Theory of the nonleptonic decays of heavy flavors

Marina Gibilisco
Dipartimento di Fisica, Università di Milano, via Celoria 16, I-20133 Milano, Italy

Giuliano Preparata
Dipartimento di Fisica, Università di Milano, via Celoria 16, I-20133 Milano, Italy and Istituto Nazionaledi Fisica Nucleare (INFN), Sezione di Milano, I-20133 Milano, Italy

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The meson wave functions of the anisotropic chromodynamics theory are used in order to calculate the matrix elements and hence the form factors for D, B mesons. Nonleptonic weak decays into two pseudoscalar (PP) or pseudoscalar-vector (PV) final states are studied in a factorization approach, and the exclusive widths are successfully computed. We find that the W-exchange contribution turns out to be very important for D^0 and D_S decays.

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I. INTRODUCTION: TWO RESEARCH PROGRAMS

The problem of nonleptonic weak decays of hadrons is a rather old one: for instance, in the late 1960s, the explanation of the observed $\Delta I=\frac{1}{2}$ enhancement in the $K^0\to\pi^+\pi^-$ and $\Lambda\to\pi N$ decays was a necessary task for any theory that wished to understand nonleptonic physics.

Thus two rival research programs have been developed in order to face these puzzling questions: the short-distance (SD) program, initiated by Wilson [1], and the long-distance (LD) program, whose basic ideas were put forward in Ref. [2]. The names of these approaches are related to their different ways of handling the current-current product which appears in the effective nonleptonic Hamiltonian in the manner of Wilson [1]:

$$H_{\rm NL}(0)=g^2\int d^4x\, D_W(x)T\left[J^{+\mu}\left[\frac{x}{2}\right]J_\mu\left[\frac{-x}{2}\right]\right], \quad (1.1)$$

where the propagation function $D_W(x)$ has an effective support $|m_W^2x^2|\le 1$.

According to Wilson, writing

$$H_{\rm NL}\sim\frac{G}{\sqrt{2}}J^{+\mu}\left[\frac{x}{2}\right]J_\mu\left[\frac{-x}{2}\right]\Bigg|_{|x|\simeq 1/m_W} =\frac{G}{\sqrt{2}}\sum_K C_K(m_W^{-2})O^K(0)\ , \quad (1.2)$$

it would happen that

$$C^{\Delta I=3/2}(m_W^{-2})\ll C^{\Delta I=1/2}(m_W^{-2}) \quad (1.3)$$

as a result of different SD anomalous dimensions; this fact would explain the very large numerical difference between $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ weak nonleptonic amplitudes.

With the discovery of the asymptotic freedom [3] of QCD, Wilson's suggestion could finally be checked, with the result [4,5]:

$$\frac{c^{1/2}(m_W^{-2})}{c^{3/2}(m_W^{-2})}\simeq\left[1+\frac{b\alpha_s(\mu)}{6\pi}\ln\left[\frac{m_W}{\mu}\right]\right]^{0.72}\simeq 4 \quad (1.4)$$

($\mu=1$ GeV, $b=33-2n_f$), which is insufficient to account for the experimentally known enhancement of the $\Delta I=\frac{1}{2}$ channel. In spite of its failure, the perturbative QCD (PQCD) inspired SD program was extended to the physics of the weak decays of heavy flavors, with the expectation that the large masses of the c and b quarks would this time ensure the complete dominance of short-distance effects [6]; then, the effective nonleptonic Hamiltonian for the Cabibbo allowed charmed particles decays was written [6] (:: denotes Wick products):

$$H_{\rm NL}^{C}=\frac{G}{\sqrt{2}}\cos^2\theta_C\{c_1{:}\bar u\gamma^\mu(1-\gamma_5)d\bar s\,\gamma_\mu(1-\gamma_5)c{:} +c_2{:}\bar u\gamma^\mu(1-\gamma_5)c\bar s\gamma_\mu(1-\gamma_5)d{:}\}\ , \quad (1.5)$$

where

$$c_{1,2}=\tfrac{1}{2}[c_+\pm c_-] \quad (1.6)$$

and

$$c_+=\left[1+\frac{g_s^2(\mu)}{3\pi^2}\ln\left[\frac{m_W}{\mu}\right]\right]^{-0.24}, \quad (1.7)$$

$$c_-=\left[1+\frac{g_s^2(\mu)}{3\pi^2}\ln\left[\frac{m_W}{\mu}\right]\right]^{0.48}. \quad (1.8)$$

In the same years, Nussinov and Preparata tried to face these problems starting from a different point of view: through a short-distance analysis [2] of the time-ordered product of two charged weak currents, they showed that one has *no short-distance* singularities. This fact strongly suggested that the physical states $|n\rangle$ between the currents could be restricted such that the momentum transfer between them and the initial and final states is $Q^2=|(p_{\rm in,out}-p_n)^2|\le 1$ GeV^2, i.e.,

$$J^{+\mu}\left[\frac{x}{2}\right]J_\mu\left[\frac{-x}{2}\right]\underset{x\to 0}{\longrightarrow}\sum_n J^{+\mu}\left[\frac{x}{2}\right]|n\rangle\langle n|J_\mu\left[\frac{-x}{2}\right]\Bigg|_{|(p_{\rm in,out}-p_n)^2|\le 1\ {\rm GeV}^2} \tag{1.9}$$

or, put differently, nonleptonic physics is *dominated by long-distance effects.* The outcome was very promising, including a "dynamical" explanation of the $\Delta I=\frac{1}{2}$ rule for hyperon weak decays [2] and nonleptonic K decays [7]. Thus the LD approach appears as the right approach to nonleptonic physics.

In this paper we wish to analyze the exclusive decays of D and B mesons within the framework of the LD program, supplemented by the anisotropic chromodynamics (ACD) calculational strategy [8]. The plan of this paper is as follows. In Sec. II we recall the basic ideas of ACD theory. In Sec. III we describe the details of the theoretical background. In Sec. IV we discuss the determination of the form factors. Sections V and VI deal with a discussion of our results for D and B mesons, respectively. A comparison with other approaches is carried out in Sec. VII, while the conclusions appear in Sec. VIII.

II. BASIC IDEAS OF ACD THEORY

Before starting to study in the LD approach the nonleptonic decays of heavy flavored mesons, we think it useful to recall the basic points of ACD theory; in fact, the meson wave functions obtained within this theoretical framework shall be used in the next sections to estimate the relevant matrix elements (a detailed discussion can be found in Ref. [8]).

Anisotropic chromodynamics is the theory of strong interactions proposed by one of us (G.P.) in 1980, with the purpose of finding an answer to an old and unsolved problem of QCD, color confinement. In fact, ACD was formulated with the belief that the only natural description of confinement emerges in the framework of two-dimensional gauge theories.

The starting point is the assumption that the dynamics of color is locally isomorphic to the dynamics of two-dimensional QCD (one spatial plus one temporal dimension). This fact requires a new geometry for space-time: thus one introduces a seven-dimensional manifold $M_4\times S_3$, where M_4 is the usual Minkowskian world and S_3 is a unit pseudosphere, spanned by a four-vector n_μ which is spacelike and normalized as $n_\mu^2=-1$.

Now, to each elementary physical event one associates a pair of Minkowskian vectors (x_μ,n_μ): in practice, we can picture this seven-dimensional continuum as a collection of usual Minkowskian worlds labeled by the variable n_μ and we shall refer to them as "n sheets."

This view seems to destroy the space isotropy, but in fact one demonstrates [8] that the hadron dynamics can be globally expressed in terms of the collective quark fields, obtained by averaging over the sheet variables n_μ.

An investigation of the structure of the ground state of QCD showed that it can be accurately approximated by the "chromomagnetic liquid" (CML) [9,10], which is formed by a large number of needle-shaped magnetic domains and which has all the suitable properties (Lorentz and rotational invariance, color confinement, freedom at short distances) expected for the real ground state of the theory.

The idea of the space breaking up into a large number of uncorrelated needles with a polarization along a given direction provides also a physical and dynamical realization of the anisotropy vector $\mathbf{n}$.

The first step in the construction of the hadronic world is the determination of the spectrum of the "primitive world": in fact, the confining Hamiltonian of ACD can be split [8] into two pieces:

$$H_C=H_C^{(0)}+H_C' , \tag{2.1}$$

where $H_C^{(0)}$ is the confining, infrared singular part and H_C' describes pair creation and has no singularity in the infrared region. Because of this peculiar behavior, one can treat H_C' *perturbatively,* whereas under no circumstance can this be done with $H_C^{(0)}$; thus the diagonalization of the first part of the Hamiltonian produces the "primitive hadronic world" where the hadrons are stable states with a definite number of $q,\bar{q}$. After the introduction of the perturbation H_C', the primitive hadrons will become unstable, as a result of the pair creation processes: however, in order to determine the meson spectrum, one does not need this piece of the Hamiltonian and one can proceed directly to diagonalize $H_C^{(0)}$.

