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Simply modeling meson heavy-quark effective theory

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A simple relativistic model of heavy-quark–light-quark mesons is proposed. In an expansion in inverse powers of the heavy-quark mass we find that all zeroth- and first-order heavy-quark symmetry relations are satisfied. The main results are the following. The difference between the meson mass and the heavy-quark mass plays a significant role even at zeroth order. The slope of the Isgur-Wise function at the zero recoil point is typically less than -1. The first-order correction to the pseudoscalar decay constant is large and negative. The four universal functions describing the first-order corrections to the semileptonic decay form factors are small. These latter corrections are quite insensitive to the choice of model parameters, and in particular to the effects of hyperfine mass splitting.

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I. INTRODUCTION

It has recently been realized that major simplifications occur in the theoretical treatment of weak decays of mesons containing a heavy quark [1]. New symmetries appear in an expansion in inverse powers of the heavy-quark mass. For example at lowest order the six form factors for the decays of pseudoscalar to pseudoscalar and vector mesons become related to a single universal form factor (the Isgur-Wise function). The latter depends only on the dynamics of the light degrees of freedom. In addition the masses of pseudoscalar and vector mesons with the same heavy-light flavor content are equal at this order, as are their decay constants.

However in the real world it is important to know the magnitude of the $1/m_Q$ corrections. It is expected that the b quark is probably heavy enough for the corrections to be small, but it is less clear for the charm quark. It is also becoming apparent that the size of the corrections depends strongly on what quantity is being calculated. We will attempt to shed more light on these questions.

We will present a model based on simple finite quark loop graphs. It is relativistic and incorporates the effects of hadronic recoil in a natural way. We demonstrate explicitly that all heavy-quark symmetry relations among the semileptonic decay form factors at zeroth and first order in $1/m_Q$ are satisfied by the model. The model also allows calculations to all orders in $1/m_Q$.

We consider our approach to be complementary to QCD sum rules [2]. The latter approach relies on a certain representation of the hadronic contribution to a three-point function. The basic input for our model is a representation of a Bethe-Salpeter amplitude for the heavy-quark–light-quark meson. As such, it may be improved by future QCD-based numerical computations of Bethe-Salpeter amplitudes. Our model is also complementary to nonrelativistic quark models [3,4] and it provides the first indication of how the $1/m_Q$ corrections are affected by hyperfine mass splitting. (Note that hyperfine mass splitting effects are not included in the Isgur-Scora-Grinstein-Wise (ISGW) model [3].)

Our model highlights the importance of the difference between a meson mass and the mass of the associated heavy quark [5]. This mass difference is needed to obtain consistent results at zeroth order in $1/m_Q$ (e.g., the Isgur-Wise function). We will show that ignoring this difference leads to very different results, and correspondingly the $1/m_Q$ corrections to this mass difference must be properly incorporated into the $1/m_Q$ corrections of other quantities.

Another result is a confirmation that the $1/m_Q$ correction to the B meson decay constant f_B is very large and negative. This was first suggested by lattice calculations [6] and it has been noted in two-dimensional calculations [7] and sum rule calculations [8,9]. This is to be contrasted with the corrections to the weak decay form factors, which we find to be relatively small. The latter observation also agrees qualitatively with sum rule results, although the actual numerical values of various $1/m_Q$ corrections differ.

We test the sensitivity of our results to the parameters of the model, and in particular we find that the form factor corrections are quite insensitive. We feel that the quantitative differences between our model and sum rules give an indication as to the true uncertainty in present theoretical determinations of these quantities.

II. DEFINITION OF THE MODEL

Before beginning we make the standard definitions of physical quantities. The meson decay constants are given by

$$\langle 0 | A_\mu | P(p) \rangle = i f_P p_\mu \text{ and } \langle 0 | V_\mu | V(p) \rangle = f_V M_V \varepsilon_\mu . \quad (1)$$

Our normalization is such that $A_\mu = \overline{q}\gamma_\mu\gamma_5 Q$. The various form factors for semileptonic decay are defined by

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$$\langle P_2(v_2)\,|V_\mu|\,P_1(v_1)\rangle = \sqrt{M_{P_1}M_{P_2}}\left[h_+(\omega)(v_1+v_2)_\mu + h_-(\omega)(v_1-v_2)_\mu\right], \tag{2}$$

$$\langle V_2(v_2)\,|V_\mu|\,P_1(v_1)\rangle = \sqrt{M_{P_1}M_{V_2}}h_V(\omega)\varepsilon_{\mu\upsilon\rho\sigma}\varepsilon_2^{*\upsilon}v_2^\rho v_1^\sigma, \tag{3}$$

$$\langle V_2(v_2)\,|A_\mu|\,P_1(v_1)\rangle = -i\sqrt{M_{P_1}M_{V_2}}\left[(\omega+1)h_{A_1}(\omega)\varepsilon_{2\mu}^* - \left(h_{A_2}(\omega)v_{1\mu} + h_{A_3}(\omega)v_{2\mu}\right)\varepsilon_2^*\cdot v_1\right], \tag{4}$$

where the v's are meson velocities, and $\omega = v_1\cdot v_2$.

