Front-form spinors in the Weinberg-Soper formalism and generalized Melosh transformations for any spin

D. V. Ahluwalia
Medium Energy Physics Division, MS H-844, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

Mikolaj Sawicki
Department of Physics, Iowa State University, Ames, Iowa 50011
(Received 25 September 1992)

Using the Weinberg-Soper formalism we construct the front-form $(j,0)\oplus(0,j)$ spinors. Explicit expressions for the generalized Melosh transformations up to spin two are obtained. The formalism, without explicitly invoking any wave equations, reproduces the spin-$\frac{1}{2}$ front-form results of Melosh, Lepage and Brodsky, and Dziembowski.

PACS number(s): 11.30.Cp, 11.10.Qr, 11.80.Cr

I. INTRODUCTION

Following the work of Weinberg [1] and Soper [2] we extend our recent work [3] and explicitly construct the front-form covariant spinors for arbitrary spin j *without* a direct reference to any wave equation. We also obtain a generalization of the Melosh transformation $\Omega(j)$ to $(j,0)\oplus(0,j)$ fields. Explicit examples for $j=\frac{1}{2},1,\frac{3}{2},2$ are given. For $j=\frac{1}{2}$, the formalism reproduces the front-form spinors of Lepage and Brodsky [4] and $\Omega(\frac{1}{2})$ coincides with the celebrated "Melosh transformation" [5,6]. This approach is in the spirit of Dirac's [7] original motivation for the high-spin wave equations for "approximate application to composite particles" and opens up the possibility of constructing a QCD-based effective field theory of hadronic resonances along the lines of the recent work of Cahill [8].

Unless otherwise indicated we follow the notation of Refs. [9,3(b)]. We use the notation $x^\mu=(x^+,x^1,x^2,x^-)$. In terms of the instant form variables $\underline{x}^\mu=(x^0,x^1,x^2,x^3)$, we have $x^\pm=x^0\pm x^3$. The evolution of a system is studied along the coordinate x^+, and as such it plays the role of "time."

II. FRONT-FORM HADRONIC SPINORS

We will work under the assumption that the center of mass of a composite hadron is best described, in the phenomenological sense, by the fields constructed from the $(j,0)\oplus(0,j)$ spinors in the front form. Following the Weinberg-Soper formalism [1,2], these spinors will be constructed from the right-handed $\phi^R(p^\mu)$ and left-handed $\phi^L(p^\mu)$ matter fields. We begin from the transformation which takes a particle from rest, $\mathring{p}^\mu=(m,0,0,m)$, to a particle moving with an arbitrary four-momentum $p^\mu=(p^+,p^1,p^2,p^-)$. [Note: for massive particles $p^+>0$.]

In the *instant form of field theory* the transformation which takes the rest momentum $\underline{\mathring{p}}^\mu=(m,0,0,0)\to\underline{p}^\mu=(p^0,\mathbf{p})$, is constructed out of the boost operator $\mathbf{K}$ and is given by

$$\underline{p}^\mu=\Lambda^\mu{}_\nu\underline{\mathring{p}}^\nu \tag{1}$$

with

$$\Lambda=\exp(i\boldsymbol{\varphi}\cdot\mathbf{K}) \ . \tag{2}$$

In Eq. (2) the boost parameter $\boldsymbol{\varphi}$ is defined as

$$\hat{\varphi}=\mathbf{p}/|\mathbf{p}|,\quad \cosh(\varphi)=E/m,\quad \sinh(\varphi)=|\mathbf{p}|/m \ . \tag{3}$$

Note that the stability group of the $x^0=0$ plane consists of the six generators $\mathbf{J}$ and $\mathbf{P}$. $\mathbf{K}$, along with P^0, generate an instant-form dynamics.

In the *front form of field theory* the transformation which takes $\mathring{p}^\mu=(m,0,0,0)\to\underline{p}^\mu=(p^0,\mathbf{p})$ is defined [2,10] by

$$p^\mu=L^\mu{}_\nu\mathring{p}^\nu \ , \tag{4}$$

with the matrix L given by

$$L=\exp(i\mathbf{v}_\perp\cdot\mathbf{G}_\perp)\exp(i\eta K_3) \ . \tag{5}$$

The parameters η and $\mathbf{v}_\perp=(v_x,v_y)$ specify a given boost. The generators $\mathbf{G}_\perp$ are defined as

$$G_1=K_1-J_2,\quad G_2=K_2+J_1 \ , \tag{6}$$

and together with

$$P_-=P_0-P_3,P_1,P_2,J_3,K_3 \ , \tag{7}$$

form the seven generators of the stability group of the $x^+=0$ plane. (Note that $P_-=P^+$.) The algebra associated with the stability group is summarized in Table I. The generators $D_1=K_1+J_2$, $D_2=K_2-J_1$, and $P_+=P_0+P_3$ generate the front-form dynamics.

It is important to note that while the front-form transformation L is specified entirely in terms of the generators of the $x^+=0$ plane stability group, the instant-form transformation Λ involves dynamical generators associated with the $x^0=0$ plane.

