_1,P_4,P_6)_{ii}\ . \quad (2.15)$$

For the off-diagonal case, $\nu_f \neq \nu_i$, Hermiticity does not put any restriction on the form factors. For the diagonal case, Hermiticity requires that all the form factor are real except for T_6 and P_1.

Under the CPT transformation, $\theta_{\alpha\beta} \xrightarrow{CPT} \theta_{\alpha\beta}$ and

$$_{CPT}\langle \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)\rangle_{CPT} = \langle \nu_i(p_i)|\theta_{\alpha\beta}|\nu_f(p_f)\rangle\ . \quad (2.16)$$

In terms of the Dirac spinor, the left-hand side of Eq. (2.16) can be written as

$$_{CPT}\langle \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)\rangle_{CPT}$$
$$=\bar{u}_{CPT}(-p_i)(\bar{\Gamma}_{\alpha\beta})_{fi}u_{CPT}(-p_f)\ , \quad (2.17)$$

and $u_{CPT}(p)$ is the CPT conjugate of the spinor $u(p)$,

$$u_{CPT}(-p)=\gamma_0 V_T^{-1}[C\bar{u}^t(p)]^*\ , \quad (2.18)$$

where V_T is the time-reversal matrix, t denotes the transpose operation, and $\bar{\Gamma}_{\alpha\beta}$ is the vertex function describing the process $\bar{\nu}_i \to \bar{\nu}_f + g$, where $\bar{\nu}$ denotes the antineutrino state.

Using Eq. (2.17) and the transformation properties of the γ matrices under the operators C and V_T in Eq. (2.16), we obtain

$$CV_T(\bar{\Gamma}_{\alpha\beta})_{fi}V_T^{-1}C^{-1} = -(\Gamma_{\alpha\beta})_{if}\ . \quad (2.19)$$

As a result of CPT invariance, we obtained the following relations among the form factors:

$$(\bar{T}_1,\bar{T}_2,\bar{T}_6,\bar{P}_1,\bar{P}_2,\bar{P}_6)_{fi}$$
$$=(-T_1,-T_2,T_6,-P_1,-P_2,P_6)_{if} \quad (2.20)$$

and

$$(\bar{T}_1,\bar{T}_3,\bar{T}_6,\bar{P}_1,\bar{P}_4,\bar{P}_6)_{ii}=(-T_1,-T_3,T_6,-P_1,P_4,P_6)_{ii} \quad (2.21)$$

Under the CP transformation, $\theta_{\alpha\beta} \xrightarrow{CP} \eta_\alpha\eta_\beta\theta_{\alpha\beta}$,

$$_{CP}\langle \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)\rangle_{CP} = \eta_\alpha\eta_\beta\langle \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)\rangle\ . \quad (2.22)$$

The left-hand side of Eq. (2.22) is given by

$$_{CP}\langle \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)\rangle_{CP} = \bar{u}_{CP}(-p_i')(\bar{\Gamma}'_{\alpha\beta})_{fi}u_{CP}(-p_f')\ , \quad (2.23)$$

where $p'_\alpha = -\eta_\alpha p_\alpha = (ip_0, -\mathbf{p})$, $\bar{\Gamma}'$ denotes the dressed vertex function with q and P replaced by q' and P', and

$$u_{CP}(-p')=\gamma_0 C\bar{u}^t(p)\ . \quad (2.24)$$

Inserting Eqs. (2.23) and (2.24) into (2.22), we obtain

$$\gamma_0 C(\bar{\Gamma}'_{\alpha\beta})^t_{fi}C^{-1}\gamma_0 = -\eta_\alpha\eta_\beta(\Gamma_{\alpha\beta})_{fi}\ . \quad (2.25)$$

If CP invariance holds, we obtain the following relations among the form factors:

$$(\bar{T}_1,\bar{T}_2,\bar{T}_6,\bar{P}_1,\bar{P}_2,\bar{P}_6)_{fi}=(-T_1,-T_2,T_6,P_1,P_2,-P_6)_{fi} \quad (2.26)$$

and

$$(\bar{T}_1,\bar{T}_3,\bar{T}_6,\bar{P}_1,\bar{P}_4,\bar{P}_6)_{ii}=(-T_1,-T_3,T_6,P_1,P_4,-P_6)_{ii} \quad (2.27)$$

For the diagonal case, it follows from CPT and CP invariance that $P_{1ii}=P_{6ii}=0$. That means in a CPT invariant theory, a Dirac neutrino cannot have the form factors P_1 and P_6 if the interaction respects CP symmetry.

