Global fit of $\pi\pi$ and πK elastic scattering in chiral perturbation theory with dispersion relations

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We apply the one-loop results of the $SU(3)_L \times SU(3)_R$ chiral perturbation theory supplemented with the inverse-amplitude method to fit the available experimental data on $\pi\pi$ and πK scattering. With essentially only three parameters we describe accurately data corresponding to six different channels: namely, $(I,J)=(0,0)$, $(2,0)$, $(1,1)$, $(\frac{1}{2},0)$, $(\frac{3}{2},0)$, and $(\frac{1}{2},1)$. In addition we reproduce the first resonances of the $(1,1)$ and $(\frac{1}{2},1)$ channel with the right mass corresponding to the ρ and the K^* (892) particles.

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INTRODUCTION

In 1979 Weinberg [1] suggested that it is possible to summarize many previous current algebra results in a phenomenological Lagrangian that incorporates all the constraints coming from the chiral symmetry of the strong interactions and QCD. This technique, also called chiral perturbation theory (ChPT), was developed some time later to the one-loop level and in great detail in a celebrated set of papers by Gasser and Leutwyler [2]. In these works the authors showed how it is possible to compute many different Green's functions involving low-energy pions and kaons as functions of the lowest powers of their momenta, their masses, and a few phenomenological parameters. In these and in further works, it was also shown that this method provides a good parametrization of many low-energy experimental data.

More recently, the strongly interacting symmetry-breaking sector of the standard model [3] has also been described phenomenologically using ChPT [4].

In both contexts one of the main obstacles found when one tries to apply ChPT to higher energies lies in the issue of unitarity. ChPT, being a consistent theory, is unitary in the perturbative sense. However, as the expansion parameters are the momenta and the Nambu-Goldstone boson masses, perturbative unitarity breaks down, sometimes even at moderate energies [5]. Different attempts to improve this behavior of ChPT and extend the applicability to higher energies have been proposed in the literature. These methods include the use of unitarization procedures such as the Padé expansion [5,6], the inverse-amplitude method [5], and the explicit introduction of more fields describing resonances [7]. All of these improve the unitarity behavior of the ChPT expansion and provide a more accurate description of the data, although there is some controversy about which of them is more appropriate.

In this paper we show the results of the application of the inverse-amplitude method to the ChPT one-loop computation of elastic $\pi\pi$ and πK scattering [2,8]. The obtained amplitude converges at low energies with that of standard ChPT, but it satisfies strictly elastic unitarity, since the contribution of the right cut in the corresponding dispersion relation is taken into account exactly in the inverse-amplitude method. Using this approach (which incidentally is connected with the formal [1,1] Padé approximant of the one-loop amplitudes), we will make a global fit of the data for elastic $\pi\pi$ and πK scattering just varying the L_1, L_2, and L_3 ChPT constants. The resulting values of these parameters will not be quite different from those obtained previously, but the range of energies and the quality of the fit will be enlarged quite amazingly.

PARTIAL WAVES IN ChPT

The elastic-scattering partial waves are defined from the corresponding isospin amplitude $T_I(s,t)$ as

$$t_{IJ}(s)=\frac{1}{32K\pi}\int_{-1}^{1} d(\cos\theta)P_J(\cos\theta)T_I(s,t)\ , \tag{1}$$

where $K=2$ or 1 depending on whether or not the two particles in the reaction are identical. For elastic $\pi\pi$ scattering, the possible isospin channels are $I=0,1,2$, while for πK we can have $I=\frac{1}{2}$ or $\frac{3}{2}$. In the first case, the isospin amplitudes T_I can be written in terms of a simple function $A(s,t,u)$ as follows:

