PHYSICAL REVIEW D VOLUME 47, NUMBER 11 1 JUNE 1993

Four-fermion interactions from n generations and minimal dynamical breaking of electroweak gauge group

Bang-Rong Zhou*
International Centre for Theoretical Physics, Trieste, Italy
(Received 21 December 1992)

The n fermion generation extension of the Nambu–Jona-Lasinio mechanism of spontaneous breaking of the electroweak gauge group $SU_L(2)\times U_Y(1)$ is proven in the bubble approximation. When both the minimal Higgs condition and the gap equation are satisfied, the explicit calculations of the propagators for bound states indicate that only a neutral massive Higgs boson and a neutral and two charged massless Goldstone bosons emerge from the theory and they are now some definte combinations of the spin-zero bound state modes consisting of the n generations of fermions. The mass of the Higgs boson is restricted between the double mass of the lightest and the heaviest fermions but more approaches the latter. We also give the inverse propagators for the charged and the neutral electroweak gauge bosons including the insertion of the three Goldstone bosons in the vacuum polarizations and display the composite Higgs mechanism generating the masses of the $W^{\mp}$ and Z^0 bosons.

PACS number(s): 12.50.Lr, 11.15.Ex, 12.15.Cc, 14.80.Gt

I. INTRODUCTION

It has been proven in a recent paper [1] that the four-fermion interactions from one-quark-like generation, based on the Nambu–Jona-Lasinio (NJL) mechanism [2], could induce the minimal dynamical breaking of the electroweak group $SU_L(2)\times U_Y(1)$. The result is the one quark-like generation extension of the current top-quark condensate scheme [3,4] when the t and b quarks are treated on an equal footing. However, such an extension includes only the very light b quarks, so it is useless for solving the unsatisfactory fine-tuning problem [1]. In order to be able to deal with this problem we must include some heavier fermions than the top quarks, together with the latter incorporated into a NJL-mechanism-based scheme to realize the minimal dynamical breaking, assuming that such fermions exist. These assumed heavy fermions could be the fourth generation of quark leptons, some exotic quarklike fermions, the fermions in the $SU_c(3)$ high-dimension representations [5], or the technifermions [6,7]. In a word, they may be in some definite representations of a colorlike group G_c, other than in a standard $SU_L(2)\times U_Y(1)$ flavor doublet [i.e., a left-handed $SU_L(2)$ doublet and two right-handed $SU_L(2)$ singlets]. Such fermions will be generally called forming a fermion generation. For exploring the possibility stated above we must further extend the NJL mechanism from one generation to $n>1$ generations. In this paper we will generally prove that such an extension is possible; i.e., the four-fermion interactions from n generations could indeed induce the minimal dynamical breaking of the electroweak gauge group.

The discussions will follow the approach used in the one-generation case [1] but with greater complexity. In Sec. II we will give the effective n-generation four-fermion Lagrangian corresponding to the minimal dynamical breaking of the electroweak group $SU_L(2)\times U_Y(1)$ and emphasize the following: how to rotate off the possible complex phase angle mixture among the generations and how to describe the minimal Higgs condition in this case. In Sec. III a general gap equation and $2n-1$ relations among the dynamical fermion masses and the four-fermion coupling constants are derived when the n generations of fermions exist. In Sec. IV it is proven that if the minimal Higgs condition and the gap equation are both satisfied, then it is still the case that only a single neutral massive composite Higgs boson and one neutral and two charged massless composite Goldstone bosons as physical modes emerge from the theory. The mass constraints on the Higgs boson are definitely given. The proof is based on the calculations of the four-point Green functions and the propagators for all spin-zero composite particles in the bubble approximation. In Sec. V, considering the vacuum polarization effects of the three massless Goldstone bosons we calculate the inverse propagators for the electroweak gauge bosons from which the composite Higgs mechanism generating the masses of $W^{\mp}$ and Z^0 gauge bosons is displayed. Finally, in Sec. VI we reach our conclusions.

II. FOUR-FERMION LAGRANGIAN FROM n GENERATIONS

Let us consider n generations of Q fermions without bare masses. They will be in n left-handed $SU_L(2)$ doublets and $2n$ right-handed $SU_L(2)$ singlets:

$$Q_{\alpha L}=\begin{bmatrix}U_\alpha\\ D_\alpha\end{bmatrix}_L,\quad Q_{\alpha R}=U_{\alpha R},D_{\alpha R},$$

$$\alpha=1,\ldots,n\ \text{(generation number)}\,, \qquad (1)$$

*Permanent address: Department of Physics, Graduate School, Academia Sinica, Beijing 100039, People's Repbulic of China.

with the general hypercharge assignments

$$Y(\bar{Q}_{\alpha L}U_{\alpha R})=+1 \quad \text{and} \quad Y(\bar{Q}_{\alpha L}D_{\alpha R})=-1 \; . \tag{2}$$

Hence once the hypercharge $Y_{Q_{\alpha L}}$ of the left-handed doublet $Q_{\alpha L}$ is specified, the ones of the corresponding right-handed fermions $U_{\alpha R}$ and $D_{\alpha R}$ are also completely fixed. In addition to the conditions (2), the Y-charge assignments of the Q fermions must make the theory $\mathrm{SU}_L(2)\times \mathrm{U}_Y(1)$ anomaly-free [8]. This can be done either by including the corresponding lepton generation if Q_α are ordinary quarks or by taking $Y_{Q_{\alpha L}}=0$ if Q_α are some exotics [7], and these two ways will lead to the anomaly cancellation within one generation of the Q fermions. One can also assign the Y charges of the Q fermions in such a way that the anomalies from different generations cancel each other. The colorlike quantum number of the Q fermions is not explicitly denoted and all the colorlike interactions will be omitted in the discussions throughout this paper.

As in Ref. [1], at low energies the effective $G_c\times \mathrm{SU}_L(2)\times \mathrm{U}_Y(1)$-invariant four-fermion Lagrangian $\mathcal{L}_{4F}$ coming from the Q fermions could be obtained through the equivalent Yukawa-form Lagrangian $\mathcal{L}_Y$ appearing in the standard electroweak theory. In order to keep the minimal dynamical breaking we will introduce only a single auxiliary static scalar field $\mathrm{SU}_L(2)$ doublet H and its conjugate $\tilde{H}$ [4], where

$$H=\begin{bmatrix} h_+ \\ h_0 \end{bmatrix} \quad \text{and} \quad \tilde{H}=i\sigma_2(H^\dagger)^T \tag{3}$$

are both in the G_c singlets, σ_2 is the Pauli matrix, and T means the transposition of an $\mathrm{SU}_L(2)$ spinor. The hypercharges of H and $\tilde{H}$ are assigned to

$$Y(H)=1 \quad \text{and} \quad Y(\tilde{H})=-1 \; . \tag{4}$$

The Lagrangian $\mathcal{L}_Y$ will consist of the $G_c\times \mathrm{SU}_L(2)\times \mathrm{U}_Y(1)$-invariant terms coming from Q and H fields. It is clear that the Y-charge assignments (2) of the Q-field products will be able to match the ones (4) of the H scalar fields. However, the assignments (2) do not specify the Y charges of the left-handed Q fermions, thus the ones of the Q fermions themselves. There are three possible cases.

(a) The n generations of the Q fermions have different Y charges, or, although part or all of them have the same Y charges, some new symmetry, e.g., a colorlike or horizontal one, would forbidden the emergence of the couplings between different generations with identical Y charges. In this case there will be no coupling terms such as $(\bar{Q}_{\alpha L}U_{\beta R})\tilde{H}$ $(\alpha\neq\beta)$ and $(\bar{Q}_{\alpha L}D_{\beta R})H$ $(\alpha\neq\beta)$, and the Yukawa-form Lagrangian can be written as

$$\mathcal{L}_Y=\sum_{\alpha=1}^{n}[\tilde{g}^0_\alpha(\bar{Q}_{\alpha L}U_{\alpha R})\tilde{H}+g^0_\alpha(\bar{Q}_{\alpha L}D_{\alpha R})H+\text{H.c.}] -m_0^2H^\dagger H \; , \tag{5}$$

where m_0 is the bare mass of the H field. The coupling constants $\tilde{g}^0_\alpha$ (g^0_α) could always be taken to be real and positive by appropriate selections of the relative phases between $\bar{Q}_{\alpha L}$ and $U_{\alpha L}$ $(D_{\alpha R})$.

