$D_s^+ \to \pi^+\eta$ and $\pi^+\eta'$ decays

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From a perspective in which the dynamical contribution of hadrons is manifested, Cabibbo-angle-favored two-body decays of charm mesons into final states involving η or η' are studied in consistency with the other charm meson decays.

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Although the decays F^+ (recently D_s^+)$\to\pi^+\eta$ and $\pi^+\eta'$ have been studied by many authors [1] the theoretical understanding of these decays has not reached any consensus. In this Brief Report, we investigate these decays from a theoretical perspective from which the other two-body decays of charm mesons have already been described well.

Our crucial starting point is the following approximate amplitudes [2,3] for three pseudoscalar (PS) meson processes, $P_1(\mathbf{p}_1)\to P_2(\mathbf{p}_2)+P_3(\mathbf{q})$,

$$M\simeq M_{\rm ETC}+M_S\ , \tag{1}$$

which can be obtained by using PCAC (partial conservation of axial-vector current) and an extrapolation $\mathbf{q}\to 0$ in the *infinite momentum frame* (IMF), i.e., $\mathbf{p}_1\to\infty$. Here the equal-time commutator (ETC) term

$$M_{\rm ETC}(P_1\to P_2P_3)=-i\left\{\frac{1}{\sqrt{2}f_{P_3}}\langle P_2(\mathbf{p}_2)|[V_{\bar P_3},H_W]|P_1(\mathbf{p}_1)\rangle+(P_2\leftrightarrow P_3)\right\} \tag{2}$$

and the surface term, which is expressed in terms of a sum of asymptotic pole amplitudes,

$$M_S(P_1\to P_2P_3)=-i\left\{\frac{1}{\sqrt{2}f_{P_3}}\left[\sum_n\left[\frac{m_2^2-m_1^2}{m_n^2-m_1^2}\right]\langle P_2|A_{\bar P_3}|n\rangle\langle n|H_W|P_1\rangle +\sum_l\left[\frac{m_2^2-m_1^2}{m_l^2-m_2^2}\right]\langle P_2|H_W|l\rangle\langle l|A_{\bar P_3}|P_1\rangle\right]+(P_2\leftrightarrow P_3)\right\}, \tag{3}$$

have to be evaluated in the IMF, where the notation is given in Refs. [2] and [3]. The above can be considered as an innovation of the old soft pion technique [4]. We note that the amplitude [Eq. (1) with Eqs. (2) and (3)] is thus expressed solely in terms of asymptotic matrix elements of the vector and axial-vector charges V_α and A_α and the effective weak Hamiltonian H_W [matrix elements taken between single-hadron (not only the ordinary $\{Q\bar Q\}$ but also hypothetical [5] four-quark $[QQ][\bar Q\bar Q]$ and $(QQ)(\bar Q\bar Q)$, glueball, hybrid $\{Q\bar Qg\}$, etc.) states with *infinite momentum*] and that the asymptotic matrix elements of V_α and A_α can be well parametrized by using the asymptotic flavor symmetry [6]. We thus realize that the essential features of nonleptonic weak decays must be found in the dynamics which will show up among the asymptotic matrix elements of H_W appearing in Eqs. (2) and (3).

In the past, we have already obtained constraints on asymptotic matrix elements of H_W responsible for nonleptonic weak decays of K and charm mesons, i.e., diagonal

$$\langle\{Q\bar Q\}_0|H_W|\{Q\bar Q\}_0\rangle$$

and nondiagonal

$$\langle[QQ][\bar Q\bar Q]|H_W|\{Q\bar Q\}_0\rangle$$

and

$$\langle\{Q\bar Q\}_0|H_W|[QQ][\bar Q\bar Q]\rangle$$

satisfy the asymptotic $|\Delta I|=\frac{1}{2}$ rule and its charm counterpart under the asymptotic $SU_f(3)$ symmetry, while $\langle(QQ)(\bar Q\bar Q)|H_W|\{Q\bar Q\}_0\rangle$ and $\langle\{Q\bar Q\}_0|H_W|(QQ)(\bar Q\bar Q)\rangle$ can violate the selection rules, where the exact $SU_I(2)$ symmetry is always assumed. To this, we have used two alternative but complementary methods, i.e., one [7,8] is algebraic and based on the asymptotic realization of commutation relations involving V_α, A_α, and H_W, and the other [3,9,10] is based on more intuitive quark-line arguments. We have also pointed out [8–10] that the s-channel contribution of four-quark mesons and a glueball to M_S is important in the charm meson decays. However, in the u channel, only charm mesons can contribute to