$$T_0(s,t,u)=3A(s,t,u)+A(t,s,u)+A(u,t,s)\ ,$$

$$T_1(s,t,u)=A(s,t,u)-A(t,s,u)\ ,$$

and

$$T_2(s,t,u)=A(s,t,u)+A(t,s,u)\ .$$

In the second, the $I=\frac{1}{2}$ amplitude can be written as

$$T_{1/2}(s,t,u)=\tfrac{3}{2}T_{3/2}(u,t,s)-\tfrac{1}{2}T_{3/2}(s,t,u)\ .$$

The Mandelstam variables s,t,u satisfy $s+t+u=2(M_\alpha^2+M_\beta^2)$, where we use the notation α and $\beta=\pi$ or K, so that we can describe both processes with the same general formulas. For $s>s_{\rm th}=(M_\alpha+M_\beta)^2$, the partial waves t_{IJ} can be parametrized as

$$t_{IJ}(s)=e^{i\delta_{IJ}(s)}\sin\delta_{IJ}(s)/\sigma_{\alpha\beta}(s)\ , \tag{2}$$

where

$$\sigma_{\alpha\beta}(s)=\left[\left[1-\frac{(M_\alpha+M_\beta)^2}{s}\right]\left[1-\frac{(M_\alpha-M_\beta)^2}{s}\right]\right]^{1/2}. \tag{3}$$

The $t_{IJ}(s)$ amplitude satisfies the elastic unitarity condition

$$\mathrm{Im}t_{IJ}=\sigma_{\alpha\beta}|t_{IJ}|^2 \tag{4}$$

on the right cut.

Using standard one-loop ChPT, it is possible to compute the above scattering amplitudes to order p^4 (as it is customary, any Mandelstam variable M_π^2 and M_K^2 will be considered of the order of p^2). The relevant functions $A(s,t,u)$ and $T_{3/2}$ were computed in [2,8]:

$$\begin{aligned}
A(s,t,u)&=(s-M_\pi^2)/F_\pi^2+B(s,t,u)+C(s,t,u)+O(E^6)\ ,\\
B(s,t,u)&=\frac{1}{F_\pi^4}\left[\frac{M_\pi^4}{18}J^r_{\eta\eta}(s)+\frac{1}{2}(s^2-M_\pi^4)J^r_{\pi\pi}(s)+\frac{1}{8}s^2J^r_{KK}(s)\right.\\
&\quad\left.+\frac{1}{4}(t-2M_\pi^2)^2J^r_{\pi\pi}(t)+t(s-u)\left[M^r_{\pi\pi}(t)+\frac{1}{2}M^r_{KK}(t)\right]+(t\leftrightarrow u)\right]\ ,\\
C(s,t,u)&=\frac{4}{F_\pi^4}\{(2L_1^r+L_3)(s-2M_\pi^2)^2+L_2^r[(t-2M_\pi^2)^2+(u-2M_\pi^2)^2]\\
&\quad+(4L_4^r+2L_5^r)M_\pi^2(s-2M_\pi^2)+(8L_6^r+4L_8^r)M_\pi^4\}\ ,
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
T^{3/2}(s,t,u)&=\frac{M_\pi^2+M_K^2-s}{2F_\pi^2}+T_4^T(s,t,u)+T_4^P(s,t,u)+T_4^U(s,t,u)\ ,\\
T_4^T(s,t,u)&=\frac{1}{16F^2}(\mu_\pi[10s-7M_\pi^2-13M_K^2]+\mu_K[2M_\pi^2+6M_K^2-4s]+\mu_\eta[5M_\pi^2+7M_K^2-6s])\ ,\\
T_4^P(s,t,u)&=\frac{2}{F_\pi^2F_K^2}\{4L_1^r(t-2M_\pi^2)(t-2M_K^2)+2L_2^r[(s-M_\pi^2-M_K^2)^2+(u-M_\pi^2-M_K^2)^2]\\
&\quad+L_3^r[(u-M_\pi^2-M_K^2)^2+(t-2M_\pi^2)(t-2M_K^2)]+4L_4^r[t(M_\pi^2+M_K^2)-4M_\pi^2M_K^2]\\
&\quad+2L_5^rM_\pi^2(M_\pi^2-M_K^2-s)+8(2L_6^r+L_8^r)M_\pi^2M_K^2\}\ ,\\
T_4^U(s,t,u)&=\frac{1}{4F_\pi^2F_K^2}[t(u-s)[2M^r_{\pi\pi}(t)+M^r_{KK}(t)]\\
&\quad+\tfrac{3}{2}\{(s-t)(L_{\pi K}(u)+L_{K\eta}(u)-u[M^r_{\pi K}(u)+M^r_{K\eta}(u)])\\
&\quad+(M_K^2-M_\pi^2)[M^r_{\pi K}(u)+M^r_{K\eta}(u)]\}+J^r_{\pi K}(s)(s-M_K^2-M_\pi^2)^2\\
&\quad+\tfrac{1}{2}(M_K^2-M_\pi^2)[K_{\pi K}(u)(5u-2M_K^2-2M_\pi^2)+K_{K\eta}(u)(3u-2M_K^2-2M_\pi^2)]\\
&\quad+\tfrac{1}{8}J^r_{\pi K}(u)[11u^2-12u(M_K^2+M_\pi^2)+4(M_K^2+M_\pi^2)^2]+\tfrac{3}{8}J^r_{K\eta}(u)[u-\tfrac{3}{2}(M_K^2+M_\pi^2)]^2\\
&\quad+\tfrac{1}{2}J^r_{\pi\pi}(t)t(2t-M_\pi^2)+\tfrac{3}{4}J^r_{KK}(t)t^2+\tfrac{1}{2}J^r_{\eta\eta}(t)M_\pi^2(t-\tfrac{8}{9}M_K^2)]\ .
\end{aligned} \tag{6}$$

