Asymptotic fermion propagator in massless three-dimensional QED

B. J. Hand
Department of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7
(Received 14 July 1992)

Massless quantum electrodynamics in two spatial and one time dimensions has a logarithmically confining static Coulomb potential, and thus nontrivial infrared behavior. We apply a technique developed for ordinary four-dimensional quantum electrodynamics in which the charged asymptotic states in the theory are dressed with soft vector bosons, in order to improve the representation of the infrared dynamics in perturbation theory. The resulting modification to the mass-shell behavior of the fermion propagator is determined, with the result that the propagator no longer possesses a mass-shell singularity.

PACS number(s): 11.10.Jj, 11.15.Tk, 12.20.Ds

I. INTRODUCTION

Quantum electrodynamics in $2+1$ dimensions has been investigated from several different viewpoints over the past 15 years. Perhaps the form of most current interest is the one that contains a Chern-Simons term $\mu\varepsilon^{\mu\nu\alpha}F_{\mu\nu}A_{\alpha}$, which results in a vector field mass that does not violate gauge invariance [1]. It is hoped that this form of three-dimensional QED (QED_3) will be a useful tool in the understanding of certain exotic phenomena such as the quantum Hall effect [2].

QED_3 with massless vector and fermion fields has also been of interest to several authors [3–5]. Since the static Coulomb potential is logarithmically confining, the theory provides a useful framework in which to study field-theoretical techniques dealing with confinement [3]. One immediate consequence of the confining nature of the massless theory is that conventional perturbation theory [6] results in severe infrared divergences at two loops [4]. For example, when the internal vector line of the one-loop fermion self-energy graph is corrected by a fermion loop (vacuum polarization), the resulting expression contains a Feynman parameter integral that is logarithmically divergent, even before any mass-shell condition is enforced. Jackiw and Templeton [4] have shown, however, that by considering the full inverse propagators of the theory instead of the individual perturbative contributions, the infrared divergences are removed in a natural way. The full fermion propagator contains a logarithm of the (dimensionful) coupling over the external momentum, and thus becomes a nonanalytic function. Their result is valid when the ratio of the coupling over the off-shell external momentum is small.

Another approach to understanding the infrared sector of QED_3 that we suggest here is to make use of the understanding of the infrared sector in ordinary $(3+1)$-dimensional quantum electrodynamics (QED_4). Here it is well known that graphs containing one or more loops can encounter infrared divergences when mass-shell conditions are enforced. For any physical cross section, the infrared divergences are canceled when soft bremsstrahlung processes are taken into account [7,8]. The origin of the infrared divergences lies in the unsuitable nature of standard perturbation theory when an infinite range interaction, the electromagnetic interaction, is present. In standard perturbation theory, incoming or outgoing fermion states are created by free-field operators, in contradiction to the fact that a physical fermion carries charge and is thus coupled to the electromagnetic field. The infinite range of the electromagnetic field invalidates the assumption that at asymptotic times the charge may be "switched off." The problem of formulating more physically suitable asymptotic states for electrodynamics was first addressed by Dollard [9] in the context of nonrelativistic quantum mechanics; the field theoretical extension has been addressed by several authors [10–15]. The more recent approaches are based on work by Kulish and Faddeev, who dress free-field states with a pseudounitary transformation derived from a Hamiltonian containing the infrared dynamics of the theory. The result is that at asymptotic times a charged fermion is surrounded by an infinite number of soft photons, and acquires a phase factor that represents the Coulombic interaction with the other charged fermions in the asymptotic state. Reference [16] contains a good summary of their approach and its problems.

With an understanding of the origin of infrared divergences in QED_4, it is not surprising that in QED_3, where the Coulomb potential is confining and thus the assumption of free fields asymptotically is even more unphysical, the infrared divergences encountered in standard perturbation theory are more severe. In this paper we attempt to improve the representation of the infrared dynamics of massless QED_3 in perturbation theory by employing asymptotic fermion states constructed with the techniques of Kulish and Faddeev. We shall determine the effect of using asymptotic states on the mass-shell behavior of the fermion propagator. The result may be considered as a supplement to the elegant nonperturbative off-shell result of Jackiw and Templeton, although it is obtained by a completely different approach. A secondary motivation is that the result may provide some guidance for the formidable task of applying the Kulish-Faddeev technique to QCD_4, which has been undertaken by several authors [17]. We begin in Sec. II by reviewing the canonical quantization of the free fermion and vector field in 2+1 dimensions. In Sec. III the asymptotic fields and states are developed. Section IV contains the derivation of the asymptotically modified fermion propagator, and Sec. V contains a discussion and our conclusions.

