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Mass formula for strange and nonstrange quark matter

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A mass formula for spherical lumps of two- and three-flavor quark matter is derived self-consistently from an asymptotic expansion within the MIT bag model, taking into account bulk, surface, curvature, and Coulomb contributions. For massless quarks the asymptotic expansion fits exceedingly well with exact mode-filling calculations. A bulk approximation to the mass formula is also discussed. The curvature energy is extremely important, adding up to $400A^{-2/3}$ MeV to the energy per baryon for baryon number A.

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I. INTRODUCTION

The possibility that strange quark matter, rather than ^{56}Fe, could be the ground state of hadronic matter even at zero temperature and pressure, has attracted a lot of attention since Witten resurrected the idea in 1984 [1] (the first notion of the idea seems to be due to Bodmer [2]; see Ref. [3] for reviews and references on strange quark matter).

Several experiments with relativistic heavy-ion collisions [4], as well as cosmic ray searches, are ongoing or planned. Therefore it is important to know the signatures to be expected for small lumps of quark matter, be it composed of up and down quarks, or (the more stable case) up, down, and strange.

One decisive property is the mass. A calculation involving mode filling in a spherical MIT bag has been performed for *ud* systems by Vasak, Greiner, and Neise [5], and for two- and three-flavor systems for a few parameter sets by Farhi and Jaffe [6] and Greiner *et al.* [7]. Such calculations, capable of showing shell effects, etc., are rather tedious. For many applications, including studies of decay modes, a global mass formula analogous to the liquid drop model for nuclei is of great use.

Such an investigation of the strangelet mass-formula within the MIT bag model was performed by Berger and Jaffe [8]. That investigation included Coulomb corrections and surface tension effects stemming from the depletion in the surface density of states due to the mass of the strange quark. Both effects were treated as perturbations added to a bulk solution with the surface contribution derived from a multiple reflection expansion.

Recently it was pointed out that another contribution to the energy, the curvature term, is dominant (and strongly destabilizing) at baryon numbers below 100 [9]. In view of this and also in order to test the perturbative approach in Ref. [8] there is a need for a detailed, self-consistent treatment of the mass formula.

One important problem however is that the density of states correction due to curvature is only known for massless quarks, whereas the surface tension is an effect of the mass, vanishing for zero quark mass. The present investigation therefore mainly focuses on two- and three-flavor lumps of massless quarks. First, a general framework will be presented. Then the bulk solutions will be derived for the purpose of comparison. Solutions for systems of massless *ud*- and *uds*-quark matter will be derived, and finally I shall comment on the consequences of a massive *s* quark. It will be clear from the results that inclusion of curvature energy is decisive. All calculations will be done for zero temperature and strong coupling constant α_s. As argued by Farhi and Jaffe [6] the latter assumption can be relaxed by a rescaling of the bag constant. Also, we shall concentrate on systems small enough ($A < 10^7$) to justify neglect of electrons, which if present to ensure charge neutrality are mainly situated outside the quark phase. Finally we neglect charge screening, an issue of negligible importance for the mass formula, but of some importance for the charge-to-mass ratio for systems of radii above 5–10 fm [10].

II. ENERGY OF A QUARK LUMP

A. General framework

In the ideal Fermi-gas approximation the energy of a system composed of quark flavors i is given by

$$E = \sum_i (\Omega_i + N_i\mu_i) + BV + E_{\rm Coul}. \quad (1)$$

Here Ω_i, N_i, and μ_i denote thermodynamic potentials, total number of quarks, and chemical potentials, respectively. B is the bag constant, V is the bag volume, and $E_{\rm Coul}$ is the Coulomb energy.

Extending the treatment of Berger and Jaffe [8] to include curvature corrections [9] one gets the following relation for the total number of quarks of flavor i, N_i, in terms of volume, surface, and curvature densities, $n_{i,V}$, $n_{i,S}$, and $n_{i,C}$:

$$N_i = n_{i,V}V + n_{i,S}S + n_{i,C}C, \quad (2)$$

where area $S = \oint dS$ ($= 4\pi R^2$ for a sphere) and curvature $C = \oint \left(\frac{1}{R_1} + \frac{1}{R_2}\right) dS$ ($= 8\pi R$ for a sphere). Curvature

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radii are denoted R_1 and R_2. For a spherical system $R_1 = R_2 = R$.