Using the matrix expressions for $\mathbf{J}=(J_1,J_2,J_3)$ and $\mathbf{K}=(K_1,K_2,K_3)$ given in Eqs. (2.65)–(2.67) of Ref. [9] we obtain an explicit expression for the boost L defined in Eq. (5):

$$[L^\mu{}_\nu]=\begin{bmatrix}\cosh(\eta)+\frac{1}{2}\mathbf{v}_\perp^2\exp(\eta) & v_x & v_y & \sinh(\eta)+\frac{1}{2}\mathbf{v}_\perp^2\exp(\eta)\\ v_x\exp(\eta) & 1 & 0 & v_x\exp(\eta)\\ v_y\exp(\eta) & 0 & 1 & v_y\exp(\eta)\\ \sinh(\eta)-\frac{1}{2}\mathbf{v}_\perp^2\exp(\eta) & v_x & v_y & \cosh(\eta)-\frac{1}{2}\mathbf{v}_\perp^2\exp(\eta)\end{bmatrix} . \tag{8}$$

Recalling that the components of the front-form momentum p^μ are defined as $p^\pm=p^0\pm p^3$ this yields

$$p^+=m\exp(\eta),$$
$$\mathbf{p}_\perp=m\exp(\eta)\mathbf{v}_\perp , \tag{9}$$
$$p^-=m\exp(-\eta)+m\exp(\eta)\mathbf{v}_\perp^2 .$$

The variables η and $\mathbf{v}_\perp$ are fixed by requiring $\underline{p}^\mu$ generated by Eq. (1) to be identical to the $\underline{p}^\mu$ produced by Eq. (4).

Given the transformation L, Eq. (5), we now wish to construct the front-form $(j,0)\oplus(0,j)$ hadronic spinors. To proceed in this direction we rewrite L by expanding the exponentials in Eq. (5) and using Table I to arrive at [11]

$$L=\exp[i(a\mathbf{v}_\perp\cdot\mathbf{G}_\perp+\eta K_3)] \tag{10}$$

with

$$a=\frac{\eta}{1-\exp(-\eta)} . \tag{11}$$

For the $(j,0)$ matter fields, $\phi^R(p^\mu)$, we have [1,9,3(g)] $\mathbf{K}=-i\mathbf{J}$. For the $(0,j)$ matter fields, $\phi^L(p^\mu)$, $\mathbf{K}=+i\mathbf{J}$. Using this observation, along with Eq. (10) and definitions (6), we obtain the transformation properties of the front form $(j,0)$ and $(0,j)$ hadronic fields

$$\phi^R(p^\mu)=\exp(+\eta\hat{\mathbf{b}}\cdot\mathbf{J})\phi^R(\mathring{p}^\mu) \tag{12}$$

and

$$\phi^L(p^\mu)=\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J})\phi^L(\mathring{p}^\mu) , \tag{13}$$

where the unit vectors $\hat{\mathbf{b}}$ and $\hat{\mathbf{b}}^*$ are given by

$$\hat{\mathbf{b}}=\eta^{-1}(av_r,-iav_r,\eta) , \tag{14}$$

$$\hat{\mathbf{b}}^*=\eta^{-1}(av_l,iav_l,\eta),\quad \hat{\mathbf{b}}\cdot\hat{\mathbf{b}}=1=\hat{\mathbf{b}}^*\cdot\hat{\mathbf{b}}^* \tag{15}$$

with $v_r=v_x+iv_y$ and $v_l=v_x-iv_y$.

We now make two observations. First, under the operation of *parity* we have $\phi^R(p^\mu)\leftrightarrow\phi^L(p^\mu)$. Second, for the particle at rest ($\mathbf{p}=\mathbf{0},p^\mu=\mathring{p}^\mu$) the concept of handedness loses its physical significance (c.f. Ref. [9] and more detailed discussion in Ref. [12]), and this, in turn, yields the relation

$$\phi^R(\mathring{p}^\mu)=\pm\phi^L(\mathring{p}^\mu) . \tag{16}$$

We now introduce the spin-j hadronic spinor

$$\psi(p^\mu)=\frac{1}{\sqrt{2}}\begin{bmatrix}\phi^R(p^\mu)+\phi^L(p^\mu)\\ \phi^R(p^\mu)-\phi^L(p^\mu)\end{bmatrix} , \tag{17}$$

and observe that the plus (minus) sign in Eq. (16) yields spinors with *even (odd) spinor parity.* We will denote the even spinor-parity spinors by $\mathcal{U}(p^\mu)$; and the odd spinor-parity spinors by $\mathcal{V}(p^\mu)$.

The transformation property for these hadronic spinors under the boost (5) is now readily obtained by using Eqs. (12) and (13). The result is

$$\psi(p^\mu)=M(L)\psi(\mathring{p}^\mu) , \tag{18}$$

with the operator $M(L)$ given by

$$M(L)=\begin{bmatrix}\exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})+\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J}) & \exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})-\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J})\\ \exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})-\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J}) & \exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})+\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J})\end{bmatrix} . \tag{19}$$

TABLE I. Algebra associated with the stability group of the $x^+=0$ plane. The commutator [element in the *first* column, element in the *first* row] = the element at the intersection of *the* row and column.

	P_1	P_2	J_3	K_3	P_-	G_1	G_2
P_1	0	0	$-iP_2$	0	0	iP_-	0
P_2	0	0	iP_1	0	0	0	iP_-
J_3	iP_2	$-iP_1$	0	0	0	iG_2	$-iG_1$
K_3	0	0	0	0	iP_-	iG_1	iG_2
P_-	0	0	0	$-iP_-$	0	0	0
G_1	$-iP_-$	0	$-iG_2$	$-iG_1$	0	0	0
G_2	0	$-iP_-$	iG_1	$-iG_2$	0	0	0

In what follows we present the explicit construction of the hadronic spinors $\mathcal{U}(p^\mu)$ and $\mathcal{V}(p^\mu)$ for $j=\frac{1}{2},1,\frac{3}{2},2$.