II. CANONICAL QUANTIZATION

We begin by quantizing the free Dirac field. In 2+1 dimensions the massless Dirac equation describes a fermion with no spin, possessing two degrees of freedom [18]. We realize the algebra as in Ref. [1]:

$$\gamma^0=\sigma^3,\quad \gamma^1=i\sigma^1,\quad \gamma^2=i\sigma^2\,,$$
$$\gamma^\mu\gamma^\nu=g^{\mu\nu}-i\varepsilon^{\mu\nu\alpha}\gamma_\alpha,\quad g^{\mu\nu}=\mathrm{diag}(1,-1,-1)\,. \tag{1}$$

The two-component free spinor field ϕ solves the equation

$$i\partial\!\!\!/\phi(x)=0\,. \tag{2}$$

Positive and negative energy solutions are

$$u_p e^{-ip\cdot x}=\frac{1}{\sqrt{2}\epsilon_p}\begin{bmatrix}\epsilon_p\\ ip_1-p_2\end{bmatrix}e^{-ip\cdot x}\,,$$
$$v_p e^{ip\cdot x}=\frac{1}{\sqrt{2}\epsilon_p}\begin{bmatrix}ip_1+p_2\\ -\epsilon_p\end{bmatrix}e^{ip\cdot x}\,, \tag{3}$$

where $\epsilon_p=\sqrt{p_1^2+p_2^2}$. These solutions have the normalizations [19]

$$u_p^\dagger u_p=v_p^\dagger v_p=1\,,$$
$$\bar u_p u_p=\bar v_p v_p=0\,, \tag{4}$$
$$u_p^\dagger u_{-p}=u_p^\dagger v_{-p}=0\,.$$

For future use we state a (2+1)-dimensional Gordan identity

$$\bar u_p\gamma\cdot k u_p=p\cdot k/\epsilon_p\,. \tag{5}$$

We make a mode decomposition for ϕ:

$$\phi(x)=\int\frac{d^2p}{2\pi}(b_p u_p e^{-ip\cdot x}+d_p^\dagger v_p e^{ip\cdot x})\,, \tag{6}$$

and impose the standard anticommutation relations

$$\{b_p,b_k^\dagger\}=\delta^2(\mathbf{p}-\mathbf{k})\,,\quad \{d_p,d_k^\dagger\}=\delta^2(\mathbf{p}-\mathbf{k})\,. \tag{7}$$

From (6) and (7) it is straightforward to verify the anticommutation relation

$$\{\phi_\alpha(x),\bar\phi_\beta(y)\}=(i\partial\!\!\!/_x)_{\alpha\beta}i\Delta(x-y)\,, \tag{8}$$

where $\Delta(x-y)$ is the three-dimensional invariant singular function:

$$i\Delta(x-y)=\int\frac{d^2p}{(2\pi)^2}\frac{1}{2\epsilon_p}(e^{-ip\cdot(x-y)}-e^{ip\cdot(x-y)})\,. \tag{9}$$

At equal times (9) reduces to the usual expression

$$\{\phi_\alpha(\mathbf{x},t),\phi_\beta^\dagger(\mathbf{y},t)\}=\delta^2(\mathbf{x}-\mathbf{y})\delta_{\alpha\beta}\,. \tag{10}$$

We turn now to the quantization of the free vector field, which will be done in the manifestly covariant Feynman gauge. The free vector field satisfies the equation of motion

$$\Box a^\mu(x)=0 \tag{11}$$

and may be given the mode decomposition

$$a_\mu(x)=\int\frac{d^2k}{2\pi}\frac{1}{(2\omega_k)^{1/2}}\sum_{\lambda=0}^{2}\varepsilon_\mu(k,\lambda)[\alpha(k,\lambda)e^{-ik\cdot x}+\alpha^\dagger(k,\lambda)e^{ik\cdot x}]\,, \tag{12}$$

where real polarization vectors are used, and $k^0=\omega_k=|\mathbf{k}|$. The creation and annihilation operators are given the commutation relations

$$[\alpha(k,\lambda),\alpha^\dagger(k',\lambda')]=-g^{\lambda\lambda'}\delta^2(\mathbf{k}-\mathbf{k}') \tag{13}$$

from which the field commutator may be evaluated:

$$[a_\mu(x),a_\nu(y)]=-ig_{\mu\nu}\Delta(x-y)\,. \tag{14}$$

The Gupta-Bleuler subsidiary condition [20] will be discussed in the next section. The Feynman propagator is

$$iD_F(x,x')_{\mu\nu}=\langle 0|T[a_\mu(x)a_\nu(x')]|0\rangle$$
$$=i\int\frac{d^3k}{(2\pi)^3}e^{-ik\cdot(x-x')}\frac{-g_{\mu\nu}}{k^2+i\varepsilon}\,. \tag{15}$$