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M_S. Among these charm mesons, exotic ones provide negligibly small contributions because of their high masses and small wave-function overlappings with the ground-state $\{Q\bar{Q}\}_0$ meson. In the $K\to\pi\pi$ decays, contributions of exotic mesons are not very important [3] unless we are interested in small effects such as the violation of the $|\Delta I|=\frac{1}{2}$ rule. A possible solution to the long-standing puzzle in the Cabibbo-angle-suppressed decays of the D^0 meson, $\Gamma(D^0\to K^+K^-)/\Gamma(D^0\to\pi^+\pi^-)\simeq 2-3$, has also been provided [8–10] preserving the overall consistency, by taking account of contributions of the ground-state $\{Q\bar{Q}\}_0$, the exotic $[QQ][\bar{Q}\bar{Q}]$ and $(QQ)(\bar{Q}\bar{Q})$ mesons, and a glueball.

On the same line as our previous papers mentioned above, we now investigate Cabibbo-angle-favored decays of charm mesons into two-body final states involving η or η'. To this, we need further information of additional asymptotic matrix elements of H_W, i.e., constraints on asymptotic matrix elements of H_W involving η or η', although we have already obtained [9,10] constraints on various asymptotic matrix elements of H_W. In the asymptotic matrix element of the charm-changing H_W, the partner which sandwiches H_W with η or η' must be a charm meson. Noting that the charm-changing H_W must be of $|\Delta C|=|\Delta S|=|\Delta I|=1$, we obtain

$$\langle\eta|H_W|D^0\rangle=\langle\eta'|H_W|D^0\rangle=0\ . \tag{4}$$

Since the excited-state mesons with charm provide negligibly small contribution to M_S as was mentioned before, we do not need to worry about nondiagonal matrix elements of H_W in the present crude discussion.

Substituting the constraints on the diagonal and nondiagonal asymptotic matrix elements of H_W obtained in Refs. [9] and [10] and also in Eq. (4) into the general form of the amplitude, Eq. (1) with Eqs. (2) and (3), we can calculate explicitly the amplitudes for the $F^+\to\pi^+\eta,\pi^+\eta'$ and the $D^0\to\bar{K}^0\eta,\bar{K}^0\eta'$ decays which involve the contributions of the four-quark $[QQ][\bar{Q}\bar{Q}]$ and $(QQ)(\bar{Q}\bar{Q})$ mesons in addition to the ground-state $\{Q\bar{Q}\}_0$ mesons:

$$\begin{aligned}M(F^+\to\pi^+\eta)\simeq{}&\frac{i}{\sqrt{2}f_\pi}\langle\bar{K}^0|H_W|D^0\rangle a_\eta^s\left\{\left(\frac{1}{2}\right)^{1/2}\frac{f_\pi}{f_\eta}e^{i\delta}+\left(\frac{1}{2}\right)^{1/2}\frac{f_\pi}{f_\eta}\left(\frac{m_F^2-m_\pi^2}{m_{F^*}^2-m_\pi^2}\right)\left(\frac{1}{2}\right)^{1/2}h\right.\\&+\left[\left(\frac{m_F^2-m_\eta^2}{m_F^2-m_{\hat{\delta}^{*s}}^2}\right)+\left(\frac{1}{2}\right)^{1/2}\frac{f_\pi}{f_\eta}\left(\frac{m_F^2-m_\pi^2}{m_F^2-m_{\hat{\delta}^{*s}}^2}\right)\right]f_a^*\\&\left.+\left[\left(\frac{m_F^2-m_\eta^2}{m_{C_\pi^{*s}}^2-m_F^2}\right)+\left(\frac{1}{2}\right)^{1/2}\frac{f_\pi}{f_\eta}\left(\frac{m_F^2-m_\pi^2}{m_{C_\pi^{*s}}^2-m_F^2}\right)\right]f_s^*\right\}+\cdots\ ,\end{aligned}\tag{5}$$