III. CONSTRUCTION OF $\mathcal{U}(p^\mu)$ and $\mathcal{V}(p^\mu)$ AND THEIR PROPERTIES

Let us first note that the front-form helicity operator

$$\mathcal{J}_3\equiv J_3+\frac{1}{P_-}(G_1P_2-G_2P_1) , \tag{20}$$

introduced by Soper [2] and discussed by Leutwyler and Stern [10], commutes with all generators of the stability group associated with the $x^+=0$ plane. The front-form helicity operator associated with the $(j,0)\oplus(0,j)$ spinors constructed above is then readily defined to be

$$\Theta=\begin{bmatrix}\mathcal{J}_3 & 0\\ 0 & \mathcal{J}_3\end{bmatrix}. \tag{21}$$

If choose a matrix representation of the **J** operators with J_3 diagonal (we follow the standard convention of Ref. [13]) then the $2(2j+1)$ element basis spinors for a particle at rest have the general form

$$\mathcal{U}_{+j}(\mathring{p}^\mu)=\begin{bmatrix}\mathcal{N}(j)\\0\\0\\\vdots\\0\end{bmatrix},\quad \mathcal{U}_{j-1}(\mathring{p}^\mu)=\begin{bmatrix}0\\\mathcal{N}(j)\\0\\\vdots\\0\end{bmatrix},\ \ldots,\quad \mathcal{V}_{-j}(\mathring{p}^\mu)=\begin{bmatrix}0\\0\\0\\\vdots\\\mathcal{N}(j)\end{bmatrix}. \tag{22}$$

The index $h=j,j-1,\ldots,-j$ on $\mathcal{U}_h(p^\mu)$ and $\mathcal{V}_h(p^\mu)$ corresponds to the eigenvalues of the front-form helicity operator Θ. The spin-dependent normalization constant $\mathcal{N}(j)$ is to be so chosen that for the *massless* particles all $\mathcal{U}_h(\mathring{p}^\mu)$ and $\mathcal{V}_h(\mathring{p}^\mu)$ vanish (there can be no massless particles at rest), and the only nonvanishing spinors are $\mathcal{U}_{h=\pm j}(p^\mu)$ and $\mathcal{V}_{h=\pm j}(p^\mu)$. The simplest choice satisfying these requirements is

$$\mathcal{N}(j)=m^j . \tag{23}$$

We now have all the details needed to construct $\mathcal{U}_h(p^\mu)$ and $\mathcal{V}_h(p^\mu)$ for any hadronic field. In this paragraph we summarize the algebraic construction used for $j=\frac{1}{2},1,\frac{3}{2},2$. Using Eqs. (A27), (A28), (A31), and (A32) of Ref. [1] along with Eqs. (9), (14), and (15) above we obtain the expansions for $\exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})$ and $\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J})$ which appear in the light-front $(j,0)\oplus(0,j)$ spinor boost matrix $M(L)$, Eq. (19). These expansions are presented in Appendix A. Using the results of Appendix A, explicit expressions for $M(L)$, Eq. (19), are then calculated as a simple, but somewhat lengthy algebraic exercise. These expressions for $M(L)$ combined with Eqs. (18) and (22) yield the hadronic spinors presented in Appendix B. The generality of the procedure for any spin is now obvious, and the procedure reduces to the well-defined algebraic manipulations.

We now introduce the useful matrices

$$\Gamma^0=\begin{bmatrix}I & 0\\ 0 & -I\end{bmatrix},\quad \Gamma^5=\begin{bmatrix}0 & I\\ I & 0\end{bmatrix}, \tag{24}$$

with $I=(2j+1)\times(2j+1)$ identity matrix. In reference to the spinors presented in Appendix B, we define

$$\bar{\psi}(p^\mu)\equiv\psi^\dagger(p^\mu)\Gamma^0 . \tag{25}$$

Using the explicit expressions for $\mathcal{U}(p^\mu)$ and $\mathcal{V}(p^\mu)$, Eqs. (B1)–(B7), we verify that

$$\overline{\mathcal{U}}_h(p^\mu)\mathcal{U}_{h'}(p^\mu)=m^{2j}\delta_{hh'} , \tag{26}$$

$$\overline{\mathcal{V}}_h(p^\mu)\mathcal{V}_{h'}(p^\mu)=-m^{2j}\delta_{hh'} , \tag{27}$$

$$\overline{\mathcal{U}}_h(p^\mu)\mathcal{V}_{h'}(p^\mu)=0=\overline{\mathcal{V}}_h(p^\mu)\mathcal{U}_{h'}(p^\mu) . \tag{28}$$

For spin $\frac{1}{2}$ the result given by Eq. (B1) corresponds to that given by Lepage and Brodsky [4, Eq. (A3)].[1] A noteworthy feature of the front-form spinors constructed here in the $(j,0)\oplus(0,j)$ representation is the observation that (i) for the massless case, $m=0$, the $\mathcal{U}_h(p^\mu)$ and $\mathcal{V}_h(p^\mu)$ identically vanish unless the associated front-form helicity $h=\pm j$, and (ii) for the massive case, $m\neq 0$, an examination of Eqs. (B3), (B4), and (B5)–(B7) yields the result that in the "high-momentum" limit, $p^+ \gg m$, the leading asymptotic behavior of $\mathcal{U}_h(p^\mu)$, and $\mathcal{V}_h(p^\mu)$ is given by $\sim(p^+)^{|h|}$. The correspondence between observations (i) and (ii) is to be perhaps fully realized when various matrix elements, of appropriate front-form operators $\mathcal{O}$, $\bar{\psi}_{h'}(p'^\mu)\mathcal{O}\psi_h(p^\mu)$, are studied for the massive case. Since the high-momentum behavior of $\mathcal{U}_h(p^\mu)$ and $\mathcal{V}_h(p^\mu)$ is $\sim(p^+)^{|h|}$, we can expect to generalize to arbitrary spin the results on helicity amplitudes presented by Lepage and Brodsky [4] for $j=\frac{1}{2}$. It is expected that the dominant *elastic-scattering* amplitudes will correspond to the helicity-nonchanging processes, while the helicity-changing transitions will be suppressed by appropriate powers of the factor (m/p^+). The fact that the front-form spinors $\mathcal{U}_{|h|<j}(p^\mu)$ and $\mathcal{V}_{|h|<j}(p^\mu)$ for massive particles do *not* identically vanish in the high-momentum limit is of profound physical significance. To see this note that, as argued by Brodsky and Lepage [14], the hadrons in $e^+ + e^- \to \gamma^* \to \mathcal{H}_A + \overline{\mathcal{H}}_B$ are produced at large Q^2 with opposite helicity $h_A + h_B = 0$ and $|h_i| \le \frac{1}{2}$. As a consequence, to give an example [14, Table I], the process $e^+ + e^- \to \underline{p}_{\pm 1/2} + \overline{\Delta}_{\pm 3/2}$ is *suppressed* relative to $e^+ + e^- \to p_{\pm 1/2} + \overline{\Delta}_{\mp 1/2}$. Here, the physically dominant degree of freedom is *not* $\Delta_{\pm 3/2}$ but $\Delta_{\pm 1/2}$—the degree of freedom which for the massless case identically vanishes.

