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Quantization of massive chiral electrodynamics reexamined

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We show that the models considered by Andrianov *et al.* [Phys. Rev. Lett. **63**, 1554 (1989); and Phys. Rev. D **44**, 2602 (1991)] are equivalent to other models where it is easily proved that the anomaly decouples and consequently the value of the chiral triangle amplitude is irrelevant for the unitarity of the S matrix.

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Recently Andrianov *et al.* [1, 2] have criticized models for chiral fermions coupled to a massive vector field based on different mechanisms for the anomaly cancellation. In particular, they mentioned the models presented in Refs. [3, 4], where the functional integral is either defined by constraining the vector field to be transversal [3] or based on a gauge group functional integration [4]. They assert that in these models unitarity is jeopardized by the radiative corrections.

In Ref. [1] they introduce what they claim is a new model which gives place to an unusual Becchi-Rouet-Stora-Tyutin (BRST) symmetry, with a charge Q satisfying $Q^3 = 0$ instead of $Q^2 = 0$. They refer to the BRST procedure to prove the unitarity of the S matrix, but in such a case this is not possible in a direct way. The order-three nilpotency of Q implies that there is not only a quartet but also a sextet of nonphysical states. This spoils the usual proof of unitarity, and its discussion following this line becomes very cumbersome. In fact, they do not prove unitarity in a convincing way.

In Ref. [2] they propose an alternative model, and on its basis they reexamine unitarity. They claim that it is corrupt because of the chiral triangle amplitude, when computed with standard ultraviolet regulators. This misleading interpretation is induced because they are using the propagator for the transverse vector field, although the expression they consider for the triangle amplitude actually corresponds to the vertex for the interaction with the complete vector field. As we will discuss later, the actual value of the triangle is irrelevant for the proof of unitarity.

The aim of this Comment is to clarify the subject. The starting classical Lagrangian is

$$\mathcal{L}_0 = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\not\partial + ie \not A P_L)\psi + \tfrac{1}{2}m^2 A_\mu A^\mu \ , \quad (1)$$

where $P_L = \frac{1}{2}(1+\gamma_5)$. Introducing the decomposition of the vector field in transversal and longitudinal components,

$$A^\mu = A^\mu_T + A^\mu_L \equiv A^\mu_T + \partial^\mu \Theta \ , \quad (2)$$

where

$$\partial \cdot A_T = 0, \quad \Box\Theta = \partial \cdot A \ , \quad (3)$$

we can write

$$\mathcal{L}_0 = -\tfrac{1}{4}F_{T\mu\nu}F_T{}^{\mu\nu} + \bar{\psi}[\not\partial + ie(\not A_T + \not\partial\Theta)P_L]\psi + \tfrac{1}{2}m^2 A_{T\mu}A_T{}^\mu + \tfrac{1}{2}m^2\partial_\mu\Theta\partial^\mu\Theta \ . \quad (4)$$

The equations of motion are

$$\partial_\mu F_T{}^{\mu\nu} + m^2 A_T{}^\nu = -J^\nu_L \ , \quad (5)$$

$$[\not\partial + ie(\not A_T + \not\partial\Theta)P_L]\psi = 0 \ , \quad (6)$$

$$m^2\Box\Theta = -\partial_\mu J^\mu_L \ , \quad (7)$$

where $J^\mu_L = ie\bar{\psi}\gamma^\mu P_L\psi$. The coupling of the physically interesting sector $(A_T, \psi, \bar{\psi})$ with the anomaly is through the scalar field Θ.

The key of the proposed approach is to introduce an additional scalar ghost field η whose role is to decouple the anomaly from the sector of fermions and transversal vector fields. To obtain such an effect they use two constructions, which give place to exactly the same S matrix for the fields $(A_T, \psi, \bar{\psi})$, as we will show in the following.

In Ref. [1] they add to $\mathcal{L}_0$ the Lagrangian

$$\mathcal{L}^{\rm I}_\eta = -\tfrac{1}{2}m^2(\partial\eta)^2 + \eta\partial\cdot J_L \ , \quad (8)$$

and in Ref. [2] they use, instead,

$$\mathcal{L}^{\rm II}_\eta = -m^2\eta\Box\Theta = m^2(\partial\eta)\cdot(\partial\Theta) \ . \quad (9)$$

Both Lagrangians, up to a divergence, are members of the single-parameter family

$$\mathcal{L} = \mathcal{L}_0 + \tfrac{1}{2}m^2\left[(b^2-1)(\partial\eta)^2 + 2b(\partial\eta)\cdot(\partial\Theta)\right] + (1-b)\partial\cdot J_L \ . \quad (10)$$

The model considered in Ref. [1] corresponds to $b = 0$, and the one of Ref. [2] to $b = 1$. In every case the transformation $\Theta \to \lambda - b\eta$ leads to the Lagrangian

$$\mathcal{L} = \mathcal{L}_0(A_T) + \tfrac{1}{2}m^2[(\partial\lambda)^2 - (\partial\eta)^2] + (\eta - \lambda)\partial\cdot J_L \ , \quad (11)$$

or introducing $\sigma = \lambda + \eta$ and $\varphi = \lambda - \eta$ [2], to

$$\mathcal{L} = \mathcal{L}_0(A_T) + \tfrac{1}{2}m^2[(\partial\sigma)\cdot(\partial\varphi) - \varphi\partial\cdot J_L] \ . \quad (12)$$

It is easy to see what happens to the anomaly at the level of the equations of motion:

$$\partial_\mu F_T{}^{\mu\nu} + m^2 A^\nu_T = -J^\nu_L \ , \quad (13)$$

$$[\not\partial + ie(A_T + \not\partial\varphi)P_L]\psi = 0\ , \tag{14}$$

$$m^2\Box\varphi = 0, \tag{15}$$

$$m^2\Box\sigma = -2\partial\cdot J_L\ . \tag{16}$$

It only acts as a source for the scalar field σ, which is decoupled from the remaining fields.