We choose to represent transition amplitudes by diagrams with heavy mesons attached to a loop involving heavy and light quarks. The essential nonperturbative physics of QCD is that contributions with large momentum flowing through the light-quark line are suppressed. We will model this physics by including factors in the vertices which damp the loop integrals when the light-quark momentum k is larger than some scale Λ set by QCD. This has two desirable effects: the integrals are made finite and we may consider k small in comparison to a meson momentum. This latter fact is the crucial ingredient that gives rise to the correct heavy-quark symmetry relations.

The effective vertices between a light quark, a heavy quark, and a pseudoscalar or vector meson are taken to be

$$\left(\frac{Z_P^2}{-k^2+\Lambda_-^2}\right)^n\gamma_5 \text{ and } -i\left(\frac{Z_V^2}{-k^2+\Lambda_+^2}\right)^n\gamma_\mu, \tag{5}$$

where k is the momentum of the light quark. The light and heavy quarks are assigned standard propagators with masses m_q and m_Q, respectively. With these choices, and with n an integer or half-integer, standard methods involving Feynman parameters may be used in the one-loop computations.

We may now consider expanding various quantities to first order in Λ/m_Q:

$$\Lambda_\pm = \Lambda\left(1+(\pm h - g)\frac{\Lambda}{m_Q}\right), \tag{6}$$

$$Z_{P,V} = A\Lambda\left(\frac{m_Q}{\Lambda}\right)^{\frac{1}{4n}}\left(1+B_{P,V}\frac{\Lambda}{m_Q}\right), \tag{7}$$

$$M_{P,V}^2 - m_Q^2 = c\Lambda m_Q\left(1+d_{P,V}\frac{\Lambda}{m_Q}\right), \tag{8}$$

$$f_{P,V} = a\frac{\Lambda^{3/2}}{{m_Q}^{1/2}}\left(1+b_{P,V}\frac{\Lambda}{m_Q}\right). \tag{9}$$

The $P\leftrightarrow V$ symmetry in (7)–(9) at zeroth order is a direct consequence of the model, as shown in Appendix A. The same is true of the nontrivial zeroth-order m_Q dependence, and in particular the standard scaling $f_{P,V}\propto m_Q^{-1/2}$. The zeroth-order constants A, c, and a will be completely determined in terms of Λ/m_q and n without further input.

The first-order constants $B_{P,V}$, $d_{P,V}$, and $b_{P,V}$ depend in addition on the values of g and h. These latter constants model the effect of hyperfine mass splitting. Through their effect on $d_{P,V}$ they determine how the pseudoscalar and vector masses approach a common value in the heavy-quark limit. To first order in $1/m_Q$ we will find that g and h dependence cancels out of certain physical quantities. As with A, c, and a, we stress that the quantities $B_{P,V}$, $d_{P,V}$, and $b_{P,V}$ are not parameters of the model and will be determined by Λ/m_q, n, g, and h.

For certain physical quantities we will need to know m_q and Λ separately. It is reasonable that the appropriate effective light-quark mass should be of order its constituent mass, in common with most successful quark models. As for Λ we note the following relation which follows from (8):

$$M_{P,V}|_{\text{zeroth order}} - m_Q \equiv \overline{\Lambda} = \frac{c}{2}\Lambda. \tag{10}$$

The first equality is the standard definition of $\overline{\Lambda}$ appearing in the literature. When necessary to choose Λ, it will be chosen so as to give reasonable values of $\overline{\Lambda}$. The parameter n determines the form and extent of the damping due to the meson vertex factors, and one of our goals will be to study the sensitivity of $1/m_Q$ corrections to the choice of vertex factors.

The various weak decay form factors may also be expanded:

$$h_i(\omega) = \alpha_i\xi(\omega) + \delta_c h_i(\omega)\frac{\Lambda}{m_c} + \delta_b h_i(\omega)\frac{\Lambda}{m_b} \tag{11}$$

with $\alpha_+ = \alpha_V = \alpha_{A_1} = \alpha_{A_3} = 1$ and $\alpha_- = \alpha_{A_2} = 0$. $\xi(\omega)$ is the Isgur-Wise function and we will calculate it in terms of Λ/m_q and n. In QCD it has been shown that numerous relations exist between the first-order corrections [5,13]. We demonstrate in Appendix B that these relations are satisfied by our model for any values of the parameters. This consistency with heavy-quark symmetry at first order in Λ/m_Q is a nontrivial test for any quark model of heavy mesons.

We will present numerical results for the first-order corrections and their first derivatives, all at $\omega = 1$. We do not treat the perturbative QCD corrections; they have been calculated elsewhere [10] and should be added to our results.

III. ZEROTH-ORDER RESULTS

We first consider the results of our model at zeroth order in Λ/m_Q. Details of these calculations may be found in Appendix A. Computation of c involves finding the zero of the meson two-point function as given in (A4). Some examples of these "mass functions" are displayed in Fig. 1. When we set the location of the zeros equal to $M^2 = m_Q^2 + c\Lambda m_Q$ we find the following values of c:

Λ/m_q	$c_{n=1}$	$c_{n=3/2}$	$c_{n=2}$
2	1.5797	1.3949	1.3006
4	1.4635	1.1976	1.0545

(12)

We see that increasing n decreases M. Λ is constrained to be greater than m_q, since otherwise we find no sensible zero of the mass function. We will see later that the two choices shown in (12), $\Lambda/m_q = 2$ and 4, imply a reasonable range for $\overline{\Lambda}$.