(b) The n generations of the Q fermions have identical Y charges but no different new quantum number to distinguish them. In this case we will have the coupling terms among the fermions from different generations such as $(\bar{Q}_{\alpha L}U_{\beta R})\tilde{H}$, $(\bar{Q}_{\alpha L}D_{\beta R})H$ $(\alpha\neq\beta)$, etc. The $G_c\times \mathrm{SU}_L(2)\times U_Y(1)$-invariant $\mathcal{L}_Y$ can be written as

$$\mathcal{L}_Y=\sum_{\alpha,\beta=1}^{n}[\tilde{g}^0_{\alpha\beta}(\bar{Q}'_{\alpha L}U'_{\beta R})\tilde{H}+g^0_{\alpha\beta}(\bar{Q}'_{\alpha L}D'_{\beta R})H +\text{H.c.}]-m_0^2H^\dagger H \; , \tag{6}$$

where the Q' fields with the primes represent their weak eigenstates. By the standard procedure [9] we can take the unitary gauge so that

$$H=\begin{bmatrix} 0 \\ h_0 \end{bmatrix} \quad \text{and} \quad \tilde{H}=\begin{bmatrix} h_0 \\ 0 \end{bmatrix} \tag{7}$$

and make the bilinear transformations of the Q fields:

$$\begin{aligned} &\bar{U}'_L B_L^{-1}=\bar{U}_L, \quad B_R U'_R=U_R \; , \\ &\bar{D}'_L A_L^{-1}=\bar{D}_L, \quad A_R D'_R=D_R \; , \end{aligned} \tag{8}$$

where $\bar{U}_L\equiv(\bar{U}_{1L},\ldots,\bar{U}_{nL})$, etc., and the Q fields without the primes represent their mass eigenstates. The transformations (8) must diagonalize the matrices

$$\tilde{g}^0\equiv(\tilde{g}^0_{\alpha\beta}) \quad \text{and} \quad g^0\equiv(g^0_{\alpha\beta}) \tag{9}$$

so that

$$\begin{aligned} &B_L\tilde{g}^0B_R^{-1}=D_U\equiv(\tilde{g}^0_\alpha\delta_{\alpha\beta}) \; , \\ &A_Lg^0A_R^{-1}=D_D\equiv(g^0_\alpha\delta_{\alpha\beta}) \; , \end{aligned} \tag{10}$$

where $\tilde{g}^0_\alpha$ and g^0_α all can be taken to be non-negative real numbers. Then by the inverse unitary gauge transformation which changes $\bar{U}_{\alpha L}h_0$ into $\bar{Q}_{\alpha L}\tilde{H}$ and $\bar{D}_{\alpha L}h_0$ into $\bar{Q}_{\alpha L}H$, etc., we will obtain a $\mathcal{L}_Y$ identical to the one given by Eq. (5), but the Q fields in it should be understood to be in their mass eigenstates. Obviously, under the present circumstances the Cabibbo-like mixture will appear in the sector of the weak charged current of the Q fermions when the Q fields' weak eigenstates are replaced by their mass eigenstates.

(c) Some of the n generations have different Y charges, and the others have identical ones but there are no different new quantum numbers to distinguish among the generations with the same Y charges. In this case, based on the above discussions of (a) and (b), the Cabibbo-like mixture will appear only among these generations with identical Y charges. However, by the same procedure as the one taken in (b) we may finally represent these Q fields by means of their mass eigenstates; hence, the resulting $\mathcal{L}_Y$ will still have the same form as the one in Eq. (5).

In brief, no matter which one of the above three cases appears, as long as we take the Q-fermion fields to be in their mass eigenstates then $\mathcal{L}_Y$ always have the form of Eq. (5). We note that $\mathcal{L}_Y$ contains $2n$ real and non-negative coupling constants $\tilde{g}^0_\alpha$ and g^0_α altogether.

Now integrating out the auxiliary scalar fields $H^\dagger$ and H from the path integral $\int \mathcal{D}H^\dagger\mathcal{D}H\exp(i\int d^4x\,\mathcal{L}_Y)$ we will obtain the desired effective four-fermion Lagrangian

$$\mathcal{L}_{4F}=\sum_{\alpha,\beta=1}^{n}\{[g_{U_\beta U_\alpha}(\bar{Q}_{\beta L}U_{\beta R})(\bar{U}_{\alpha R}Q_{\alpha L})+g_{D_\beta D_\alpha}(\bar{Q}_{\beta L}D_{\beta R})(\bar{D}_{\alpha R}Q_{\alpha L})]+[g_{U_\beta D_\alpha}(\bar{U}_{\beta R}Q^T_{\beta L}i\sigma_2)(\bar{D}_{\alpha R}Q_{\alpha L})+\text{H.c.}]\}\ , \tag{11}$$

where the four-fermion coupling constants are defined by

$$g_{U_\beta U_\alpha}=g^{1/2}_{U_\beta U_\beta}g^{1/2}_{U_\alpha U_\alpha},\quad g_{D_\beta D_\alpha}=g^{1/2}_{D_\beta D_\beta}g^{1/2}_{D_\alpha D_\alpha},$$
$$g_{U_\beta D_\alpha}=g^{1/2}_{U_\beta U_\beta}g^{1/2}_{D_\alpha D_\alpha}\quad(\alpha=1,\ldots,n)\ . \tag{12}$$

Among these coupling constants, only $2n$ "diagonal" ones with the definitions

$$g_{U_\alpha U_\alpha}\equiv(\tilde{g}^0_\alpha/m_0)^2\quad\text{and}\quad g_{D_\alpha D_\alpha}\equiv(g^0_\alpha/m_0)^2 \tag{13}$$

are in fact independent ones. They are real and nonnegative and respectively correspond to the $2n$ coupling constants $\tilde{g}^0_\alpha$ and g^0_α $(\alpha=1,\ldots,n)$ in $\mathcal{L}_Y$. Equation (12) may be compactly written as

$$g_{Q'Q}=g^{1/2}_{Q'Q'}g^{1/2}_{QQ},\quad Q',Q=U_\alpha,D_\alpha\quad(\alpha=1,\ldots,n)\ . \tag{14}$$

Equation (14) is exactly the minimal Higgs condition in the case with the n generations of the Q fermions because it results from the fact that $\mathcal{L}_Y$ in Eq. (5) contains only a single static scalar field $SU_L(2)$ doublet H. After using Eq. (1) and the definitions

$$Q_{\substack{L\\R}}=\tfrac{1}{2}(1\mp\gamma_5)Q\ , \tag{15}$$

we may rewrite the $\mathcal{L}_{4F}$ in Eq. (11) as

$$\mathcal{L}_{4F}=\mathcal{L}^{N_S}_{4F}+\mathcal{L}^{N_P}_{4F}+\mathcal{L}^{C}_{4F}\ . \tag{16}$$

It is now divided into three independent sectors: the neutral scalar sector

$$\mathcal{L}^{N_S}_{4F}=\frac{1}{4}\sum_{Q,Q'}g_{Q'Q}(\bar{Q}'Q')(\bar{Q}Q)\ , \tag{16a}$$

where henceforth it is always understood that the sum of Q runs over the full n generation, i.e., $Q=U_\alpha,D_\alpha$ $(\alpha=1,\ldots,n)$; the neutral pseudoscalar sector

$$\mathcal{L}^{N_P}_{4F}=-\frac{1}{4}\sum_{Q,Q'}g'_{Q'Q}(\bar{Q}'\gamma_5Q')(\bar{Q}\gamma_5Q) \tag{16b}$$

with the definition

$$g'_{Q'Q}\equiv(-1)^{I^3_{Q'}-I^3_Q}g_{Q'Q} \tag{16b'}$$

where I^3_Q is the third component of the left-handed weak isospin of the Q fermions, and the charged sector

$$\mathcal{L}^{C}_{4F}=\sum_{i,j}\sum_{\alpha,\beta}g^{\bar{C}_\beta C_\alpha}_{ji}(\bar{D}_\beta\Gamma_jU_\beta)(\bar{U}_\alpha\Gamma_iD_\alpha) \tag{16c}$$

with the latin indices $i,j=1,5$, the greek indices $\alpha,\beta=1,\ldots,n$, and the definitions

$$\Gamma_1\equiv1,\quad\Gamma_5\equiv\gamma_5,\quad C_\alpha=(\bar{U}_\alpha D_\alpha)\ ,\quad\bar{C}_\beta=(\bar{D}_\beta U_\beta) \tag{16c'}$$

and

$$g^{\bar{C}_\beta C_\alpha}_{ji}=\eta^\beta_j\eta^\alpha_i(-1)^{\delta_{i5}},\quad\eta^\alpha_1=\tfrac{1}{2}(g^{1/2}_{U_\alpha U_\alpha}-g^{1/2}_{D_\alpha D_\alpha})\ ,$$
$$\eta^\alpha_5=\tfrac{1}{2}(g^{1/2}_{U_\alpha U_\alpha}+g^{1/2}_{D_\alpha D_\alpha})\ . \tag{16c''}$$

We note that in the n generation case the coupling constants $g_{Q'Q}$, $g'_{Q'Q}$, and $g^{\bar{C}_\beta C_\alpha}_{ji}$ become the entries of the corresponding $2n\times2n$ matrices. For $g^{\bar{C}_\beta C_\alpha}_{ji}$, the row and column of the corresponding matrix are denoted respectively by $\binom{\beta}{j}$ and $\binom{\alpha}{i}$. Similar to the one generation case [1], we may have the relation

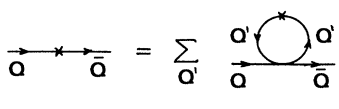
$$g^{\alpha\beta}_{11}g^{\alpha\beta}_{55}-g^{\alpha\beta}_{15}g^{\alpha\beta}_{51}=0 \tag{17}$$

being valid as a result of the minimal Higgs condition (14).

