# Vector-meson magnetic moments and radiative two-pseudoscalar decays

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We analyze the effects of the magnetic moments of charged vector mesons into the $V \to PP\gamma$ radiative decays.

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Projected $\tau$ factories are expected to produce in the near future a large amount of $\tau$ leptons and (very) accurate data for their various decay modes and the corresponding decay products. Among the latter, the $\tau \to \rho\nu$ and $\tau \to K^*\nu$ decays—whose branching ratios are known to be [1] $(22.7\pm0.8)\%$ and $(1.4\pm0.2)\%$, respectively—will furnish a reasonably large number of charged vector mesons in an otherwise purely leptonic context. In this sense, the situation resembles the clean production of their neutral partners in the well understood $e^+e^- \to \rho^0$ $\omega$, and $\varphi$ processes and therefore, the possibility of measuring the properties of charged $\rho$'s and $K^*$'s in an accurate and similarly clean way seems fully justified. The purpose of this Brief Report is to analyze the subsequent $\rho^+ \to \pi^+\pi^0\gamma$, $K^{*+} \to K^0\pi^+\gamma$, and $K^{*+} \to K^+\pi^0\gamma$ radiative decays (and their corresponding charge conjugates), which depend on the magnetic moment of the initial vector meson thus in principle allowing for a measurement of this unknown and important static property.

The magnetic moment of a positively charged $(e>0)$ vector meson generated exclusively by the usual Abelian gauge coupling with the photon is given by $3/2M_V$, where $M_V$ stands for the meson mass. The total magnetic moment

$$\mu=(1+\kappa)\frac{e}{2M_V} \tag{1}$$

can also contain an anomalous, $\kappa$-dependent piece. For $\rho$ mesons coupled through non-Abelian SU(2) Yang-Mills terms, one predicts $\kappa=1$ and $\mu=e/M_V$. These values can be considered [2] as the canonical or natural ones for the spin-one systems, in analogy with the value for the $W^+$ magnetic moment in the standard electroweak theory. The total magnetic moment of charged $K^*$ is similarly predicted to be the canonical one if the triple-$\rho$ coupling in the SU(2) Yang-Mills model is generalized to SU(3). However, substantial deviations from these canonical values can be expected since the Yang-Mills nature of the $\rho$ mesons (and, *a fortiori,* any generalization for $K^*$'s) is far from being proved. Similarly, one can expect a $\kappa$ dependence for the electrically neutral $K^*$'s magnetic moment deviating from the globally vanishing canonical prediction.

The gauge invariant amplitudes for the various $V^+ \to P^+P^0\gamma$ radiative decays previously mentioned are then easily found to be

$$\mathcal{M} \equiv \sqrt{2}Ceg(\mathcal{M}^{\mathrm{C}}+\mathcal{M}^{\mathrm{NC}}) , \tag{2}$$

where

$$\mathcal{M}^{\mathrm{C}}=\frac{(p^+-p^0)^\alpha}{Q\cdot k}[\epsilon_\alpha(Q\cdot\epsilon^*)+\epsilon^*_\alpha(k\cdot\epsilon)-k_\alpha(\epsilon^*\cdot\epsilon)]+(p^0-p^+-k)\cdot\epsilon\left[\frac{p^+\cdot\epsilon^*}{p^+\cdot k}\right]+\epsilon\cdot\epsilon^* , \tag{3}$$

$$\mathcal{M}^{\mathrm{NC}}=(1-\kappa)\frac{(p^+-p^0)^\alpha}{2Q\cdot k}\left[k_\alpha(\epsilon^*\cdot\epsilon)-\epsilon^*_\alpha(k\cdot\epsilon)-\frac{(p^++p^0)\alpha}{M_V^2}(Q\cdot\epsilon^* k\cdot\epsilon-\epsilon\cdot\epsilon^* Q\cdot k)\right] , \tag{4}$$

and $Q$ $(\epsilon)$ stands for the initial $V^+$ four-momentum (polarization), $k$ $(\epsilon^*)$ for the final photon one(s), and $p^+$ and $p^0$ are the four-momenta of the charged and neutral pions or kaons (in the above lines, $Q=p^++p^0+k$ and $V^+=\rho^+, K^{*+}$). The Clebsch-Gordan coefficient $C$ is 1, $1/\sqrt{2}$, and 1/2 for $\rho^+ \to \pi^+\pi^0\gamma$, $K^{*+} \to K^0\pi^+\gamma$, and $K^{*+} \to K^+\pi^0\gamma$, respectively.