The corresponding thermodynamical potentials are related by

$$\Omega_i = \Omega_{i,V}V + \Omega_{i,S}S + \Omega_{i,C}C, \tag{3}$$

where $\partial\Omega_i/\partial\mu_i = -N_i$, and $\partial\Omega_{i,j}/\partial\mu_i = -n_{i,j}$. The volume terms $\Omega_{i,V}$ are given by [6]

$$\Omega_{u,V} = -\frac{\mu_u^4}{4\pi^2},$$

$$\Omega_{d,V} = -\frac{\mu_d^4}{4\pi^2}, \tag{4}$$

$$\Omega_{s,V} = -\frac{\mu_s^4}{4\pi^2}\left((1-\lambda^2)^{1/2}(1-\frac{5}{2}\lambda^2) + \frac{3}{2}\lambda^4 \ln\frac{1+(1-\lambda^2)^{1/2}}{\lambda}\right), \tag{5}$$

defining $\lambda \equiv m_s/\mu_s$, with corresponding densities

$$n_{u,V} = \frac{\mu_u^3}{\pi^2},$$
$$n_{d,V} = \frac{\mu_d^3}{\pi^2}, \tag{6}$$
$$n_{s,V} = \frac{\mu_s^3}{\pi^2}(1-\lambda^2)^{3/2}.$$

The surface contribution from massive s quarks is [8]

$$\Omega_{s,S} = \frac{3}{4\pi}\mu_s^3\left\{\frac{1-\lambda^2}{6} - \frac{\lambda^2(1-\lambda)}{3} - \frac{1}{3\pi}\left(\arctan\left[\frac{(1-\lambda^2)^{1/2}}{\lambda}\right] - 2\lambda(1-\lambda^2)^{1/2} + \lambda^3\ln\left[\frac{1+(1-\lambda^2)^{1/2}}{\lambda}\right]\right)\right\}, \tag{7}$$

$$n_{s,S} = -\frac{3}{4\pi}\mu_s^2\left\{\frac{(1-\lambda^2)}{2} - \frac{1}{\pi}\left(\arctan\left[\frac{(1-\lambda^2)^{1/2}}{\lambda}\right] + \lambda(1-\lambda^2)^{1/2}\right)\right\}. \tag{8}$$

For massless quarks $\Omega_{i,S} = n_{i,S} = 0$, whereas [9]

$$\Omega_{i,C} = \frac{\mu_i^2}{8\pi^2}, \tag{9}$$

$$n_{i,C} = -\frac{\mu_i}{4\pi^2}. \tag{10}$$

The curvature terms are not known for massive quarks.

With these prescriptions the differential of $E(V,S,N_i)$ is given by

$$dE = \left[\sum_i \Omega_{i,V} + B\right]dV + \left[\sum_i \Omega_{i,S} + \sum_i \Omega_{i,C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]dS + \sum_i \mu_i dN_i + dE_{\mathrm{Coul}}. \tag{11}$$

Minimizing the total energy at fixed N_i by taking $dE = 0$ gives the pressure equilibrium constraint

$$B = -\sum_i \Omega_{i,V} - \left[\sum_i \Omega_{i,S} + \sum_i \Omega_{i,C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]\frac{dS}{dV} - \frac{dE_{\mathrm{Coul}}}{dV}. \tag{12}$$

In general, the differential dS/dV depends on the curvature radii

$$\frac{dS}{dV} = \frac{1}{R_1} + \frac{1}{R_2}; \tag{13}$$

so, for a spherical volume, the constraint is

$$B = -\sum_i \Omega_{i,V} - \frac{2}{R}\sum_i \Omega_{i,S} - \frac{4}{R^2}\sum_i \Omega_{i,C} - \frac{dE_{\mathrm{Coul}}}{dV}, \tag{14}$$

with

$$E_{\mathrm{Coul}} = \frac{\alpha Z_V^2}{10R} + \frac{\alpha Z^2}{2R}, \tag{15}$$

$$\frac{dE_{\mathrm{Coul}}}{dV} = -\frac{\alpha Z_V^2}{40\pi R^4} - \frac{\alpha Z^2}{8\pi R^4}, \tag{16}$$

where $Z_V = \sum_i q_i n_{i,V} V$ is the volume part of the total charge Z, whereas charge $Z - Z_V = \sum_i q_i(n_{i,S}S + n_{i,C}C)$ is distributed on the surface. The quark charges are $q_u = 2/3$, $q_d = q_s = -1/3$. Eliminating B from Eq. (1) then gives the energy for a spherical quark lump as

$$E = \sum_i (N_i\mu_i + \tfrac{1}{3}\Omega_{i,S}S + \tfrac{1}{3}\Omega_{i,C}C) + \tfrac{4}{3}E_{\mathrm{Coul}}. \tag{17}$$