A. Meson wave functions of ACD

The calculation of the ACD meson wave functions in an arbitrary moving frame has been performed in Ref. [11]: basically, such moving-frame functions are the solutions of a Schrödinger-like equation (see Fig. 1):

$$\left\{\left[\left[\frac{\mathbf{p}}{2}+\mathbf{k}\right]^2+m_q^2\right]^{1/2}+\left[\left[\frac{\mathbf{p}}{2}-\mathbf{k}\right]^2+m_{\bar{q}}^2\right]^{1/2}\right\}\Psi_{rsn}(\mathbf{p},\mathbf{k})+\int\frac{d^3K'}{(2\pi)^3}V_{rs}^{r's'}(\mathbf{p},\mathbf{k},\mathbf{k}')\Psi_{r's'n}(\mathbf{p},\mathbf{k}')=E_n\Psi_{rsn}(\mathbf{p},\mathbf{k}) , \tag{2.2}$$

where r,s denote the quark helicities and

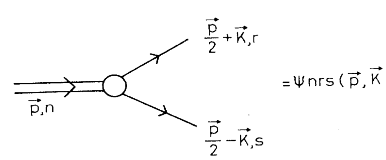


FIG. 1. Kinematics of the wave function $\psi_{nrs}(\mathbf{p},\mathbf{k})$; r and s denote the quark helicities.

$$V_{rs}^{r's'}(\mathbf{p},\mathbf{k},\mathbf{k}')=\langle p,k,r,s|H_C^{(0)}|p,k',r',s'\rangle \ . \qquad (2.3)$$

$V(\mathbf{0},\mathbf{k},\mathbf{k}')$ has been computed [8] and decomposed into four distinct contributions, depending in a different way on the spin and flavor.

This potential is inserted in (2.2) at $\mathbf{p}=0$ in order to find the meson spectrum in terms of five parameters only, the string tension μ^2 and the quark masses m_u, m_d, m_s, m_c, m_b: the case $\mathbf{p}=0$ is solved in a simple way, and then the problem one has to face is the determination of the potential V at $\mathbf{p}\neq 0$. A direct calculation is not easy, involving an elaborate analysis of the structure of color currents for quarks, which must be performed after a boost with velocity equal to $|\mathbf{p}|/E$. In order to achieve this, one uses an approximate Lorentz transformation establishing a connection between $V(\mathbf{p},\mathbf{k},\mathbf{k}')$ and $V(\mathbf{0},\mathbf{k},\mathbf{k}')$, the form of which can be found in Ref. [11]. In this way one succeeds in obtaining the meson wave functions in an arbitrary moving frame.

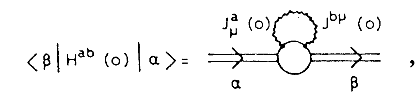
III. THEORY OF NONLEPTONIC DECAYS

Now, within the framework of the LD program and ACD theory, we can study nonleptonic decays of heavy flavors in order to calculate the exclusive decay widths; we can also test the effectiveness of the ACD meson wave functions. We start by calculating matrix elements of operators of the type

$$H^{ab}=\frac{G}{\sqrt{2}}:J_\mu^a J^{\mu b}: \ , \qquad (3.1)$$

where $J_\mu^{a,b}$ are general weak currents.

Let us then consider the matrix element



, (3.2)

where $|\alpha\rangle$ and $|\beta\rangle$ are two generic hadronic states.

From a purely topological point of view, we can classify all diagrams contributing to (3.2) in two classes: (a) the disconnected diagrams (D class) and (b) the connected diagrams (C class).

To the D class belong those contributions for which the individual hadrons appearing in $|\alpha\rangle$ and $|\beta\rangle$ separate in two *disconnected* groups. It is clear that these diagrams can be generated in *only* two ways: (1) by inserting the vacuum between the two currents $J_\mu^a(0)$ and $J^{\mu b}(0)$ (see Fig. 2) and (2) by inserting the vacuum between any "Fierz rearrangement" of the product of the two currents $J_\mu^a(0)$ and $J^{\mu b}(0)$ (see Fig. 3):

$$:J_\mu^a(0)J^{\mu b}(0):=\sum F_{K_1K_2}^{ab}:J_{K_1}(0)J_{K_2}(0): \ . \qquad (3.3)$$

The C class, on the other hand, comprises all the contributions which cannot be so decomposed; as an illustration, in Fig. 4 we report typical contributions of both classes to the decay of $K^+\to\pi^+\pi^0$.

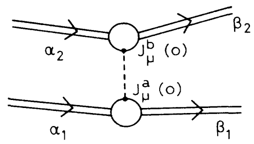


FIG. 2. Vacuum insertion between the two currents $J_\mu^a(0)$ and $J^{\mu b}(0)$.

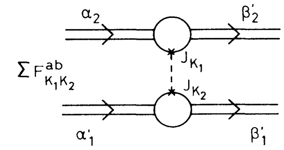


FIG. 3. Vacuum insertion between any "Fierz rearrangement" of the product of the two currents $J_\mu^a(0)$ and $J^{\mu b}(0)$.

It is clear that the great theoretical advantage of the diagrams of the D class is their complete factorization in terms of semileptonic processes, which thus can be taken as input in the calculation of such nonleptonic amplitudes.

On the contrary, the evaluation of C-class diagrams is a difficult hadrodynamical problem, which in general cannot be solved at the present time. However, we should remark here on a fundamental difference between the two types of contributions: whereas D-class contributions do closely reflect the internal (isospin, ΔS, etc.) structure of the current-current Hamiltonian, the C-class amplitudes are much more sensitive to the peculiar dynamics of hadrons; examples of the latter fact are to be found in the already mentioned dynamical explanation of the $\Delta I=\frac{1}{2}$ rule in hyperon nonleptonic decays [2] and in the calculation of the nonleptonic K decays of Ref. [7].

In order to evaluate the diagrams of the D class for the decays of heavy flavored mesons, we start by calculating the weak form factors that appear in those diagrams. Let us consider first the D (or B) decay into two pseudoscalar mesons (see Fig. 5); this amplitude is given by the product of the matrix elements

$$\text{(i)}\ \ \langle P(p_2)|V_\mu|D(p_1)\rangle = f^+(q^2)(p_1+p_2)_\mu + f^-(q^2)(p_1-p_2)_\mu \ , \qquad (3.4)$$

$$\text{(ii)}\ \ \langle P(q)|A_\mu|0\rangle = -if_P q_\mu \ . \qquad (3.5)$$

For a final state that comprises a pseudoscalar and a vector meson, we have two possible configurations (see Fig. 6); the relevant matrix elements of the weak currents are

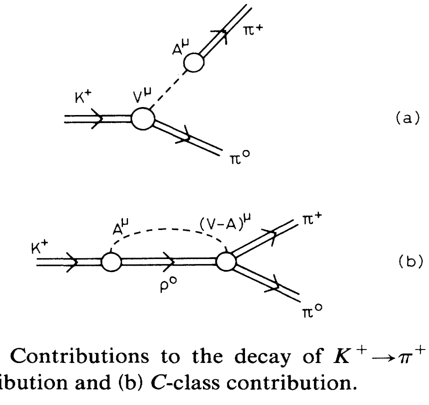


FIG. 4. Contributions to the decay of $K^+\to\pi^+\pi^0$: (a) D-class contribution and (b) C-class contribution.

$$\text{(iii)} \quad \langle V(p_2)|J_\mu|D(p_1)\rangle = i\left[\epsilon_\mu(M_D+M_V)A_1(q^2) - \epsilon^\nu(p_1-p_2)_\nu(p_1+p_2)_\mu \frac{A_2(q^2)}{M_D+M_V} + \epsilon^\nu(p_1-p_2)_\nu(p_1-p_2)_\mu \frac{2M_V[A_0(q^2)-A_3(q^2)]}{q^2}\right] + \frac{2\epsilon_{\mu\nu\alpha\beta}}{(M_D+M_V)}\epsilon^\nu p_1^\alpha p_2^\beta V(q_2) , \quad (3.6)$$

$$\text{(iv)} \quad \langle V(q)|V_\mu|0\rangle = f_V\epsilon_\mu \quad (3.7)$$

(where ϵ_μ is the polarization of the vector meson).

Thus the contribution of Fig. 6(a) is obtained by calculating the product of the matrix elements (3.5) and (3.6) and the one of Fig. 6(b) from (3.4) and (3.7); the first step in carrying this out is the evaluation of all matrix elements by using the ACD meson wave functions discussed in the previous section.

IV. DETERMINATION OF THE FORM FACTORS

The evaluation of the matrix elements containing the vacuum state has been carried out in Ref. [12], where the decay constants f_P, f_V for various mesons are obtained in satisfactory agreement with the experimental data; however, one is still working in order to improve the theoretical determination of f_P, f_V [13].

