How constant is the Fermi coupling constant?

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We discuss various astrophysical limits on the spatial and time variation of the Fermi coupling constant G_F. We consider two cases: (a) G_F and the fermion masses vary through a change in the vacuum expectation value of the Higgs field; (b) G_F varies while the fermion masses are held constant. In the former case, redshift measurements probe both the spatial and time variation of G_F through changes in the electron mass: the agreement between measurements of hyperfine and optical lines in distant galaxies and quasars indicates that G_F varies by less than 0.04% on cosmological length scales. Such measurements also show that G_F varies by less than 0.2% back to a redshift of $z=3.4$. If G_F varies without any change in the fermion masses, the best constraints on spatial variations in G_F come from supernova light curves, whose slopes depend upon the lifetime of ^{56}Co. The similarities between light curves argue that the Fermi coupling constant G_F varies by less than 5% on cosmological scales. Big bang nucleosynthesis indicates that the Fermi coupling constant at $t\sim 1$ sec differed by less than $\sim 10-20\,\%$ from the contemporary terrestrial value, with the exact limits depending on which model we choose for the variation in G_F. Variation in G_F would allow big bang nucleosynthesis to produce a lower ^{4}He abundance without changing significantly any of the other element abundances.

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Over 50 years ago, Dirac speculated that the fundamental constants of nature are not constant but are time variable [1]. This speculation has continued to spur scientists to consider the possibility of the temporal and spatial variation of the fundamental constants [2]. Astrophysical observations have placed important limits on the variation of these constants. For example, solar system dynamics constrain the time variation of the gravitational constant [3], while observations of atomic lines in cosmological objects constrain the time variation of the electromagnetic couplings and the electron-proton mass ratio [4].

In this paper, we present several independent astrophysical observations that constrain the spatial and temporal variation of the weak interaction. We express our limits as constraints on the spatial and temporal variation of $\beta=G_F'/G_F$, where G_F is the Fermi constant measured on earth at the present, and G_F' is the (possibly different) value of the Fermi constant measured at some other time or place. (Of course, only variations of fundamental constants that change dimensionless ratios have physical meaning.)

In fact, the Fermi constant is not a fundamental coupling constant in the same sense as the fine structure constant. (This was noted previously in Ref. [5].) The Fermi constant is given by

$$\frac{G_F}{\sqrt{2}}=\frac{g^2}{8M_W^2}\ , \tag{1}$$

where g is the SU(2) gauge coupling constant and M_W is the mass of the W. However, M_W^2 is also proportional to g^2:

$$M_W^2=\tfrac{1}{4}g^2\langle\phi\rangle^2\ , \tag{2}$$

where $\langle\phi\rangle$ is the vacuum expectation value of the Higgs field; $\langle\phi\rangle=246$ GeV. Thus, G_F is actually independent of the coupling constant g at the tree level and depends only on $\langle\phi\rangle^2$:

$$\frac{G_F}{\sqrt{2}}=\frac{1}{2\langle\phi\rangle^2}\ . \tag{3}$$

Therefore, G_F is a measure of the magnitude of the electroweak symmetry breaking. It is considerably more natural to allow for the possibility of a time or spatial variation in the vacuum expectation value of a field than for the variation in a fundamental coupling constant.

As noted by Dixit and Sher [5], any change in $\langle\phi\rangle$ will also alter the fermion masses. The electron mass is proportional to $\langle\phi\rangle$, and thus varies as $G_F^{-1/2}$, while the variation in the neutron-proton mass difference (important for primordial nucleosynthesis) is more complicated [see Eq. (4) below]. It is possible to change G_F without altering the fermion masses by simultaneously changing the Yukawa couplings, but this is rather contrived. However, for completeness we will consider both possibilities: that the Yukawa couplings are fixed so that both G_F and the fermion masses vary in parallel, or that the Yukawa couplings vary so that G_F changes without any corresponding change in the fermion masses.