The masses M_α and the decay constant F_α appearing in these equations are the physical values. The relation with the corresponding constants appearing in the chiral Lagrangian and the functions μ_α can be found in [8]. The transcendental functions $M^r_{\alpha\beta}$, $L^r_{\alpha\beta}$, and $J^r_{\alpha\beta}$ are defined in [2]. The first terms in the above amplitudes reproduce the well-known Weinberg low-energy theorems. The L_i^r constants can be considered as phenomenological parameters that up to constant factors are the renormalized coupling constants of the chiral Lagrangian renormalized conventionally at the m_η scale. Their relation with the corresponding bare constants L_i and their evolution with the renormalization scale can be found in [2]. Using Eqs. (1) and (5), it is possible to obtain the corresponding partial-wave amplitudes. In the general framework of ChPT they can be obtained as a series with increasing number of p^2 powers: i.e.,

$$t_{IJ}=t_{IJ}^{(0)}+t_{IJ}^{(1)}+\cdots\ , \tag{7}$$

where $t_{IJ}^{(0)}$ is of order p^2 and corresponds to the low-energy theorem and $t_{IJ}^{(1)}$ is of order p^4. In general, the real part of $t_{IJ}^{(1)}$ cannot be expressed in terms of elementary functions, but it can be computed numerically. The amplitudes in Eq. (5) have been used in the literature, without further elaboration, to fit the low-energy $\pi\pi$ and πK scattering data [2,8,9].

DISPERSION RELATIONS AND THE INVERSE-AMPLITUDE METHOD

One very important point concerning the partial-wave amplitudes computed from ChPT to one loop is the fact that they have the appropriate cut structure, namely, the left cut and the right or unitarity cut. However, they only satisfy the unitarity condition on the right cut in a perturbative sense, i.e.,

$$\mathrm{Im}t_{IJ}^{(1)}=\sigma_{\alpha\beta}|t_{IJ}^{(0)}|^2 \ . \tag{8}$$

Let us show now how, with the use of dispersion theory, it is possible to build up a completely unitarized amplitude for the $\pi\pi$ and πK amplitudes starting from the one-loop ChPT result above. Let us start writing a three subtracted dispersion relation for the partial wave t_{IJ}:

$$t_{IJ}(s)=C_0+C_1 s+C_2 s^2+\frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty}\frac{\mathrm{Im}t_{IJ}(s')ds'}{s'^3(s'-s-i\epsilon)} +X_{\mathrm{LC}}(t_{IJ}) \ , \tag{9}$$

where $X_{\mathrm{LC}}(t_{IJ})$ represent the left-cut contribution. In particular, this equation is satisfied by the standard one-loop ChPT result. Note that three subtractions are needed to ensure the convergence of the integrals in this approximation since the one-loop ChPT amplitudes are second-order polynomials modulo log factors (higher-order ChPT amplitudes require more subtractions, but we do not know how many are needed for the exact result). To the one-loop level, the imaginary part of the amplitude can be written on the right-cut integral as $\mathrm{Im}t_{IJ}\simeq\mathrm{Im}t_{IJ}^{(1)}=\sigma t_{IJ}^{(0)2}$ (from the left cut, there is a generic $t_{IJ}^{(1)}$ contribution).