All of the following calculations based on $\mathcal{L}_{4F}$ will be made in the bubble approximation.

III. GAP EQUATION AND MASS RELATIONS

The gap equation could be contributed by the terms coming from the neutral scalar Lagrangian $\mathcal{L}^{N_S}_{4F}$. Suppose that $\mathcal{L}^{N_S}_{4F}$ will lead to formation of the G_c-invariant vacuum condensates $\langle\bar{U}_\alpha U_\alpha\rangle$ and $\langle\bar{D}_\alpha D_\alpha\rangle$ $(\alpha=1,\ldots,n)$ and generation of the dynamical masses m_{U_α} and m_{D_α} $(\alpha=1,\ldots,n)$; then we can obtain the coupled equations expressed graphically as



$Q=U_\alpha,D_\alpha\quad(\alpha=1,\ldots,n)$ (18)

or by means of the Feynman rule from $\mathcal{L}^{N_S}_{4F}$ algebraically as

$$m_Q=-\frac{1}{2}\sum_{Q'}g_{QQ'}\langle\bar{Q}'Q'\rangle$$
$$=-\frac{1}{2}g^{1/2}_{QQ}\sum_{Q'}g^{1/2}_{Q'Q'}\langle\bar{Q}'Q'\rangle\ ,$$
$$Q=U_\alpha,D_\alpha\ (\alpha=1,\ldots,n)\ , \tag{18'}$$

where Eq. (14) has been used in the second equality. It is seen from Eq. (18') that all of the fermion masses m_Q $(Q=U_\alpha,D_\alpha,\ \alpha=1,\ldots,n)$ are not equal to zeros simultaneously only if the combined condensate

$$\sum_Q g^{1/2}_{QQ}\langle\bar{Q}Q\rangle\neq0\ . \tag{19}$$

By means of the expression for the condensates $\langle\bar{Q}Q\rangle$,

$$\langle\bar{Q}Q\rangle=-2m_QI_Q\ ,$$
$$I_Q=2d_Q(R)\int\frac{id^4l}{(2\pi)^4}\frac{1}{l^2-m_Q^2}$$
$$=\frac{d_Q(R)\Lambda^2}{8\pi^2}\left[1-\frac{m_Q^2}{\Lambda^2}\ln\frac{\Lambda^2+m_Q^2}{m_Q^2}\right]\ , \tag{20}$$

where $d_Q(R)$ is the dimension of the G_c group representation of the Q fermions and Λ is the momentum cutoff of the loop integration, the coupled equations (18′) can be divided into the gap equation

$$\sum_Q g_{QQ} I_Q = 1 \tag{21}$$

and $2n-1$ independent relations between the ratios of the masses and the ones of the coupling constants:

$$m_Q/m_{Q'} = g_{QQ}^{1/2}/g_{Q'Q'}^{1/2}\ , \quad Q,Q'=U_\alpha,D_\alpha\ (\alpha=1,\ldots,n)\ . \tag{22}$$

Equation (22) indicates that the heavier fermion Q is related to the stronger coupling constant g_{QQ}. By the relations coming from Eqs. (20) and (22),

$$I_Q/I_{Q'} = d_Q(R)m_Q^2/d_{Q'}(R)m_{Q'}^2 = d_Q(R)g_{QQ}/d_{Q'}(R)g_{Q'Q'} \tag{23}$$

the gap equation (21) can also be written as

$$g_{Q_0Q_0}\left[1+\sum_{Q\neq Q_0} d_Q(R)g_{QQ}^2/d_{Q_0}(R)g_{Q_0Q_0}^2\right] I_{Q_0} = 1\ , \tag{24}$$

where Q_0 can be, in general, any fermion flavor among the n generations. However, the most stringent condition ensuring Eq. (24) to have a solution emerges only if Q_0 is taken to be the heaviest fermion flavor Q_h among the n generations. In this case the corresponding strongest coupling constant $g_{Q_hQ_h}$ must obey the inequality

$$g_{Q_hQ_h}\left[1+\sum_{Q\neq Q_h} d_Q(R)g_{QQ}^2/d_{Q_h}(R)g_{Q_hQ_h}^2\right] > 8\pi^2/d_{Q_h}(R)\Lambda^2\ . \tag{24′}$$

IV. HIGGS AND GOLDSTONE BOSONS

We will prove that, in the case with the n generations of the Q fermions, the symmetry breakdown induced by the condensate (19),

$$SU_L(2)\times U_Y(1)\to U_{\hat{Q}}(1),\quad \hat{Q}=I_L^3+\tfrac{1}{2}Y\ , \tag{25}$$

is accompanied by only a single massive composite Higgs boson and three massless composite Goldstone bosons corresponding to the three broken group generators.

To start with, we may attempt to read out the configurations of these composite particles as the combinations of the spin-zero bound-state modes $(\bar{Q}Q')$ and $(\bar{Q}\gamma_5Q')$ $(Q,Q'=U_\alpha,D_\alpha,\ \alpha=1,\ldots,n)$ by means of the rule indicated in Ref. [1]. For this purpose let us go back to the Yukawa-form Lagrangian $\mathcal{L}_Y$ in Eq. (5). Considering the definition (13) of g_{QQ} we can rewrite $\mathcal{L}_Y$ as

$$\begin{aligned}\mathcal{L}_Y &= H^\dagger C + C^\dagger H - m_0^2 H^\dagger H\ ,\\ C^\dagger &= \tfrac{1}{2}m_0 G^{1/2}(\sqrt{2}\phi^-,\phi_S^0 - i\phi_P^0)\ ,\end{aligned} \tag{26}$$

where

$$G=\sum_Q g_{QQ} \tag{27}$$

is the sum of the $2n$ coupling constants g_{QQ} and

$$\phi_S^0=\sum_Q(\phi_S^0)_Q(\bar{Q}Q),\quad (\phi_S^0)_Q=G^{-1/2}g_{QQ}^{1/2}\ , \tag{28a}$$

$$\phi_P^0=\sum_Q(\phi_P^0)_Q(\bar{Q}i\gamma_5Q),\quad (\phi_P^0)_Q=iG^{-1/2}(-1)^{I_Q^3}g_{QQ}^{1/2}\ , \tag{28b}$$

$$\begin{aligned}\phi^- &= \sum_{i,\alpha}(\phi^-)_i^{C_\alpha}(\bar{U}_\alpha\Gamma_iD_\alpha),\\ (\phi^-)_i^{C_\alpha} &= -\sqrt{2}G^{-1/2}\eta_i^\alpha(-1)^{\delta_{i5}}\ .\end{aligned} \tag{28c}$$

In addition, the Hermitian conjugate of ϕ^- can be rewritten as

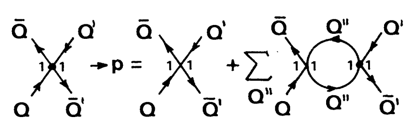
$$\begin{aligned}\phi^+ &= (\phi^-)^\dagger = \sum_{i,\alpha}(\bar{\phi}^-)_i^{\bar{C}_\alpha}(\bar{D}_\alpha\Gamma_iU_\alpha),\\ (\bar{\phi}^-)_i^{\bar{C}_\alpha} &= (\phi^-)_i^{C_\alpha}(-1)^{\delta_{i5}}\ .\end{aligned} \tag{28d}$$

Correspondingly, the four-fermion Lagrangian $\mathcal{L}_{4F}$ in Eq. (16) will become

$$\mathcal{L}_{4F}=\frac{1}{m_0^2}C^\dagger C=\frac{G}{4}[2\phi^-\phi^+ + (\phi_S^0)^2+(\phi_P^0)^2]\ . \tag{29}$$

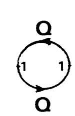
If the rule given in Ref. [1] remains valid in the n generation case, then ϕ_S^0, ϕ_P^0, and $\phi^\mp$ should be the configurations of the allowed Higgs and Goldstone bosons. We will show by explicit calculations for the four-point Green functions that this is the case indeed.