The optimal composition for fixed baryon number A can be found by minimizing the energy with respect to

N_i at fixed V, S, giving

$$0 = dE = \sum_i \left(\mu_i + \frac{\partial E_{\rm Coul}}{\partial N_i}\right) dN_i. \quad (18)$$

B. Massless quarks—Bulk limit

For uncharged bulk quark matter Eq. (17) reduces to the usual result for the energy per baryon:

$$\epsilon^0 = A^{-1} \sum_i N_i^0 \mu_i^0, \quad (19)$$

where a superscript 0 denotes bulk values. The energy minimization, Eq. (12), corresponds to

$$B = -\sum_i \Omega_{i,V}^0 = \sum_i \frac{(\mu_i^0)^4}{4\pi^2}. \quad (20)$$

The last equality assumes massless quarks. In the bulk limit the baryon number density is given by

$$n_A^0 = \frac{1}{3} \sum_i \frac{(\mu_i^0)^3}{\pi^2}, \quad (21)$$

and one may define a bulk radius per baryon as

$$R^0 = (3/4\pi n_A^0)^{1/3}. \quad (22)$$

1. Three flavor quark matter

For quark matter composed of massless u, d, and s quarks, the Coulomb energy vanishes at equal number densities due to the fact that the sum of the quark charges is zero. Thus it is energetically most favorable to have equal chemical potentials for the three flavors. From the equations above one may derive the following bulk expressions for 3-flavor quark matter (defining $B_{160}^{1/4} \equiv B^{1/4}/160$ MeV):

$$\mu_i^0 = \left(\frac{4\pi^2 B}{3}\right)^{1/4} = 1.905 B^{1/4} = 304.7 \text{ MeV} B_{160}^{1/4}, \quad (23)$$

$$n_A^0 = (\mu_i^0)^3/\pi^2 = 0.700 B^{3/4}, \quad (24)$$

$$R^0 = (3/4\pi n_A^0)^{1/3} = 0.699 B^{-1/4}. \quad (25)$$

And the energy per baryon is

$$\epsilon^0 = 3\mu_i^0 = 5.714 B^{1/4}. \quad (26)$$

Following Berger and Jaffe [8] one may to first order regard Coulomb, surface (and here, correspondingly, curvature) energies as perturbations on top of the bulk solution. In this approach one gets

$$\begin{aligned} \frac{E}{A} &= \epsilon^0 + A^{-1} \sum_i \Omega_{i,C}^0 C^0 \\ &= \epsilon^0 + \frac{3^{13/12} B^{1/4}}{\pi^{1/6} 2^{1/6} A^{2/3}} \\ &\approx \left[914 \text{ MeV} + 387 \text{ MeV} A^{-2/3}\right] B_{160}^{1/4}. \end{aligned} \quad (27)$$

2. Two flavor quark matter

For two-flavor quark matter composed of u and d quarks the Coulomb term does not vanish, but the contribution to the energy per baryon is negligible (less than 4.8 MeV, with a broad maximum for A near 800; cf. Fig. 1). For small systems the chemical potentials of u and d quarks are roughly equal (thereby minimizing the curvature energy), but in the bulk limit, where curvature is negligible, chemical potentials adjust to allow charge neutrality, $\mu_d^0 = 2^{1/3}\mu_u^0$. With this relation for the bulk limit one finds

$$\mu_u^0 = \left(\frac{4\pi^2 B}{1+2^{4/3}}\right)^{1/4} = 1.830 B^{1/4} = 292.8 \text{ MeV} B_{160}^{1/4}, \quad (28)$$

$$n_A^0 = (\mu_u^0)^3/\pi^2 = 0.621 B^{3/4}, \quad (29)$$

$$R^0 = (3/4\pi n_A^0)^{1/3} = 0.727 B^{-1/4}. \quad (30)$$

And the energy per baryon is

$$\epsilon^0 = (1+2^{4/3})\mu_u^0 = 6.441 B^{1/4}. \quad (31)$$

Adding again the curvature energy as a correction to the bulk solution leads to

$$\begin{aligned} \frac{E}{A} &= \epsilon^0 + A^{-1} \sum_i \Omega_{i,C}^0 C^0 \\ &\approx \left[1031 \text{ MeV} + 321 \text{ MeV} A^{-2/3}\right] B_{160}^{1/4}. \end{aligned} \quad (32)$$