In a first step [14] of our work, the pseudoscalar form factors $f^+(q^2), f^-(q^2)$ have been calculated from (3.4) in the Breit system, choosing the z axis of the coordinates along the direction of the spatial component of the momentum; the results thus obtained were quite good for the $D\to K$ case, but they were not really satisfactory (because they were small) for the $D\to\pi$ transition. Then we thought that an improvement of this calculation could come from the insertion into (3.4) of an *additional* current term δJ_μ. The origin of such a term lies in the fact that we work in a well-defined approximation (we truncate our theory by treating quark-pair creation perturbatively), and in general, it is not possible to guarantee that the "naive current"

$$J_\mu^{(0)} = \int \frac{d^3q\,d^3q'}{(2\pi)^3}\left[\frac{m_q m_{\bar q}}{E_q E_{\bar q}}\right]^{1/2}[\bar u(q)b^+(q)+\bar v(q)d(q)] \times\gamma_\mu(1-\gamma_5)[u(q')b(q')+v(q')d^+(q')] \quad (4.1)$$

is conserved (or partially conserved) [11]. The additional term acts in the following way:

$$\langle b|q^\mu(J_\mu^{(0)}+\delta J_\mu)|a\rangle = \langle b|D|a\rangle , \quad (4.2)$$

where we call $|a\rangle$ and $|b\rangle$ the generic initial and final states, respectively, $q_\mu=(p_1-p_2)_\mu$, and D is the divergence:

$$\langle b|D|a\rangle = (m_a^{(0)}-m_b^{(0)})\langle b|\bar u u|a\rangle - (m_a^{(0)}+m_b^{(0)})\langle b|\bar u\gamma_5 u|a\rangle . \quad (4.3)$$

The additional piece δJ_μ restores the current conservation and can be evaluated in a simple way from (4.2) provided we know the current quark masses $m^{(0)}$ in (4.3).

For this purpose we use the values determined in Ref. [15] from an analysis of the (approximate) chiral symmetry of the ACD Hamiltonian:

$$m_u^{(0)} \simeq m_d^{(0)} = 18 \text{ MeV} , \quad (4.4)$$

$$m_s^{(0)} = 123 \text{ MeV} , \quad (4.5)$$

$$m_c^{(0)} = 1100 \text{ MeV} , \quad (4.6)$$

$$m_b^{(0)} = 4400 \text{ MeV} . \quad (4.7)$$

By parametrizing the components 0 and 3 of δJ_μ as

$$\delta J_0 = F\cos\theta(p) , \quad \delta J_3 = F\sin\theta(p) , \quad (4.8)$$

and minimizing their size, we can calculate separately δJ_0 and δJ_3. For $q^2=0$, the minimizing angle θ is equal to $3\pi/4$, and this means that the components 0 and 3 of the current correction have the same weight but an opposite sign.

In Table I we report the form factors obtained in our approach at $q^2=0$, while in Table II one can find a comparison with the results of some different theoretical approaches, using, for instance, QCD sum rules [17] or a lattice calculation [18]. We do not indicate the theoretical error which affects our form factors because it comes mainly from the determination of the meson wave functions; thus, it is not directly computable. However we estimate that it would be about 10%.

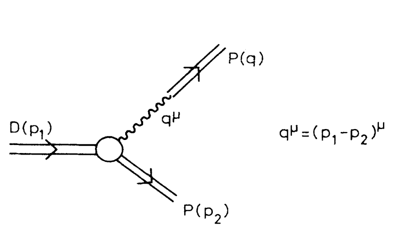


FIG. 5. D decay into two pseudoscalar mesons.

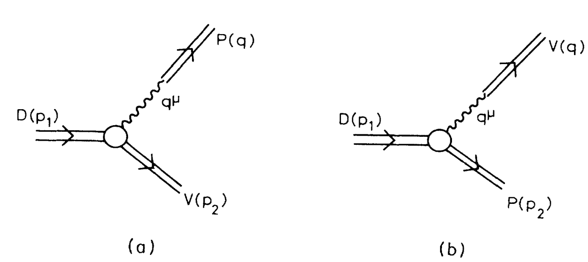


FIG. 6. D decay into a pseudoscalar and a vector meson in the two possible configurations.

TABLE I. Pseudoscalar form factors.

Case	$f^+(0)$	$f^-(0)$	Remarks
$D\to\eta$	0.98	−0.39	
$D\to K$	0.77	−0.06	$f^+_{D\to K}(0)_{\rm expt}=0.69\pm0.04$ [16]
$D\to\pi$	0.78	−0.34	
$D_s\to\eta$	0.70	−0.01	
$D_s\to K$	0.63	0.08	
$B\to D$	0.75	−0.79	
$B\to K$	0.63	−0.74	
$B\to\pi$	0.58	−0.83	

Looking at Tables I and II one can make the following remarks.

(a) Our value of $f^+_{D\to K}(0)$ is in satisfactory agreement with the experimental world average [16]:

$$f^+_{D\to K}(0)=0.69\pm0.04\ . \tag{4.9}$$

Moreover, we agree also with the values calculated in other theoretical approaches: in general, the theoretical value falls within the range 0.60–0.75 and the model dependence seems to be weak.

(b) The experimental determination of $f^+_{D\to\pi}(0)$ is not available yet. Thus we perform only a comparison among the theoretical predictions: our ACD value is in good agreement with the Dominguez-Paver [17] and the Crisafulli-Martinelli-Sachrajda [18] ones, obtained, respectively, by using QCD sum rules and lattice calculations.

Note that the insertion of the additional current δJ_μ *really increases* the value of $f^+_{D\to\pi}(0)$ as compared to our previous determination of Ref. [14], thus setting it in the theoretically expected range.

(c) As for the D_s form factors, we obtain some smaller values than the ones for D mesons: this fact satisfies also the theoretical expectations that predict an (approximate) inverse proportionality with the mass difference between initial and final states.

Finally, the B form factors calculated in ACD are roughly in the range suggested in Refs. [20,21].

A relevant test of the accuracy of our form factors can be given by the results that we have obtained for the exclusive semileptonic widths; in this paper we have decided not to discuss these important decays, to which we shall devote a future work, but we just present in Table III some results and some experimental data. In general, the agreement is satisfactory and this fact confirms that our form factors have the correct size. Note, however, that we find

$$\Gamma(D^+\to\bar K^0e^+\nu_e)=\Gamma(D^0\to K^-e^+\nu_e)\ , \tag{4.10}$$

as one can expect considering that exactly the *same process* and the *same diagram* are involved in these cases. Thus the experimental data for these channels seem very puzzling and a reexamination of this important question is necessary.

Finally, note our prediction of $\Gamma(B^-\to\pi^0e^-\bar\nu_e)$, a width for which we have only an upper limit from the Crystal Ball Collaboration [23].

TABLE II. Comparison with some different theoretical models.

Group	$f^+_{D\to K}(0)_{\rm calc}$	Model
Gibilisco-Preparata	0.77	ACD theory
Dominguez-Paver	0.75±0.05	QCD sum rules [17]
Aliev-Eletskii-Kogan	0.60±0.10	QCD sum rules [17]
Crisafulli-Martinelli-Sachrajda	0.74±0.17	Lattice calculation [18]
Lubicz-Martinelli-Sachrajda	0.63±0.08	Lattice calculation [18]
Gibilisco-Preparata	0.77	ACD theory
Domingueg-Paver	0.75±0.05	QCD sum rules [17]
Aliev-Eletskii-Kogan	0.60±0.10	QCD sum rules [17]
Crisafulli-Martinelli-Sachrajda	0.74±0.17	Lattice calculation [18]
Lubicz-Martinelli-Sachrajda	0.63±0.08	Lattice calculation [18]
Bauer-Stech-Wirbel	0.75–0.82	Constituent quark model [19]
Group	$f^+_{D\to\pi}(0)_{\rm calc}$	**Model**
Gibilisco-Preparata	0.78	ACD theory
Dominguez-Paver	0.75±0.05	QCD sum rules [17]
Crisafulli-Martinelli-Sachrajda	0.70±0.20	Lattice calculation [18]
Lubicz-Martinelli-Sachrajda	0.58±0.09	Lattice calculation [18]
Group	$f^+_{B\to D}(0)_{\rm calc}$	**Model**
Gibilisco-Preparata	0.75	ACD theory
Ovchinnikov-Slobodenyuk	1.0±0.2	QCD sum rules [20]
Group	$f^+_{B\to\pi}(0)_{\rm calc}$	**Model**
Gibilisco-Preparata	0.58	ACD theory
Dominguez-Paver	0.4±0.1	QCD sum rules [21]

TABLE III. Results: semileptonic decays $D,B \to Pe\nu_e$.

Channel	Γ_{cal} (10^{11} sec^{-1})	Γ_{expt} (10^{11} sec^{-1})	Group
$D^+ \to \bar{K}^0 e^+ \nu_e$	0.86	$(0.56 \pm 0.08 \pm 0.15)$	E691
		$(0.61^{+0.15}_{-0.10} \pm 0.07)$	Mark III
		$(0.56 \pm 0.09 \pm 0.12)$	E653
		(0.79 ± 0.07)	PDG
$D^0 \to K^- e^+ \nu_e$	0.86	$(0.81 \pm 0.12 \pm 0.09)$	Mark III
		$(0.88 \pm 0.07 \pm 0.14)$	CLEO
		$(0.91 \pm 0.07 \pm 0.17)$	E691
$D^0 \to \pi^- e^+ \nu_e$	0.076	$(0.09^{+0.05}_{-0.03} \pm 0.01)$	Mark III
$D_s^+ \to K^0 e^+ \nu_e$	0.038	(0.045)[a]	
$D_s^+ \to \eta e^+ \nu_e$	0.59	(0.46)[a]	
$\bar{B}^0 \to D^+ e^- \bar{\nu}_e$	0.165	0.13 ± 0.03	PDG
		$0.17 \pm 0.03 \pm 0.02$	CLEO
		$0.15 \pm 0.05 \pm 0.04$	ARGUS
$B^- \to D^0 e^- \bar{\nu}_e$	0.165	$0.135 \pm 0.05^{+0.08}_{-0.05}$	CLEO
$B^- \to \pi^0 e^- \bar{\nu}_e$	0.004	<0.016	Crystal Ball

[a]Estimates of the expected rates (see Ref. [22]).