First, let us discuss the sector of the neutral scalar modes $(\bar{Q}Q)$. The interactions governing $(\bar{Q}Q)$ are expressed by $\mathcal{L}_{4F}^{N_S}$ in Eq. (16a). The four-point functions for the transitions from $(\bar{Q}Q)$ to $(\bar{Q}'Q')$ obey the graphical coupled equations

$\Gamma_S^{\bar{Q}'Q'\bar{Q}Q}(p^2)\equiv$ 

$\to p,\quad Q,Q'=U_\alpha,D_\alpha\ (\alpha=1,\ldots,n)\ ,$ (30)

where the digit 1 at a vertex represents the unit matrix and p denotes the four-momentum of the center of mass of the corresponding bound states which are presently $(\bar{Q}Q)$, $(\bar{Q}''Q'')$, and $(\bar{Q}'Q')$. By means of the Feynman rule coming from Eq. (16a) and the expression for the loop integration,



$$\begin{aligned}&= -2iN_Q(p^2),\\ N_Q(p^2)&=I_Q+\left[2m_Q^2-\frac{p^2}{2}\right]K_Q(p^2)\end{aligned} \tag{31}$$

with I_Q given by Eq. (20) and $K_Q(p^2)$ represented by the formula

$$K_Q(p^2)=2d_Q(R)\int\frac{id^4l}{(2\pi)^4}\frac{1}{(l^2-m_Q^2)[(l+p)^2-m_Q^2]} \tag{32}$$

$$=-\frac{d_Q(R)}{8\pi^2}\int_0^1 dx\left[\ln\frac{\Lambda^2+M_Q^2}{M_Q^2}-\frac{\Lambda^2}{\Lambda^2+M_Q^2}\right],$$

$$M_Q^2=m_Q^2-p^2x(1-x),$$

we may translate the graphical equations (30) into the algebraic ones

$$\sum_{Q''}\Gamma_S^{\bar{Q}'Q'\bar{Q}''Q''}(p^2)[\delta_{Q''Q}-N_{Q''}(p^2)g_{Q''Q}]=\frac{i}{2}g_{Q'Q}\,,$$

$$Q,Q'=U_\alpha,D_\alpha\ (\alpha=1,\ldots,n)\,. \tag{33}$$

When the minimal Higgs condition (14) is used Eq. (33) can be rewritten as

$$\sum_{Q''}\Gamma_S^{\bar{Q}'Q'\bar{Q}''Q''}(p^2)[\delta_{Q''Q}-\tilde{N}_{Q''}(p^2)g_{QQ}^{1/2}]=\frac{i}{2}g_{Q'Q'}^{1/2}g_{QQ}^{1/2}\,,$$

$$Q,Q'=U_\alpha,D_\alpha\ (\alpha=1,\ldots,n) \tag{33'}$$

$$\tilde{N}_{Q''}(p^2)=N_{Q''}(p^2)g_{Q''Q''}^{1/2}\ \ (\text{no}\ \textstyle\sum\ \text{of}\ Q'')\,.$$

It is noted that the $2n\times 2n$ coefficient determinant in Eq. (33′),

$$\Delta(p^2)=\det[\delta_{Q''Q}-\tilde{N}_{Q''}(p^2)g_{QQ}^{1/2}]\,, \tag{34}$$

has a special feature; i.e., when we take off all the 1's in its diagonal elements, the remaining determinant and its subdeterminants are all equal to zero. This is due to the fact that in these determinants any two rows or any two columns are always proportional to each other. This fact will greatly simplize the expansion of the determinant $\Delta(p^2)$. The nonzero contributions to $\Delta(p^2)$ come from only its diagonal elements and the result is that

$$\Delta(p^2)=1-\sum_Q\tilde{N}_Q(p^2)g_{QQ}^{1/2}=1-\sum_Q N_Q(p^2)g_{QQ}\,. \tag{35}$$

Considering that the relevant determinants encountered in solving Eq. (33′) possess the same features as $\Delta(p^2)$, we can easily obtain the solution of Eq. (33′) as

$$\Gamma_S^{\bar{Q}'Q'\bar{Q}Q}(p^2)=ig_{Q'Q}/\Delta_S(p^2)\,,$$

$$Q,Q'=U_\alpha,D_\alpha(\alpha=1,\ldots,n) \tag{36}$$

where

$$\Delta_S(p^2)=\sum_Q g_{QQ}K_Q(p^2)(p^2-4m_Q^2)\,. \tag{37}$$

It is emphasized that when deriving this expression we must have used the gap equation (21). A remarkable fact is that the four-point functions given by Eq. (36) could be represented by the numerical components of ϕ_S^0 as

$$\Gamma_S^{\bar{Q}'Q'\bar{Q}Q}(p^2)=iG(\phi_S^0)_{Q'}(\phi_S^0)_Q^*/\Delta_S(p^2)\,. \tag{38}$$

Such a form make it very easy for us to derive the propagators for all the possible configurations consisting of $(\bar{Q}Q)$. In fact, the propagator for ϕ_S^0 may be calculated from Eqs. (28a) and (38) and the result is that

$$\Gamma^{\phi_S^0}(p^2)=\sum_{Q',Q}(\phi_S^0)_{Q'}^*\Gamma_S^{\bar{Q}'Q'\bar{Q}Q}(p^2)(\phi_S^0)_Q=iG/\Delta_S(p^2) \tag{39}$$

where the normalization condition of the configuration ϕ_S^0,

$$\sum_Q(\phi_S^0)_Q^*(\phi_S^0)_Q=1\,, \tag{40}$$

has been used. Since ϕ_S^0 is merely one of the $2n$ independent combinations of the neutral scalar models $(\bar{Q}Q)$ ($Q=U_\alpha,D_\alpha$, $\alpha=1,\ldots,n$), there must be the other $2n-1$ neutral scalar configurations orthogonal to ϕ_S^0. Denote them as

$$\phi_{S,k}^0=\sum_Q(\phi_{S,k}^0)_Q(\bar{Q}Q),\quad k=1,\ldots,2n=1 \tag{41}$$

whose components satisfy the orthogonal conditions

$$\sum_Q(\phi_{S,k}^0)_Q^*(\phi_S^0)_Q=0,\quad k=1,\ldots,2n-1\,. \tag{42}$$

By means of Eqs. (39), (41), and (42), it immediately follows that the propagators for $\phi_{S,k}^0$,

$$\Gamma^{\phi_{S,k}^0}(p^2)=0,\quad k=1,\ldots,2n-1\,. \tag{43}$$

This implies that these configurations do not exist. The ϕ_S^0 given by (28a) is the only neutral scalar bound state with a nonzero propagator. It is seen from Eqs. (39) and (37) that ϕ_S^0 is a massive particle. In addition, it has precisely the same configurations as the nonzero condensate (19). Therefore, ϕ_S^0 could be correctly identified with the Higgs boson in the minimal dynamical breaking scheme of the group $SU_L(2)\times U_Y(1)$. The mass of the Higgs boson ϕ_S^0 is determined by the condition $\Delta_S(p^2)=0$ which, by Eq. (37), gives

$$m_{\phi_S^0}^2=p^2=\sum_Q 4m_Q^2 g_{QQ}K_Q(p^2)\Big/\sum_Q g_{QQ}K_Q(p^2)$$

$$=\sum_Q 4m_Q^4K_Q(p^2)\Big/\sum_Q m_Q^2K_Q(p^2)\,. \tag{44}$$

In the last equality in Eq. (44) we have used the relation

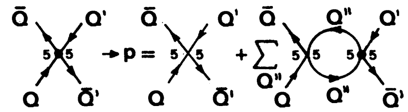
$$g_{QQ}/G=m_Q^2\Big/\sum_Q m_Q^2 \tag{45}$$

which can be derived from Eq. (22). We deduce from Eq. (44) that $m_{\phi_S^0}$ must obey the restrictions

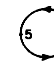
$$2(m_Q)_{\min}\le m_{\phi_S^0}\le 2(m_Q)_{\max}\,, \tag{46}$$

where $(m_Q)_{\min}$ and $(m_Q)_{\max}$ respectively represent the masses of the lightest and the heaviest ones among the $2n$ flavors of the Q fermions. However, in the inequalities $m_{\phi_S^0}$ will be more inclined toward heavy mass because in the sum of Q in Eq. (44) the heavier fermions have the larger weights. Equation (46) is a definite limitation on the Higgs-boson mass in the bubble approximation.

Next let us discuss the sector of the neutral pseudoscalar modes $(\bar{Q}i\gamma_5 Q)$. The interactions dominating them are expressed by $\mathcal{L}_{4F}^{N_P}$ in Eq. (16b). The four-point functions for the transitions from $(\bar{Q}i\gamma_5 Q)$ to $(\bar{Q}'i\gamma_5 Q')$ obey the graphical coupled equations

$-\Gamma_P^{\bar{Q}'Q'\bar{Q}Q}(p^2) \equiv$  $\to p, \quad Q,Q'=U_\alpha,D_\alpha \ (\alpha=1,\ldots,n)$ (47)

where the number 5 at a vertex represents the γ_5 matrix. By means of the Feynman rule coming from Eq. (16b) and the expression for the loop integration,

 $=2iN_Q^0(p^2), \quad N_Q^0(p^2)=I_Q-\frac{p^2}{2}K_Q(p^2)$. (48)

Equation (47) can be translated into the algebraic ones

$$\sum_{Q''}\Gamma_P^{\bar{Q}'Q'\bar{Q}''Q''}(p^2)[\delta_{Q''Q}-N_{Q''}^0(p^2)g'_{Q''Q}]=\frac{i}{2}g'_{Q'Q}, \quad Q,Q'=U_\alpha,D_\alpha \ (\alpha=1,\ldots,n) \tag{49}$$

or

$$\sum_{Q''}\Gamma_P^{\bar{Q}'Q'\bar{Q}''Q''}(p^2)[\delta_{Q''Q}-\tilde{N}_{Q''}^0(p^2)\tilde{g}_{QQ}^{1/2}]=\frac{i}{2}(-1)^{I_{Q'}^3}g_{Q'Q'}^{1/2}\tilde{g}_{QQ}^{1/2}, \quad Q,Q'=U_\alpha,D_\alpha \ (\alpha=1,\ldots,n) \tag{49'}$$

where we have used the notations

$$\tilde{N}_Q^0(p^2)=(-1)^{I_Q^3}g_{QQ}^{1/2}N_Q^0(p^2) \text{ and } \tilde{g}_{QQ}^{1/2}=(-1)^{-I_Q^3}g_{QQ}^{1/2} . \tag{50}$$