C. Massless quarks: Self-consistent asymptotic solution

Self-consistent solutions can be obtained as follows. For massless quarks the number of quarks of flavor i depends only on the product $\mu_i R$, with

$$N_i = \frac{4}{3\pi}\mu_i^3 R^3 - \frac{2}{\pi}\mu_i R, \quad (33)$$

or

$$f(N_i) \equiv \mu_i R = \frac{1}{2}\{ [3\pi N_i + (9\pi^2 N_i^2 - 8)^{1/2}]^{1/3} + [3\pi N_i - (9\pi^2 N_i^2 - 8)^{1/2}]^{1/3}\}. \quad (34)$$

Another important property, only valid for massless quarks, is that the energy is simply given by

$$E = 4BV + \sum_i \frac{\mu_i^2 R}{\pi}. \quad (35)$$

(This can be shown explicitly from the equations above and is valid regardless of Coulomb energies. It is not valid for massive quarks.) The radius can be expressed in terms of B using Eq. (12), giving

$$R = \frac{1}{(2\pi)^{1/2}} \left[\sum_i (f(N_i)^4 - 2f(N_i)^2) + \frac{\alpha\pi Z_V^2}{10} + \frac{\alpha\pi Z^2}{2} \right]^{1/4} B^{-1/4}; \quad (36)$$

$Z_V = \frac{4}{3\pi} \sum_i q_i f(N_i)^3$, $Z = \sum_i q_i N_i$. Thus for fixed composition the energy is proportional to $B^{1/4}$:

$$E = \frac{16\pi}{3} BR^3 + \sum_i \frac{f(N_i)^2}{\pi R} \propto B^{1/4}. \quad (37)$$

The preferred composition for a spherical strangelet of radius R and baryon number $A = \sum_i N_i/3$ can be found from Eq. (18). The results of such numerical calculations are shown in Figs. 1 and 2 and compared to the bulk-limit approximations from Eqs. (32) and (27). The bulk-limit fits are very good for *uds*-quark matter, with significant deviations only for $A < 10$. In fact one can show explicitly that Eq. (27) gives the first terms in an expansion in powers of $A^{-2/3}$. Somewhat larger deviations occur for *ud* matter due to the change in chemical potentials from $\mu_u \approx \mu_d$ $(A \to 1)$ to $\mu_d \approx 2^{1/3}\mu_u$ $(A \to \infty)$.

For three-flavor quark matter there is no Coulomb energy. All quark flavors have the same chemical potential μ_q and the energy becomes

$$E = 3\frac{\mu_q^4}{\pi^2} V - \frac{5\mu_q^2}{\pi} R. \quad (38)$$

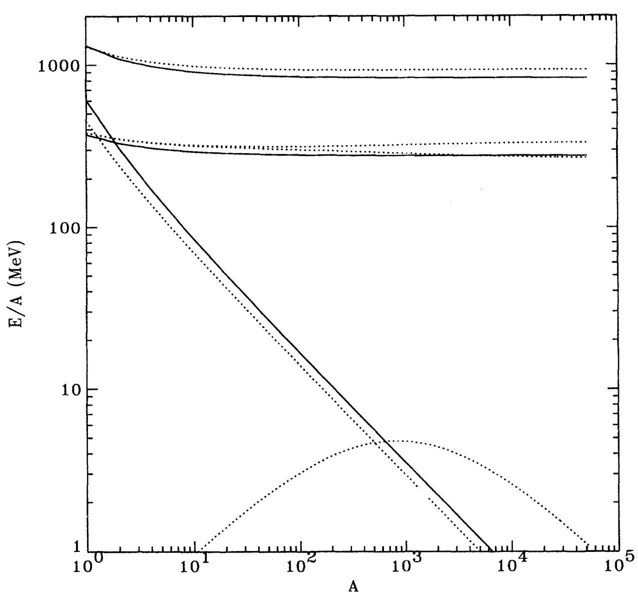

FIG. 1. Total energy (upper set of two curves), curvature energy (diagonal curves), and Coulomb energy (parabolalike curve), all in MeV per baryon, plotted as a function of baryon number for *ud*- (dotted curves) and *uds*-quark matter (solid curves). Also shown are the chemical potentials, all equal for *uds*, whereas $\mu_d \to 2^{1/3}\mu_u$ for $A \to \infty$ in the case of *ud* matter. The value $B = (145 \text{ MeV})^4$ was assumed; all energies scale like $B^{1/4}$.