The subtraction terms appearing in the above dispersion relation can be expanded in powers of M_α^2, and they have contributions from $t_{IJ}^{(0)}$ and from $t_{IJ}^{(1)}$ that can be written as $a_0+a_1 s$ and $b_0+b_1 s+b_2 s^2$ and also from higher-order terms of the ChPT series, i.e., $C_0=a_0+b_0+\cdots$ and $C_1=a_1+b_1+\cdots$. The a and b constants depend also on the L_i^r parameters. Then, in the one-loop ChPT approximation, the exact amplitude $t(s)$ can be written as

$$t_{IJ}(s)\simeq t_{IJ}^{(0)}(s)+t_{IJ}^{(1)}(s) \ , \tag{10}$$

where

$$\begin{aligned} t_{IJ}^{(0)}&=a_0+a_1 s \ , \\ t_{IJ}^{(1)}&=b_0+b_1 s+b_2 s^2 \\ &\quad+\frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty}\frac{\sigma t_{IJ}^{(0)}(s')^2 ds'}{s'^3(s'-s-i\epsilon)}+X_{\mathrm{LC}}(t_{IJ}) \ , \end{aligned} \tag{11}$$

where in the left-cut contribution we have to integrate here $\mathrm{Im}t_{IJ}^{(1)}(s)$ as an approximation to $t_{IJ}(s)$. In some sense we can understand Eqs. (9)–(11) in a rather different way that will be useful later: We can assume that the dispersion relation in Eq. (9) is satisfied by the exact amplitude $t(s)$, and we solve this equation approximately by introducing inside the left- and right-cut integrals the one-loop prediction for $\mathrm{Im}t(s)$ to find again Eq. (10). However, we would like to stress again that this result does not satisfy the elastic unitarity condition in Eq. (4) but only the perturbative version in Eq. (8). In fact, this happens to any order in the ChPT expansion since a polynomial can never satisfy Eq. (4). However, there are other ways to use the information contained in the ChPT series apart from the direct comparison with the experiment of the truncated series. Instead of using the dispersion relation for the loop ChPT amplitude, we can try a three subtracted dispersion relation for the inverse of t_{IJ} or more exactly for the auxiliary function $G(s)=t_{IJ}^{(0)2}/t_{IJ}$: namely,

$$G(s)=G_0+G_1 s+G_2 s^2+\frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty}\frac{\mathrm{Im}G(s')ds'}{s'^3(s'-s-i\epsilon)} +X_{\mathrm{LC}}(G)+X_{\mathrm{PC}} \ , \tag{12}$$

where $X_{\mathrm{LC}}(G)$ is the left-cut contribution and X_{PC} is the pole contribution that eventually could appear due to possible zeros of $t_{IJ}(s)$. Now, on the right cut, we have

$$\begin{aligned} \mathrm{Im}G&=t_{IJ}^{(0)2}\,\mathrm{Im}(1/t_{IJ}) \\ &=-t_{IJ}^{(0)2}\,\mathrm{Im}t_{IJ}/|t_{IJ}|^2=-t_{IJ}^{(0)2}\sigma \ . \end{aligned}$$

This means that the right-cut integral appearing in the dispersion relation for G is the same as that appearing in Eqs. (10) and (11) for the one-loop dispersion relation for t, and hence it can be obtained from the one-loop result. The left-cut integral of the dispersion relation in Eq. (12) cannot be computed exactly, but we can use the one-loop ChPT result to write

$$\mathrm{Im}G=-t_{IJ}^{(0)2}\,\mathrm{Im}t_{IJ}/|t_{IJ}|^2\simeq-\mathrm{Im}t_{IJ}^{(1)} \ .$$