TABLE IV. (a) Vector and axial form factors for the D meson. (b) Vector and axial form factors for the D_s meson. (c) Vector and axial form factors for the B meson.

(a)

Channel	Form factor	Gibilisco-Preparata	Different models
			on lattice
	V_0	1.78	0.86 ± 0.10
$D \to \bar{K}^{0*}$	A_1	0.77	0.53 ± 0.03
	A_2	0.49	0.19 ± 0.21
			(Martinelli and co-workers [18])
	V_0	1.62	
$D \to \omega$	A_1	0.54	
	A_2	0.54	
	V_0	1.61	0.78 ± 0.10
$D \to \rho$	A_1	0.54	0.45 ± 0.04
	A_2	0.54	0.02 ± 0.26
			(Martinelli and co-workers [18])

(b)

Channel	Form factor	our model
	V_0	1.92
$D_s \to \phi$	A_1	0.73
	A_2	0.50
	V_0	1.67
$D_s \to K^{*+}$	A_1	0.65
	A_2	0.52

(c)

Channel	Form factor	our model
	V_0	0.33
$B \to D^{*+}$	A_1	0.11
	A_2	0.18
	V_0	0.56
$B \to \omega$	A_1	0.43
	A_2	0.18
	V_0	0.57
$B \to \rho$	A_1	0.43
	A_2	0.18

Turning now to the vector mesons' form factors, we report in Table IV the results obtained in ACD theory for D, D_s, and B and the values determined by a lattice calculation [18] for the D case [the term containing the form factors A_0, A_3 in (3.6) is negligible in the zero lepton mass limit; thus, we need actually $A_1(q^2)$, $A_2(q^2)$, and $V(q^2)$ only].

In general, from Table IV(a) one finds that our values of the vector mesons' form factors are systematically *larger* than those of Martinelli and co-workers [18] and the difference is noteworthy especially for A_2. It is interesting to recall also the E691 experimental determination [24] of $A_1(0)$, $A_2(0)$, and $V(0)$ for $D \to K^*$; it has given *very surprising results,* because they *contradict* almost all the theoretical predictions. In Tables V(a) and V(b) we perform an extensive comparison between the theoretical and experimental values of the $D \to K^*$ form factors. Let us remark that the ACD results agree with the theoretical ones of Refs. [25–28]: there is *no evidence* from our calculation of a value $A_2(0)_{D \to K^*}$ very near to zero, as suggested by the E691 data and by some lattice calculations [18].

The ratio

$$\Gamma_L(D^+ \to \bar{K}^{0*} e^+ \nu_e)/\Gamma_T(D^+ \to \bar{K}^{0*} e^+ \nu_e) ,$$

derived by E691 disagrees also with the results coming from the WA82 [29] and Mark III [30] Collaborations [see Table V(c)]. Note, however, that the experimental errors are large; thus, this disagreement could be reduced by a more accurate determination of this ratio. In any case, either Γ_L/Γ_T or Γ_+/Γ_- obtained in our approach agrees with all these measurements.

V. NONLEPTONIC DECAYS OF D's: RESULTS

As we have emphasized in Sec. III, in order to calculate the decay widths for heavy flavored mesons D,B, we

TABLE V. (a) Form factors for $D\to K^*$. Here $R_2=A_2(0)/A_1(0)$ and $R_V=V(0)/A_1(0)$. (b) E691 experimental form factors for $D\to K^*$ (see [24]). (c) Ratios Γ_L/Γ_T and Γ_+/Γ_- for the decay $D^+\to\bar{K}^{0*}e^+\nu_e$.

(a)

Model	$A_1(0)$	$A_2(0)$	$V(0)$	R_2	R_V
Our model	0.77	0.49	1.78	0.64	2.31
Isgur-Scora [25]	0.8	0.8	1.1	1.0	1.4
Bauer-Wirbel [26]	0.9	1.2	1.3	1.3	1.4
Gilman-Singleton [27]	0.8	0.6	1.5	0.75	1.9
Korner-Schuler [28]	1.0	1.0	1.0	1.0	1.0
Lubicz-Martinelli-Sachrajda [18]	0.53 ± 0.03	0.19 ± 0.02	0.86 ± 0.01	0.36	1.62

(b)

Form factors	E691 data [24]
$A_1(0)$	$0.46\pm0.55\pm0.05$
$A_2(0)$	$0.0\pm0.2\pm0.1$
$V(0)$	$0.9\pm0.3\pm0.1$
$R_2=A_2(0)/A_1(0)$	$0.0\pm0.5\pm0.2$
$R_V=V(0)/A_1(0)$	$2.0\pm0.6\pm0.3$

(c)

Model	Γ_L/Γ_T	Γ_+/Γ_-
Our model	1.08	0.09
Isgur-Scora [25]	1.1	
Bauer-Wirbel [26]	0.9	
Gilman-Singleton [27]	1.2	
Korner-Schuler [28]	1.2	
Lubicz-Martinelli-Sachrajda [18]	1.51 ± 0.27	
E691 expt. [24]	$1.8^{+0.6}_{-0.4}\pm0.3$	$0.15^{+0.07}_{-0.05}\pm0.03$
WA82 expt. [29]	$0.6\pm0.3^{+0.3}_{-0.1}$	
Mark III expt [30]	$0.5^{+1.0+0.1}_{-0.1-0.2}$	

must distinguish between two classes of contributions: the *disconnected* and the *connected* ones. Here we will examine in a more detailed way these amplitudes and the topology of the diagrams related to them (see Fig. 7).

The disconnected diagram [Fig. 7(a)] is usually referred to as the "spectator" contribution: it can be evaluated in a simple way, after the insertion of the additional current, by means of the product matrix elements (3.4) and (3.5) (*PP* case), (3.6) and (3.5) [*PV* case in the configuration of Fig. 6(a)], and (3.4) and (3.7) [*PV* case in the configuration of Fig. 6(b)].

The connected diagram [Fig. 7(b)] has been called "essentially disconnected" as a consequence of its reducibility to a disconnected one through a Fierz transformation. The D^+ decays receive a contribution only from (a) and (b) topologies: thus, for instance, the amplitude for $D^+\to\bar{K}^0\pi^+$ in the rest frame of D^+ meson is written as

$$A(D^+\to\bar{K}^0\pi^+)=\frac{G}{\sqrt{2}}V_{cs}V_{ud}\{f_\pi[f^+_{D\to K}(0)(m_D^2-m_K^2)+f^-_{D\to K}(0)(m_\pi^2)] -\tfrac{1}{3}f_K[f^+_{D\to\pi}(0)(m_D^2-m_\pi^2)+f^-_{D\to\pi}(0)(m_K^2)]\}\ . \quad (5.1)$$

Note that, after the Fierz transformation, we have a *color-suppression factor* $\frac{1}{3}$ and a minus sign: in Sec. VII we shall discuss the importance of this factor.

The "properly connected" diagram [Fig. 7(c)] involves the exchange of a W boson, and in our model it has a considerable weight for D^0,D_s decays, its amplitude being comparable with the disconnected one. We are convinced indeed that this contribution is the cause of the difference of lifetimes between D^+ and D^0,D_s mesons.

Note, however, that the W-exchange amplitude is usually considered negligible, owing to its SD helicity suppression. In the LD program, this is in general not

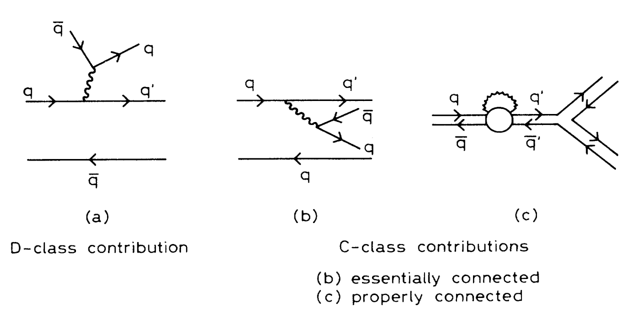


FIG. 7. Different contributions to heavy flavor meson decays.

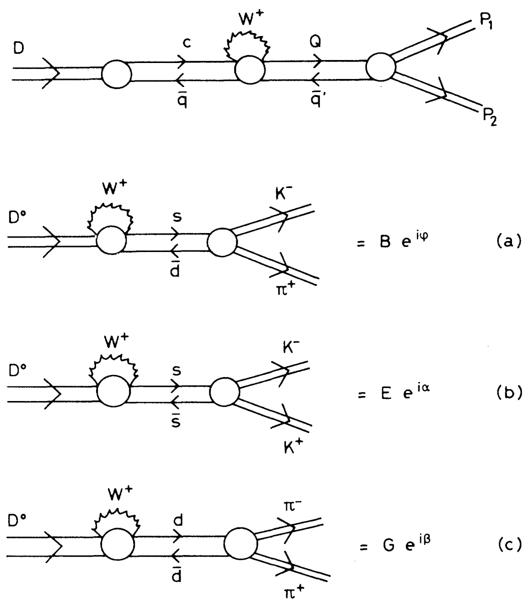


FIG. 8. Parametrization of the properly connected contributions to the decay $D^0 \to PP$.

the case, and in Sec. VII we shall make a few comments about how the LD program avoids such suppression. Unfortunately, we are not yet in a position to calculate the properly connected contributions; thus, we choose for them a suitable parametrization in terms of a modulus and a phase angle (see Fig. 8). Using this parametrization, we perform a fit of those channels that have the smallest experimental errors; subsequently, we use the parameters thus obtained in order to calculate the remaining widths.