The best constraints on the time variation of the fundamental constants (i.e., the constraints valid over the longest range in time) come from primordial nucleosynthesis. Rozenthal [6] argued that if the strong force had been weaker during nucleosynthesis, then deuterium would not have been stable and the production of heavy elements would have been blocked. Rozenthal concluded that the strength of the strong force during the nucleosynthetic era could not have been more than 10–15 % less than the contemporary value. The abundances of the light elements produced in the big bang are also very sensitive to the strength of the weak interaction. Rozenthal discussed this effect qualitatively [6]. Kolb, Perry, and Walker [7] examined the changes in the primordial element abundances produced when the fine-structure constant, the gravitational constant, and the Fermi constant are all varied simultaneously due to changes in the size of extra dimensions in superstring and Kaluza-Klein models. They found that the size of these extra dimensions could not have varied by more than 0.5–1 % between the epoch of primordial nucleosynthesis and today. This limit was subsequently refined by Barrow [8] and Dixit and Sher [5]. Here we isolate the variation in G_F and examine it alone.

A change in G_F has a number of effects on primordial nucleosynthesis. The neutron decay rate and all of the other weak $n \leftrightarrow p$ rates are proportional to G_F^2; increasing G_F^2 increases these rates, allowing them to stay in thermal equilibrium longer and thus decreasing the final ^{4}He abundance, while decreasing G_F^2 has the opposite effect [7]. [Since all of these rates scale inversely with the neutron lifetime, altering G_F is just equivalent to changing the neutron lifetime by the factor G_F^{-2}]. There is also a small effect on the freeze-out temperatures of the neutrinos; increasing G_F^2 allows the neutrinos to stay in thermal equilibrium longer, increasing their temperature relative to the photon temperature because they participate more fully in the heating due to e^+e^- annihilation [9]. This is a much smaller effect than the basic effect on the $n \leftrightarrow p$ rates, so we have ignored it. If G_F alone varies, then these are the only effects on primordial nucleosynthesis. Two further effects come into play if we allow for the fact that altering $\langle \phi \rangle$ changes the fermion masses [5]. The electron mass varies as $G_F^{-1/2}$, changing both the weak $n \leftrightarrow p$ rates and the temperature at which the e^+e^- pairs annihilate. The neutron-proton mass difference also changes; we follow Dixit and Sher [5] and take

$$m_n - m_p = 2.193 \text{ MeV } \beta^{-1/2} - 0.9 \text{ MeV} . \quad (4)$$

This alters the equilibrium neutron-proton ratio and changes the ^{4}He abundance in the opposite direction from the effect of changing the weak rates: as G_F is increased, $m_n - m_p$ decreases, giving a larger n/p ratio at a fixed temperature, and thus, more ^{4}He. We have not included changes in the other nuclear masses in our calculations; we expect these changes to have a much smaller effect than the change in the neutron-proton mass difference.

We have incorporated these effects into the primordial nucleosynthesis code of Wagoner [10] with a neutron lifetime of $\tau_n = 888.6$ s [11], for η (the baryon to photon ratio) in the range 2×10^{-10}–10^{-9}, varying β between 0.7 and 1.1 for each value of η. We first examined the case where only G_F is allowed to vary, with no changes in the fermion masses. In this case, we see as expected that increasing G_F decreases the final ^{4}He abundance. Varying G_F has relatively less effect on the abundances of elements other than ^{4}He, since these other abundances depend primarily on the rates for the nuclear fusion reactions. However, there is some effect because lowering the neutron to proton ratio decreases the number of neutrons which can be incorporated into these elements; we find that increasing G_F decreases the D, D+^{3}He, and ^{7}Li abundances slightly. When we include the changes in both the electron mass and the neutron-proton mass difference, we find that the change in the electron mass has a negligible effect, while the change in the neutron-proton mass difference dominates the direct change in the weak rates. Thus, ^{4}He increases with increasing G_F (in agreement with the conclusions of Dixit and Sher [5]), rather than decreasing as in the case where only G_F varies. Again, the other elements vary a small amount in the same direction as ^{4}He; increasing G_F increases the D, D+^{3}He, and ^{7}Li abundances slightly.