We display in (13) the results for the zeroth-order parameters A and a appearing in $Z_{P,V}$ and $f_{P,V}$, respectively, and given by (A7) and (A9):

Λ/m_q	$A_{n=1}$	$A_{n=3/2}$	$A_{n=2}$	$a_{n=3/2}$	$a_{n=2}$
2	0.9181	0.9184	0.9059	0.25	0.17
4	1.1461	1.1358	1.1077	0.29	0.19

(13)

We do not include in (13) results for $a_{n=1}$ because in this case the dependence of f_P on Λ is not of the form of (9) (it is logarithmic), while f_V diverges. For $n = 3/2$, $m_b = 4.8$ GeV and $\Lambda = 500$ MeV and 1 GeV we find $f_B = f_{B^*} =$ 40 MeV and 132 MeV, respectively, at zeroth order. The strong Λ dependence reflects $f_{P,V} \propto \Lambda^{3/2}$ at zeroth order. $f_{P,V}$ become smaller for larger n.

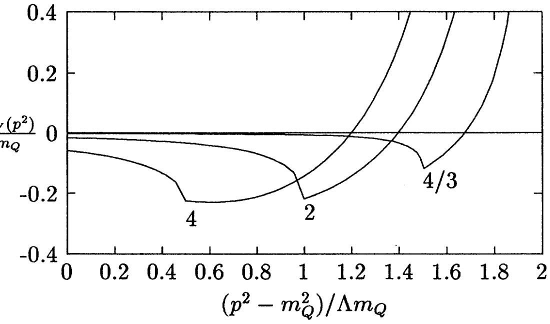

FIG. 1. Normalized lowest-order mass functions for $n = 3/2$ and $\Lambda/m_q = 4, 2, 4/3$.

The Isgur-Wise function $\xi(\omega)$ of our model is given by (A13). It depends only on the index n and the ratio $\alpha \equiv m_q/\Lambda$. Let us see the effect of ignoring the zeroth-order mass difference $\overline{\Lambda}$. Putting $c = 0$ in (A13) allows one to perform the relevant integrations explicitly, yielding a linear combination

$$\xi^{c=0}(\omega) = \frac{f_1(\alpha)\xi_1(\omega) + f_2(\alpha)\xi_2(\omega)}{f_1(\alpha)\xi_1(1) + f_2(\alpha)\xi_2(1)} \tag{14}$$

of two functions independent of n and α,

$$\xi_1(\omega) = \frac{2}{1+\omega} \text{ and } \xi_2(\omega) = \frac{\ln\left(\omega + \sqrt{\omega^2 - 1}\right)}{\sqrt{\omega^2 - 1}}. \tag{15}$$

The coefficients are given by

$$f_1(\alpha) = \left(\frac{\partial}{\partial t}\right)^{2n-1} \frac{\pi}{\sqrt{t} + \alpha}\Bigg|_{t=1} \text{ and } f_2(\alpha) = \left(\frac{\partial}{\partial t}\right)^{2n-1} \frac{\alpha \ln(t/\alpha^2)}{t - \alpha^2}\Bigg|_{t=1}. \tag{16}$$

Expression (A13) for $\xi(\omega)$ with nonzero c must be evaluated numerically. We plot the result in Fig. 2 together with the $c = 0$ form in (14). The difference is quite significant; this demonstrates the importance of retaining the zeroth-order mass difference $\overline{\Lambda}$. A simple quark-loop model was recently described by de Rafael and Taron [11] in which they obtained $\xi(\omega) = \xi_2(\omega)$. But that model, like our model with $c = 0$, does not correctly take into account a nonzero $\overline{\Lambda}$.

It is interesting to see how $\xi(\omega)$ with nonzero c depends on α. We plot in Fig. 3 the first and second derivatives of $\xi(\omega)$ at $\omega = 1$, for the case $n = 3/2$. We see that there is a range of α over which they are quite insensitive to the value of α . Our two choices $\alpha = 1/2$ and $\alpha = 1/4$ more or less fall within the region of insensitivity.

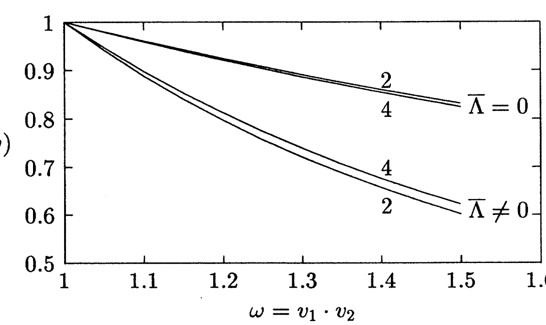

FIG. 2. Isgur-Wise functions for $n = 3/2$ and $\Lambda/m_q = 4, 2$. Also shown are results for the inconsistent case $\overline{\Lambda} = 0$.