IV. GENERALIZED MELOSH TRANSFORMATION: THE CONNECTION WITH THE INSTANT FORM

In a representation appropriate for comparison with the front-form spinors $\mathcal{U}_h(p^\mu)$ and $\mathcal{V}_h(p^\mu)$, the instant form hadronic spinors $u_\sigma(\underline{p}^\mu)$ and $v_\sigma(\underline{p}^\mu)$, $\sigma=j,j-1,\ldots,-j$, were recently constructed explicitly

[1]Note, however, a slightly different normalization and convention chosen by Lepage and Brodsky for the odd spinor parity spinors: $u_\uparrow=\sqrt{2}\,\mathcal{U}_{1/2}$, $u_\downarrow=\sqrt{2}\,\mathcal{U}_{-1/2}$, $v_\uparrow=\sqrt{2}\,\mathcal{V}_{-1/2}$, $v_\downarrow=-\sqrt{2}\,\mathcal{V}_{1/2}$.

(following Weinberg [1] and Ryder [9]), in Refs. [3(a)–3(c), 3(g)]. A brief report, sufficient for the present discussion, can be found in Ref. [3(b)]. Here we only remark that the construction of instant-form spinors follows the steps outlined in Eqs. (10)–(19), above, with the only difference that one starts with transformation Λ of Eq. (2) rather than L of Eq. (10).

The instant-form hadronic spinors of Refs. [3(a)–3(c), 3(g)] satisfy the normalization properties

$$\overline{u}_\sigma(\underline{p}^\mu)u_{\sigma'}(\underline{p}^\mu)=m^{2j}\delta_{\sigma\sigma'} , \tag{29}$$

$$\overline{v}_\sigma(\underline{p}^\mu)v_{\sigma'}(\underline{p}^\mu)=-m^{2j}\delta_{\sigma\sigma'} , \tag{30}$$

$$\overline{u}_\sigma(\underline{p}^\mu)v_{\sigma'}(\underline{p}^\mu)=0=\overline{v}_\sigma(\underline{p}^\mu)u_{\sigma'}(\underline{p}^\mu) , \tag{31}$$

where

$$\overline{u}_\sigma(\underline{p}^\mu)=[u_\sigma(\underline{p}^\mu)]^\dagger\gamma^0,\quad \overline{v}_\sigma(\underline{p}^\mu)=[v_\sigma(\underline{p}^\mu)]^\dagger\gamma^0 \tag{32}$$

with γ^0 having the form identical to Γ^0 of Eq. (24).

In what follows we assume that p^μ and $\underline{p}^\mu$ correspond to the same physical momentum. The connection between the front-form and instant-form spinors is then established by noting that on general algebraic grounds we can express the instant-form hadronic spinors as a linear combination of the front-form hadronic spinors. That is,

$$u_\sigma(\underline{p}^\mu)=\Omega^{(u\mathcal{U})}_{\sigma h}\mathcal{U}_h(p^\mu)+\Omega^{(u\mathcal{V})}_{\sigma h}\mathcal{V}_h(p^\mu) , \tag{33}$$

$$v_\sigma(\underline{p}^\mu)=\Omega^{(v\mathcal{U})}_{\sigma h}\mathcal{U}_h(p^\mu)+\Omega^{(v\mathcal{V})}_{\sigma h}\mathcal{V}_h(p^\mu) , \tag{34}$$

where the sum on the repeated indices is implicit.

We now multiply Eq. (33) by $\overline{\mathcal{U}}_{h'}(p^\mu)$ from the left, and using the orthonormality relations, Eqs. (26)–(28), we get

$$\Omega^{(u\mathcal{U})}_{\sigma h}=\frac{1}{m^{2j}}[\overline{\mathcal{U}}_h(p^\mu)u_\sigma(\underline{p}^\mu)] . \tag{35}$$

Similarly by multiplying Eq. (34) from the left by $\overline{\mathcal{V}}_{h'}(p^\mu)$ and again using the orthonormality relations, Eqs. (26)–(28), we obtain

$$\Omega^{(v\mathcal{V})}_{\sigma h}=-\frac{1}{m^{2j}}[\overline{\mathcal{V}}_h(p^\mu)v_\sigma(\underline{p}^\mu)] . \tag{36}$$

Further it is readily verified, e.g., by using the results presented in Appendix B here and explicit expressions for $u_\sigma(\underline{p}^\mu)$ and $v_\sigma(\underline{p}^\mu)$ found in Refs. [3(a)-3(c),3(g)], that

$$\overline{\mathcal{V}}_h(p^\mu)u_\sigma(\underline{p}^\mu)=0=\overline{\mathcal{U}}_h(p^\mu)v_\sigma(\underline{p}^\mu) , \tag{37}$$

which yields

$$\Omega^{(u\mathcal{V})}_{\sigma h}=0=\Omega^{(v\mathcal{U})}_{\sigma h} . \tag{38}$$