Using the standard constraints on the primordial element abundances [12],

$$(\text{D/H})_p > 1.8\times10^{-5} , \quad (5)$$

$$(\text{D}+{}^3\text{He})/\text{H}_p < 1\times10^{-4} , \quad (6)$$

$$0.22 < Y_p < 0.24 , \quad (7)$$

where Y_p is the primordial mass fraction of ^{4}He, and the other element abundances are expressed in terms of their number densities relative to hydrogen, we obtain the allowed regions shown in Figs. 1 and 2. Figure 1 is for the case in which G_F varies in parallel with changes in the fermion masses through a change in the vacuum expectation value of the Higgs field, while Fig. 2 includes only the change in G_F. The abundances of D and D+^{3}He,

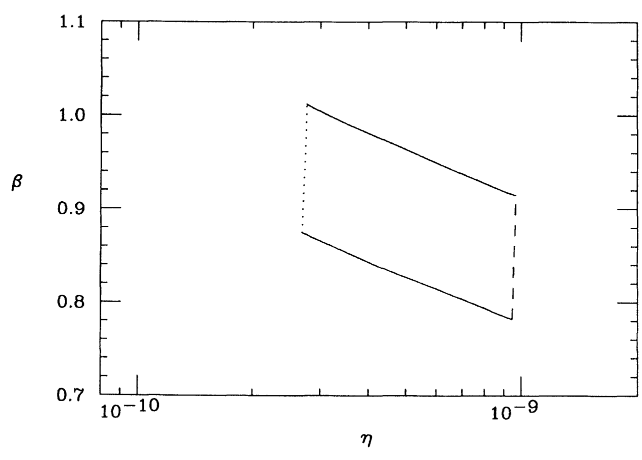

FIG. 1. The allowed region for η (the baryon to photon ratio) and β (the factor by which G_F is altered relative to its present value), for the case in which G_F varies in parallel with changes in the fermion masses through a change in the vacuum expectation value of the Higgs field. Dashed curve is from the lower bound on D/H, dotted curve is from the upper bound on (D+^{3}He)/H, and solid curves are from upper and lower bounds on the abundance of ^{4}He.

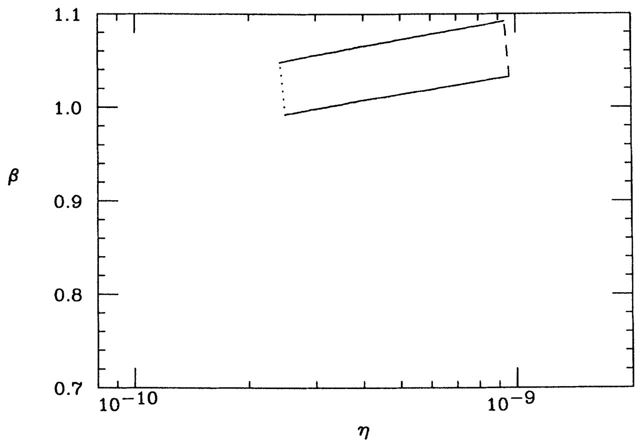

FIG. 2. As Fig. 1, where G_F alone varies and the fermion masses are held fixed.

which are nearly independent of β, set the upper and lower bounds on η, respectively, and the primordial ^{4}He abundance determines the limits on β for each value of η.

Because the predicted standard model values for Y_p are very close to the observed upper limit on Y_p in Eq. (7), there is very little room to increase Y_p, while there is considerably more freedom to decrease Y_p. For the case where both G_F and the fermion masses vary, we find that

$$0.78 < \beta < 1.01 \; . \tag{8}$$

The inclusion of limits on ^{7}Li production would increase the lower bound on β to 0.85. If only G_F varies, the limits on β are

$$0.99 < \beta < 1.09 \; . \tag{9}$$

The inclusion of limits on ^{7}Li production would decrease the upper bound on β to 1.06.

We note that changing G_F at the epoch of nucleosynthesis (with or without changes in the fermion masses) is one of several methods (although perhaps not the most plausible method) for decreasing the primordial ^{4}He abundance without altering any of the other predictions of standard primordial nucleosynthesis (see Ref. [13] for a discussion of other such mechanisms); this is of interest because the predictions of standard big bang nucleosynthesis are already close to the observed upper bounds on Y_p [12–14].