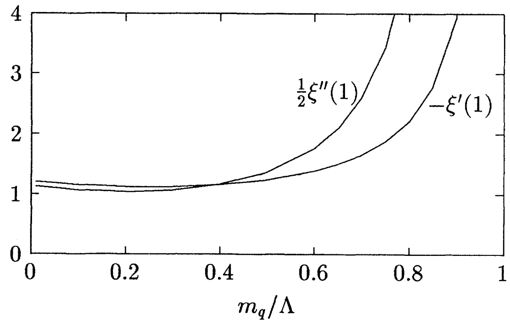

FIG. 3. First and second derivatives of the Isgur-Wise function at $\omega = 1$ for $n = 3/2$.

Although we cannot obtain $\xi(\omega)$ analytically we have checked numerically that it has the correct behavior at large ω, namely, it tends to zero. We have also computed the derivatives of $\xi(\omega)$ at $\omega = 1$:

$\frac{\Lambda}{m_q}, n$	$\xi(1)$	$\xi'(1)$	$\frac{\xi''(1)}{2}$	$\frac{\xi'''(1)}{3!}$	$\frac{\xi''''(1)}{4!}$
$2, \frac{3}{2}$	1	−1.241	1.370	−1.459	1.536
$4, \frac{3}{2}$	1	−1.118	1.047	−0.917	0.777
$2, 1$	1	−1.359	1.691	−2.094	2.631

(17)

Our values for $\xi'(1)$ are consistent with the present experimental determinations [12]. [Note that these determinations assume some functional form for $\xi(\omega)$ which is not identical to ours.] We see that our model, like most other models, does not satisfy the constraints on $\xi'(1)$ and $\xi''(1)$ recently argued for in [11]. It is amusing that the constraints *are* satisfied by the inconsistent $c = 0$ model.

IV. ORDER-Λ/m_Q CORRECTIONS

The purpose of this section is to find the $O(\Lambda/m_Q)$ corrections and to explore their dependence on the parameters n, Λ, g, and h. For $(\Lambda/m_q, n)$ we take the three sets of values $(2, \frac{3}{2})$, $(4, \frac{3}{2})$, and $(2, 1)$, and for each quantity we calculate we will indicate explicitly the g and h dependences.

We remark that just as the zeroth-order Λ dependence of the meson masses (i.e., c) plays an important role in the zeroth-order results, the next-to-leading-Λ dependence of the meson masses (i.e., $d_{P,V}$) plays an important role in the first-order corrections. The meson masses enter the loop calculations via the on-shell external momenta $p^2 = M_{P,V}^2$. Λ dependence from meson masses also originates in the factor M_V in the definition of the vector decay constant (1) and in the factors $\sqrt{M_{P_1} M_{P_2}}$ and $\sqrt{M_{P_1} M_{V_2}}$ in the definition of the form factors (2)–(4).

Our results for the corrected masses $M_{P,V}$ and the corrected vertex normalizations $Z_{P,V}$ (i.e., the parameters $d_{P,V}$ and $B_{P,V}$) are

$\Lambda/m_q, n$	d_P	d_V
$2, \frac{3}{2}$	$0.379 - (h+g)0.646$	$0.335 + (h-g)0.646$
$4, \frac{3}{2}$	$0.385 - (h+g)0.886$	$0.262 + (h-g)0.886$
$2, 1$	$0.461 - (h+g)0.801$	$0.365 + (h-g)0.801$

(18)

$\Lambda/m_q, n$	B_P	B_V
$2, \frac{3}{2}$	$0.266 - (h+g)1.387$	$0.301 + (h-g)1.387$
$4, \frac{3}{2}$	$0.193 - (h+g)0.989$	$0.274 + (h-g)0.989$
$2, 1$	$0.404 - (h+g)1.341$	$0.535 + (h-g)1.341$

(19)

The values of the correction coefficients for the decay constants as defined in (9) are displayed in (20). Note in particular the large negative values of b_P for the pseudoscalar. (From our parameter fit at the end we find that both g and h are positive.) The implication is that the first-order corrections to f_B are nearly of the same order (with opposite sign) as the zeroth-order values. We conclude that the Λ/m_Q expansion for f_B is breaking down:

$\Lambda/m_q, n$	b_P	b_V
$2, \frac{3}{2}$	$-2.92 - (h+g)1.82$	$-1.37 + (h-g)1.82$
$4, \frac{3}{2}$	$-3.82 - (h+g)1.52$	$-1.43 + (h-g)1.52$

(20)

We now turn to the first-order corrections to the weak decay form factors. It is standard [2] to write the first-order terms in (11) as

$$\gamma_i(\omega)\xi(\omega) \equiv \delta_c h_i(\omega)\frac{\Lambda}{m_c} + \delta_b h_i(\omega)\frac{\Lambda}{m_b}. \qquad (21)$$

In QCD the first-order terms may be expressed in terms of the universal functions $\chi_{1,2,3}(\omega)$ and $\xi_3(\omega)$ in addition to $\xi(\omega)$ [5,13]. These relations are reproduced in Appendix B. Also in Appendix B we demonstrate that these relations are satisfied by our model for any values of the parameters. We also verify in detail Luke's theorem [13] which reads $\chi_1(1) = \chi_3(1) = 0$.