Finally, we exploit the facts

$$\{\Gamma^5,\Gamma^0\}=0 ,\quad \Gamma^{5\dagger}=\Gamma^5 , \tag{39}$$

and

$$(\Gamma^5)^2=I ,$$

to conclude that $\mathcal{V}_h(p^\mu)v_\sigma(\underline{p}^\mu)=-\overline{\mathcal{U}}_h(p^\mu)u_\sigma(\underline{p}^\mu)$. Thus, the matrix which connects the instant-form spinors with front-form spinors reads

$$\Omega(j)=\begin{bmatrix} B(j) & 0 \\ 0 & B(j) \end{bmatrix} , \tag{40}$$

where $B(j)$ is a $(2j+1)\times(2j+1)$ matrix with elements $B_{\sigma h}=\overline{\mathcal{U}}_h(p^\mu)u_\sigma(\underline{p}^\mu)=\Omega^{(u\mathcal{U})}_{\sigma h}=\Omega^{(v\mathcal{V})}_{\sigma h}$.

The explicit expressions for $\Omega(j)$ are presented in Appendix C. For spin $\frac{1}{2}$ the transformation matrix $\Omega(\frac{1}{2})$ computed by us coincides with the celebrated "Melosh transformation" given by Melosh in [5, Eq. (26)] and by Dziembowski in [6, Eq. (A8)]. As formally demonstrated by Kondratyuk and Terent'ev [15], the transformation matrix $\Omega(j)$ represents a pure rotation of the spin basis. However, since $\Omega(j)$ has block zeros off diagonal, what manifestly emerges here is that this rotation does not mix the even and odd spinor-parity spinors.

V. SUMMARY

Within the framework of the Weinberg-Soper formalism [1,2] we constructed explicit hadronic spinors for arbitrary spin in the front form, and established their connection with the instant-form fields. For a given spin j there are $2j+1$ hadronic spinors with *even* spinor parity, $\mathcal{U}_h(p^\mu)$, and $2j+1$ hadronic spinors with *odd* spinor parity, $\mathcal{V}_h(p^\mu)$. The normalization of these spinors is so chosen that for the *massless* particles $\mathcal{U}_h(\mathring{p}^\mu)$ and $\mathcal{V}_h(\mathring{p}^\mu)$ identically vanish, and only $\mathcal{U}_{h=\pm j}(p^\mu)$ and $\mathcal{V}_{h=\pm j}(p^\mu)$ survive. The simplest choice of this normalization is given by Eq. (23). Next we constructed the matrix $\Omega(j)$ which provides the connection between the front-form hadronic spinors with the more familiar [i.e., more "familiar" at least for the spin-$\frac{1}{2}$ case) instant-form hadronic spinors. We verified that the transformation matrix $\Omega(\frac{1}{2})$ coincides with the well-known "Melosh transformation" [5,6], and the spin-$\frac{1}{2}$ spinors are in agreement with the previous results of Lepage and Brodsky [4]. Explicit results for $\mathcal{U}_h(p^\mu)$, $\mathcal{V}_h(p^\mu)$, and $\Omega(j)$ up to spin 2 are found in Appendixes B and C here.

ACKNOWLEDGMENTS

Stan Brodsky and Terry Goldman helped us clear the discussion of the massless case and the high-momentum limit for massive case; for this we thank them. D.V.A. extends his thanks to Ovid C. Jacob, Mikkel B. Johnson, and R. M. Thaler for insightful conversations on the general subject of this work. He acknowledges an indebtedness to Dave Ernst for our previous collaborative work on this subject, and the insights which are carried over into the present work. He also thankfully acknowledges financial support from the Los Alamos National Laboratory. Part of this work was done while one of the authors (M.S.) was at the Department of Physics of the Texas A&M University. He would like to thank Professor David J. Ernst and members of the Department of Physics for warm hospitality extended to him during his stay. The work of D.V.A. was done under the auspices of the U.S. Department of Energy. The work of M. S. was supported by the U. S. National Science Foundation under Grant No. PHY-8907852.

APPENDIX A: EXPANSIONS FOR THE $\exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})$ AND $\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J})$ UP TO SPIN 2

This appendix provides the expansions for the $\exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})$ and $\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J})$, up to $j=2$, which appear in the spinor-boost matrix $M(L)$, Eq. (19):

$j=\frac{1}{2}$:

$$\exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})=\cosh(\eta/2)I+(\hat{\mathbf{b}}\cdot\boldsymbol{\sigma})\sinh(\eta/2)\ , \tag{A1}$$

$$\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J})=\cosh(\eta/2)I-(\hat{\mathbf{b}}^*\cdot\boldsymbol{\sigma})\sinh(\eta/2)\ . \tag{A2}$$

$j=1$:

$$\exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})=I+2(\hat{\mathbf{b}}\cdot\mathbf{J})^2\sinh^2(\eta/2)+2(\hat{\mathbf{b}}\cdot\mathbf{J})\cosh(\eta/2)\sinh(\eta/2)\ , \tag{A3}$$

$$\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J})=I+2(\hat{\mathbf{b}}^*\cdot\mathbf{J})^2\sinh^2(\eta/2)-2(\hat{\mathbf{b}}^*\cdot\mathbf{J})\cosh(\eta/2)\sinh(\eta/2)\ . \tag{A4}$$

$j=\frac{3}{2}$:

$$\exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})=\cosh(\eta/2)[I+\tfrac{1}{2}\{(2\hat{\mathbf{b}}\cdot\mathbf{J})^2-I\}\sinh^2(\eta/2)]+(2\hat{\mathbf{b}}\cdot\mathbf{J})\sinh(\eta/2)[I+\tfrac{1}{6}\{(2\hat{\mathbf{b}}\cdot\mathbf{J})^2-I\}\sinh^2(\eta/2)]\ , \tag{A5}$$

$$\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J})=\cosh(\eta/2)[I+\tfrac{1}{2}\{(2\hat{\mathbf{b}}^*\cdot\mathbf{J})^2-I\}\sinh^2(\eta/2)]-(2\hat{\mathbf{b}}^*\cdot\mathbf{J})\sinh(\eta/2)[I+\tfrac{1}{6}\{(2\hat{\mathbf{b}}^*\cdot\mathbf{J})^2-I\}\sinh^2(\eta/2)]\ . \tag{A6}$$