C. Gravitational form factors of a Majorana neutrino

Under the CPT transformation a Majorana neutrino ν^M transforms as [9]

$$CPT|\nu^M(\mathbf{p},s)\rangle = \eta^s_{CPT}|\nu^M(\mathbf{p},-s)\rangle\ , \quad (2.28)$$

where η^s_{CPT} is a phase factor that depends on the spin of the particle, with $\eta^s_{CPT} = -\eta^{-s}_{CPT}$. Assuming CPT invariance for the energy-momentum tensor matrix element, we have

$$_{CPT}\langle \nu^M_f(p_f)|\theta_{\alpha\beta}|\nu^M_i(p_i)\rangle_{CPT} = \langle \nu^M_i(p_i)|\theta_{\alpha\beta}|\nu^M_f(p_f)\rangle\ . \quad (2.29)$$

For a Majorana neutrino, the left-hand side of Eq. (2.29) can be written as

$$_{CPT}\langle \nu^M_f(p_f)|\theta_{\alpha\beta}|\nu^M_i(p_i)\rangle_{CPT} = \bar{u}_{PT}(p_f)(\Gamma_{\alpha\beta})_{fi}u_{PT}(p_i)\ , \quad (2.30)$$

where $u_{PT}(p)=\gamma_0 V_T^{-1}u^*(p)$. This implies that

$$V_T^{-1}(\Gamma^t_{\alpha\beta})_{fi}V_T = (\Gamma_{\alpha\beta})_{if}\ . \quad (2.31)$$

Using the transformation properties of the γ matrices under the operator V_T in Eq. (2.31), then as a result of CPT invariance, we obtain the following relations among the form factors:

$$(T_1,T_2,T_6,P_1,P_2,P_6)_{fi}=(T_1,T_2,T_6,P_1,P_2,P_6)_{if} \quad (2.32)$$

and

$$(T_1,T_3,T_6,P_1,P_4,P_6)_{ii}=(T_1,T_3,T_6,P_1,-P_4,P_6)_{ii}\ . \quad (2.33)$$

For the same neutrino species, CPT invariance implies that $P_4=0$; that is, a Majorana neutrino cannot have the gravitational anapole moment form factor.

Under CP transformation, a Majorana neutrino transforms as

$$CP|\nu^M(\mathbf{p},s)\rangle = \eta^*_{CP}|\nu^M(-\mathbf{p},s)\rangle\ , \quad (2.34)$$

where η^*_{CP} is the CP parity of the Majorana neutrino with $\eta^*_{CP}=\pm i$. Assuming CP invariance we have

$$_{CP}\langle \nu^M_f(p_f)|\theta_{\alpha\beta}|\nu^M_i(p_i)\rangle_{CP} = \eta_\alpha\eta_\beta\langle \nu^M_f(p_f)|\theta_{\alpha\beta}|\nu^M_i(p_i)\rangle\ . \quad (2.35)$$

The left-hand side of Eq. (2.35) can be written as

$$ {}_{CP}\langle \nu_f^M(p_f)|\theta_{\alpha\beta}|\nu_i^M(p_i)\rangle_{CP} = \bar{u}_P(p_f')(\Gamma'_{\alpha\beta})_{fi}u_P(p_i') \ , \quad (2.36)$$

where $u_P(p')=\gamma_0 u(p)$. Using Eqs. (2.35) and (2.36), we obtain

$$\eta^i\eta^f\gamma_0(\Gamma'_{\alpha\beta})_{fi}\gamma_0=\eta_\alpha\eta_\beta(\Gamma_{\alpha\beta})_{fi} \ , \quad (2.37)$$

where $\eta_{CP}=i\eta$. As a result of CP invariance, we obtain the following relations among the form factors:

$$\eta^i\eta^f(T_1,T_2,T_6,P_1,P_2,P_6)_{fi} = (T_1,T_2,T_6,-P_1,-P_2,-P_6)_{fi} \quad (2.38)$$

and

$$(T_1,T_3,T_6,P_1,P_4,P_6)_{ii}=(T_1,T_3,T_6,-P_1,-P_4,-P_6)_{ii} \ . \quad (2.39)$$