TABLE VI. (a) Pseudoscalar meson decay constants. (b) Vector meson decay constants. (c) Constituent quark masses. (d) Cabibbo-Kobayashi-Maskawa matrix elements.

	(a) Decay constants [12,13]	
$f_\pi = 0.133$ GeV		$f_K = 0.162$ GeV
$f_D = 0.191$ GeV		$f_{D_s} = 0.226$ GeV
$f_\eta = 0.068$ GeV		
	(b) Decay constants [12,13]	
$f_\omega = 0.167$ GeV^2		$f_{K^*} = 0.198$ GeV^2
$f_{\rho^0} = 0.118$ GeV^2		$f_{\rho^+} = 0.167$ GeV^2
$f_\phi = 0.30$ GeV^2		
$f_{D^*} = 0.592$ GeV^2		$f_{D^{*0}} = 0.577$ GeV^2
$f_{J/\Psi} = 1.111$ GeV^2		$f_\Psi = 0.971$ GeV^2
	(c) Quark masses	
$m_u = 0.100$ GeV		$m_c = 1.12$ GeV
$m_d = 0.100$ GeV		$m_b = 4.35$ GeV
$m_s = 0.171$ GeV		
	(d) Matrix elements employed	
$V_{ud} = 0.976$	$V_{us} = 0.221$	$V_{ub} = 0.007$ [31]
$V_{cd} = -0.221$	$V_{cs} = 0.975$	$V_{cb} = 0.046$ [31]

TABLE VII. Results: $D^+ \to PP$.

Channel	$\Gamma_{\rm calc}$ (10^{11} sec^{-1})	$\Gamma_{\rm expt}$ (10^{11} sec^{-1})	Group
$D^+ \to \bar{K}^0 \pi^+$	0.33	0.24 ± 0.04	PDG
		$0.23 \pm 0.04 \pm 0.06$	E691
$D^+ \to \pi^+ \pi^0$	0.012	< 0.050	PDG
$D^+ \to K^+ \bar{K}^0$	0.071	$0.064 \pm 0.019 \pm 0.012$	E691
$D^+ \to \eta \pi^+$	0.011	0.062 ± 0.021	PDG

In Table VI we report the decay constants, the quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements employed in our calculation; in Tables VII–XII one can find our results for D^+, D^0, and D_s exclusive decays compared to the experimental widths (the inclusive ones have been discussed in Ref. [32]).

A few remarks are now in order.

(a) Our approach, in general, gives good results: we will emphasize that for the D^+ decays our model is *completely parameter free,* thus allowing us to successfully calculate the widths of many channels, as for instance, $\Gamma(D^+ \to \bar{K}^0 \pi^+)$, $\Gamma(D^+ \to K^+ \bar{K}^0)$, $\Gamma(D^+ \to K^+ \bar{K}^{0*})$, and $\Gamma(D^+ \to \bar{K}^{0*} \pi^+)$.

Note, however, that our value of $\Gamma(D^+ \to \phi \pi^+)$ is out (by a factor 2) of the experimental range stated by E691 [33], and this failure does not depend on the value of $f^+_{D\to\pi}(0)$ calculated in our approach. With regard to this problem, we think that a pole diagram (see Fig. 9), which we have so far neglected, this time could be important. However, in order to evaluate such diagram, we need a knowledge of $\Gamma(D^{0*} \to D\pi)$, which is presently unavailable.

We must remark that Buccella *et al.* [34], in a rescattering model based on the production of resonances in SU(3) symmetry, also obtain a somewhat small width, in agreement with our result:

$$\Gamma(D^+ \to \phi \pi^+) = 0.290 \times 10^{10} \text{ sec}^{-1} ,$$

$$\Gamma_{\rm expt}(D^+ \to \phi \pi^+) = (0.64 \pm 0.06 \pm 0.11) \times 10^{10} \text{ sec}^{-1}$$

(see Ref. [33]).

(b) As for the D^0 and D_s decays, we list all experimental widths fitted in Table XIII(a), while in Table XIII(b) we report the value of the parameters thus obtained, subsequently reemployed in the computation of the properly connected contributions to other amplitudes.

Note that we succeed in limiting the number of channels fitted by *assuming* that the size B of the properly connected contribution is nearly comparable to the

TABLE VIII. Results: $D^0 \to PP$.

Channel	$\Gamma_{\rm calc}$ (10^{11} sec^{-1})	$\Gamma_{\rm expt}$ (10^{11} sec^{-1})	Group
$D^0 \to \bar{K}^0 K^0$	0.023	$0.031^{+0.017}_{-0.012} \pm 0.005$	CLEO
		0.026 ± 0.09	PDG
$D^0 \to \eta \bar{K}^0$	0.12	$0.20 \pm 0.03 \pm 0.04$	CLEO
$D^0 \to \pi^0 \pi^0$	0.012	$0.021 \pm 0.005 \pm 0.005$	CLEO

TABLE IX. Results: $D^+ \to PV$.

Channel	Γ_{calc} (10^{11} sec^{-1})	Γ_{expt} (10^{11} sec^{-1})	Group
$D^+ \to \bar{K}^{0*}\pi^+$	0.22	$0.55 \pm 0.18 \pm 0.23$	Mark III
		0.18 ± 0.06	PDG
$D^+ \to \bar{K}^{0*}K^+$	0.036	$0.041 \pm 0.018 \pm 0.010$	Mark III
		0.044 ± 0.008	PDG
$D^+ \to \rho^+\bar{K}^0$	0.87	$0.64 \pm 0.07 \pm 0.21$	Mark III
		0.62 ± 0.16	PDG
$D^+ \to K^{*+}\bar{K}^0$	0.050		
$D^+ \to \omega\pi^+$	0.001	< 0.056	PDG
$D^+ \to \phi\pi^+$	0.030	$0.064 \pm 0.006 \pm 0.011$	E691
		0.056 ± 0.007	PDG
$D^+ \to \rho^0\pi^+$	0.002	$0.007 \pm 0.005 \pm 0.001$	E691

disconnected amplitude, as suggested by our theoretical approach.

The calculation of all remaining widths gives good results, thus showing that our handling of the W-exchange (or W-annihilation) diagrams is correct.

We obtain, for instance, a satisfactory agreement with the most recent CLEO II data for $\Gamma(D^0 \to \bar{K}^0 K^0)$, $\Gamma(D^0 \to \pi^0\pi^0)$; our results for $\Gamma(D^0 \to \bar{K}^{0*}\pi^0)$, $\Gamma(D^0 \to \phi\bar{K}^0)$, and $\Gamma(D^0 \to \omega\bar{K}^0)$ seem also to agree very well with the experimental widths. For $\Gamma(D^0 \to K^{*+}K^-)$ we can determine a range of values only, by requesting the existence of the phase angle α of our parametrization; in any case, we agree with the experimental domain suggested by the E691 and the Particle Data Group (PDG) Collaborations.

A very important result is obtained in our approach for the channel $D_s^+ \to \phi\pi^+$ (see Table XI): for this decay one has *no W-exchange contribution,* thus allowing us a complete, parameter-free calculation of the amplitudes; we must note that the calculated width is in very good agreement with *all* experimental data. This is a clear indication, in our opinion, that when we are able to calculate all contributions, the model works very well.

Let us remark that we obtain also a ratio $\Gamma(D_s \to K^{*+}K^0)/\Gamma(D_s \to \phi\pi^+)$ in agreement with the experimental data (but they are still affected by large errors).

A more exact experimental determination of $\Gamma(D_s \to K^0\pi^+)$, a channel for which we have the disconnected contribution only (see Table XII), will give us another important test of the effectiveness of our model.

Concluding this section, in spite of some problems, we believe that our approach altogether provides a satisfactory picture of nonleptonic decays of D mesons.

VI. NONLEPTONIC DECAYS OF B's: RESULTS

The same analysis just carried out for D mesons can be successfully applied to determine the exclusive decay widths of B mesons. However, a fundamental difference does exist between D and B decays: while D mesons through the W-exchange amplitude can be mixed to well-defined, almost degenerate light-quark resonances, B's cannot, as we can easily note from the values of their masses. Indeed, we believe that it is the closeness to a resonance that makes the W-exchange amplitude important for D^0 and D_s. Thus we expect *no significant contribution* from the properly connected amplitude for B and *no remarkable difference* between the lifetimes of $B^\pm$ and B^0, as the experimental data confirm.

In Tables XIV–XVI, we list our results for the exclusive widths concerning the $B \to PP$ and the $B \to PV$ channels, obtained from the evaluation of the spectator contributions only.

For the CKM matrix elements and the lifetimes, we have used the values

TABLE X. Results: $D^0 \to PV$.