In addition, the subtraction constants can be expanded in terms of M_α^2/F_β^2 powers so that $G_0=a_0-b_0+O((M_\alpha^2/F_\beta^2)^3)$, $F_\alpha^2 G_1=a_1-b_1+O((M_\alpha^2/F_\beta^2)^2)$, and $F_\alpha^2 F_\beta^2 G_2=-b_2+O(M_\alpha^2/F_\beta^2)$. Therefore, neglecting the pole contribution, the dispersion relation in Eq. (12) can be written approximately as

$$\frac{t_{IJ}^{(0)2}}{t_{IJ}}\simeq a_0+a_1 s-b_0-b_1 s-b_2 s^2 -\frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty}\frac{\sigma t_{IJ}^{(0)2}(s')ds'}{s'^3(s'-s-i\epsilon)}+X_{\mathrm{LC}}(G) \ , \tag{13}$$

with $X_{\mathrm{LC}}(G)$ computed with the $\mathrm{Im}G$ approximated by $-\mathrm{Im}t_{IJ}^{(1)}$ or, in other words,

$$\frac{t_{IJ}^{(0)2}}{t_{IJ}}\simeq t_{IJ}^{(0)}-t_{IJ}^{(1)} \tag{14}$$

or, what it is the same,

$$t_{IJ}\simeq\frac{t_{IJ}^{(0)2}}{t_{IJ}^{(0)}-t_{IJ}^{(1)}} \ . \tag{15}$$

Remarkably, to derive this result the one-loop ChPT approximation has been used only inside the left-cut integral but not inside the right-cut integral, which was computed exactly. This is in contrast with the one-loop

ChPT result in Eq. (10) when considered from the point of view of the dispersion relation that t_{IJ} has to satisfy. In this case, in addition to the above approximations used to derive Eq. (15), we have done a much stronger one concerning the right-cut contribution since the $\mathrm{Im}t_{IJ}$ was also approximated by the one-loop result. This fact is crucial because the right-cut contribution is responsible for the very strong rescattering effects present in these reactions. As a consequence of that, Eq. (15) satisfies the unitarity condition $\mathrm{Im}t_{IJ}=\sigma|t_{IJ}|^2$ exactly and not only perturbatively as it was the case of Eq. (10). Of course, both results [Eqs. (10) and (15)] provide the same answers at low energies, but it is expected that Eq. (15) will provide more realistic results at higher energies and this is in fact what we find when comparing with the experimental data.

When zeros are present in the partial waves, they need to be included and the final result is not so simple as in Eq. (15). This can be done taking into account their contribution to Eq. (12) or just making a new subtraction in these points. However, in practical cases, it can happen that the corresponding residues are very small and Eq. (15) is still a good approximation outside the region around the position of these zeros. This is, for instance, the case of the $I=J=0$ and $I=2,J=0$ channels [6] which have zeros close to $M_\pi^2/2$ and $2M_\pi^2$ respectively. However, for a given channel IJ there is no way to know *a priori* if zeros will be present or not since ChPT only provides a low-energy expansion of $t(s)$ and not the whole amplitude. Nevertheless, in practice, a simple inspection of the low-energy behavior of the amplitudes can make sensible the hypothesis that zeros are not present in the low-energy region, but of course this must not be necessarily true for all the channels.

We could also ask about the possibility of extending the above method to higher orders of the ChPT series, for instance, to the two-loop computation. In principle, this can be done in a straightforward way. We start writing a four subtracted dispersion relation for the two-loop amplitude since it is a third-order polynomial modulo logarithms. As in the one-loop case, this dispersion relation can be interpreted as an approximation to the exact amplitude t_{IJ}. Then we write a four subtracted dispersion relation for the auxiliary function $G=t_{IJ}^{(0)2}/t_{IJ}$. The integrals of this dispersion relation can be related modulo higher-order terms to the ChPT by approaching $\mathrm{Im}G$ by its low-energy expansion. The subtraction constants can be treated similarly as we did in the one-loop case, and finally, we arrive at

$$t_{IJ}\simeq\frac{t_{IJ}^{(0)2}}{t_{IJ}^{(0)}-t_{IJ}^{(1)}+t_{IJ}^{(1)2}/t_{IJ}^{(0)}-t_{IJ}^{(2)}}\ . \tag{16}$$