The $2n\times 2n$ coefficient determinant $\Delta'(p^2)$ of Eq. (49′) is found out to be

$$\Delta'(p^2)=\det[\delta_{Q''Q}-\tilde{N}_{Q''}^0(p^2)\tilde{g}_{QQ}^{1/2}]=1-\sum_Q\tilde{N}_Q^0(p^2)\tilde{g}_{QQ}^{1/2}=1-\sum_Q N_Q^0(p^2)g_{QQ} \tag{51}$$

seeing that $\Delta'(p^2)$ possesses the same feature as $\Delta(p^2)$ in Eq. (34). Then when a similar feature of relevant determinants is considered and the gap equation (21) is used we obtain the solution of Eq. (49′) as

$$\Gamma_P^{\bar{Q}'Q'\bar{Q}Q}(p^2)=ig'_{Q'Q}/\Delta_P(p^2) \tag{52}$$

$$=iG(\phi_P^0)_{Q'}(\phi_P^0)_Q^*/\Delta_P(p^2) ,$$

$$Q,Q'=U_\alpha,D_\alpha \ (\alpha=1,\ldots,n) , \tag{53}$$

with the notation

$$\Delta_P(p^2)=\sum_Q g_{QQ}K_Q(p^2)p^2 , \tag{54}$$

where, similar to the case of the scalar sector, the solutions have been expressed by the numerical components of ϕ_P^0 in Eq. (28b). From Eqs. (28b) and (53) and from the normalization

$$\sum_Q(\phi_P^0)_Q^*(\phi_P^0)_Q=1 \tag{55}$$

we may obtain the propagator for ϕ_P^0 to be

$$\Gamma^{\phi_P^0}(p^2)=iG/\Delta_P(p^2) . \tag{56}$$

For the other $2n-1$ neutral pseudoscalar configurations which are defined by

$$\phi_{P,k}^0=\sum_Q(\phi_{P,k}^0)_Q(\bar{Q}i\gamma_5 Q) , \quad k=1,\ldots,2n-1 \tag{57}$$

and satisfy the orthogonal conditions

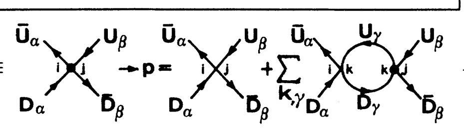
$$\sum_Q(\phi_{P,k}^0)_Q^*(\phi_P^0)_Q=0, \quad k=1,\ldots,2n-1 \tag{58}$$

it is proven that their propagators

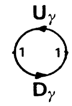
$$\Gamma^{\phi_{P,k}^0}(p^2)=0, \quad k=1,\ldots,2n-1 . \tag{59}$$

Therefore, ϕ_P^0 given by Eq. (28b) will be the only physical neutral pseudoscalar configuration with a nonzero propagator. The expression (54) for $\Delta_P(p^2)$ indicates that ϕ_P^0 is a massless particle and, hence, can be identified with the neutral pseudoscalar Goldstone boson in this symmetry-breaking scheme.

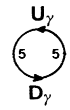
Lastly, let us turn to the sector of the charged bound-state modes $(\bar{Q}Q')$ and $(\bar{Q}\gamma_5 Q')$ $(Q\neq Q')$. The interactions governing them are expressed by $\mathcal{L}_{4F}^C$ in Eq. (16c). The four-point functions for the transitions from $(\bar{U}_\alpha\Gamma_i D_\alpha)$ to $(\bar{U}_\beta\Gamma_j D_\beta)$ obey the graphical coupled equations

$\Gamma_{ji}^{\bar{C}_\beta C_\alpha}(p^2)\equiv$  $\to p, \quad i,j=1,5, \quad \alpha,\beta=1,\ldots,n$. (60)

By means of the Feynman rule obtained from Eq. (16c) and the expressions for the loop integrations



$$=-iL_1^\gamma(p^2)=-2i[A_\gamma(p^2)+B_\gamma(p^2)] \ ,$$



$$=-iL_5^\gamma(p^2)=2i[A_\gamma(p^2)-B_\gamma(p^2)] \ , \tag{61}$$

$$A_\gamma(p^2)=I_{U_\gamma}+m_{D_\gamma}^2K_\gamma(p^2)-p^2J_{U_\gamma D_\gamma}(p^2)=I_{D_\gamma}+m_{U_\gamma}^2K_\gamma(p^2)-p^2J_{D_\gamma U_\gamma}(p^2) \ ,$$

$$B_\gamma(p^2)=m_{U_\gamma}m_{D_\gamma}K_\gamma(p^2) \ ,$$

with the notations

$$\left.\begin{matrix} J_{U_\gamma D_\gamma}(p^2) \\ K_\gamma(p^2) \end{matrix}\right\}=2d_{Q_\gamma}(R)\int_0^1 dx\int\frac{i\,d^4l}{(2\pi)^4}\frac{1}{(l^2-M_{U_\gamma D_\gamma}^2)^2}\times\begin{cases}1-x\\1\end{cases}$$

$$=-\frac{d_{Q_\gamma}(R)}{8\pi^2}\int_0^1 dx\left[\ln\frac{\Lambda^2+M_{U_\gamma D_\gamma}^2}{m_{U_\gamma D_\gamma}^2}-\frac{\Lambda^2}{\Lambda^2+M_{U_\gamma D_\gamma}^2}\right]\times\begin{cases}1-x\\1\end{cases} \ , \tag{62}$$

$$M_{U_\gamma D_\gamma}^2=(m_{U_\gamma}^2-p^2x)(1-x)+m_{D_\gamma}^2x \ ,$$

Eq. (60) can be translated into the algebraic equations

$$\sum_{k,\gamma}\Gamma_{jk}^{\bar{C}_\beta C_\gamma}(p^2)[\delta_{ki}\delta^{\gamma\alpha}-L_k^\gamma(p^2)g_{ki}^{\bar{C}_\gamma C_\alpha}]=ig_{ji}^{\bar{C}_\beta C_\alpha}, \quad i,j=1,5, \ \alpha,\beta=1,\dots,n \ . \tag{63}$$

When Eq. (16c'') is taken into account, Eq. (63) may be written as

$$\sum_{k,\gamma}\Gamma_{jk}^{\bar{C}_\beta C_\gamma}(p^2)[\delta_{ki}\delta^{\gamma\alpha}-\tilde{L}_k^\gamma(p^2)\eta_{\alpha i}(-1)^{\delta_{i5}}]=i\eta_j^\beta\eta_i^\alpha(-1)^{\delta_{i5}}, \quad i,j=1,5, \ \alpha,\beta=1,\dots,n \tag{63'}$$

where

$$\tilde{L}_k^\gamma(p^2)=L_k^\gamma(p^2)\eta_k^\gamma \ . \tag{64}$$

Seeing that the matrix corresponding to the coefficient determinant

$$\Delta_c(p^2)=\det[\delta_{ki}\delta^{\gamma\alpha}-\tilde{L}_k^\gamma(p^2)\eta_i^\alpha(-1)^{\delta_{i5}}] \tag{65}$$

has rows and the columns denoted by $\binom{\gamma}{k}$ and $\binom{\alpha}{i}$ respectively, $\Delta_c(p^2)$ will have the same features as $\Delta(p^2)$ in Eq. (34). This means that we may directly write down the result

$$\Delta_c(p^2)=1-\sum_{k,\gamma}\tilde{L}_k^\gamma(p^2)\eta_k^\gamma(-1)^{\delta_{k5}}$$

$$=1-\sum_{k,\gamma}L_k^\gamma(p^2)(\eta_k^\gamma)^2(-1)^{\delta_{k5}} \ . \tag{66}$$

Substituting expression (61) for $L_k^\gamma(p^2)$ into Eq. (66) and using the gap equation (21) and the relation (22) we may obtain

$$\Delta_c(p^2)=\sum_\alpha[g_{U_\alpha U_\alpha}J_{U_\alpha D_\alpha}(p^2)+g_{D_\alpha D_\alpha}J_{D_\alpha U_\alpha}(p^2)]p^2 \ . \tag{67}$$

Then the solution of Eq. (63) or (63′) is given by

$$\Gamma_{ji}^{\bar{C}_\beta C_\alpha}(p^2)=ig_{ji}^{\bar{C}_\beta C_\alpha}/\Delta_c(p^2) \tag{68}$$

$$=iG(\bar{\phi}^-)_j^{\bar{C}_\beta}(\phi^-)_i^{C_\alpha}/2\Delta_c(p^2) \ . \tag{69}$$