$$\begin{aligned}M(D^0\to\bar{K}^0\eta)\simeq{}&-\frac{i}{\sqrt{2}f_K}\left(\frac{1}{2}\right)^{1/2}\langle\bar{K}^0|H_w|D^0\rangle a_\eta^0\left[\left\{1-\left[1-\left(\frac{1}{2}\right)^{1/2}\frac{f_K}{f_\eta}\right]\frac{\sqrt{2}a_\eta^s}{a_\eta^0}\right\}e^{i\delta}\right.\\&+\left[\left\{\left(\frac{m_D^2-m_\eta^2}{m_D^2-m_{K^*}^2}\right)-\left(\frac{1}{2}\right)^{1/2}\frac{f_K}{f_\eta}\left(\frac{m_D^2-m_K^2}{m_D^2-m_{K^*}^2}\right)\right\}\frac{\sqrt{3}a_\eta^{(8)}}{a_\eta^0}\right.\\&\left.+\left(\frac{1}{2}\right)^{1/2}\frac{f_K}{f_\eta}\left(\frac{m_D^2-m_K^2}{m_{D^*}^2-m_K^2}\right)\right]\left(\frac{1}{2}\right)^{1/2}h\\&+\left[\left(\frac{m_D^2-m_\eta^2}{m_D^2-m_{\hat{K}^*}^2}\right)+\left(\frac{1}{2}\right)^{1/2}\frac{f_K}{f_\eta}\left(\frac{m_D^2-m_K^2}{m_D^2-m_{\hat{K}^*}^2}\right)\right]f_a^*\\&\left.+\left[\left(\frac{m_D^2-m_\eta^2}{m_D^2-m_{C_K^*}^2}\right)+\left(\frac{1}{2}\right)^{1/2}\frac{f_K}{f_\eta}\left(\frac{m_D^2-m_K^2}{m_D^2-m_{C_K^*}^2}\right)\right]f_s^*\right]+\cdots\ .\end{aligned}\tag{6}$$

The amplitudes $M(F^+\to\pi^+\eta')$ and $M(D^0\to\bar{K}^0\eta')$ are obtained by replacing η by η' in Eqs. (5) and (6), respectively. h denotes the asymptotic invariant matrix element of A_α taken between the ground-state meson states and is defined by $h=\langle\rho^0|A_{\pi^+}|\pi^-\rangle$. In the above, we have taken the usual η-η' mixing, $|\eta\rangle=a_\eta^{(8)}|\eta^{(8)}\rangle+a_\eta^{(0)}|\eta^{(0)}\rangle=a_\eta^0|\eta_0\rangle+a_\eta^s|\eta_s\rangle$ and $|\eta'\rangle=a_{\eta'}^{(8)}|\eta^{(8)}\rangle+a_{\eta'}^{(0)}|\eta^{(0)}\rangle=a_{\eta'}^0|\eta_0\rangle+a_{\eta'}^s|\eta_s\rangle$, where $\eta^{(0)}$ and $\eta^{(8)}$ denote the singlet and the eighth component of the octet of $SU_f(3)$ and η_0 and η_s denote the $(u\bar{u}+d\bar{d})/\sqrt{2}$ and

$(s\bar{s})$ components, respectively. In Eqs. (5) and (6), the terms proportional to $e^{i\delta}$, h, f_a^*, and f_s^* arise from $M_{\rm ETC}$, contributions of the ground-state $\{Q\bar{Q}\}_0$, the s-channel contributions of the exotic $[QQ][\bar{Q}\bar{Q}]$ and $(QQ)(\bar{Q}\bar{Q})$ mesons to M_S, respectively, and ellipses denote the neglected contributions of excited $\{Q\bar{Q}\}$, possible $J^{PC}=1^{-+}$ hybrid $\{Q\bar{Q}g;1^{-+}\}$, four-quark mesons with charm, etc. The contribution of the orbitally excited $\{Q\bar{Q}\}_{L\neq 0}$ will be small since matrix elements of H_W taken between $\{Q\bar{Q}\}_{L\neq 0}$ and the $\{Q\bar{Q}\}_0$ are expected to be small [8]. The four-quark mesons with charm will provide only a small contribution through the u channel as stated before. If the matrix element $\langle\{Q\bar{Q}g\}|H_W|\{Q\bar{Q}\}\rangle$ is described by the diagram in Fig. 1, we can obtain $\langle\{Q\bar{Q}g;1^{-+}\}|H_W|\{Q\bar{Q}\}_0\rangle=0$, since $J^{PC}=1^{-+}$ cannot be realized by the $\{Q\bar{Q}\}$ states.