It is very easy to show that this amplitude, which incidentally corresponds to the formal [1,2] Padé approximant, satisfies the elastic unitarity condition in Eq. (4), but in this case the right-cut integral appearing in the dispersion relation for G cannot be computed exactly as it was in the one-loop case. Unfortunately, as there is not any two-loop computation available, we cannot confront the above equation with the experimental data.

DETERMINATION OF THE CHIRAL PARAMETERS AND DISCUSSION

First of all, we have used the set of low-energy chiral parameters L_i proposed in [10] for $i=1,2,3$ and those give in [2] for $i=4-8$, which will not be changed in this work. Applying those values to the proposed unitarized partial waves of Eq. (15), we have found the appropriate qualitative behavior for both reactions. Moreover, we find two resonances in the partial-wave amplitudes corresponding to the (1,1) and $(\frac{1}{2},1)$ channels. These resonances can be identified naturally as the ρ and K^* particles. The phase shifts cross the $\pi/2$ value in their corresponding channels. In addition, in the chiral limit where an analytical computation of the partial waves is possible, two poles are found in the second Riemann sheet according to these resonances. In the other channels, where no physical resonances exist below 1 GeV, they do not appear in our numerical results. We consider this fact as strong support for the use of the unitarized partial waves in Eq. (15).

To fit completely the data with these formulas, the next step has been to tune the (1,1) $\pi\pi$ channel to give the correct mass for the ρ resonance. Since this partial wave is almost only sensible to the relation $L_3+2L_1^r-L_2^r$, then setting the ρ mass to 774 MeV means fixing a value of this special combination of parameters.

Nevertheless, there are still two degrees of freedom. Slightly varying the initial values, we can fit the $\pi\pi$ (2,0) and (0,0) channels, thus obtaining L_1^r and L_2^r. Unfortunately, the experimental data coming from these channels allow some small uncertainty in the parameters, rending a K^* mass between 850 and 950 MeV. So we use the mass of this resonance for a further parameter tuning, and finally we obtain, at the renormalization

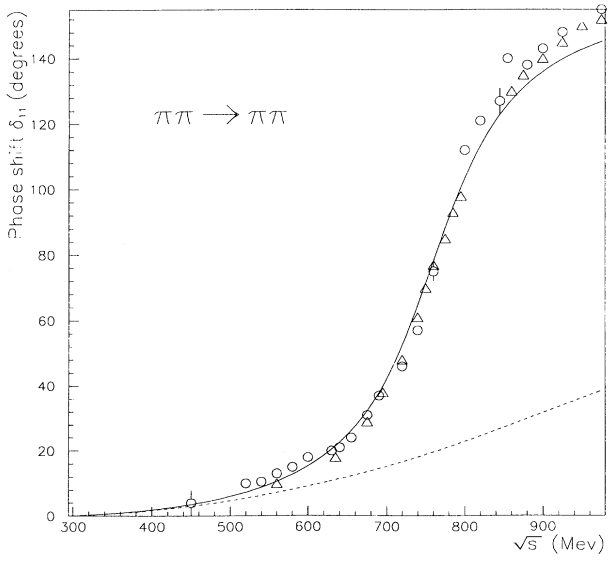

FIG. 1. (1,1) phase shift for $\pi\pi$ scattering. The solid line corresponds to our fit using Eq. (15). The dashed line is the result coming from nonunitarized ChPT with the $\bar{l}_i$ parameters proposed in [11]. The experimental data come from [12] (○) and [13] (△).