At $\omega = 1$ we find the following results in which we temporarily set $g = h = 0$:

$\frac{\Lambda}{m_q}, n$	$\chi_1(1)$	$\chi_2(1)$	$\chi_3(1)$	$\xi_3(1)$
$2, \frac{3}{2}$	0	−0.153	0	−0.002
$4, \frac{3}{2}$	0	−0.132	0	−0.008
$2, 1$	0	−0.148	0	−0.005

(22)

$\frac{\Lambda}{m_q}, n$	$\chi_1'(1)$	$\chi_2'(1)$	$\chi_3'(1)$	$\xi_3'(1)$
$2, \frac{3}{2}$	−0.368	0.212	−0.058	−0.097
$4, \frac{3}{2}$	−0.307	0.161	−0.057	−0.116
$2, 1$	−0.419	0.218	−0.073	−0.101

(23)

When g and h are nonzero the results in (22) do not change. The g and h dependence of various preceding quantities has canceled out. Only two of the first derivatives are changed by the following amounts:

$$\delta\chi_1'(1) = x(h-2g), \tag{24}$$

$$\delta\chi_3'(1) = -\frac{1}{2}xh. \tag{25}$$

$x = 0.193$, 0.003, and 0.155 for $(\Lambda/m_q, n) = (2, \frac{3}{2})$, $(4, \frac{3}{2})$, and $(2,1)$, respectively. In fact $\chi_2(\omega)$ and $\xi_3(\omega)$ are independent of g and h for any ω, as demonstrated in Appendix B.

We may translate these results into values for the $\gamma_i(1)$'s. At this stage we are finally forced to make a choice for m_q. We choose $m_q = 250$ MeV as a representative value of an effective constituent quark mass in our loops. Using $m_b = 4.8$ GeV, $m_c = 1.44$ GeV, and $g = h = 0$ we obtain the following results for $\gamma_i(1)$ and $\gamma_i'(1)$ expressed as percentages of unity:

Λ, n	$\gamma_+(1)$	$\gamma_-(1)$	$\gamma_V(1)$	$\gamma_{A_1}(1)$	$\gamma_{A_2}(1)$	$\gamma_{A_3}(1)$
0.5 GeV, $\frac{3}{2}$	0	−8.5	15.8	0	−19.5	11.1
1.0 GeV, $\frac{3}{2}$	0	−14.8	27.1	0	−31.6	17.5
0.5 GeV, 1	0	−9.7	17.9	0	−21.8	12.3

(26)

Λ, n	$\gamma_+'(1)$	$\gamma_-'(1)$	$\gamma_V'(1)$	$\gamma_{A_1}'(1)$	$\gamma_{A_2}'(1)$	$\gamma_{A_3}'(1)$
0.5 GeV, $\frac{3}{2}$	−12.9	−1.7	−8.4	−1.2	8.4	−2.2
1.0 GeV, $\frac{3}{2}$	−20.8	−3.6	−11.3	0.7	14.0	0.5
0.5 GeV, 1	−20.1	−2.1	−11.2	−3.2	9.3	−3.9

(27)

Again the only effect of nonzero g and h are in the following first derivatives:

$$\delta\gamma_V'(1) = \delta\gamma_{A_1}'(1) = \delta\gamma_{A_3}'(1) = -2\overline{\Lambda}x\left(\frac{g-h}{m_c} + \frac{g+h}{m_b}\right), \tag{28}$$

$$\delta\gamma_+'(1) = -2\overline{\Lambda}x(h+g)\left(\frac{1}{m_c} + \frac{1}{m_b}\right). \tag{29}$$

x takes the same values as above.

An important point is that the zero recoil values of the four universal functions in (22) and the corresponding first-order corrections to the form factors in (26) are independent of hyperfine mass splitting, as modeled by g and h. Another is the weak dependence of the results in (22) and (23) on the parameters n and Λ. The results in (26) and (27) show more variation and reflect the fact that they are proportional to $\overline{\Lambda}$. Corresponding to the three rows of these tables we have $\overline{\Lambda} \approx 350$, 600, and 400 MeV, respectively.

In our discussion thus far we have been more concerned with the sensitivity of the results to the parameters rather than trying to find an optimal set of parameters. The latter could be accomplished in the following way. We may calculate a Λ_+ for each of B^* and D^* and a Λ_- for each of B and D by fitting the zeros of the full mass functions to the physical meson masses. By fitting these four $\Lambda_\pm$'s to the first-order form in (6) we find, for $n = 1$, $\Lambda = 667$ MeV, $g = 0.057$, $h = 0.37$, and for $n = 3/2$, $\Lambda = 818$ MeV, $g = 0.047$, $h = 0.32$. In both cases $\overline{\Lambda} = 504$ MeV which coincides with a sum rule estimate [2,8].

Our results for the corrections are somewhat different from a sum rule calculation [2] which gives $\xi_3(1) = 0.33$ and $\chi_2(1) = 0$ and from an improved sum rule calculation [14] which gives $\chi_2(1) = -0.038$. Note that in QCD $\chi_2(1)$ measures the effect of a chromomagnetic moment operator insertion [13]. Our values for $\gamma_i(1)$ and $\gamma_i'(1)$ may be compared directly with the corresponding results in [2].