The Lagrangian for the coupled theory is

$$L=-\tfrac14F_{\mu\nu}(x)F^{\mu\nu}(x)+i\bar\psi(x)\partial\!\!\!/\psi(x)$$
$$+e\bar\psi(x)A\!\!\!/(x)\psi(x)-\tfrac12(\partial\cdot A(x))^2\,. \tag{16}$$

The quantization of the coupled theory proceeds in a manner completely similar to the (3+1)-dimensional case, and for the sake of brevity we shall not present the details.

We will make the usual assumption that there exists a unitary transformation between the free fields and the full (interpolating) fields:

$$\phi(\mathbf{x},t)=Z(t)\psi(\mathbf{x},t)Z^{-1}(t)\,, \tag{17}$$

$$a_\mu(\mathbf{x},t)=Z(t)A_\mu(\mathbf{x},t)Z^{-1}(t)\,. \tag{18}$$

Using the Heisenberg relation,

$$i[H(t),\psi(\mathbf{x},t)]=\dot\psi(\mathbf{x},t)\,, \tag{19}$$

it is straightforward to show that

$$i\dot Z(t)Z^{-1}(t)=H_I(t)+\epsilon_0(t)\,, \tag{20}$$

where $H_I(t)$ is the interaction Hamiltonian

$$H_I(t)=-e\int d^2x\,\bar\phi(\mathbf{x},t)\gamma\cdot a(\mathbf{x},t)\phi(\mathbf{x},t)\,, \tag{21}$$

and $\epsilon_0(t)$ is an arbitrary time-dependent c number.

III. ASYMPTOTIC STATES AND FIELDS

Now we come to a crucial step. We wish to construct asymptotic fermion states in such a way that the infrared dynamics is sufficiently represented. That is, a charged fermion at an asymptotic time must be accompanied by a low-frequency electromagnetic field. To accomplish this we start by considering the large-time behavior of the coupled theory. Let us decompose the interaction part of the Hamiltonian into

$$H_I(t)=H_I^{\rm as}(t)+H_I'(t)\ , \tag{22}$$

where $H_I^{\rm as}(t)$ is the dominant term of $H_I(t)$ at asymptotic times, $|t|\rightarrow\infty$. To find an expression for $H_I^{\rm as}(t)$, one first substitutes the mode expansions (6) and (12) into (21). Consider a typical contribution to $i\dot{Z}Z^{-1}$:

$$\eta=-e\int\frac{d^2x}{(2\pi)^2}\frac{d^2p}{2\pi}d^2q\frac{d^2k}{\sqrt{2\omega_k}}b_p^\dagger b_q\bar{u}_p\gamma\cdot\varepsilon(k,\lambda)u_q[\alpha(k,\lambda)e^{ix\cdot(p-k-q)}+\alpha^\dagger(k,\lambda)e^{ix\cdot(p+k-q)}]$$

$$=-e\int\frac{d^2p}{2\pi}\frac{d^2k}{\sqrt{2\omega_k}}[b_p^\dagger b_{p-k}\bar{u}_p\gamma\cdot\varepsilon(k,\lambda)u_{p-k}\alpha(k,\lambda)e^{it(\epsilon_p-\epsilon_{p-k}-\omega_k)}$$

$$+b_p^\dagger b_{p+k}\bar{u}_p\gamma\cdot\varepsilon(k,\lambda)u_{p+k}\alpha^\dagger(k,\lambda)e^{it(\epsilon_p-\epsilon_{p-k}+\omega_k)}]\ . \tag{23}$$