These limits are less stringent but cover a longer time scale than constraints from the analysis of ores from the Oklo uranium mine, where a natural fission reactor operated about 2×10^9 years ago. Shlyakhter [15] argued that shifting the difference in the ground states of ^{150}Sm and ^{149}Sm by more than 0.02 eV would be inconsistent with the isotope ratios in these ores. He calculated the contribution of the weak interaction to the nuclear binding energy to be $\sim 2\times10^{-7} m_p$ and concluded that G_F could not have changed by more than 0.1% in the past 2 Gyr. These limits have been reexamined recently yielding similar conclusions [16].

A number of astrophysical observations constrain spatial variations of G_F. Consider first the case where both G_F and the fermion masses vary. In this case, the change in the electron mass provides a stringent constraint on the spatial variation of G_F from the agreement between the redshift determined from the observations of the 21-cm HI hyperfine line and the redshift determined from observations of optical resonance lines from the same object. This agreement has been used to constrain temporal and spatial variation in the fine-structure constant, the electron to proton mass ratio and the product of these quantities times the proton gyromagnetic moment, $\alpha^2 g_p m_e/m_p$ [4]. Of the quantities in this ratio, both m_e and m_p will change as G_F is varied; however, the fractional change in the proton mass will be much smaller than the fractional change in the electron mass, so the former can be ignored. The most stringent limits come from the agreement between the measured optical and ratio redshifts of better than 2 parts in 10^4 for a quaser absorption system at $z=1.77$ [17]. This agreement implies that $\Delta \ln(\alpha^2 g_p m_e/m_p) < 2\times10^{-4}$ [17]. Since $m_e \propto G_F^{-1/2}$, this implies that the Fermi constant does not vary by more than 0.04% over cosmological scales. A constraint at even higher redshift comes from the agreement between the optical and the radio redshifts (3.395 ± 0.005 and 3.3968 ± 0.0004) for a distant galaxy recently detected by Uson, Bagri, and Cornwell [18]. This agreement implies that G_F has varied by less than 0.2% back to a redshift of 3.4. (Incidentally, this observation also provides a constraint on the variation of the fine structure constant at $z=3.4$.)

If only G_F varies, then a bound on spatial variations comes from the shape of the light curves of type IA supernovae. The light curve is determined by the lifetime of ^{56}Co, whose radioactive decay is the dominant heating source for the supernova remnant [19]. The radioactive lifetimes vary as β^{-2}. Observations of supernovae in nearby galaxies show that the light curves of type IA supernovae are remarkably similar [20]. Variations in the slope of $\sim 10\%$ would be readily apparent in these light curves. This implies that β is not varying by more than 5% on a scale of 30 Mpc. The observations of *SN* 1988*U* at $z=0.3$ hint at the promise of dedicated supernova searches as probes of the time evolution of physical constants, including the Hubble constant [21]. Our limit, however, could be weakened by selection effects: if β were spatially varying, astronomers might have classified as anomalous any supernova that went off in a region with a slightly different value of β, so that variations in β could have been missed.

Our constraints on spatial variations in G_F are considerably more stringent if fermion masses also vary through the change in the vacuum expectation value of the Higgs field than if G_F varies alone: in the former case redshift measurements indicate that G_F cannot vary by more than $\sim 0.04\%$ on cosmological length scales, and G_F must be within 0.2% of its present value at $z=3.4$, while in the latter case G_F can vary by at most 5% on cosmological scales. Primordial nucleosynthesis yields less stringent limits valid back to earlier times: G_F must have been within $\sim 10-20\,\%$ of its present value when the neutron-proton weak reactions froze out at $t \sim 1$ s ($T \sim 1$ MeV). (The exact limits depend on which model we consider.) We note that limits on the time variation of G_F prior to the electroweak phase transition at $T \sim 100$ GeV are meaningless; this still leaves a factor $\sim 10^{10}$ in time over which variations in G_F are unconstrained by the results presented here.

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