We observe that the amplitude for the process $\nu_i^M \to \nu_f^M + g$ depends on the relative CP parity of the initial and final neutrino states. For instance, if $\eta^i\eta^f=1$, a Majorana neutrino has a tensorial type of transition form factors, while for $\eta^i\eta^f=-1$, a Majorana neutrino has a pseudotensorial type of transition form factors.

III. SUMMARY

It is shown that the invariant amplitude for the process $\nu_i \to \nu_f + g$ is characterized by six gravitational form factors (three tensor and three pseudotensor types). The Hermiticity condition requires that four of the form factors are real. As a result of CPT and CP invariance, a Dirac neutrino of the same species has four gravitational form factors. A Majorana neutrino has five form factors (no gravitational anapole moment form factor) as a result of CPT invariance, which agrees with the result given in Ref. [7]. In a CP invariant theory, if the initial and final Majorana neutrinos have the same (opposite) CP parity, then only tensor-(pseudotensor-)type transitions are allowed. For the same neutrino species, the energy-momentum matrix element for a Majorana neutrino is characterized by tensor couplings only [7].

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APPENDIX

In this Appendix we present the identities among the various types of coupling that were employed in our calculation.

Tensor-type couplings

$$\bar{u}_f\{\sigma qq\}u_i=i\bar{u}_f\{qP-M_{if}q\gamma\}u_i \ , \quad (A1)$$

$$\bar{u}_f\{\sigma qP\}u_i=i\bar{u}_f\{PP-M_{if}P\gamma\}u_i \ , \quad (A2)$$

$$\bar{u}_f\{\sigma q\gamma\}u_i=i\bar{u}_f(\{q\gamma\}-2\delta m_{fi}\eta_{\alpha\beta})u_i \ , \quad (A3)$$

$$\bar{u}_f\{\sigma PP\}u_i=i\bar{u}_f\{qP-\delta m_{fi}P\gamma\}u_i \ , \quad (A4)$$

$$\bar{u}_f\{\sigma Pq\}u_i=i\bar{u}_f(qq-\{\delta m_{fi}q\gamma\})u_i \ , \quad (A5)$$

$$\bar{u}\{\sigma P\gamma\}=i\bar{u}_f(\{P\gamma\}-2M_{if}\eta_{\alpha\beta})u_i \ . \quad (A6)$$

Pseudotensor-type couplings

$$\bar{u}_f\gamma_5\{\sigma qq\}u_i=i\bar{u}_f\gamma_5\{Pq+\delta m_{fi}q\gamma\}u_i \ , \quad (A7)$$

$$\bar{u}_f\gamma_5\{\sigma qP\}u_i=i\bar{u}_f\gamma_5\{PP+\delta m_{fi}P\gamma\}u_i \ , \quad (A8)$$

$$\bar{u}_f\gamma_5\{\sigma q\gamma\}u_i=i\bar{u}_f\gamma_5(\{q\gamma\}+2M_{if}\eta_{\alpha\beta})u_i \ , \quad (A9)$$

$$\bar{u}_f\gamma_5\{\sigma PP\}u_i=i\bar{u}_f\gamma_5\{qP+M_{if}P\gamma\}u_i \ , \quad (A10)$$

$$\bar{u}_f\gamma_5\{\sigma Pq\}u_i=i\bar{u}_f\gamma_5(qq+\{M_{if}q\gamma\})u_i \ , \quad (A11)$$

$$\bar{u}_f\gamma_5\{\sigma P\gamma\}=i\bar{u}_f\gamma_5(\{P\gamma\}+2\delta m_{fi}\eta_{\alpha\beta})u_i \ , \quad (A12)$$

where $\delta m_{fi}=m_f-m_i$ and $M_{if}=m_i+m_f$.

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