One now retains only low-frequency contributions to the momentum integrals. For $|t|\rightarrow\infty$, these occur when the momentum factors multiplying the time t in the exponents are small. That is, when $\epsilon_p-\epsilon_{p-k}\pm\omega_k$ approaches zero. This in turn requires ω_k to be small, and so the vector-field momentum integration is restricted to a small sphere centered around $\mathbf{k}=0$. Then the approximations $u_{p\pm k}\approx u_p$, $b_{p\pm k}\approx b_p$ are made, which allows the γ-matrix dependence to be eliminated via the Gordon identity (5). Finally, terms of order $(\omega_k/\epsilon_p)^2$ in the exponentials are neglected. This reduces (23) to

$$\eta=-e\int\frac{d^2p}{2\pi}\frac{d^2k}{\sqrt{2\omega_k}}b_p^\dagger b_p\frac{p\cdot\varepsilon(k,\lambda)}{\epsilon_p}[\alpha(k,\lambda)e^{-itk\cdot p/\epsilon_p}+\alpha^\dagger(k,\lambda)e^{itk\cdot p/\epsilon_p}]\ . \tag{24}$$

When all contributions to $\dot{Z}Z^{-1}$ are reduced in this manner, the following expression for $H_I^{\rm as}(t)$, valid only at asymptotic times, is obtained [12]:

$$H_I^{\rm as}(t)=-e\int\frac{d^2p}{2\pi}\frac{d^2k}{\sqrt{2\omega_k}}\rho(p)\frac{p\cdot\varepsilon(k,\lambda)}{\epsilon_p}[\alpha(k,\lambda)e^{-itk\cdot p/\epsilon_p}+\alpha^\dagger(k,\lambda)e^{itk\cdot p/\epsilon_p}]\ , \tag{25}$$

where

$$\rho(p)=b_p^\dagger b_p-d_p^\dagger d_p\ . \tag{26}$$

Consider now the large time behavior of Eq. (20). Define the operator $U(t)$ to satisfy the relation

$$i\dot{U}(t)U^{-1}(t)=H_I^{\rm as}(t),\quad |t|\rightarrow\infty\ . \tag{27}$$

In other words, $U(t)$ is the operator that allows a free field to be mapped into an interacting field, suitable for use at asymptotic times only. A significant result is that with the expression for $H_I^{\rm as}(t)$ found above, a closed form solution to (27) exists [21]:

$$U(t)=\exp\left\{-i\int_{t_0}^{t}H_I^{\rm as}(t_1)dt_1+\tfrac{1}{2}(-i)^2\int_{t_0}^{t}dt_1\int_{t_0}^{t_1}dt_2[H_I^{\rm as}(t_1),H_I^{\rm as}(t_2)]\right\}\ , \tag{28}$$

where $U(t_0)=1$. The commutator in (28) may be evaluated by using relations (7) and (13):

$$[H_I^{\rm as}(t_1),H_I^{\rm as}(t_2)]=\int\frac{d^2p}{2\pi}\frac{d^2q}{2\pi}\rho(p)\rho(q)\frac{p\cdot q}{\epsilon_p\epsilon_q}i\Delta[(\mathbf{p}/\epsilon_pt_1,t_1)-(\mathbf{q}/\epsilon_qt_2,t_2)]\ . \tag{29}$$

This commutator is called the Coulomb phase operator, and it does not have any significant effect in modifying the mass-shell behavior of the fermion propagator [12]. In what follows we shall therefore neglect its contributions. This makes the $U(t)$ operator formally unitary.

We may now construct asymptotic states of the theory by operating on free-field states with $U(t)$. For example, incoming and outgoing fermion states are represented by

$$|p\rangle_{\rm in}=U^{-1}(t_{\rm in})\int d^2x\,\phi^\dagger(\mathbf{x},t_{\rm in})N(\mathbf{x},t_{\rm in})|0\rangle\ , \tag{30}$$

$$_{\rm out}\langle q|=\int d^2y\,N^\dagger(\mathbf{y},t_{\rm out})\langle 0|\phi(\mathbf{y},t_{\rm out})U(t_{\rm out})\ , \tag{31}$$

where $N(\mathbf{x},t)$ is a normalizable solution of the free massless Dirac equation (2) and $|0\rangle$ is the vacuum in the space of the free-field operators. The fermion operator dependence in U and U^{-1} can be anticommuted by the operators in ϕ and $\phi^\dagger$ to give

$$|p\rangle_{\rm in}=\left[\exp\left[+ie\int_{t_0}^{t_{\rm in}}dt_1\frac{p\cdot a(\mathbf{p}/\epsilon_p t_1,t_1)}{\epsilon_p}\right]\right]b_p^\dagger|0\rangle\ , \tag{32}$$

$$_{\rm out}\langle q|=\langle 0|b_q\left[\exp\left[-ie\int_{t_0}^{t_{\rm out}}dt_1\frac{q\cdot a(\mathbf{q}/\epsilon_q t_1,t_1)}{\epsilon_q}\right]\right]\ . \tag{33}$$

We see that the effect of the $U(t)$ operator is to dress incoming and outgoing free fermions with a cloud of soft vector bosons.