Channel	Γ_{calc} (10^{11} sec^{-1})	Γ_{expt} (10^{11} sec^{-1})	Group
$D^0 \to \rho^+K^-$	1.49	1.74 ± 0.26	PDG
$D^0 \to \omega\bar{K}^0$	0.57	0.59 ± 0.12	PDG
		$0.81 \pm 0.21 \pm 0.24$	CLEO
$D^0 \to \bar{K}^{0*}\pi^0$	0.40	0.50 ± 0.24	PDG
$D^0 \to \rho^0\bar{K}^0$	0.45	0.15 ± 0.07	PDG
$D^0 \to \phi\bar{K}^0$	0.27	0.21 ± 0.03	PDG
$D^0 \to \eta\bar{K}^{0*}$	0.075	< 0.33	E691
		0.50 ± 0.29	PDG
$D^0 \to K^{*+}K^-$	$0.002 \le \Gamma \le 0.15$[a]	$(0.16^{+0.08}_{-0.06} \pm 0.04)$	E691
		0.083 ± 0.019	PDG

[a]In order to have $|\cos\alpha| < 1$.

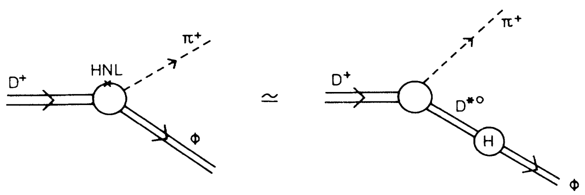


FIG. 9. Pole model for the channel $D^+ \to \phi\pi^+$.

$$V_{cb}=0.046,\ V_{ub}=0.007 \tag{6.1}$$

(see Ref. [31]),

$$\tau_{B^\pm}=1.35\times10^{-12}\ \text{sec} \tag{6.2}$$

(see Ref. [35]),

$$\tau_{B^0}=1.42\times10^{-12}\ \text{sec} \tag{6.3}$$

(see Ref. [35]). The values (6.1) are (probably) slightly high; however, the error in V_{cb}, V_{ub} is not very important at this stage of experimental knowledge.

In particular, we can consider noteworthy the results obtained for $\Gamma(B^- \to D^0\pi^-)$, $\Gamma(\bar{B}^0 \to D^+\pi^-)$, $\Gamma(\bar{B}^0 \to D^+D_s^-)$ (see Table XIV), $\Gamma(B^- \to J/\psi K^-)$, $\Gamma(B^- \to D^0 D_s^{*-})$ (Table XV), and $\Gamma(\bar{B}^0 \to D^+\rho^-)$, $\Gamma(\bar{B}^0 \to J/\psi \bar{K}^0)$ (Table XVI).

In general, we can predict many values of the widths, because frequently the experimental data are upper limits only. The calculation of the semileptonic channel is also very satisfactory (see Table III). However, we must wait for more precise measurements before judging our results, but we believe that one can see already a basic agreement with the presently known data.

VII. COMPARISON WITH OTHER APPROACHES

As we have emphasized in Sec. V, a color-suppression factor equal to $\frac{1}{3}$ is present in the (Fierz transformed) essentially disconnected amplitude. The value of this factor is a very fundamental question concerning the study of exclusive nonleptonic decays of D and B mesons; in fact, it affects in a remarkable way the cancellation between the different contributions to the overall amplitude.

The Bauer-Stech-Wirbel (BSW) model [19], a milestone in this research domain, adopts an effective Lagrangian which is a Wick-ordered product of hadronic currents weighted via two scale-independent coefficients a_1, a_2. At the scale of the decaying heavy quarks, they are connected with the coefficients c_1, c_2 of Eq. (1.6) through the relations

TABLE XI. Results: $D_s^+ \to PV$. We calculate also $\Gamma(D_s^+ \to K^{*+}\bar{K}^0)/\Gamma(D_s^+ \to \phi\pi^+)=1.04$, ratio (PDG)$=1.18 \pm 0.32$, and ratio (CLEO)$=1.2\pm0.25$.

Channel	Γ_{calc} (10^{11} sec^{-1})	Γ_{expt} (10^{11} sec^{-1})	Group
		0.78±0.27	CLEO
		0.74±0.36±0.09	TASSO
		0.72±0.16±0.11	ARGUS
$D_s^+ \to \phi\pi^+$	0.66	>0.76	Mark III
	(B=2.9%)	0.74±0.22	HRS
		0.79±0.18	E691
		0.62±0.11	PDG

$$\begin{aligned} a_1 &\simeq c_1+\xi c_2|_{\mu\simeq m_c, m_b}\,, \\ a_2 &\simeq c_2+\xi c_1|_{\mu\simeq m_c, m_b}\,, \end{aligned} \tag{7.1}$$

where ξ is the color factor equal to $1/N$ (N=number of colors).

An isospin analysis gives, for D mesons,

$$a_1 \sim 1.3\pm0.1\ ,\quad a_2 \sim -0.55\pm0.1\ . \tag{7.2}$$

On the other hand, at the mass scale $\mu \sim 1.5$ GeV, one has

$$c_1 \sim 1.21\ ,\quad c_2 \sim -0.42\ . \tag{7.3}$$

Thus, from (7.1)–(7.3), one deduces that $\xi=0$, while a value $\xi=\frac{1}{3}$ is disastrous because it yields

$$a_1 \sim 1.07\ ,\ a_2 \sim 0\ , \tag{7.4}$$

which are definitely *unable* to reproduce the experimental branching ratios.

Thus one faces a problematic situation: adopting the coefficients (7.2) and using relativistic oscillator wave functions at infinite momentum, in the BSW model [19] one obtains a satisfactory agreement with the experimental data, but at the cost of admitting the debatable value $\xi=0$.

Buras, Gérard, and Rückl [36] tried to justify this peculiar ξ value on the basis of the $1/N$ expansion; however, a careful study of this question, carried out by one of us (G.P.) [37], showed that the correct value of ξ must be $\frac{1}{3}$. In fact, one proves that the first-order coefficient of the a_1 expansion in powers of $g_s^2(\mu)$ must necessarily vanish and this fact requires a value $\xi=\frac{1}{3}$. As a consequence, the BSW approach is probably vitiated, in our opinion, by an incorrect value of the color factor.

Another important question arises about the helicity suppression of the W-exchange contributions: in fact, they involve the decay of a pseudoscalar state ($J^P=0^-$) into a pair of fermions. This process implies the equality

TABLE XII. Results: $D_s^+ \to PP$.

Channel	$\Gamma_{\text{calc}}/\Gamma(D_s^+ \to \phi\pi^+)$	$\Gamma_{\text{expt}}/\Gamma(D_s^+ \to \phi\pi^+)_{\text{CLEO}}$	Group
$D_s^+ \to \eta\pi^+$	0.86	0.53±0.14	PDG
$D_s^+ \to K^0\pi^+$ [a]	0.091	≤0.21	PDG

[a]Here we have the disconnected contribution only.

TABLE XIII. (a) Channels fitted. (b) Parameters obtained from the fit of the experimental widths.

(a)

(i) $D^0 \to PP$ widths

$\Gamma(D^0 \to K^- \pi^+) = (0.88 \pm 0.06) \times 10^{11}$ sec^{-1} (PDG)

$\Gamma(D^0 \to \bar{K}^0 \pi^0) = (0.45 \pm 0.09 \pm 0.05) \times 10^{11}$ sec^{-1} (Mark III)

$\Gamma(D^0 \to K^- K^+) = (0.109 \pm 0.007 \pm 0.012) \times 10^{11}$ sec^{-1} (CLEO)

$\Gamma(D^0 \to \pi^+ \pi^-) = (0.040 \pm 0.007 \pm 0.007) \times 10^{11}$ sec^{-1} (ARGUS)

(ii) $D^0 \to PV$ width

$\Gamma(D^0 \to K^{*-} \pi^+) = (1.09 \pm 0.14) \times 10^{11}$ sec^{-1} (PDG)

(We assume $A_{\text{prop. conn.}} \simeq A_{\text{disc.}}$)

(iii) $D_s^+ \to PP$ width

$\Gamma(D_s^+ \to \bar{K}^0 K^+) = (0.76 \pm 0.13) \times 10^{11}$ sec^{-1} (CLEO)

(iv) $D_s^+ \to PV$ width

$\Gamma(D_s^+ \to \bar{K}^{0*} K^+) = (0.66 \pm 0.24 \pm 0.17) \times 10^{11}$ sec^{-1} (Mark III)

(We assume $A_{\text{prop. conn.}} \simeq A_{\text{disc.}}$)

(b)

Parameters	$D^0 \to PP$	$D^0 \to PV$
B	0.42×10^{-5} GeV	-0.48×10^{-5} GeV
E	0.097×10^{-5} GeV	-0.11×10^{-5} GeV
G	-0.097×10^{-5} GeV	0.11×10^{-5} GeV
α	100°	
ϕ	130°	110°
β	135°	

Parameters	$D_s \to PP$	$D_s \to PV$
B	-0.36×10^{-5} GeV	0.54×10^{-5} GeV
E		
G		
α		
ϕ	110°	125°
β		

of the $q, \bar{q}$ helicities, and because left-handed quarks only can interact weakly with the W boson, one has that the annihilation amplitude is proportional to the masses of noncharmed quarks, thus being negligible. The existence of some decay channels such as $D^0 \to \bar{K}^0 \phi$ [$B = (0.88 \pm 0.12)\%$ [38]] and $D^0 \to \bar{K}^0 K^0$ [$B = (0.11 \pm 0.04)\%$ [38]] with a significant width shows that the helicity-suppression mechanism must be *somehow overcome*: in fact, these decays can only proceed through a W-exchange process.