They have not realized that both versions of the model are equivalent to the one discussed in Ref. [3], as is apparent from the Lagrangian (1) in Ref. [2]. Furthermore, when they assert in this paper that the theory is nonunitary their argument is based on a inconsistent treatment of the model. They are using the propagators corresponding to the transversal vector field, but the expression for the triangle anomaly is calculated using the complete vector field [5].

Turning now to quantum theory, Demarco *et al.* [6] proved that in the theory defined by the Lagrangian (12) the anomaly is also decoupled from the physical fields. Moreover, because of the BRST treatment the equations of motion in the physical subspace are manifestly stable under renormalization. In addition, we can easily prove that this Lagrangian is equivalent to the nonlocal model proposed by Thompson *et al.* [4], as we show in the following.

The generating functional for Green's functions is

$$\mathcal{Z}[j^\mu,\eta,\bar\eta] = \int \mathcal{D}A\ \mathcal{D}\bar\psi\ \mathcal{D}\psi\ \mathcal{D}\eta \exp\left\{i\int d^4x(\mathcal{L} + j\cdot A + \bar\eta\psi + \bar\psi\eta)\right\}\ . \tag{17}$$

Decomposing the vector field in longitudinal and transversal components, where

$$A_L{}^\mu = \partial^\mu\frac{1}{\Box}\partial\cdot A,\quad A_T{}^\mu = A^\mu - \partial^\mu\frac{1}{\Box}\partial\cdot A\ , \tag{18}$$

we have

$$\mathcal{Z}[j^\mu,\xi,\bar\xi] = \int \mathcal{D}A_T\ \mathcal{D}\Theta\ \mathcal{D}\bar\psi\ \mathcal{D}\psi\ \mathcal{D}\eta\ \exp\Big\{i\int d^4x[\,\mathcal{L}_0(A_T) + \frac{1}{2}m^2(\partial\Theta)^2 + m\eta\Box\Theta - \bar\psi e\ \not\partial\Theta P_L\psi + j\cdot A_T + j\cdot\partial\Theta + \bar\xi\psi + \bar\psi\xi]\Big\}\ . \tag{19}$$

If we now use $\delta(\Box\Theta) = \det^{-1}\Box\ \delta(\Theta)$ and perform the integration over Θ, only $\mathcal{L}_0(A_T)$ remains, which is the Lagrangian considered by Thompson and Zhang [4]. As they demonstrated, it has the gauge symmetry

$$\delta A_\mu = \partial_\mu\omega,\quad \delta\psi = 0,\quad \Delta\bar\psi = 0\ . \tag{20}$$

This is a manifestation of the fact quoted by Konopleva and Popov [7]: if the Lorentz condition is satisfied the massive theory has a gauge invariance (even if it is a non-Abelian one).

Although it is possible to compute the propagator

$$\langle A_\mu A_\nu\rangle = -i\int\frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot(x-y)}}{k^2-m^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\ , \tag{21}$$

it is rather difficult to prove the unitarity of the theory, because there is a nonlocal interaction involving the time coordinate [8]:

$$\mathcal{L}_{\rm int} = -eJ\cdot A_T = -eJ\cdot A - e\partial\cdot J\frac{1}{\Box}\partial\cdot A\ . \tag{22}$$

When we have a nonanomalous theory the nonlocal term is irrelevant. We can always perform a fermion transformation

$$\psi \to e^{ie\frac{\partial\cdot A}{\Box}P_L}\psi,\quad \bar\psi \to \bar\psi e^{-ie\frac{\partial\cdot A}{\Box}P_L}\ , \tag{23}$$

with a Jacobian one, which cancels this term. In our case this transformation induces an anomalous Jacobian and the nonlinearity persists. Its only effect is to replace

$$\partial\cdot J\frac{1}{\Box}\partial\cdot A \to G(A)\frac{1}{\Box}\partial\cdot A\ , \tag{24}$$

where $G(A)\propto\epsilon_{\mu\nu\sigma\tau}F^{\sigma\tau}F^{\mu\nu}$ is the anomalous divergence of the chiral current J_L. This last nonlocal form of the theory has a gauge symmetry defined by the transformations

$$A_\mu \to A_\mu + \partial_\mu\omega,\quad \psi\to e^{-i\omega P_L}\psi,\quad \bar\psi\to\bar\psi e^{-i\omega P_L}\ . \tag{25}$$

To analyze the unitarity of the theory it is convenient to use a local version for $\mathcal{L}$. This can be achieved in several ways. For example we can use

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{2}m^2[(\partial\Theta)^2 - A\cdot\partial\Theta] + \alpha(\partial\cdot A - \Box\Theta) - e\Theta G(A) + \frac{b^2}{2\lambda} + b\partial\cdot A - i\partial_\mu\bar c\partial^\mu c\ , \tag{26}$$

which corresponds to the Faddeev-Popov construction, or alternately

$$\mathcal{L} = \mathcal{L}_0 - \frac{\lambda}{2}(\partial\cdot A)^2 + \Theta(\partial\cdot A - \Box\phi) - e\phi G(A)\ , \tag{27}$$

which reproduces the nonlocal term when we integrate with respect to Θ and ϕ, and generates a Stueckelberg-type term. For both versions a BRST treatment exists that proves the unitarity of the theory (see Refs. [9] and [6], respectively).

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