In this paper we have presented the zeroth- and first-order model results for various quantities of interest in the meson heavy-quark effective theory. The same model allows the calculation of physical quantities to all orders in the $1/m_Q$ expansion. This comparison, to be pursued elsewhere [15], should shed further light on the usefulness of the heavy-quark quark expansion in B and D meson physics.

ACKNOWLEDGMENT

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APPENDIX A

We treat in this appendix the zeroth-order pieces of (7)–(9).

1. Masses and normalizations

We begin with the pseudoscalar and vector mass functions $\Gamma_{P,V}(p^2)$ defined by setting the self-energy graphs equal to $i\Gamma_P$ and $-ig_{\mu\nu}\Gamma_V + \cdots$, respectively, where the ellipsis denotes $p_\mu p_\nu$ terms. The light-quark momentum k may be chosen to be the same as the loop momentum, and the heavy-quark momentum is then $vM_{P,V} + k$ where $v^2 = 1$. In the heavy-quark limit we may ignore k compared with $M_{P,V}$ and use (8) to show that the heavy-quark propagator becomes (after scaling $k \to \Lambda k$)

$$\frac{-i}{\Lambda}\frac{\not{v}+1}{-2k\cdot v - c_{P,V}}. \tag{A1}$$

We have temporarily allowed the constants $c_{P,V}$ describing the lowest-order difference between quark and meson mass to be different for the pseudoscalar and vector. Our first task is to show that they are in fact equal to a common constant c.

The relevant traces are

$$\mathrm{Tr}\left\{\begin{array}{c}\gamma_5\\-i\gamma_\mu\end{array}\right\}(\not v+1)\left\{\begin{array}{c}\gamma_5\\-i\gamma_\nu\end{array}\right\}(\not k+\alpha), \tag{A2}$$

where $\alpha = m_q/\Lambda$. The translation is $k \to k - xv$ where x is a Feynman parameter; the integral linear in k then vanishes. We may anticommute the resulting v leftwards past the gamma matrices since terms proportional to v_ν do not contribute in the vector case. The results are

$$(x+\alpha)\mathrm{Tr}\left\{\begin{array}{c}\gamma_5\\-i\gamma_\mu\end{array}\right\}(\not v+1)\left\{\begin{array}{c}\gamma_5\\-i\gamma_\nu\end{array}\right\}=\bar\xi\left\{\begin{array}{c}4\\-4g_{\mu\nu}\end{array}\right\}, \tag{A3}$$

where $\bar\xi \equiv -k\cdot v + \alpha$ is equivalent with $x+\alpha$. We thus find

$$\Gamma_{P,V}(M^2_{P,V}) = \int \frac{d^4k}{(2\pi)^4}\frac{4in_cZ^{4n}_{P,V}\Lambda^{2-4n}}{[-k^2+1]^{2n}}\frac{\bar\xi}{[-k^2+\alpha^2][-2k\cdot v - c_{P,V}]}. \tag{A4}$$

We are regarding these mass functions as the full two-point functions of the mesons; i.e., there are no tree level contributions. Thus the meson masses are defined by the locations of the zeros of the real parts of these mass functions:

$$\mathrm{Re}\,\Gamma_{P,V}(M^2_{P,V}) = 0. \tag{A5}$$

Notice that imaginary parts are present in any quantity which is evaluated on the meson mass shell. This is because the difference $\overline{\Lambda}$ between the meson mass and the heavy-quark mass is greater than the light-quark mass m_q, and thus the meson is above the threshold for two free quarks. But note that our $\overline{\Lambda}$ is consistent with that of other approaches [2,8]. We will not consider these imaginary parts further. We also find and then drop an overall, physically irrelevant minus sign [15].

We regard c_P and c_V as variables whose value is fixed by (A4) and (A5). It is then obvious that they are equal to a common value c because the functions $\Gamma_{P,V}(M^2_{P,V})$ differ at most by multiplicative factors. This demonstrates that $M_P = M_V$ at zeroth order [see (8)].

The mass functions are also required to satisfy the normalization condition

$$\Gamma'_{P,V}(M^2_{P,V}) = 1. \tag{A6}$$

This immediately implies that the normalization factors $Z_{P,V}$ are equal to one another in the heavy-quark limit. The value of the zeroth-order constant A defined in (7) is thus fixed by

$$1 = \frac{\bar\xi}{D_k^{2n}D_\alpha D^2}, \tag{A7}$$

where $D_k = -k^2+1$, $D_\alpha = -k^2+\alpha^2$, $D = -2k\cdot v - c$. Here and in the following we will adopt the convention that an overall factor $4in_cA^{4n}$ and $\int d^4k/(2\pi)^4$ are understood whenever the denominator is written explicitly as a product of D's.

By the Ward identity the above determination of A is equivalent to the normalization of the Isgur-Wise function. This will be verified explicitly below.