In Eq. (69) we have also used the numerical components of ϕ^- in Eq. (28c) to represent the solution. The propagator for ϕ^- can be calculated by

$$\Gamma^{\phi^-}(p^2)=\sum_{j,\beta}\sum_{i,\alpha}(\phi^-)_j^{C_\beta}(-1)^{\delta_{j5}}\Gamma_{ji}^{\bar{C}_\beta C_\alpha}(p^2)(\phi^-)_i^{C_\alpha}$$

$$=iG/2\Delta_c(p^2) \tag{70}$$

where the definition of $(\bar{\phi}^-)_i^{C_\alpha}$ in Eq. (28d) and the normalizations

$$\sum_{j,\beta}(\bar{\phi}^-)_j^{\bar{C}_\beta}(\bar{\phi}^-)_j^{\bar{C}_\beta}=\sum_{i,\alpha}(\phi^-)_i^{C_\alpha}(\phi^-)_i^{C_\alpha}=1 \tag{71}$$

have been used. It is indicated that the normalizations (71) are different from the ones used in Ref. [1] by a factor 2, so it is now no longer necessary to add an extra factor $\frac{1}{4}$ when the propagator for ϕ^- is calculated, as shown in Eq. (70). For the other $2n-1$ configurations which are defined by

$$\phi_k^-=\sum_{i,\alpha}(\phi_k^-)_i^{C_\alpha}(\bar{U}_\alpha\Gamma_iD_\alpha), \quad k=1,\dots,2n-1 \tag{72}$$

and satisfy the orthogonal conditions

$$\sum_{i,\alpha}(\phi_k^-)_i^{C_\alpha}(\phi^-)_i^{C_\alpha}=\sum_{i,\alpha}(\bar{\phi}_k^-)_i^{\bar{C}_\alpha}(\bar{\phi}^-)_i^{\bar{C}_\alpha}=0\ ,$$
$$k=1,\ldots,2n-1\ , \quad (73)$$

it is straightforward to prove that they have null propagators, i.e.,

$$\Gamma^{\phi_k^-}(p^2)=0\ ,\quad k=1,\ldots,2n=1\ . \quad (74)$$

As for the Hermitian conjugates of ϕ^- and ϕ_k^- $(k=1,\ldots,2n-1)$—ϕ^+ and ϕ_k^+ $(k=1,\ldots,2n-1)$, it is clear that their propagators will be identical to the ones represented by Eqs. (70) and (74) respectively.

In this way we have proven that only ϕ^- and ϕ^+ expressed by Eqs. (28c) and (28d) are physical charged spin-zero bound states with nonzero propagators. In view of expression (70) for their propagators and formula (67) for $\Delta_c(p^2)$, $\phi^\mp$ must be massless particles and could be identified with the two charged Goldstone bosons.

In brief summary, when the minimal Higgs conditions (14) is satisfied and the gap equation (21) has a solution, the four-fermion Lagrangian (5) coming from the n generations of the Q fermions will remain to lead to the result that only a single massive Higgs boson ϕ_S^0 and three massless Goldstone bosons ϕ_P^0 and $\phi^\mp$ emerge from the theory. These bosons are precisely the products of the minimal dynamical breaking of the global $SU_L(2)\times U_Y(1)$ electroweak group.

V. MASS GENERATION OF ELECTROWEAK GAUGE BOSONS

Just as in the top-quark condensate scheme [4], once the electroweak gauge interactions are opened, the three massless composite Goldstone bosons $\phi^\mp$ and ϕ_P^0 will enter the vacuum polarizations of the propagators for the electroweak gauge bosons $W^\mp$ and Z^0 and become their longitudinal components. This will lead to the generation of the masses of these gauge bosons, i.e., realization of the composite Higgs mechanism. We will show this result by explicit calculations of the inverse propagators for the electroweak gauge bosons in the n-generation case.

First, we discuss the charged gauge boson sector. The Lagrangian of the charged current interactions between the n generations of the Q fermions and the $W^\mp$ gauge bosons may be written as

$$\mathcal{L}_{\text{gauge}}^C=-\frac{1}{\sqrt{2}}(j_{\mu L}^- W^{\mu+}+j_{\mu L}^+ W^{\mu-})\ , \quad (75)$$

where

$$j_{\mu L}^-=\sum_\alpha \bar{U}_{\alpha L}\gamma_\mu D_{\alpha L}=(j_{\mu L}^+)^\dagger \quad (76)$$

are the charged weak currents of the Q fermions. In order to avoid the complexity caused by a possible Cabibbo-like mixture we assume that no couplings among different generations exist due to different assignments of Y charge or colorlike or horizontal quantum numbers so that the weak eigenstates of the Q fermions in Eq. (76) could be identified with their mass eigenstates. In momentum space, the inverse propagator for the W boson becomes

$$\frac{1}{g_2^2}D_{\mu\nu}^W(p)^{-1}=\frac{1}{g_2^2}(p_\mu p_\nu-g_{\mu\nu}p^2) -\frac{1}{2}\int d^4x\, e^{ip\cdot x}\langle Tj_{\mu L}^-(0)j_{\nu L}^+(x)\rangle\ , \quad (77)$$

where g_2 is the $SU_L(2)$ gauge coupling constant and we have taken the kinetic energy term of the gauge fields to be of the form $(-1/4g_2^2)F_{\mu\nu}^a F^{a\mu\nu}$. When calculating the second term in Eq. (77), we must include the contributions of not only the one-loop vacuum polarizations induced by the n generations of the Q fermions but also the insertions in them of the intermediate states represented by the massless charged Goldstone bosons $\phi^\mp$ whose configurations and propagators have been given by Eqs. (28c), (28d), and (70), respectively. By means of the Feynman rule coming from Eq. (75) we may obtain the expression

$$\begin{aligned}\frac{1}{g_2^2}D_{\mu\nu}^W(p)^{-1}=&\frac{1}{g_2^2}(p_\mu p_\nu-g_{\mu\nu}p^2)\\ &-\frac{1}{8}\Bigg\{\sum_\alpha d_{Q_\alpha}(R)\int\frac{d^4l}{(2\pi)^4}\text{tr}[\gamma_\mu(1-\gamma_5)(\not l-m_{U_\alpha})^{-1}\gamma_\nu(1-\gamma_5)(\not l+\not p-m_{D_\alpha})^{-1}]\\ &+\sum_{i,\alpha}d_{Q_\alpha}(R)\int\frac{d^4l}{(2\pi)^4}\text{tr}[\gamma_\mu(1-\gamma_5)(\not l-m_{U_\alpha})^{-1}(\phi^-)_i^{C_\alpha}\Gamma_i(\not l+\not p-m_{D_\alpha})^{-1}]\frac{iG}{2\Delta_c(p^2)}\\ &\times\sum_{j,\beta}d_{Q_\beta}(R)\int\frac{d^4q}{(2\pi)^4}\text{tr}[\gamma_\nu(1-\gamma_5)(\not q+\not p-m_{D_\beta})^{-1}(\bar{\phi}^-)_j^{\bar{C}_\beta}\Gamma_j(\not q-m_{U_\beta})^{-1}]\Bigg\}\ . \quad (78)\end{aligned}$$

Considering Eqs. (28c), (28d), (67), and the relation (45), by direct calculation we obtain the inverse propagator for the W boson expressed as follows (we have taken the same notation as used in Ref. [4]):

$$\frac{1}{g_2^2}D_{\mu\nu}^W(p)^{-1}=i(p_\mu p_\nu/p^2-g_{\mu\nu})[p^2/\bar{g}_2^2(p^2)-\bar{f}^2(p^2)]\ , \quad (79)$$

where

$$\frac{1}{\bar{g}_2^2(p^2)}=\frac{1}{g_2^2}-\sum_\alpha E_{U_\alpha D_\alpha}(p^2),\quad \bar{f}^2(p^2)=-\frac{1}{2}\sum_\alpha[m_{U_\alpha}^2 J_{U_\alpha D_\alpha}(p^2)+m_{D_\alpha}^2 J_{D_\alpha U_\alpha}(p^2)]\ , \tag{80}$$

$$\begin{aligned}E_{U_\alpha D_\alpha}(p^2)&=2d_{Q_\alpha}(R)\int_0^1 dx\int\frac{i\,d^4l}{(2\pi)^4}\frac{x(1-x)}{(l^2-M_{U_\alpha D_\alpha}^2)^2}\\&=-\frac{d_{Q_\alpha}(R)}{8\pi^2}\int_0^1 dx\,x(1-x)\left[\ln\frac{\Lambda^2+M_{U_\alpha D_\alpha}^2}{M_{U_\alpha D_\alpha}^2}-\frac{\Lambda^2}{\Lambda^2+M_{U_\alpha D_\alpha}^2}\right]\end{aligned} \tag{81}$$

and $J_{U_\alpha D_\alpha}$ ($J_{D_\alpha U_\alpha}$) and $M_{U_\alpha D_\alpha}^2$ given by Eq. (62). The transversity of the inverse propagator corresponds to a gauge-invariant Higgs mechanism, as was indicated in Ref. [4]. The mass of the W boson is determined by the condition in which the inverse propagator is equal to zero, i.e., by the equation