For this purpose, in the standard theory, one admits [19] the possibility of an exchange of momentum and an-

TABLE XIV. Results: $B \to PP$.

Channel	Γ_{calc} (sec^{-1})	Γ_{expt} (sec^{-1})	Group
		$(0.14 \pm 0.08) \times 10^{11}$	PDG
$B^- \to D^0 D_s^-$	0.068×10^{11}	$(0.21 \pm 0.10) \times 10^{11}$	CLEO
		$(0.18 \pm 0.09 \pm 0.03) \times 10^{11}$	ARGUS[a]
$B^- \to D^0 \pi^-$	0.020×10^{11}	$(0.028 \pm 0.008) \times 10^{11}$	PDG
		$(0.015 \pm 0.006 \pm 0.004) \times 10^{11}$	ARGUS
$B^- \to \pi^- \pi^0$	1.82×10^7	$< 0.18 \times 10^9$	PDG
		$(0.056 \pm 0.035) \times 10^{11}$	PDG
$\bar{B}^0 \to D^+ D_s^-$	0.065×10^{11}	$(0.084 \pm 0.049) \times 10^{11}$	CLEO
		$(0.120 \pm 0.091 \pm 0.042) \times 10^{11}$	ARGUS[a]
$\bar{B}^0 \to D^+ \pi^-$	0.041×10^{11}	$(0.022 \pm 0.005) \times 10^{11}$	PDG
		$(0.034 \pm 0.007 \pm 0.007) \times 10^{11}$	ARGUS
		$(0.036 \pm 0.019 \pm 0.010) \times 10^{11}$	CLEO
$\bar{B}^0 \to D_s^+ \pi^-$	0.13×10^9	$< 0.91 \times 10^9$	PDG
$\bar{B}^0 \to \pi^+ \pi^-$	0.83×10^8	$< 0.63 \times 10^8$	PDG
$\bar{B}^0 \to K^- \pi^+$	0.61×10^7	$< 0.63 \times 10^8$	PDG
		$< 0.91 \times 10^8$	ARGUS
$\bar{B}^0 \to \pi^0 \pi^0$	0.23×10^7	$< 0.32 \times 10^7$	Crystal Ball[a]

[a]Preliminary data.

TABLE XV. Results: $B^- \to PV$. We use $\tau_{B^\pm}=13.5\times10^{-13}$ sec, $\tau_{\bar{B}^0}=14.2\times10^{-13}$ sec [35].

Channel	Γ_{calc} (sec^{-1})	Γ_{expt} (sec^{-1})	Group
$B^- \to D^0\rho^-$	0.11×10^{10}	$(0.96\pm0.30\pm0.30)\times10^{10}$	ARGUS
		$(0.96\pm0.44)\times10^{10}$	PDG
$B^- \to D^0 D_s^{*-}$	0.73×10^{10}	$(1.2\pm0.9\pm0.2)\times10^{10}$	ARGUS[a]
$B^- \to D_s^- D^{0*}$	0.24×10^{8}	$(1.0\pm0.7\pm0.1)\times10^{10}$	ARGUS[a]
$B^- \to D^{0*}\pi^-$	0.10×10^{10}	$(0.29\pm0.10\pm0.09)\times10^{10}$	ARGUS
		$(0.38\pm0.11)\times10^{10}$	PDG
$B^- \to J/\psi K^-$	0.086×10^{10}	$(0.38^{+0.24}_{-0.17}\pm0.08)\times10^{10}$	LEP[a]
		$(0.05\ \pm0.02\pm0.01)\times10^{10}$	ARGUS
		$(0.057\pm0.015)\times10^{10}$	PDG
$B^- \to \psi(2S)K^-$	0.022×10^{10}	$(0.13\pm0.06\pm0.03)\times10^{10}$	ARGUS
		$<0.015\times10^{10}$	PDG
$B^- \to \rho^0\pi^-$	3.93×10^{8}	$<1.11\times10^{8}$	PDG
$B^- \to K^-\rho^0$	0.61×10^{7}	$<0.52\times10^{8}$	PDG

[a]Preliminary data.

gular momentum between the quarks and gluons inside hadrons or a possible gluon emission during the decay process; in this way, the wave function of the D meson would be a linear combination of $|q\bar{q}\rangle$, $|qG\bar{q}\rangle$, and $|qGG\bar{q}\rangle$ states (G=gluon field), and thus the helicity-suppression mechanism does not act [39].

Actually, in our LD approach, a simple argument avoids resorting to the presence of soft gluons inside hadrons in order to overcome this suppression effect.

Let us consider, for instance, the long-distance part of the amplitude of a D^0 at rest dissociating by weak interactions into an $s\bar{d}$ pair [40]:

$$a(D^0\to s\bar{d})=\sum_{n,|\mathbf{q}|}\langle s\bar{d}|\bar{s}\gamma_\mu(1-\gamma_5)c|n,\mathbf{q}\rangle \times\langle n,\mathbf{q}|\bar{u}\gamma^\mu(1-\gamma_5)d|D^0\rangle \tag{7.5}$$

for $|\mathbf{q}|\leq 1$ GeV. This condition ensures that we are considering the long-distance part of the weak matrix element.

In (7.5) we can indeed restrict the sum over the intermediate states $|n\rangle$ to D^+,D^{*+}, the only ones which are important.

Now let us examine the vector current $V_k=\bar{u}\gamma_k d$: the calculation shows that in the small momentum transfer region it is suppressed by a factor $o(|\mathbf{q}|/m_D)$. This fact is the crucial point of our discussion: substituting the D,D^* meson by their on-shell quark lines and performing a Fierz transformation, we no longer have the V_k current in this amplitude, which is thus helicity unsuppressed (see Fig. 10): therefore the W-exchange contribution turns out to be very important in the computation of the decay widths.

Recently, the so-called "heavy-quark effective theory" (HQET) has gained the interest of many theorists [41] as a powerful method for investigating heavy flavored meson physics. This theory, proposed by Isgur and Wise [42], is based on the limit of $m_b,m_c\to\infty$; in this way, one acquires two new, important, symmetries: a heavy flavor symmetry SU(N) due to the unimportance of the flavor in the limit $m_q\to\infty$ and a spin symmetry due to the decou-

TABLE XVI. Results: $\bar{B}^0\to PV$.

Channel	Γ_{cal} (sec^{-1})	Γ_{expt} (sec^{-1})	Group
$\bar{B}^0\to D^+D_s^{*-}$	0.08×10^{11}	$(0.19\pm0.12\pm0.03)\times10^{11}$	ARGUS[a]
$\bar{B}^0\to D^+\rho^-$	0.095×10^{11}	$(0.063\pm0.042)\times10^{11}$	PDG
		$(0.06\pm0.03\pm0.02)\times10^{11}$	ARGUS
$\bar{B}^0\to D_s^- D^{*+}$	0.24×10^{8}	$(0.099\pm0.070\pm0.021)\times10^{11}$	ARGUS[a]
		$(0.11\pm0.08)\times10^{11}$	PDG
$\bar{B}^0\to D^{*+}\pi^-$	0.196×10^{8}	$(0.022\pm0.005)\times10^{11}$	PDG
		$(0.020\pm0.006\pm0.004)\times10^{11}$	ARGUS
		$(0.020\pm0.010\pm0.006)\times10^{11}$	CLEO
$\bar{B}^0\to J/\psi\bar{K}^0$	0.009×10^{11}	$(0.004\pm0.002)\times10^{11}$	PDG
$\bar{B}^0\to D^0\rho^0$	0.007×10^{11}	$<0.004\times10^{11}$	PDG
		$<0.004\times10^{11}$	CLEO
$\bar{B}^0\to\rho^+\pi^-$	0.003×10^{11}	$<0.036\times10^{10}$	PDG
$\bar{B}^0\to\rho^-\pi^+$	0.016×10^{10}	$<0.036\times10^{10}$	PDG
$\bar{B}^0\to K^{*-}\pi^+$	0.11×10^{8}	$<0.31\times10^{9}$	PDG
$\bar{B}^0\to\bar{K}^0\rho^0$	0.34×10^{6}	$<0.22\times10^{9}$	PDG

[a]Preliminary data.

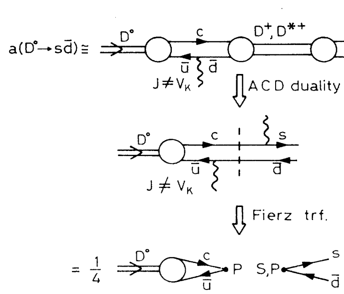


FIG. 10. Long-distance part of the amplitude of a D^0 at rest dissociating by weak interactions into an $s\bar{d}$ pair.

pling of the spin degrees of freedom in the heavy- quark limit.

Applying these symmetries, one can in principle simplify the calculation of the matrix elements, thus allowing, for instance, the determination of heavy flavored baryon form factors also [43].

However, in our opinion, the complete analysis of the corrections to the limit $m_q \to \infty$ is now awaited in order to describe the real world of the finite-mass heavy flavored mesons and to disclose the potentialities of this approach.