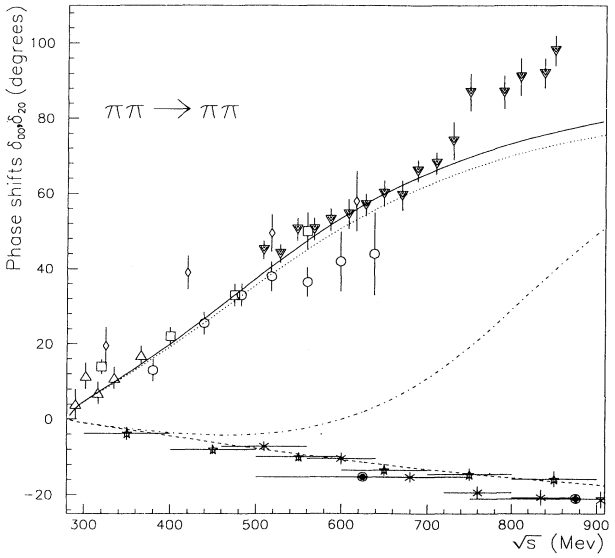

FIG. 2. Phase shift for $\pi\pi$ scattering. The results coming from the fit proposed in this paper [Eq. (15)] are shown as the solid line which represents the (0,0) phase shift and the dashed line which corresponds to that of (2,0). The dotted and dot-dashed lines are the (0,0) and (2,0) phase shifts, respectively; they were obtained with nonunitarized ChPT and the parameters given in [11]. The experimental data correspond to [14] (△), [15] (○), [16] (□), [17] (◇), [12] (▽), [18] (*), [19] (×), and [20] (●).

scale $\mu = M_\eta = 548.8$ MeV,

$$L_1^r = 0.61\times10^{-3}\ ,\quad L_2^r = 1.61\times10^{-3}\ ,$$
$$L_3 = -3.80\times10^{-3}\ . \tag{17}$$

These values are well inside the errors quoted in [10]:

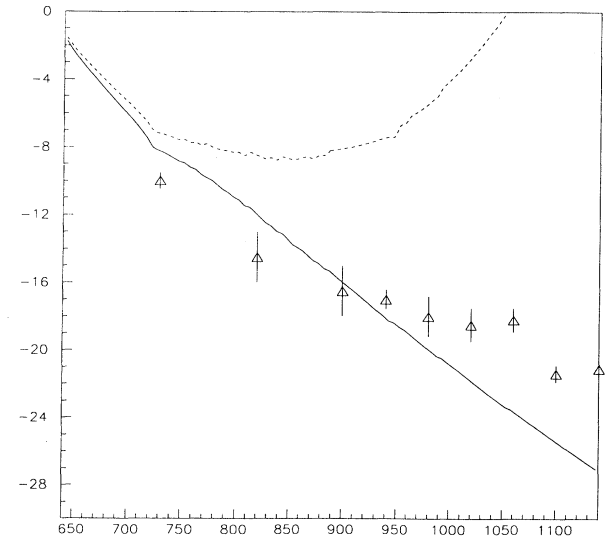

FIG. 3. Phase shift of the $(\frac{3}{2},0)$ channel for πK scattering. The solid line is the result of the inverse-amplitude method [Eq. (15)] with the parameters proposed in this paper, whereas the dashed line is nonunitarized ChPT with the parameters proposed in [2] and [10]. Data correspond to Ref. [21].

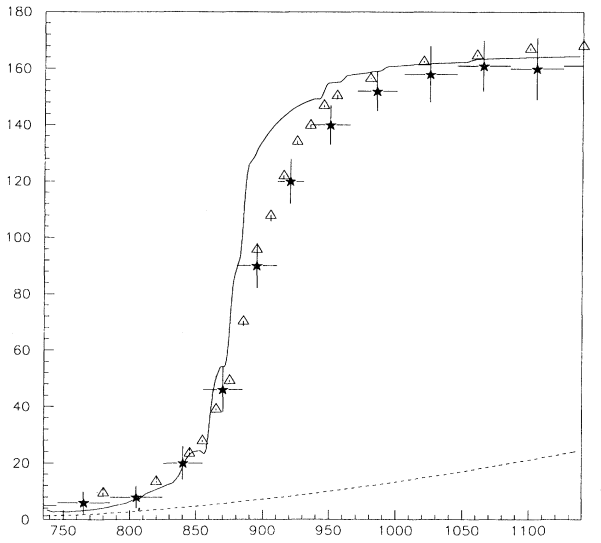

FIG. 4. Same as Fig. 3 but for the $(\frac{1}{2},1)$ partial wave. Data comes from [21] (△) and [22] (*).