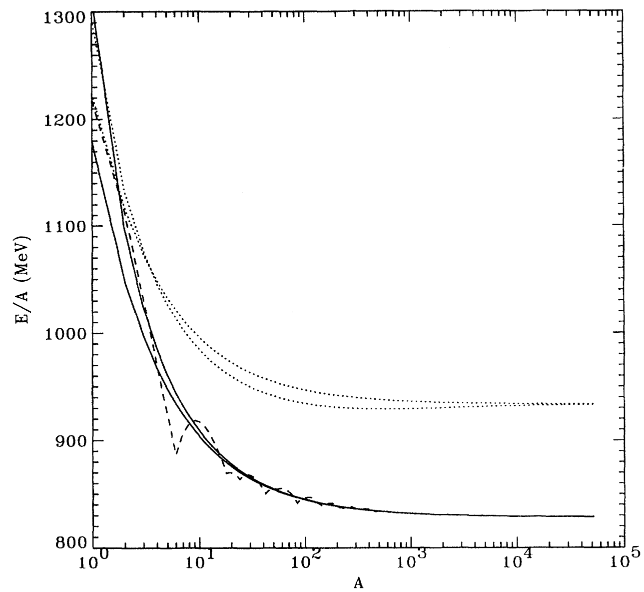

FIG. 2. Total energy per baryon for *ud*- (dotted curves) and *uds*-quark matter (solid curves), compared to the "bulk approximations" of Eqs. (32) and (27). The bulk approximations undershoot as $A \to 1$. Also shown are results of an exact mode-filling calculation for *uds*-quark matter (dashed curve). The value $B = (145 \text{ MeV})^4$ was assumed; all energies scale like $B^{1/4}$.

D. Massless quarks: Exact solution

Also shown in Fig. 2 is the exact solution for three massless quark flavors, which has the form

$$E = \frac{16\pi}{3} BR_E^3 = 401.66 \text{ MeV} B_{160}^{1/4} \left[\left(\sum \omega_{\kappa n}\right)^2 - \sum \omega_{\kappa n}^2 \right]^{3/8}, \quad (39)$$

$$R_E = \left[\left(\sum \omega_{\kappa n}\right)^2 - \sum \omega_{\kappa n}^2 \right]^{1/8} / (4\pi B)^{1/4}, \quad (40)$$

where the sums are to be taken over all $3A$ quark levels, and the numbers $\omega_{\kappa n}$ are tabulated in Ref. [5]. The mass formula derived above from the asymptotic expansion fits these exact quantum mechanical calculations exceedingly well, apart from the "wiggles" due to shell effects.

E. Limits on B from the stability of nuclei

The fact that nuclei are composed of neutrons and protons rather than *ud*-quark matter allows a lower limit to be placed on the bag constant, B. An estimate can be obtained from the inequality $E/A < 930$ MeV for massive nuclei with $A \approx 240$ (similar limits were presented in [5, 6]). Using Eq. (32) one gets a permitted range of $B^{1/4} > 142.5$ MeV. Taking the exact numerical results strengthens the limit slightly, to $B^{1/4} > 145$ MeV, due to the existence of a minimum in E/A for *ud* matter in the range of the heaviest nuclei (this minimum is an effect of the Coulomb energy). The limits are weakened by roughly 1 MeV when noticing that *ud* matter formed from decay of a nucleus will not have the energetically most favorable composition, since this would require a high-order weak interaction. At the time of formation flavor conservation should be invoked; only later weak interactions will take the system to the ground state of *ud*, or more likely *uds* quark matter.

III. CONCLUSIONS

The present investigation has focused on the mass formula for two- and three-flavor quark matter composed of massless quarks. As expected from previous investigations, three-flavor quark matter is energetically favored in bulk, and could be absolutely stable relative to ^{56}Fe for 144 MeV $< B^{1/4} <$ 163 MeV. The lower limit corresponds to experimentally excluded stability of *ud* quark matter, whereas the upper limit corresponds to a bulk energy per baryon of *uds* matter of 930 MeV.

Within the MIT bag model finite-size systems are strongly destabilized by the curvature energy, with a magnitude of almost 400 MeV$A^{-2/3}B_{160}^{1/4}$ for three quark flavors. This may pose problems for the experimental attempts of producing strange quark matter, since these experiments so far can only hope to produce quark lumps of baryon number $A <$ 10–20. Further destabilization occurs for finite-mass *s* quarks, where the surface tension (exactly zero for massless quarks) adds another 100 MeV$A^{-1/3}$ to the energy [8, 9]. Since the curvature correction for massive quarks is not known, the present investigation has concentrated on massless quarks, where the energy including curvature and Coulomb contributions can be calculated self-consistently. A similar treatment for massive quarks will be attempted in the near future. In this context, where exact mode-filling calculations are tedious, the nice agreement between mode-filling calculations and the asymptotic expansion demonstrated in Fig. 2 is very reassuring.

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