$j=2$:

$$\begin{aligned}\exp(\eta\hat{\mathbf{b}}\cdot\mathbf{J})=&I+2(\hat{\mathbf{b}}\cdot\mathbf{J})^2\sinh^2(\eta/2)+\tfrac{2}{3}(\hat{\mathbf{b}}\cdot\mathbf{J})^2\{(\hat{\mathbf{b}}\cdot\mathbf{J})^2-I\}\sinh^4(\eta/2)\\&+2(\hat{\mathbf{b}}\cdot\mathbf{J})\cosh(\eta/2)\sinh(\eta/2)+\tfrac{4}{3}(\hat{\mathbf{b}}\cdot\mathbf{J})\{(\hat{\mathbf{b}}\cdot\mathbf{J})^2-I\}\cosh(\eta/2)\sinh^3(\eta/2)\ .\end{aligned} \tag{A7}$$

$$\begin{aligned}\exp(-\eta\hat{\mathbf{b}}^*\cdot\mathbf{J})=&I+2(\hat{\mathbf{b}}^*\cdot\mathbf{J})^2\sinh^2(\eta/2)+\tfrac{2}{3}(\hat{\mathbf{b}}^*\cdot\mathbf{J})^2\{\hat{\mathbf{b}}^*\cdot\mathbf{J})^2-I\}\sinh^4(\eta/2)\\&-2(\hat{\mathbf{b}}^*\cdot\mathbf{J})\cosh(\eta/2)\sinh(\eta/2)-\tfrac{4}{3}(\hat{\mathbf{b}}^*\cdot\mathbf{J})\{(\hat{\mathbf{b}}^*\cdot\mathbf{J})^2-I\}\cosh(\eta/2)\sinh^3(\eta/2)\ .\end{aligned} \tag{A8}$$

In Eqs. (A1) and (A2) the $\boldsymbol{\sigma}$ are the standard [9] Pauli matrices. In this appendix, I are the $(2j+1)\times(2j+1)$ identity matrices.

APPENDIX B: FRONT-FORM HADRONIC SPINORS UP TO SPIN 2

We begin with collecting together front-form hadronic spinors up to spin 2 *for even spinor parity,* first. In what follows we use the notation $p_r=p_x+ip_y$ and $p_l=p_x-ip_y$, cf. Eq.(15).

Spin-$\frac{1}{2}$ hadronic spinors with even spinor parity:

$$\mathcal{U}_{+1/2}(p^\mu)=\frac{1}{2}\left[\frac{1}{p^+}\right]^{1/2}\begin{bmatrix}p^++m\\p_r\\p^+-m\\p_r\end{bmatrix}\quad \mathcal{U}_{-1/2}(p^\mu)=\frac{1}{2}\left[\frac{1}{p^+}\right]^{1/2}\begin{bmatrix}-p_l\\p^++m\\p_l\\-p^++m\end{bmatrix}\ . \tag{B1}$$

Spin-1 hadronic spinors with even spinor parity:

$$\mathcal{U}_{+1}(p^\mu)=\frac{1}{2}\begin{bmatrix}p^++(m^2/p^+)\\\sqrt{2}p_r\\p_r^2/p^+\\p^+-(m^2/p^+)\\\sqrt{2}p_r\\p_r^2/p^+\end{bmatrix},\quad \mathcal{U}_0(p^\mu)=m\sqrt{1/2}\begin{bmatrix}-p_l/p^+\\\sqrt{2}\\p_r/p^+\\p_l/p^+\\0\\p_r/p^+\end{bmatrix},\quad \mathcal{U}_{-1}(p^\mu)=\frac{1}{2}\begin{bmatrix}p_l^2/p^+\\-\sqrt{2}p_l\\p^++(m^2/p^+)\\-p_l^2/p^+\\\sqrt{2}p_l\\-p^++(m^2/p^+)\end{bmatrix}\ . \tag{B2}$$

Spin-$\frac{3}{2}$ hadronic spinors with even spinor parity:

$$\mathcal{U}_{+3/2}(p^\mu)=\frac{1}{2}\left[\frac{1}{p^+}\right]^{1/2}\begin{bmatrix}(p^+)^2+(m^3/p^+)\\ \sqrt{3}p_r p^+\\ \sqrt{3}p_r^2\\ p_r^3/p^+\\ (p^+)^2-(m^3/p^+)\\ \sqrt{3}p_r p^+\\ \sqrt{3}p_r^2\\ p_r^3/p^+\end{bmatrix},\quad \mathcal{U}_{+1/2}(p^\mu)=\frac{m}{2}\left[\frac{1}{p^+}\right]^{1/2}\begin{bmatrix}-\sqrt{3}mp_l/p^+\\ p^++m\\ 2p_r\\ \sqrt{3}p_r^2/p^+\\ \sqrt{3}mp_l/p^+\\ p^+-m\\ 2p_r\\ \sqrt{3}p_r^2/p^+\end{bmatrix}, \tag{B3}$$

$$\mathcal{U}_{-1/2}(p^\mu)=\frac{m}{2}\left[\frac{1}{p^+}\right]^{1/2}\begin{bmatrix}\sqrt{3}p_l^2/p^+\\ -2p_l\\ p^++m\\ \sqrt{3}mp_r/p^+\\ -\sqrt{3}p_l^2/p^+\\ 2p_l\\ -p^++m\\ \sqrt{3}mp_r/p^+\end{bmatrix},\quad \mathcal{U}_{-3/2}(p^\mu)=\frac{1}{2}\left[\frac{1}{p^+}\right]^{1/2}\begin{bmatrix}-p_l^3/p^+\\ \sqrt{3}p_l^2\\ -\sqrt{3}p_l p^+\\ (p^+)^2+(m^3/p^+)\\ p_l^3/p^+\\ -\sqrt{3}p_l^2\\ \sqrt{3}p_l p^+\\ -(p^+)^2+(m^3/p^+)\end{bmatrix}. \tag{B4}$$