The first term in the canonical piece of the amplitude, $\mathcal{M}^{\mathrm{C}}$ (as well as the whole noncanonical one, $\mathcal{M}^{\mathrm{NC}}$) comes from a diagram containing the radiation of the photon from the initial charged vector meson $V^+$, the second term in $\mathcal{M}^{\mathrm{C}}$ accounts for the radiation from the final $P^+$, and the third $\epsilon\cdot\epsilon^*$ term is the contact term required to preserve gauge invariance. The strong coupling constant $g$ appears in the gauged version of the $VPP$ Lagrangian

$$\mathcal{L}=ig\,\mathrm{Tr}(V_\mu PD^\mu P-V_\mu D^\mu PP) ,$$

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TABLE I. Branching ratios (in %) for radiative decays of charged vector mesons as a function of the minimum photon energy $E_{\min}$ and the vector-meson magnetic moment $\mu=(1+\kappa)e/2M_V$.

| | $\kappa$ | $E_{\min}=10$ | 30 | 60 | 90 MeV |
|---|---|---|---|---|---|
| $B(\rho^+\to\pi^+\pi^0\gamma)$ in % | $-1$ | 1.56 | 0.942 | 0.587 | 0.404 |
| | 0 | 1.52 | 0.896 | 0.541 | 0.358 |
| | 1 | 1.51 | 0.892 | 0.537 | 0.354 |
| | 2 | 1.55 | 0.931 | 0.576 | 0.391 |
| $B(K^{*+}\to K^0\pi^+\gamma)$ in % | $-1$ | 0.735 | 0.389 | 0.199 | 0.108 |
| | 0 | 0.737 | 0.391 | 0.201 | 0.109 |
| | 1 | 0.750 | 0.404 | 0.214 | 0.121 |
| | 2 | 0.776 | 0.429 | 0.238 | 0.144 |
| $B(K^{*+}\to K^+\pi^0\gamma)$ in % | $-1$ | 0.225 | 0.130 | 0.077 | 0.050 |
| | 0 | 0.212 | 0.117 | 0.064 | 0.038 |
| | 1 | 0.210 | 0.116 | 0.063 | 0.036 |
| | 2 | 0.221 | 0.126 | 0.073 | 0.045 |

where $V_\mu=\lambda_a V_\mu^a/\sqrt{2}$ and $P=\lambda_a P^a/\sqrt{2}$ ($\lambda_a$ are Gell-Mann matrices) stand for the SU(3) octet of vector and pseudoscalar mesons, respectively, $\sqrt{2}g=6.0$ from $\Gamma(\rho\to\pi\pi)=149$ MeV and the photonic covariant derivative in its matrix form is $D^\mu P=\partial^\mu P+ie[Q,P]A^\mu$, with $Q=\mathrm{diag}(2/3,-1/3,-1/3)$, the quark-charge matrix. From the above amplitudes (2) the radiative partial widths are obtained by performing a numerical integration of

$$\Gamma(V^+\to P^+P^0\gamma)=\frac{1}{192\pi^3 M_V}\int dE\int dE^+\sum_{\mathrm{pol}}|\mathcal{M}|^2 , \quad (5)$$

where $E^+$ and $E$ stand for the charged pseudoscalar and photon energies. In order to avoid the logarithmic divergence when integrating over $E$, one introduces a minimum energy cutoff $E_{\min}$, and the integration limits become