$$m_W^2=p^2=\bar{g}_2^2(m_W^2)\bar{f}^2(m_W^2)\ . \tag{82}$$

Noting that on the classical level we have the relation

$$\frac{G_F}{\sqrt{2}}=\frac{g_2^2}{8m_W^2}=\frac{1}{2v^2},\quad \text{when } p^2\ll m_W^2 \text{ or } p\to 0\ , \tag{83}$$

where v is the standard-model Higgs vacuum expectation value. Correspondingly, on the quantum level, in the zero-momentum limit, we may have the relation

$$\frac{G_F}{\sqrt{2}}=\frac{1}{8\bar{f}^2(0)}\ . \tag{84}$$

Equation (84) is a basic relation in this kind of model. It provides a connection between the fermion masses and the momentum cutoff Λ, as was shown in the top-quark condensate scheme [4]. In the present case, from Eqs. (80) and (61) we can put down the explicit expression for $\bar{f}^2(0)$:

$$\bar{f}^2(0)=\sum_\alpha\frac{d_{Q_\alpha}(R)}{16\pi^2}\int_0^1 dx[m_{U_\alpha}^2(1-x)+m_{D_\alpha}^2x]\left[\ln\frac{\Lambda^2+m_{U_\alpha}^2(1-x)+m_{D_\alpha}^2x}{m_{U_\alpha}^2(1-x)+m_{D_\alpha}^2x}-\frac{\Lambda^2}{\Lambda^2+m_{U_\alpha}^2(1-x)+m_{D_\alpha}^2x}\right]\ . \tag{85}$$

After integrating out x, the basic relation (84) will become

$$\begin{aligned}\frac{4\sqrt{2}\pi^2}{G_F}=\sum_\alpha d_{Q_\alpha}(R)&\left\{\frac{1}{m_{U_\alpha}^2-m_{D_\alpha}^2}\left[m_{U_\alpha}^4\ln\left[1+\frac{\Lambda^2}{m_{U_\alpha}^2}\right]-m_{D_\alpha}^4\ln\left[1+\frac{\Lambda^2}{m_{D_\alpha}^2}\right]\right]\right.\\&\left.+\Lambda^2\left[\frac{\Lambda^2}{m_{U_\alpha}^2-m_{D_\alpha}^2}\ln\frac{1+m_{U_\alpha}^2/\Lambda^2}{1+m_{D_\alpha}^2/\Lambda^2}-1\right]\right\}\ .\end{aligned} \tag{86}$$

Next let us turn to the neutral gauge boson sector. The Q fermions interact with both the $SU_L(2)$ gauge field $A^{3\mu}$ and the $U_Y(1)$ gauge field B^μ and the interaction Lagrangian can be written as

$$\mathcal{L}_{\text{gauge}}^N=-\tfrac{1}{2}j_\mu^3A^{3\mu}-\tfrac{1}{2}j_\mu^0B^\mu\ , \tag{87}$$

where the neutral currents j_μ^3 and j_μ^0 are defined by

$$j_\mu^3=\sum_Q\bar{Q}_L\gamma_\mu Q_L\delta_Q\ , \tag{88}$$

$$j_\mu^0=\sum_Q(\bar{Q}\gamma_\mu QY_{Q_L}+\tfrac{1}{2}\bar{Q}_R\gamma_\mu Q_R\delta_Q) \tag{89}$$

and δ_Q is a sign function defined by

$$\delta_Q=\begin{cases}+1 & \text{for } Q=U_\alpha\\ -1, & Q=D_\alpha\end{cases}\quad(\alpha=1,\ldots,n)\ . \tag{90}$$

In writing down the expression (89) of j_μ^0 we have used the general relation (2) of the hypercharge assignments of the Q

fermions. In momentum space and in the space spanned by the gauge fields $A^{3\mu}$ and B^{μ}, the inverse propagators for the neutral gauge bosons A^3 and B can be expressed by the 2×2 matrix

$$\frac{i}{g_i g_j} D^0_{\mu\nu}(p)^{-1} = i(p_\mu p_\nu - g_{\mu\nu}p^2)\begin{bmatrix} 1/g_2^2 & 0 \\ 0 & 1/g_1^2 \end{bmatrix} - \frac{1}{4}\int d^4x\, d^{ip\cdot x}\begin{bmatrix} \langle Tj^3_\mu(0)j^3_\nu(x)\rangle & \langle Tj^3_\mu(0)j^0_\nu(x)\rangle \\ \langle Tj^0_\mu(0)j^3_\nu(x)\rangle & \langle Tj^0_\mu(0)j^0_\nu(x)\rangle \end{bmatrix}, \tag{91}$$

where g_1 is the $U_Y(1)$ gauge coupling constant. When calculating the second term in Eq. (91), in addition to the one-loop vacuum polarizations induced by the n generations of the Q fermions, we must include the contribution of the intermediate state represented by the massless pseudoscalar Goldstone boson ϕ^0_P. The latter is equivalent to the insertion of the pseudoscalar four-point Green functions $\Gamma_P^{\bar{Q}'Q'\bar{Q}Q}(p^2)$ given by Eqs. (52) and (54) in the one-loop vacuum polarization diagrams. By means of the Feynman rule coming from Eq. (87) we will have the expressions

$$\begin{aligned} -\frac{1}{4}\int d^4x\, e^{ip\cdot x}\langle Tj^3_\mu(0)j^3_\nu(x)\rangle \\ = -\frac{1}{4}\Bigg\{ \sum_Q \frac{d_Q(R)}{4}\int \frac{d^4l}{(2\pi)^4}\mathrm{tr}[\gamma_\mu(1-\gamma_5)(\not{l}+\not{p}-m_Q)^{-1}\gamma_\nu(1-\gamma_5)(\not{l}-m_Q)^{-1}] \\ + \sum_{Q,Q'}\frac{d_Q(R)\delta_Q}{2}\int\frac{d^4l}{(2\pi)^4}\mathrm{tr}[\gamma_\mu(1-\gamma_5)(\not{l}+\not{p}-m_Q)^{-1}i\gamma_5(\not{l}-m_Q)^{-1}]\frac{ig'_{Q'Q}}{\Delta_P(p^2)} \\ \times\frac{d_{Q'}(R)\delta_{Q'}}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}[i\gamma_5(\not{q}+\not{p}-m_{Q'})^{-1}\gamma_\nu(1-\gamma_5)(\not{q}-m_{Q'})^{-1}]\Bigg\}, \end{aligned} \tag{92a}$$

$$\begin{aligned} -\frac{1}{4}\int d^4x\, e^{ip\cdot x}\langle Tj^3_\mu(0)j^0_\nu(x)\rangle \\ = -\frac{1}{4}\Bigg\{ \sum_Q \frac{d_Q(R)}{4}\int \frac{d^4l}{(2\pi)^4}\mathrm{tr}[\gamma_\mu(1-\gamma_5)(\not{l}+\not{p}-m_Q)^{-1}\gamma_\nu(1+\gamma_5+2Y_{Q_L}\delta_Q)(\not{l}-m_Q)^{-1}] \\ + \sum_{Q,Q'}\frac{d_Q(R)\delta_Q}{2}\int\frac{d^4l}{(2\pi)^4}\mathrm{tr}[\gamma_\mu(1-\gamma_5)(\not{l}+\not{p}-m_Q)^{-1}i\gamma_5(\not{l}-m_Q)^{-1}]\frac{ig'_{Q'Q}}{\Delta_P(p^2)} \\ \times\frac{d_{Q'}(R)\delta_{Q'}}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}[i\gamma_5(\not{q}+\not{p}-m_{Q'})^{-1}\gamma_\nu(1+\gamma_5)(\not{q}-m_{Q'})^{-1}]\Bigg\}, \end{aligned} \tag{92b}$$

and

$$-\frac{1}{4}\int d^4x\, e^{ip\cdot x}\langle Tj^0_\mu(0)j^3_\nu(x)\rangle = -\frac{1}{4}\int d^4x e^{-ip\cdot x}\langle Tj^3_\nu(0)j^0_\mu(x)\rangle \tag{92c}$$

as the consequence of the translation invariance and

$$\begin{aligned} -\frac{1}{4}\int d^4x\, e^{ip\cdot x}\langle Tj^0_\mu(0)j^0_\nu(x)\rangle \\ = -\frac{1}{4}\Bigg\{ \sum_Q \frac{d_Q(R)}{4}\int \frac{d^4l}{(2\pi)^4}\mathrm{tr}[\gamma_\mu(1+\gamma_5+2Y_{Q_L}\delta_Q)(\not{l}+\not{p}-m_Q)^{-1}\gamma_\nu(1+\gamma_5+2Y_{Q_L}\delta_Q)(\not{l}-m_Q)^{-1}] \\ + \sum_{Q,Q'}\frac{d_Q(R)\delta_Q}{2}\int\frac{d^4l}{(2\pi)^4}\mathrm{tr}[\gamma_\mu(1+\gamma_5)(\not{l}+\not{p}-m_Q)^{-1}i\gamma_5(\not{l}-m_Q)^{-1}]\frac{ig'_{Q'Q}}{\Delta_P(p^2)} \\ \times\frac{d_{Q'}(R)\delta_{Q'}}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}[i\gamma_5(\not{q}+\not{p}-m_{Q'})^{-1}\gamma_\nu(1+\gamma_5)(\not{q}-m_{Q'})^{-1}]\Bigg\}. \end{aligned} \tag{92d}$$