2. Decay constants

We again choose the light-quark momentum to coincide with the loop momentum. The traces relevant to the decay constants as defined by (1) at lowest order are

$$\mathrm{Tr}\left\{\begin{array}{c}\gamma_\mu\gamma_5\\\gamma_\mu\end{array}\right\}(\not v+1)\left\{\begin{array}{c}\gamma_5\\-i\gamma_\nu\end{array}\right\}(\not k+\alpha)$$
$$=\bar\xi\,\mathrm{Tr}\left\{\begin{array}{c}\gamma_\mu\gamma_5\\\gamma_\mu\end{array}\right\}(\not v+1)\left\{\begin{array}{c}\gamma_5\\-i\gamma_\nu\end{array}\right\}$$
$$=\bar\xi\left\{\begin{array}{c}-4v_\mu\\-4ig_{\mu\nu}\end{array}\right\}. \tag{A8}$$

Use of the lowest-order pieces of the expansions (6)–(8) immediately shows that the decay constants are equal at lowest order and are given by

$$f_{P,V} = \frac{\Lambda^{3/2}}{m_Q^{1/2}}\times\frac{-A^{-2n}\bar\xi}{D_k^nD_\alpha D}. \tag{A9}$$

3. Form factors

We turn to the zeroth-order results for the meson form factors defined by (2)–(4). The relevant traces are

$$\mathrm{Tr}\left\{\begin{array}{c}\gamma_5\\-i\gamma_\nu\end{array}\right\}(\not v_2+1)\Gamma_\mu(\not v_1+1)\gamma_5(\not k+\alpha). \tag{A10}$$

The translation is $k \to k - xv_1 - yv_2$ with $v_i^2 = 1$, where x and y are Feynman parameters. The term linear in k vanishes, and v_1 and v_2 may be anticommuted to the left and right, respectively, to yield the standard traces

$$(x+y+\alpha)\mathrm{Tr}\left\{\begin{array}{c}\gamma_5\\-i\gamma_\nu\end{array}\right\}(\not v_2+1)\Gamma_\mu(\not v_1+1)\gamma_5. \tag{A11}$$

Use of the lowest-order pieces of expansions (6)–(8) and evaluation of the standard traces immediately yield the relations

$$h_+(\omega) = h_V(\omega) = h_{A_1}(\omega) = h_{A_3}(\omega) \equiv \xi(\omega)$$
$$\text{and}\ \ h_-(\omega) = h_{A_2}(\omega) = 0. \tag{A12}$$

The quantity $x+y+\alpha$ is equivalent to $\tilde\xi \equiv -(1+\omega)^{-1}k\cdot(v_1+v_2)+\alpha$, and we find for the Isgur-Wise function

$$\xi = \frac{\tilde{\xi}}{D_k^{2n} D_\alpha D_1 D_2}, \tag{A13}$$

where $D_i = -2k \cdot v_i - c$. The normalization $\xi(1) = 1$ is verified by setting $v_1 = v_2 = v$ and comparing with (A7).

APPENDIX B

The first-order correction terms to the weak decay form factors [defined in (11) and (21)] may be expressed in terms of the universal functions as follows [5,13]:

$$\gamma_+ \xi = \left(\frac{\overline{\Lambda}}{m_b} + \frac{\overline{\Lambda}}{m_c}\right)(\chi_1 + 2[1-\omega]\chi_2 + 6\chi_3), \tag{B1}$$

$$\gamma_- \xi = \left(\frac{\overline{\Lambda}}{m_c} - \frac{\overline{\Lambda}}{m_b}\right)\left(\xi_3 - \frac{1}{2}\xi\right), \tag{B2}$$

$$\gamma_V \xi = \frac{\overline{\Lambda}}{2m_c}(\xi + 2\chi_1 - 4\chi_3) + \frac{\overline{\Lambda}}{2m_b}(\xi - 2\xi_3 + 2\chi_1 + 4[1-\omega]\chi_2 + 12\chi_3), \tag{B3}$$

$$\gamma_{A_1}\xi = \frac{\overline{\Lambda}}{2m_c}\left(\frac{\omega-1}{\omega+1}\xi + 2\chi_1 - 4\chi_3\right) + \frac{\overline{\Lambda}}{2m_b}\left(\frac{\omega-1}{\omega+1}[\xi - 2\xi_3] + 2\chi_1 + 4[1-\omega]\chi_2 + 12\chi_3\right), \tag{B4}$$

$$\gamma_{A_2}\xi = \frac{\overline{\Lambda}}{m_c}\left(-\frac{1}{\omega+1}[\xi + \xi_3] + 2\chi_2\right), \tag{B5}$$

$$\gamma_{A_3}\xi = \frac{\overline{\Lambda}}{2m_c}\left(\frac{1}{\omega+1}[\{\omega-1\}\xi - 2\xi_3] + 2\chi_1 - 4\chi_2 - 4\chi_3\right) + \frac{\overline{\Lambda}}{2m_b}(\xi - 2\xi_3 + 2\chi_1 + 4[1-\omega]\chi_2 + 12\chi_3). \tag{B6}$$