We may define the asymptotic fields

$$\phi^{\rm as}(\mathbf{x},t)=U^{-1}(t)\phi(\mathbf{x},t)U(t)\ ,$$
$$a_\mu^{\rm as}(\mathbf{x},t)=U^{-1}(t)a_\mu(\mathbf{x},t)U^{-1}(t)\ , \tag{34}$$

and the Lehmann-Symanzik-Zimmermann [22] asymptotic conditions

$$\mathop{\text{w-lim}}_{t\to t_{\rm as}}\int d^2x\,N^\dagger(\mathbf{x},t)\psi(\mathbf{x},t)=\mathop{\text{w-lim}}_{t\to t_{\rm as}}\int d^2x\,N^\dagger(\mathbf{x},t)\phi^{\rm as}(\mathbf{x},t)\ , \tag{35}$$

$$\mathop{\text{w-lim}}_{t\to t_{\rm as}}\int d^2x\,f^*(\mathbf{x},t)\overleftrightarrow{\partial}_t A_\mu(\mathbf{x},t)=\mathop{\text{w-lim}}_{t\to t_{\rm as}}\int d^2x\,f^*(\mathbf{x},t)\overleftrightarrow{\partial}_t a_\mu^{\rm as}(\mathbf{x},t)\ , \tag{36}$$

where $f(\mathbf{x},t)$ is an arbitrary normalizable solution of the Klein-Gordon equation. The notation "w-lim" (weak limit) denotes a matrix element relation, not an operator relation. The states enclosing a w-lim relation are the asymptotic states, which we assign to be the true physical states of the theory.

At this point some discussion of a Gupta-Bleuler subsidiary condition is warranted. Since our physical states are no longer created by free-field operators, a modification of the usual subsidiary condition should be expected. We follow Haller [23] and construct a subsidiary condition that is consistent weakly with Gauss's law, at asymptotic times [24]. Gauss's law is represented by the equation of motion

$$\Box A^0+e\psi^\dagger\psi=\partial\cdot\dot{A}\ , \tag{37}$$

where the fields are the full interacting fields. Requiring that a physical state $|\nu\rangle$ satisfy the matrix relation

$$\langle\nu|\partial\cdot A^{(-)}=\partial\cdot A^{(+)}|\nu\rangle=0 \tag{38}$$

at asymptotic times, implies that

$$\langle\nu|\Box A^0+e\psi^\dagger\psi|\nu\rangle=\langle\nu|\partial\cdot\dot{A}|\nu\rangle=0\ , \tag{39}$$

which is consistent with Gauss's law. If desired, one may integrate (38) and (39) with a normalizable solution of the free, massless Klein-Gordon equation to obtain equivalent relations in terms of (time-dependent) creation and annihilation operators. The subsidiary condition (38) has the *appearance* of the usual condition found in standard texts, but is inherently different in that the vector field is not a free field. Unfortunately, the physical charged states identified in (32) and (33) (the asymptotic states) do not satisfy (38). Swanson [15] has shown how to remedy this shortcoming by dressing the asymptotic states with yet another transformation. We shall not proceed with this, however, since strict compliance with the Gupta-Bleuler condition is not necessary, if we confine our interest to the mass-shell behavior of the fermion propagator [15].

IV. ASYMPTOTIC FERMION PROPAGATOR

Let us consider the perturbative evaluation of the two-point function

$$i\Gamma^{(2)}_{\alpha\beta}(x_1,x_2)=\langle 0|T\{\psi_\alpha(x_1)\bar\psi_\beta(x_2)\}|0\rangle\ . \tag{40}$$