If the intermediate meson mass in M_S is very close to that of the external meson, the width of the intermediate meson will be important. This is possible in charm meson decays, i.e., the masses of some exotic mesons which can contribute to the s-channel intermediate states have been predicted [11] to be very close to the parent charm meson masses, for example, $(m_F^2-m_\pi^2)/(m_F^2-m_{\hat{\delta}^{*s}}^2)\gg 1$ (in the narrow-width limit) in Eq. (5). As long as the value of the width $\Gamma_{\rm exotic}$ of the exotic mesons is taken to be very large ($\Gamma_{\rm exotic}\simeq 0.3-0.5$ GeV), our calculated two-body decay rates are not very sensitive to the value of $\Gamma_{\rm exotic}$ in contrast with the phases of the amplitudes under consideration. We take the following values of the parameters involved, $\delta\simeq 50°$, $h\simeq 1.0$, $f_a^*\simeq f_s^*\simeq 0.05$, and $\Gamma_{\rm exotic}\simeq 0.4$ GeV, which have already reproduced [10] the observed values of two-body decay rates of charm mesons and the phase difference [12], $\Delta_{\pi\bar{K}}=(\delta_{1/2}-\delta_{3/2})_{\pi\bar{K}}\simeq 80°$, consistent with the approximate $|\Delta I|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays. Substituting into the amplitudes Eqs. (5) and (6) and those obtained by exchanging η by η' in these equations, the above values of the parameters, and the masses of the four-quark mesons given in Ref. [11], we can compute the decay rates for the $F^+\to\pi^+\eta$, $\pi^+\eta'$ and $D^0\to\bar{K}^0\eta$, $\bar{K}^0\eta'$, in which the remaining parameters are the asymptotic matrix element $\langle\bar{K}^0|H_W|D^0\rangle$, the decay constant f_η or $f_{\eta'}$, and the η-η' mixing parameters.

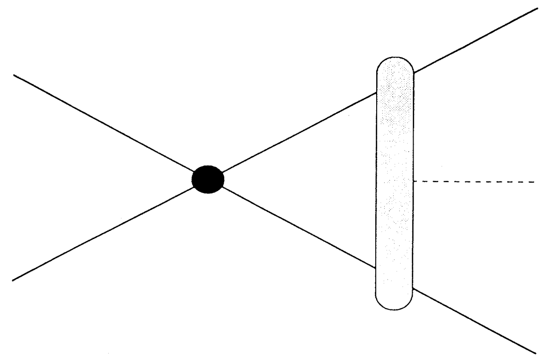

FIG. 1. The quark-line diagram describing the matrix elements $\langle\{Q\bar{Q}g\}|H_W|\{Q\bar{Q}\}\rangle$. The solid lines represent a quark and an antiquark, the dashed line a constituent gluon, the solid circle the weak vertex in the $m_W\to\infty$ limit, and the gray box the strong interactions.

In the orthonormal η-η' mixing with the mixing angle θ_P, where $a_\eta^{(8)}=a_{\eta'}^{(0)}=\cos\theta_P$ and $a_\eta^{(0)}=-a_{\eta'}^{(8)}=-\sin\theta_P$, the mixing parameters a_η^s and a_η^0 which appear in Eqs. (5) and (6) are given by $a_\eta^s=-(\sqrt{\frac{2}{3}}\cos\theta_P+\sqrt{\frac{1}{3}}\sin\theta_P)$ and $a_\eta^0=(\sqrt{\frac{1}{3}}\cos\theta_P-\sqrt{\frac{2}{3}}\sin\theta_P)$. Here we take $\theta_P\simeq -18°$ estimated by Pham [13]. The decay constants f_η and $f_{\eta'}$ have recently been estimated [14] as $f_\eta=91\pm 6$ MeV and $f_{\eta'}=78\pm 5$ MeV by fitting the calculated $\eta\gamma^*\gamma$ and $\eta'\gamma^*\gamma$ transition form factors to the observed ones. Using these values of the mixing parameters and the decay constants, we obtain the branching ratios (v) listed in Table I. Our result reproduces well the observed values [15,16] of $B(F^+\to\pi^+\eta)$, $B(F^+\to\pi^+\eta')$, $B(D^0\to\bar{K}^0\eta)$,