Spin-2 hadronic spinors with even spinor parity:

$$\mathcal{U}_{+2}(p^\mu)=\frac{1}{2}\begin{bmatrix}(p^+)^2+(m^4/(p^+)^2)\\ 2p_r p^+\\ \sqrt{6}p_r^2\\ 2p_r^3/p^+\\ p_r^4/(p^+)^2\\ (p^+)^2-(m^4/(p^+)^2)\\ 2p_r p^+\\ \sqrt{6}p_r^2\\ 2p_r^3/p^+\\ p_r^4/(p^+)^2\end{bmatrix},\quad \mathcal{U}_{+1}(p^\mu)=\frac{m}{2}\begin{bmatrix}-2m^2p_l/(p^+)^2\\ p^++(m^2/p^+)\\ \sqrt{6}p_r\\ 3p_r^2/p^+\\ 2p_r^3/(p^+)^2\\ 2m^2p_l/(p^+)^2\\ p^+-(m^2/p^+)\\ \sqrt{6}p_r\\ 3p_r^2/p^+\\ 2p_r^3/(p^+)^2\end{bmatrix}, \tag{B5}$$

$$\mathcal{U}_{0}(p^\mu)=\frac{m^2}{2}\begin{bmatrix}\sqrt{6}p_l^2/(p^+)^2\\ -\sqrt{6}p_l/p^+\\ 2\\ \sqrt{6}p_r/p^+\\ \sqrt{6}p_r^2/(p^+)^2\\ -\sqrt{6}p_l^2/(p^+)^2\\ \sqrt{6}p_l/p^+\\ 0\\ \sqrt{6}p_r/p^+\\ \sqrt{6}p_r^2/(p^+)^2\end{bmatrix},\quad \mathcal{U}_{-1}(p^\mu)=\frac{m}{2}\begin{bmatrix}-2p_l^3/(p^+)^2\\ 3p_l^2/p^+\\ -\sqrt{6}p_l\\ p^++(m^2/p^+)\\ 2m^2p_r/(p^+)^2\\ 2p_l^3/(p^+)^2\\ -3p_l^2/p^+\\ \sqrt{6}p_l\\ -p^++(m^2/p^+)\\ 2m^2p_r/(p^+)^2\end{bmatrix}, \tag{B6}$$

$$\mathcal{U}_{-2}(p^\mu)=\frac{1}{2}\begin{bmatrix} p_l^4/(p^+)^2 \\ -2p_l^3/p^+ \\ \sqrt{6}p_l^2 \\ -2p_lp^+ \\ (p^+)^2+(m^4/(p^+)^2) \\ -p_l^4/(p^+)^2 \\ 2p_l^3/p^+ \\ -\sqrt{6}p_l^2 \\ 2p_lp^+ \\ -(p^+)^2+(m^4/(p^+)^2) \end{bmatrix} . \tag{B7}$$

An examination of the spinor-boost matrix $M(L)$, Eq. (19), implies that the *odd* spinor parity hadronic spinor can be obtained from the hadronic spinor of the *even* spinor parity via the simple relation

$$\mathcal{V}_h(p^\mu)=\Gamma^5\mathcal{U}_h(p^\mu) , \tag{B8}$$

where the matrix Γ^5 defined in Eq. (24) interchanges the top $(2j+1)$ elements with the bottom $(2j+1)$ elements of the hadronic spinors.

APPENDIX C: THE EXPLICIT EXPRESSIONS FOR $\Omega(j)$, THE GENERALIZED MELOSH TRANSFORMATION, UP TO SPIN 2

In this appendix we present explicit expressions for the matrix $\Omega(j)$, Eq. (40), which connects the front-form hadronic spinors with the instant-form hadronic spinors via Eqs. (33) and (34). As in Appendix B, $p_r=p_x+ip_y$ and $p_l=p_x-ip_y$, in what follows.

For spin-$\frac{1}{2}$, the matrix connecting the instant-form hadronic spinors with the front-form spinors is

$$\Omega(\tfrac{1}{2})=\frac{1}{[2(E+m)p^+]^{1/2}}\begin{bmatrix} \beta(\frac{1}{2}) & 0 \\ 0 & \beta(\frac{1}{2}) \end{bmatrix} , \tag{C1}$$

where the 2×2 block matrix $\beta(\frac{1}{2})$ is defined as

$$\beta(\tfrac{1}{2})=\begin{bmatrix} p^++m & -p_r \\ p_l & p^++m \end{bmatrix} . \tag{C2}$$

For spin 1, the matrix connecting the instant-form hadronic spinors with the front-form spinors is

$$\Omega(1)=\frac{1}{2(E+m)p^+}\begin{bmatrix} \beta(1) & 0 \\ 0 & \beta(1) \end{bmatrix} , \tag{C3}$$

where the 3×3 block matrix $\beta(1)$ is defined as

$$\beta(1)=\begin{bmatrix} (p^++m)^2 & -\sqrt{2}(p^++m)p_r & p_r^2 \\ \sqrt{2}(p^++m)p_l & 2[(E+m)p^+-p_rp_l] & -\sqrt{2}(p^++m)p_r \\ p_l^2 & \sqrt{2}(p^++m)p_l & (p^++m)^2 \end{bmatrix} . \tag{C4}$$