By means of the Z and U operators this may be expressed in terms of asymptotic fields:

$$i\Gamma^{(2)}(x_1,x_2)_{\alpha\beta}=\langle 0|Z_0^{-1}U_oT\{U_o^{-1}Z_oZ_1^{-1}U_1\phi_\alpha^{\rm as}(x_1)U_1^{-1}Z_1Z_2^{-1}U_2\bar\phi_\beta^{\rm as}(x_2)U_2^{-1}Z_2Z_i^{-1}U_i\}U_i^{-1}Z_i|0\rangle\ , \tag{41}$$

where the time of the U and Z operators is displayed by subscripts, with o representing $t_{\rm out}(+\infty)$ and i representing $t_{\rm in}(-\infty)$. Because of the use of the U operator, (41) is valid only for large $|t_1|$ and $|t_2|$. The first-order contribution to (41) is called the asymptotic fermion propagator [13]:

$$iS_F^{\rm as}(x_1,x_2)=\langle 0|T\{\phi_\alpha^{\rm as}(x_1)\bar\phi_\beta^{\rm as}(x_2)\}|0\rangle\ . \tag{42}$$

Noting that $U^{-1}(t)|0\rangle=|0\rangle$, we have

$$iS_F^{\rm as}(x_1,x_2)_{\alpha\beta}=\langle 0|\phi_\alpha(x_1)U(t_1)U^{-1}(t_2)\bar\phi_\beta(x_2)|0\rangle\theta(t_1-t_2)-\langle 0|\bar\phi_\beta(x_2)U^{-1}(t_2)U(t_1)\phi_\alpha(x_1)|0\rangle\theta(t_2-t_1)\ . \tag{43}$$

Let us consider $t_1 > t_2$. The fermion operator dependence in U may be anticommuted past the operators in the free field expansions to give

$$iS_F^{\rm as}(x_1,x_2)_{\alpha\beta}=\int \frac{d^2p}{2\pi}\frac{d^2q}{2\pi}u_q^\alpha \bar{u}_p^\beta e^{-iq\cdot x_1+ip\cdot x_2}\delta^2(\mathbf{q}-\mathbf{p})$$
$$\times\left\langle 0\left|\exp\left[ie\int_{t_0}^{t_1}dt\frac{q\cdot a(\mathbf{q}/\epsilon_q t,t)}{\epsilon_q}\right]\exp\left[-ie\int_{t_0}^{t_2}dt'\frac{p\cdot a(\mathbf{p}/\epsilon_p t',t')}{\epsilon_p}\right]\right|0\right\rangle$$
$$=i\int\frac{d^3p}{(2\pi)^3}\frac{\not p}{p^2+i\varepsilon}e^{-ip\cdot(x_1-x_2)}$$
$$\times\left\langle 0\left|\exp\left[ie\int_{t_0}^{t_1}dt\frac{p^\mu}{\epsilon_p}[a_\mu^{(-)}(\mathbf{p}/\epsilon_p t,p^0/\epsilon_p t)+a_\mu^{(+)}(\mathbf{p}/\epsilon_p t,p^0/\epsilon_p t)]\right]\right.\right.$$
$$\left.\left.\times\exp\left[-ie\int_{t_0}^{t_2}dt'\frac{p^\mu}{\epsilon_p}[a_\mu^{(-)}(\mathbf{p}/\epsilon_p t',p^0/\epsilon_p t')+a_\mu^{(+)}(\mathbf{p}/\epsilon_p t',p^0/\epsilon_p t')]\right]\right|0\right\rangle, \tag{44}$$

where we have split the vector field into creation and annihilation components denoted by a superscript minus and plus, respectively. Repeated application of the Baker-Campbell-Hausdorff relation

$$e^{A+B}=e^Ae^Be^{-1/2[A,B]} \tag{45}$$

to more exponentiated creation operators to stand against the left vacuum, and exponentiated annihilation operators to the right vacuum, yields

$$iS_F^{\rm as}(x_1,x_2)=i\int\frac{d^3p}{(2\pi)^3}\frac{\not p}{p^2+i\varepsilon}e^{-ip\cdot(x_1-x_2)}$$
$$\times\exp\left[\frac{e^2}{2}\frac{p^2}{\epsilon_p^2}\left[\int_{t_0}^{t_1}dt\int_{t_0}^{t_1}dt'+\int_{t_0}^{t_2}dt\int_{t_0}^{t_2}dt'-2\int_{t_0}^{t_1}dt\int_{t_0}^{t_2}dt'\right]\right.$$
$$\left.\times\Delta_+[(\mathbf{p}/\epsilon_p t,p^0/\epsilon_p t)-(\mathbf{p}/\epsilon_p t',p^0/\epsilon_p t')]\right], \tag{46}$$

where Δ_+ is the positive frequency part of the invariant singular function (9):