$$L_1^r = (0.88\pm0.47)\times10^{-3}\ ,$$
$$L_2^r = (1.61\pm0.38)\times10^{-3}\ , \tag{18}$$
$$L_3 = (-3.62\pm1.31)\times10^{-3}\ .$$

The K^* mass thus obtained is 800 MeV. In Figs. 1 and 2 can be seen the results of this global fit for $\pi\pi$ scattering and those from nonunitarized ChPT with the $\bar{l}_i$ parameters proposed in [11]. Figures 3–5 represent our global fit curves (solid lines) for πK scattering in three different channels. The dashed lines are the nonunitarized ChPT predictions from the L_i given in [2] and [10].

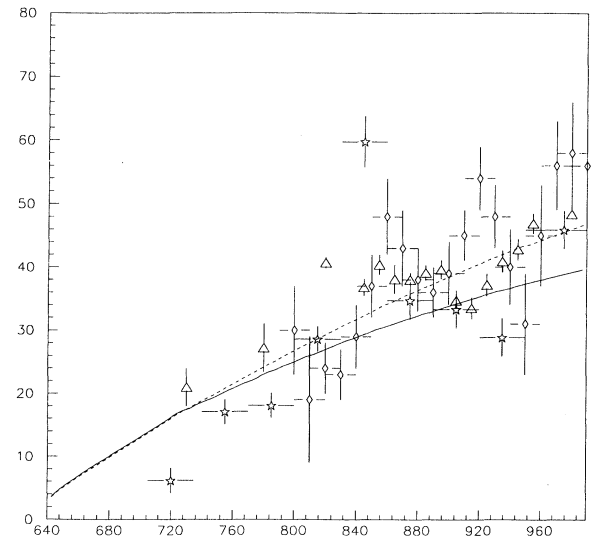

FIG. 5. As in Fig. 3 but for the $(\frac{1}{2},0)$ channel. The experimental data correspond to [21] (△), [23] (◇), and [24] (*).

Figure 3 is the $(\frac{3}{2},0)$ channel, and Figs. 4 and 5 are the $(\frac{1}{2},1)$ and $(\frac{1}{2},0)$ channels, respectively.

CONCLUSIONS

The inverse-amplitude method applied to the one-loop result coming from ChPT produces a simple way to unitarize the Goldstone-boson elastic-scattering amplitudes, which takes into account, exactly, the strong rescattering effects. Incidentally, this method is formally equivalent to the [1,1] Padé approximant applied to the one-loop ChPT result, provided that the exact partial-wave amplitude has no zeros in the first sheet. Note that since the one-loop result is not, strictly speaking, a polynomial, the equivalence is only formal (this is not the case if one considers the amplitudes as polynomials in $1/F_\pi^2$).

The unitarized amplitudes (with the previously fitted parameters for the standard one-loop ChPT result) give rise to the appearance of two resonances in the (1,1) and $(\frac{1}{2},1)$ channels that have to be understood as the ρ and K^*, but not any more are found in other channels where no physical resonances exist below 1 GeV. Therefore the existence of these resonances is a highly nontrivial prediction of the approach followed here and they do not need to be introduced by hand in the data fit. Just tuning slightly only three parameters, namely, L_1, L_2, and L_3, we are able to obtain the right value for the masses of the ρ and K^* resonances. In addition, we provide a fit for six channels in remarkable agreement with the experiment.

The results obtained in this and previous works strongly suggest that the range of validity of the one-loop ChPT can be enlarged by a consistent treatment of the analyticity and unitarity constraints summarized in the dispersion relations, but a great deal of work still remains to be done in this direction. As a final comment, we think that the recently proposed large-N approximation to ChPT (with N being the number of Nambu-Goldstone bosons) [25] (see also [26]) could provide a new extension of the one-loop ChPT results (see also [26] for applications to the $\gamma\gamma \to \pi\pi$ reaction [27]).

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