The first and the second sums in Eqs. (92a)–(92d) respectively represent the contributions of the one Q-fermion loops and the insertions of the massless Goldstone boson ϕ^0_P. Note that in the calculations of the one-loop integrations we may use the formula

$$\partial_{l_\mu}\frac{l_\nu}{l^2-M_Q^2} = -\frac{2l_\mu l_\nu - g_{\mu\nu}l^2 + g_{\mu\nu}M_Q^2}{(l^2-M_Q^2)^2} \tag{93}$$

so as to remove out the square ultraviolet divergent sectors since the loop integrations have been regularized by taking the momentum cutoff. Noting the relation

$$(-)^{I^3_{Q'}-I^3_Q} = \delta_{Q'}\delta_Q \tag{94}$$

after a direct and lengthy calculation we ultimately obtain the expression for the inverse propagators for the neutral gauge bosons as (still take the denotations in Ref. [4])

$$\frac{i}{g_i g_j} D^0_{\mu\nu}(p)^{-1} = i \left[\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} \right] \times \left\{ \begin{bmatrix} 1/g_2^2(p^2) & 0 \\ 0 & 1/g_1^2(p^2) \end{bmatrix} p^2 - \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} f^2(p^2) \right\} \quad (95)$$

where

$$\frac{1}{g_2^2(p^2)} = \frac{1}{g_2^2} - \sum_Q \tfrac{1}{2}(Y_{Q_L}\delta_Q + 1)E_Q(p^2) , \quad (96)$$

$$\frac{1}{g_1^2(p^2)} = \frac{1}{g_1^2} - \sum_Q (Y_{Q_L}^2 + \tfrac{3}{2}Y_{Q_L}\delta_Q + \tfrac{1}{2})E_Q(p^2) , \quad (97)$$

$$f^2(p^2) = -\frac{1}{4}\sum_Q m_Q^2 K_Q(p^2) - \frac{p^2}{2}\sum_Q Y_{Q_L}\delta_Q E_Q(p^2) ; \quad (98)$$

$$E_Q(p^2) = 2d_Q(R)\int_0^1 dx\, x(1-x)\int \frac{id^4 l}{(2\pi)^4} \frac{1}{(l^2 - M_Q^2)^2} = -\frac{d_Q(R)}{8\pi^2}\int_0^1 dx\, x(1-x)\left[\ln\frac{\Lambda^2 + M_Q^2}{M_Q^2} - \frac{\Lambda^2}{\Lambda^2 + M_Q^2} \right] ,$$

$$M_Q^2 = m_Q^2 - x(1-x)p^2 \quad (99)$$

and the expression for $K_Q(p^2)$ has been given by Eq. (32). It is not difficult to verify that when we take $Q=t,b$ quarks only and $\Lambda \gg m_t$, the results (95)–(98) are coincided with the ones given by the formulas (A12)–(A14) and (A16) in Ref. [4].

The 2×2 matrix in the inverse propagator (95) can be diagonalized by appropriate linear combinations of $A^{3\mu}$ and B^μ and the resulting diagonalized matrix has the diagonal entries

$$\lambda_\pm(p^2) = \frac{1}{2}\left[\frac{p^2}{g_1^2(p^2)} + \frac{p^2}{g_2^2(p^2)} - 2f^2(p^2) \right] \pm \frac{1}{2}\left\{ \left[\frac{p^2}{g_1^2(p^2)} - \frac{p^2}{g_2^2(p^2)} \right]^2 + 4f^4(p^2) \right\}^{1/2} . \quad (100)$$

The squared momentums at which $\lambda_\pm(p^2)$ are equal to zeroes will determine the masses of corresponding neutral gauge bosons. In fact, it is found that

$$\lambda_+(p^2) = 0 \quad \text{when } p^2 = 0 \quad (101)$$

and

$$\lambda_-(p^2) = 0 \quad \text{when } p^2 = f^2(p^2)[g_1^2(p^2) + g_2^2(p^2)] . \quad (102)$$

Therefore, the two combinations of $A^{3\mu}$ and B^μ corresponding to $\lambda_+(p^2)$ and $\lambda_-(p^2)$ could be respectively identified with the massless photon and the massive Z^0 gauge boson. The mass of the latter is determined by the equation

$$m_Z^2 = p^2 = f^2(m_Z^2)[g_1^2(m_Z^2) + g_2^2(m_Z^2)] . \quad (103)$$

Up to now we have eventually completed our arguments of that the minimal composite Higgs mechanism remains to work when the four-fermion interactions to carry out the NJL mechanism come from the n generations of the Q fermions.

It should be emphasized that the whole discussions above correspond to only the minimal dynamical breaking of the electroweak gauge group. An important sign about this statement is that we finally obtain only a single composite Higgs boson emerging from the theory. The same result could not appear if the four-fermion interactions of the n generations would not obey the minimal Higgs condition (14). For instance, if the four-fermion interactions of each generation would correspond to a separate Yukawa-form Lagrangian which contains respective static scalar field doublet, then the calculations of the inverse propagators for the gauge bosons could show the same results as (79) and (95). However, this case does not correspond to the minimal dynamical breaking since we would obtain n Higgs scalar bosons with the configurations $g_{U_\alpha U_\alpha}^{1/2}(\bar{U}_\alpha U_\alpha) + g_{D_\alpha D_\alpha}^{1/2}(\bar{D}_\alpha D_\alpha)$ ($\alpha = 1,\ldots,n$), and not a single one.

VI. CONCLUSIONS

In this paper we have proven by explicit calculations in the bubble approximation that the n generation extension of the NJL mechanism leading to dynamical breaking of the electroweak gauge group $\mathrm{SU}_L(2)\times \mathrm{Y}_Y(1)$ is completely feasible, especially when the minimal Higgs condition is satisfied it will still lead to the minimal dynamical breaking. The configurations of the resulting single massive Higgs boson and three massless Goldstone bosons will be some linear combinations of the spin-zero modes coming from the n generations of Q fermions. These configurations look a little complicated but are completely understandable from the vantage point of quantum theory. The mass of the Higgs boson is restricted between the double mass of the lightest and the heaviest Q fermions but is closer to the latter. Once the electroweak gauge interactions are opened, the three Goldstone bosons will enter the vacuum polarizations of the electroweak gauge bosons and lead to generation of the masses of the $W^\mp$ and Z^0 bosons and the realization of the composite Higgs mechanism.

The many generation extension of the gauged NJL mechanism will provide the possibility to include heavier fermions than the top quarks in the same minimal dynamical breaking scheme of the gauge group $\mathrm{SU}_L(2)\times \mathrm{U}_Y(1)$ so as to tackle the fine-tuning problem encountered in the top-quark condensate scheme. The detailed discussions about this topic will be given elsewhere.

ACKNOWLEDGMENTS

The author is grateful to Professor Y. Nambu for useful conversations and Professor S. Randjbar-Daemi for reading the manuscript. He would also like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. This work was partially supported by Grant No. LWTZ-1298 of the Chinese Academy of Sciences and by the National Natural Science Foundation of China under Grant No. 19175045.

[1] Bang-Rong Zhou, Commun. Theor. Phys. (to be published).
[2] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).
[3] Y. Nambu, in *New Theories in Physics,* Proceedings of the XI International Symposium on Elementary Particle Physics, Kazimierz, Poland, 1988, edited by Z. Ajduk, S. Pokorski, and A. Trautman (World Scientific, Singapore, 1989); V. A. Miransky, M. Tanabashi, and K. Yamawaki, Mod. Phys. Lett. A **4**, 1043 (1989); Phys. Lett. B **221**, 177 (1989).
[4] W. A. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. D **41**, 1647 (1990).
[5] W. J. Marciano, Phys. Rev. D **21**, 2425 (1980); Zhou Bang-Rong and Chang Chao-Hsi, Phys. Lett. **127B**, 209 (1983); Zhou Bang-Rong and J. L. Lucio M., *ibid.* **132B**, 83 (1983); **149B**, 152 (1984); G. Zoupanos, *ibid.* **129B**, 315 (1983); B. R. Zhou, Mod. Phys. Lett. A **1**, 43 (1986).
[6] S. Weinberg, Phys. Rev. D **19**, 1277 (1979); L. Susskind, *ibid.* **20**, 2619 (1979).
[7] E. Farhi and L. Susskind, Phys. Rep. **74**, 277 (1981).
[8] S. L. Adler, Phys. Rev. **177**, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento **60A**, 47 (1969); S. L. Adler and W. A. Bardeen, Phys. Rev. **182**, 1517 (1969); W. A. Bardeen, *ibid.* **184**, 1848 (1969); J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971); H. Georgi and S. L. Glashow, Phys. Rev. D **6**, 429 (1972).
[9] For example, see K. S. Huang, *Quarks, Leptons and Gauge Fields* (World Scientific, Singapore, 1982), p. 113.