TABLE I. Branching ratios (%) for the $F^+\to\pi^+\eta,\pi^+\eta'$ and $D^0\to\bar{K}^0\eta,\bar{K}^0\eta'$ decays. (i) Bauer-Stech-Wirbel (BSW), (ii) Blok-Shifman (BS), (iii) Kamal-Sinha-Sinha (KSS), and (iv) Pham denote the theoretical predictions in Ref. [1], where the observed branching ratio [15] $B(F^+\to\pi^+\phi)_{\rm expt}\simeq 2.8\%$ has been used as the input in (iv). (v) is the present result, which contains the contributions of the ground-state $\{Q\bar{Q}\}_0$ and the exotic $[QQ][\bar{Q}\bar{Q}]$ and $(QQ)(\bar{Q}\bar{Q})$ mesons ($f_a^*=f_s^*=0.05$), where the observed value $B(D^0\to\pi^+K^-)_{\rm expt}\simeq 3.7\%$ given in Ref. [15] is used as the input.

Decays	(i) BSW	(ii) BS	(iii) KSS	(iv) Pham	(v)	Experiments
$F^+\to\pi^+\eta$	2.9	3.2	3.8	1.0	1.7	1.5 ± 0.4[a]
$F^+\to\pi^+\eta'$	1.7	0.28	4.1	1.1	3.4	3.7 ± 1.2[a]
						$1.6\pm 0.6\pm 0.4$[b]
$D^0\to\bar{K}^0\eta$	0.31	0.4			0.7	$1.4\pm 0.5\pm 0.3$[c]
						$0.85\pm 0.15\pm 0.16$[d]
						$3.3\pm 0.8\pm 1.0$[b]
$D^0\to\bar{K}^0\eta'$	0.12	1.2			1.5	$1.9\pm 0.4\pm 0.3$[c]
						$2.5\pm 0.5\pm 0.6$[d]

[a]World average of observed branching ratios given by the Particle Data Group [15].
[b]As measured by the Mark III Collaboration, summarized in [16].
[c]As measured by the ARGUS Collaboration, summarized in [16].
[d]As measured by the CLEO II Collaboration, summarized in [16].

and $B(D^0 \to \bar{K}^0\eta')$ simultaneously and is much better than the predictions by the other authors, as seen in Table I, where the observed value [15] $B(D^0 \to K^-\pi^+)_{\text{expt}} \simeq 3.7\%$ has been used as the input.

In summary, we have studied the Cabibbo-angle-favored two-body decays of charm mesons into the final states involving η or η' from the perspective in which contributions of various kinds of hadrons are explicitly taken into account. The contributions of the ground-state $\{Q\bar{Q}\}_0$ and the exotic four-quark mesons to M_S in addition to M_{ETC} representing the continuum contribution have been important. Choosing reasonable values of the parameters involved, we have reproduced well the observed values of the branching ratios for these decays, preserving the consistency with the other two-body decays of charm mesons.

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[6] Asymptotic flavor symmetry implies that $a_\beta(\mathbf{k})$'s (the annihilation operators of physical hadrons such as π, K, η, $\ldots$) can be transformed as $[V_\alpha, a_\beta(\mathbf{k})] = i\sum_\gamma f_{\alpha\beta\gamma} a_\gamma(\mathbf{k}) + \delta f_{\alpha\beta}(\mathbf{k})$, where $\delta f_{\alpha\beta}(\mathbf{k}) \to 0$ as $\mathbf{k} \to \infty$, i.e., in the IMF. As long as asymptotic matrix elements are concerned with, the result from asymptotic flavor symmetry is the same as that from the usual recipe of flavor symmetry plus mixings. Asymptotic flavor symmetry and its fruitful results were reviewed in S. Oneda and K. Terasaki, Prog. Theor. Phys. Suppl. **82**, 1 (1985). See also S. Oneda and Y. Koide, *Asymptotic Symmetry and Its Implication in Elementary Particle Physics* (World Scientific, Singapore, 1991). The measure of the accuracy of the asymptotic flavor symmetry is given by the value of the form factor $f_+(0)$ of the relevant vector current at the zero-momentum-transfer squared. The estimated values $f_+^{\pi K}(0) \simeq 1$ and $f_+^{KD}(0) \simeq 0.7$ suggest that the asymptotic $\text{SU}_f(3)$ is very accurate, while the asymptotic $\text{SU}_f(4)$ may make about 30% errors. For the $f_+^{KD}(0)$, see Z. Bai *et al.*, Phys. Rev. Lett. **66**, 1011 (1991).

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