For spin-$\frac{3}{2}$, the matrix connecting the instant-form hadronic spinors with the front-form spinors is

$$\Omega(3/2)=\frac{1}{[2(E+m)p^+]^{3/2}}\begin{bmatrix} \beta(3/2) & 0 \\ 0 & \beta(3/2) \end{bmatrix} , \tag{C5}$$

where the 4×4 block matrix $\beta(\frac{3}{2})$ is defined as

$$\beta(\tfrac{3}{2})=\begin{bmatrix}(p^++m)^3 & -\sqrt{3}(p^++m)^2p_r & \sqrt{3}(p^++m)p_r^2 & -p_r^3\\ \sqrt{3}(p^++m)^2p_l & [(p^++m)^2-2p_rp_l](p^++m) & -[2(p^++m)^2-p_rp_l]p_r & \sqrt{3}(p^++m)p_r^2\\ \sqrt{3}(p^++m)p_l^2 & [2(p^++m)^2-p_rp_l]p_l & [(p^++m)^2-2p_rp_l](p^++m) & -\sqrt{3}(p^++m)^2p_r\\ p_l^3 & \sqrt{3}(p^++m)p_l^2 & \sqrt{3}(p^++m)^2p_l & (p^++m)^3\end{bmatrix}. \tag{C6}$$

For spin 2, the matrix connecting the instant-form hadronic spinors with the front-form spinors is

$$\Omega(2)=\frac{1}{[2(E+m)p^+]^2}\begin{bmatrix}\beta(2) & 0\\ 0 & \beta(2)\end{bmatrix}, \tag{C7}$$

where 5×5 block matrix $\beta(2)$ is defined via the five columns

$$\beta(2)_{\alpha,1}=\begin{bmatrix}(p^++m)^4\\ 2(p^++m)^3p_l\\ \sqrt{6}(p^++m)^2p_l^2\\ 2(p^++m)p_l^3\\ p_l^4\end{bmatrix},\quad \beta(2)_{\alpha,2}=\begin{bmatrix}-2(p^++m)^3p_r\\ 2[(E+m)p^+-2p_rp_l](p^++m)^2\\ \sqrt{6}[(E+m)p^+-p_rp_l](p^++m)p_l\\ [6p^+(E+m)-4p_rp_l]p_l^2\\ 2(p^++m)p_l^3\end{bmatrix},$$

$$\beta(2)_{\alpha,3}=\begin{bmatrix}\sqrt{6}(p^++m)^2p_r^2\\ \sqrt{6}[-(E+m)p^++p_rp_l](p^++m)p_r\\ 2[2(p^+)^2(E+m)^2-3(p^++m)^2p_rp_l]\\ \sqrt{6}[(E+m)p^+-p_rp_l](p^++m)p_l\\ \sqrt{6}(p^++m)^2p_l^2\end{bmatrix},$$

$$\beta(2)_{\alpha,4}=\begin{bmatrix}-2(p^++m)p_r^3\\ [6p^+(E+m)-4p_rp_l]p_r^2\\ \sqrt{6}[-(E+m)p^++p_rp_l](p^++m)p_r\\ 2[(E+m)p^+-2p_rp_l](p^++m)^2\\ 2(p^++m)^3p_l\end{bmatrix},\quad \beta(2)_{\alpha,5}=\begin{bmatrix}p_r^4\\ -2(p^++m)p_r^3\\ \sqrt{6}(p^++m)^2p_r^2\\ -2(p^++m)^3p_r\\ (p^++m)^4\end{bmatrix}. \tag{C8}$$

[1] S. Weinberg, Phys. Rev. **133**, B1318 (1964).
[2] D. E. Soper, SLAC Report No. 137, 1971 (unpublished).
[3] (a) D. V. Ahluwalia, Ph.D. thesis, Texas A&M University, 1991 (unpublished); (b) D. V. Ahluwalia and D. J. Ernst, Phys. Rev. C **45**, 3010 (1992); (c) in *Computational Quantum Physics, Nashville, 1991,* Proceedings of the Computational Quantum Physics, edited by A. S. Umar, V. E. Oberacker, M. R. Strayer, and C. Bottcher, AIP Conf. Proc. No. 260 (AIP, New York, 1992); (d), Phys. Lett. B **287**, 18 (1992); (e) Mod. Phys. Lett. A **7**, 1967 (1992); (f) *Classical and Quantum Systems: Foundations and Symmetries,* Proceedings of the II International Wigner Symposium, Goslar, Germany, 1991 (World Scientific, Singapore, in press); (g) Int. J. Mod. Phys. E (to be published).
[4] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).
[5] H. J. Melosh, Phys. Rev. D **9**, 1095 (1974).
[6] Z. Dziembowski, Phys. Rev. D **37**, 768 (1987).
[7] P. A. M. Dirac, Proc. R. Soc. London **A155**, 447 (1936); also see Rev. Mod. Phys. **21**, 392 (1949).
[8] R. T. Cahill, Nucl. Phys. **A543**, 63c (1992), and references therein.
[9] L. H. Ryder, *Quantum Field Theory* (Cambridge University Press, Cambridge, England, 1987) [Lewis Ryder gives much credit to Feza Gursey for teaching these matters to him at Ankara in 1965 (private communication)].
[10] H. Leutwyler and J. Stern, Ann. Phys. (N.Y.) **112**, 94 (1978).
[11] M. Sawicki, Texas A&M University Report No. CTP-TAMU-17/92, 1992 (unpublished).
[12] D. V. Ahluwalia, C. Burgard, D. J. Ernst, and M. B. Johnson (unpublished).
[13] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968).
[14] S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 2848 (1981).
[15] L. A. Kondratyuk and M. V. Terent'ev, Yad. Fiz. **31**, 1087 (1980) [Sov. J. Nucl. Phys. **31**, 561 (1980)].