VIII. CONCLUSIONS

As we discussed in the Introduction, in the past the study of nonleptonic decays of heavy flavored mesons has been carried out following two rival research approaches: the short-distance and the long-distance programs. In this paper we compare their basic points and conclude that nonleptonic physics of heavy flavors is dominated by long-distance effects.

In the framework of the LD approach and by using the meson wave functions of the anisotropic chromodynamics theory (Sec. II), we evaluated the matrix elements and the form factors parametrizing the decay amplitudes involved in these processes (Secs. III and IV). We showed that the diagrams contributing to nonleptonic decays are separable in two classes: the disconnected and the connected ones. The diagrams of the first class (called also "spectators") were evaluated in a simple way by using the form factors just calculated; the latter were parametrized in a suitable way, the parameters being obtained by a fit of the experimental widths and then reemployed in the calculation of some other channels. In this way, we obtained a satisfactory determination (Secs. V and VI) of some exclusive nonleptonic decay widths of D, D_s, and B mesons, in very good agreement with the experimental data. A list of some semileptonic decays calculated in our model was also given.

Finally, we discussed (Sec. VII) the value of the color-suppression factor ξ in our model and in the Bauer-Stech-Wirbel one, and we concluded that $\xi=\frac{1}{3}$ must be its correct value.

We believe that our approach provides an extensive and powerful description of nonleptonic weak decays of heavy flavors, thus demonstrating also the correctness of the ACD meson wave functions in studying many important physical processes.

Note added in proof. We must stress that the Fierz transformation of the product of two currents [Eq. (3.3)], while completely straightforward in the short-distance program, in the long-distance approach, which we have followed in this paper, requires some care. Indeed, we write the weak nonleptonic Hamiltonian between two hadronic states as

$$\langle\beta|H_{\rm NL}|\alpha\rangle = ig^2\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2-M_W^2}\int\langle\beta|T(J_\mu(x)J^{+\mu}(0))|\alpha\rangle e^{iqx}d^4x$$
$$=g^2\int\frac{d^4q_E}{(2\pi)^4}\frac{1}{q_E^2+M_W^2}T_{\alpha\to\beta}(iq_{E_0},\mathbf{q}) \ ,$$

where the last step follows from Wick rotation to the Euclidean space and $T_{\alpha\to\beta}(iq_{E_0},\mathbf{q})$ is given by

$$T_{\alpha\to\beta}(iq_{E_0},\mathbf{q})=i\sum_n\left[\frac{\langle\beta|J_\mu(0)|n\rangle\langle n|J^{+\mu}(0)|\alpha\rangle}{E_\alpha-E_n-iq_{E_0}}+\frac{\langle\beta|J_\mu^+(0)|n\rangle\langle n|J^\mu(0)|\alpha\rangle}{E_\alpha-E_n+iq_{E_0}}\right] .$$

By integrating the denominators over q_{E_0} we obtain

$$\int_{-\infty}^{+\infty}\frac{dq_{E_0}}{(2\pi)}\frac{1}{E_\alpha-E_n-iq_{E_0}}=\frac{1}{2}\epsilon(E_\alpha-E_n) \ ,$$

showing that the sign of the contributions of the products of current matrix elements is positive for $E_\alpha-E_n>0$ and negative for $E_\alpha-E_n<0$.

In the long-distance program the sum is dominated by $E_n<E_\alpha$ ($n=K,K^*,K^{**}$ for D decays and $n=D,D^*$ for B decays) while in the short-distance program the opposite is true.

Thus the Fierz-transformation coefficients in Eq. (3.3) in the long-distance approach are *opposite* to what one (naively) computes in the short-distance approach.

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[1] K. Wilson, Phys. Rev. **179**, 1499 (1969).
[2] S. Nussinov and G. Preparata, Phys. Rev. **175**, 2180 (1968).
[3] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33**, 108 (1974).
[4] G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351 (1974).
[5] J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. **B100**, 313 (1975).
[6] N. Cabibbo and L. Maiani, Phys. Lett. **73B**, 418 (1978).
[7] G. Nardulli, G. Preparata, and D. Rotondi, Phys. Rev. D **27**, 557 (1983); G. Preparata, Phys. Lett. **34B**, 412 (1971).
[8] J. L. Basdevant and G. Preparata, Nuovo Cimento A **67**, 19 (1982); J. L. Basdevant, P. Colangelo, and G. Preparata, *ibid.* **71**, 445 (1982).
[9] G. Preparata, Phys. Lett. B **201**, 139 (1988); Nucl. Phys. **B279**, 235 (1987).
[10] G. Preparata, Nuovo Cimento A **103**, 1073 (1990).
[11] L. Cosmai, M. Pellicoro, and G. Preparata, Nucl. Phys. **B228**, 31 (1983); Phys. Lett. **121B**, 272 (1983).
[12] P. Cea, P. Colangelo, G. Nardulli, and G. Preparata, Phys. Lett. **115B**, 310 (1982).
[13] G. Preparata and M. Scorletti, in *Hadron '91,* Proceedings of the Conference, College Park, Maryland, 1991, edited by S. Oneda and D. C. Peaslee (World Scientific, Singapore, 1992).
[14] M. Gibilisco and G. Preparata, Phys. Lett. B **264**, 207 (1991).
[15] P. Castorina, P. Cea, P. Colangelo, G. Nardulli, and G. Preparata, Phys. Lett. **115B**, 487 (1982).
[16] J. C. Anjos *et al.*, Phys. Rev. Lett. **62**, 1587 (1989); J. Adler *et al.*, *ibid.* **62**, 1821 (1989).
[17] C. Dominguez, and N. Paver, Phys. Lett. B **207**, 499 (1988); T. M. Aliev, V. L. Eletskii, and Y. I. Kogan, Yad. Fiz. **40**, 823 (1984) [Sov. J. Nucl. Phys. **40**, 527 (1984)].
[18] M. Crisafulli, G. Martinelli, V. J. Hill, and C. T. Sachrajda, Phys. Lett. B **223**, 90 (1989); V. Lubicz, G. Martinelli, and C. T. Sachrajda, Nucl. Phys. **B356**, 301 (1991).
[19] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987); **29**, 637 (1985).
[20] A. A. Ovchinnikov and V. A. Slobodenyuk, Z. Phys. C **44**, 433 (1989).
[21] C. Dominguez and N. Paver, Z. Phys. C **41**, 217 (1988).
[22] R. H. Schindler, in *Weak Interactions and Neutrinos,* Proceedings of the International Workshop, Ginosar, Israel, 1989, edited by P. Singer and B. Gad Eilam [Nucl. Phys. B (Proc. Suppl.) **13**, 89 (1990)].
[23] Crystal Ball Collaboration, D. Antreasyan *et al.*, Z. Phys. C **48**, 553 (1990).
[24] J. C. Anjos *et al.*, Phys. Rev. Lett. **65**, 2630 (1990).
[25] N. Isgur and D. Scora, Phys. Rev. D **40**, 1491 (1989).
[26] M. Bauer and M. Wirbel, Z. Phys. C **42**, 671 (1989).
[27] F. J. Gilman and R. L. Singleton, Phys. Rev. D **41**, 142 (1990).
[28] G. Korner and G. A. Schuler, Z. Phys. C **38**, 511 (1988).
[29] WA82 Collaboration, M. Adamovich *et al.*, Phys. Lett. B **268**, 142 (1991).
[30] Mark III Collaboration, Z. Bai *et al.*, Phys. Rev. Lett. **66**, 1011 (1991).
[31] L3 Collaboration, B. Adeva *et al.*, Phys. Lett. B **270**, 111 (1992).
[32] L. Angelini, L. Nitti, M. Pellicoro, and G. Preparata, Nuovo Cimento A **99**, 45 (1988).
[33] J. C. Anjos *et al.*, Phys. Rev. Lett. **60**, 897 (1988).
[34] F. Buccella, M. Forte, G. Miele, and G. Ricciardi, Z. Phys. C **48**, 47 (1990).
[35] DELPHI Collaboration, P. Abreu *et al.*, Z. Phys. C **53**, 567 (1992).
[36] A. J. Buras, J. M. Gérard, and R. Rückl, Nucl. Phys. **B268**, 16 (1986).
[37] G. Preparata, Phys. Lett. B **193**, 312 (1987).
[38] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).
[39] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **44**, 7 (1980); W. Bernreuther, O. Nachtmann, and B. Stech, Z. Phys. C **4**, 257 (1980); H. Fritzsch and P. Minkowski, Phys. Lett. **90B**, 455 (1980).
[40] G. Preparata, in *Heavy Flavors,* Proceedings of the Conference, San Miniato, Italy, 1987, edited by F. L. Navarria [Nucl. Phys. B (Proc. Suppl.) **1B**, 403 (1987)].
[41] J. D. Bjorken, in *Proceedings of the 25th International Conference on High Energy Physics,* Singapore, 1990, edited by K. K. Phua and Y. Yamaguchi (World Scientific, Singapore, 1991), p. 1195.
[42] N. Isgur and M. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
[43] T. Mannel, W. Roberts, and Z. Ryzak, Phys. Lett. B **259**, 485 (1991); Nucl. Phys. **B355**, 38 (1991).