$$E_{\min}\le E\le[M_V^2-(m_{P^+}+m_{P^0})^2]/2M_V .$$

Then Eq. (5) depends essentially on this $E_{\min}$, $\kappa$, and $g^2$. The latter $g^2$ dependence can be further avoided by computing $B(V^+\to P^+P^0\gamma)$, i.e., by simply dividing $\Gamma(V^+\to P^+P^0\gamma)$ in Eq. (5) by $\Gamma(V^+\to\text{all})$, which for $V^+=\rho^+$, $K^{*+}$ is completely dominated by two-pseudoscalar decays. Introducing the values of the masses [1], reasonable values for physical $E_{\min}$ and allowing for deviations around the canonical value $\kappa=1$, leads to the set of branching ratios quoted (in %) in Table I. The main feature of these results is the expected dominance of the emission of soft (bremsstrahlung) photons.

However, the soft-photon dominated amplitudes (3) are not the only ones contributing to our $V^+\to P^+P^0\gamma$ decays. They can also proceed through the decay chain $V\to VP\to PP\gamma$ involving a $VVP$ and a $VP\gamma$ (magnetic dipolar) transition, closely related by vector-meson dominance (VMD). An analysis in the VMD context of the radiative decays of neutral vector mesons into two neutral pseudoscalars, $V^0\to P^0P^0\gamma$, has been recently performed [3]. From this analysis one can easily deduce an $\omega(783)$-dominated contribution to $\rho^+\to\pi^+\pi^0\gamma$ given by $B(\rho^+\to\pi^+\pi^0\gamma)_{\mathrm{VMD}}\simeq B(\rho^0\to\pi^0\pi^0\gamma)_{\mathrm{VMD}}=11\times10^{-6}$ and even smaller VMD contribution for the analogous $K^{*+}$ decays. We have explicitly checked these expectations recalculating $B(\rho^+\to\pi^+\pi^0\gamma)_{\mathrm{VMD}}\simeq7\times10^{-6}$ and observing the dominance of hard-photon emission (as also observed in Ref. [3]). As a result, one can safely conclude that these VMD contributions represent essentially only negligible corrections to the values quoted in Table I. We can finally proceed to the discussion of their $\kappa$ dependence, the central point of our present analysis.

The sensitivity of our predictions on the value of $\kappa$ is seen to be rather scarce for small values of $E_{\min}$ ($\sim10$ MeV) and increases, as expected, for higher values of this cutoff energy. This fact, as well as the experimental difficulties in detecting low energy photons (a detection which is also essential in order to reconstruct the Breit-Wigner shape of the vector mesons in $\tau\to\rho\nu$, $K^*\nu$ decays), suggests the use of larger values of the cutoff, $E_{\min}\sim90$ MeV. In spite of this, the $B(\rho^+\to\pi^+\pi^0\gamma)$ is still only weakly dependent on $\kappa$ around the canonical value $\kappa=1$. The situation improves when going to $B(K^{*+}\to K^0\pi^+\gamma$ or $K^+\pi^0\gamma)$, but then the smallness of these branching ratios as well as the one for $\tau\to K^*\nu$ considerably reduces the expected number of events. A clear sensitivity on $\kappa$ could be obtained if working with neutral $K^*$'s, for which we roughly predict $B(K^{*0}\to K^0\pi^0\gamma)\simeq5\times10^{-5}\kappa^2$, but both the smallness of this branching ratio and the fact that neutral $K^*$'s cannot be produced in clean $\tau$ (or similar) decays makes this measurement also problematic.

In summary, even if the measurement of the magnetic moments of charged vector mesons is in principle feasible in $\tau^+\to V^+\bar\nu\to P^+P^0\gamma\bar\nu$ decays, our analysis indicates the high degree of difficulty of such an experiment. For the charged $\rho$ case, the sensitivity on $\kappa$ is so small that other methods should be preferred (see, for instance, Ref. [4]). For charged $K^*$'s the sensitivity on $\kappa$ improves but a very large number of $\tau$'s is then required.

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