$$i\Delta(x-y)=\Delta_+(x-y)-\Delta_-(x-y)\ . \tag{47}$$

Defining the real function [6]

$$\Delta_1(x-y)=\Delta_+(x-y)+\Delta_-(x-y) \tag{48}$$

we shall write the third term in the exponential in (46) in terms of Δ_1 and $i\Delta$, and then drop the $i\Delta$ term since its contributing to the propagator is only a phase [recall the discussion following (29)]. For the second term, since t_1 and t_2 are at asymptotic times we may take $t_1\sim -t_2$. The three terms may then be summed into a single, real term:

$$iS_F^{\rm ss}(x_1,x_2)=i\int\frac{d^3p}{(2\pi)^3}\frac{\not p}{p^2+i\varepsilon}e^{-ip\cdot(x_1-x_2)}\exp\left[\frac{e^2}{2}\frac{p^2}{\epsilon_p^2}\int_{t_0}^{t_1}dt\int_{t_2}^{t_1}dt'\Delta_1[(\mathbf{p}/\epsilon_p t,p^0/\epsilon_p t)-(\mathbf{p}/\epsilon_p t',p^0/\epsilon_p t')]\right]. \tag{49}$$

Implicit in obtaining this result is the requirement that the time integrals cannot depend on the value of t_0 [25]. This stems from the fact that the expression for the U operator (28) cannot contain contributions from the integral of $H_I^{\rm as}$ at finite (nonasymptotic) times. This requirement is not well understood in the literature, and we have nothing to add here towards a clarification. We shall follow Kulish and Faddeev [12] and drop the contribution from the lower limit t_0 for the first time integral in (49):

$$\frac{1}{2\epsilon_p^2}\int_{t_0}^{t_1}dt\int_{t_2}^{t_1}dt'\Delta_1[(\mathbf{p}/\epsilon_p t,p^0/\epsilon_p t)-(\mathbf{p}/\epsilon_p t',p^0/\epsilon_p t')]$$
$$=\frac{1}{\epsilon_p^2}\int_{t_0}^{t_1}dt\int_{t_2}^{t_1}dt'\int_{\Omega^s}\frac{d^2k}{(2\pi)^2 2\omega_k}\cos[k\cdot p(t-t')/\epsilon_p]=\int_{\Omega^s}\frac{d^2k}{(2\pi)^2 2\omega_k}\frac{1-\cos[k\cdot p(t_1-t_2)/\epsilon_p]}{(k\cdot p+i\varepsilon)^2}\ . \tag{50}$$

The subscript Ω^s on the k integration indicates that the integration is to be over soft momenta only, in order to be consistent with the derivation leading to Eq. (25). The coupling e^2 is a natural scale provided by the theory, which we shall use as the momentum cutoff.

A similar calculation may be performed for the case $t_1 < t_2$. The complete asymptotic propagator may then be written as

$$iS_F^{as}(x_1,x_2)=i\int\frac{d^3p}{(2\pi)^3}\frac{\not p}{p^2+i\varepsilon}e^{-ip\cdot(x_1-x_2)}\exp\left[ie^2\int_{\Omega^s}\frac{d^3k}{(2\pi)^3}\frac{p^2[1-\cos(\sigma k\cdot p)]}{(k^2+i\varepsilon)(k\cdot p+i\varepsilon)^2}\right], \tag{51}$$

where $\sigma=|t_1-t_2|/\epsilon_p$, and where the contour of the k^0 integral is to be completed in the lower half k^0 plane for $t_1>t_2$, and in the upper half k^0 plane for $t_1<t_2$.

Before attempting to evaluate the k integration, a short digression concerning the analogous case in four dimensions is worthwhile. There dimensional arguments indicate that the integral will be of the form of a logarithm of the momentum p. This logarithm leads to the well-known result that the simple pole of the conventional fermion propagator is modified into a branch point [11].