In the notation of (11) and Appendix A we obtain

$$\delta_b h_i = \alpha_i\left\{\left(2B_P - \frac{c}{4}\right)\xi + \frac{2(g+h)\tilde{\xi}}{D_k^3 D_\alpha D_1 D_2} + \frac{(k^2 - \frac{c^2}{2} + c d_P)\tilde{\xi}}{D_k^2 D_\alpha D_1^2 D_2} + \left(\frac{\alpha}{2} + \frac{c}{4}\right)\frac{(1+\omega)^{-1} k\cdot(v_1+v_2)}{D_k^2 D_\alpha D_1 D_2}\right\} + \frac{X_i}{D_k^2 D_\alpha D_1 D_2}, \tag{B7}$$

$$\delta_c h_i = \alpha_i\left\{\left(2B_{P,V} - \frac{c}{4}\right)\xi + \frac{2(g\pm h)\tilde{\xi}}{D_k^3 D_\alpha D_1 D_2} + \frac{(k^2 - \frac{c^2}{2} + c d_{P,V})\tilde{\xi}}{D_k^2 D_\alpha D_1 D_2^2} + \left(\frac{\alpha}{2} + \frac{c}{4}\right)\frac{(1+\omega)^{-1} k\cdot(v_1+v_2)}{D_k^2 D_\alpha D_1 D_2}\right\} + \frac{Y_i}{D_k^2 D_\alpha D_1 D_2}, \tag{B8}$$

where P or V quantities are used in the expression for $\delta_c h_i$ according to whether the final state is pseudoscalar or vector, and X_i and Y_i are given by

(B9)

i	α_i	X_i	Y_i
$+$	1	$-\frac{1}{2}k^2 - \frac{\alpha c}{4}$	X_+
$-$	0	$\frac{1}{2}k^2 + \frac{\alpha c}{4}$	$-X_-$
V	1	0	0
A_1	1	$-\frac{1}{1+\omega}k^2 - \frac{c\alpha}{2(1+\omega)}$	$\frac{2}{1+\omega}S + X_{A_1}$
A_2	0	0	$-2T + \alpha(1+\omega)^{-1}k\cdot(v_1+v_2)$
A_3	1	0	$-2U$

The quantities S, T, and U are defined by

$$\frac{k_\mu k_\nu}{D_k^2 D_\alpha D_1 D_2} = \frac{S g_{\mu\nu} + T(v_{1\mu}v_{1\nu} + v_{2\mu}v_{2\nu}) + U(v_{1\mu}v_{2\nu} + v_{2\mu}v_{1\nu})}{D_k^2 D_\alpha D_1 D_2}. \tag{B10}$$

On the other hand, proper normalization of the vector meson "charge" graph at $q^2 = 0$ gives in the notation of Appendix A

$$0 = 2B_V - \frac{c}{4} + \frac{2(g-h)\overline{\xi}}{D_k^3 D_\alpha D^2} + \frac{(k^2 - \frac{c^2}{2} + c d_V)\overline{\xi}}{D_k^2 D_\alpha D^3} + \left(\frac{\alpha}{2} + \frac{c}{4}\right)\frac{k\cdot v}{D_k^2 D_\alpha D^2} + \frac{\overline{Y}_V}{D_k^2 D_\alpha D^2}, \tag{B11}$$

where

$$\overline{Y}_V = S - \frac{1}{2}k^2 - \frac{c\alpha}{4}. \quad \text{(B12)}$$

Setting $v_1 = v = v_2$ in (B8) and (B9) and comparing with (B11) and (B12) shows that $\delta_c h_{A_1} = 0$ at $\omega = 1$.

An analogous argument using the normalization of the pseudoscalar meson charge graph shows that $\delta_b h_{A_1} = 0$. Luke's theorem, $\chi_1(1) = \chi_3(1) = 0$, is therefore satisfied for arbitrary m_c and m_b with no extra constraints between model parameters.

Returning to arbitrary ω we find that all the required relations which are supposed to hold between the $\delta_b h_i$ follow straightforwardly from the model. The following two relations which need to be checked for $\delta_c h_i$ are less trivial:

$$\delta_c h_V - \delta_c h_{A_1} = \frac{c\xi}{2(1+\omega)}, \quad \text{(B13)}$$

$$\delta_c[h_{A_1} - h_{A_2} - h_{A_3} - 2(1+\omega)^{-1}h_-] = \frac{c\xi}{(1+\omega)}. \quad \text{(B14)}$$

These are satisfied only if the following nontrivial identities are satisfied:

$$\frac{\frac{2}{c}S - \frac{1}{c}k^2 - \frac{k\cdot(v_1+v_2)}{2(1+\omega)}}{D_k^2 D_\alpha D_1 D_2} = 0, \quad \text{(B15)}$$

$$\frac{S + (1+\omega)(T+U) - \frac{\alpha}{2}k\cdot(v_1+v_2)}{D_k^2 D_\alpha D_1 D_2} = \frac{c}{2}\xi. \quad \text{(B16)}$$

We have confirmed these relations numerically and via a Taylor series expansion around $\omega = 1$.

The functions χ_2 and ξ_3 are independent of g and h. We find

$$\xi_3 = \frac{-\frac{1}{c}k^2 - \frac{k\cdot(v_1+v_2)}{2(1+\omega)}}{D_k^2 D_\alpha D_1 D_2}, \quad \text{(B17)}$$

$$\chi_2 = \frac{\xi + \xi_3}{2(1+\omega)} + \frac{-\frac{2T}{c} + \frac{\alpha}{c}\frac{k\cdot(v_1+v_2)}{1+\omega}}{D_k^2 D_\alpha D_1 D_2}. \quad \text{(B18)}$$

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