In three dimensions, however, dimensional considerations dictate a different form of k integral. One finds

$$X(p,\sigma)\equiv ie^2\int_{\Omega^s}\frac{d^3k}{(2\pi)^3}\frac{p^2[1-\cos(\sigma k\cdot p)]}{(k^2+i\varepsilon)(k\cdot p+i\varepsilon)^2}\approx-\frac{e^2(-p^2-i\varepsilon)^{1/2}\sigma}{4\pi}\ln[e^2(-p^2-i\varepsilon)^{1/2}\sigma], \tag{52}$$

valid for large $e^2(-p^2)^{1/2}\sigma\gg1$. To determine the $-p^2\to0$ behavior of the fermion propagator, it is sufficient to consider the behavior of the integral [11]:

$$\Gamma^{as}(p)=-i\int_0^\infty d\sigma\,\exp(i\sigma p^2)\exp[X(p,\sigma)]\,. \tag{53}$$

In contrast with the four-dimensional case, this integral is not straightforward to evaluate. Also, as in the discussion following (23) and (49), only large σ contributions, specifically $\sigma\sim1/(-p^2)$, should be admitted toward the evaluation of $\Gamma^{as}(p)$. Hence we may approximate $\Gamma^{as}(p)$ by

$$\Gamma^{as}(p)\approx-i\int_0^\infty d\sigma\,\exp(i\sigma p^2)\exp[X(p,1/(-p^2))]=\frac{1}{p^2+i\varepsilon}\exp\left[-\frac{e^2}{4\pi(-p^2-i\varepsilon)^{1/2}}\ln\left[\frac{e^2}{(-p^2-i\varepsilon)^{1/2}}\right]\right]. \tag{54}$$

Thus as $-p^2\to0$, $\Gamma^{as}(p)\to0$. Within our approximations the fermion propagator does not possess a singularity at the mass shell $p^2=0$. This result may be rather surprising, but as we shall briefly discuss it is consistent with the nature of a confining theory.

V. DISCUSSION

It should be emphasized that the use of the asymptotic fields for S-matrix element calculations does not alter the fundamental theory, which is governed by the full (interpolating) equations of motion and the commutation relations of the full fields. The use of asymptotic fields in place of free fields provides an alternative method of perturbatively evaluating Green's functions. We believe that this approach should provide a better representation of the true physical behavior of the theory, at least near the mass shell. For the fermion propagator, our results indicate that the usual mass-shell singularity obtained in conventional perturbation theory is eliminated. The nonsingular behavior implies that the one-particle fermion states do not have a well-defined energy-momentum relationship. This agrees with a conclusion by Cornwall [3], obtained by arguing on more general grounds, that the fermion propagator in a confining theory does not necessarily admit a mass shell characterizing a particle with well-defined properties. Our result is also in line with the belief that the Green's functions of a massless theory should be less singular than in the corresponding massive theory [11].

A softening of the infrared behavior of near-massless QED_3 has also been found by lattice-gauge simulations, with N flavors of fermions [5]. By placing the fermions in doublets, chirality may be defined in QED_3 [3], and an area of significant interest is the relation of the infrared behavior of the fermion propagator to chiral symmetry breaking. The analogous result to (54) for N flavors arranged in fermion doublets would be relevant to the issue of chiral symmetry breaking. The most direct way to study the effect of the modified mass-shell behavior on chiral symmetry breaking would be to use the form of (54) as input to the Schwinger-Dyson equation for the fermion propagator, in the mass-shell domain. This is, however, beyond the scope of this paper.

ACKNOWLEDGMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada. I wish to thank John Moffat for a critical reading of the manuscript.

[1] See, for example, S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) **140**, 372 (1982); J. Schonfeld, Nucl. Phys. **B185**, 157 (1981).

[2] *The Quantum Hall Effect,* edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1990).

[3] J. M. Cornwall, Phys. Rev. D **22**, 1452 (1980), and references therein; see also R. Feynman, Nucl. Phys. **B188**, 478 (1981), for a study of confinement in QCD_{2+1}.

[4] R. Jackiw and S. Templeton, Phys. Rev. D **23**, 2291 (1981); T. Appelquist and R. D. Pisarski, *ibid.* **23**, 2305

(1981); T. Appelquist and U. Heinz, *ibid.* **24**, 2169 (1981).
[5] E. Dagotto, A. Kocić, and J. B. Kogut, Nucl. Phys. **B334**, 279 (1990), and references therein.
[6] See, for example, J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
[7] F. Bloch and A. Nordsieck, Phys. Rev. **52**, 54 (1937).
[8] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. (N.Y.) **13**, 379 (1961).
[9] J. D. Dollard, J. Math. Phys. **5**, 729 (1964).
[10] V. Chung, Phys. Rev. **140**, B1110 (1965).
[11] T. W. B. Kibble, J. Math. Phys. **9**, 315 (1968); Phys. Rev. **173**, 1527 (1968); **174**, 1892 (1968); **175**, 1624 (1968).
[12] P. P. Kulish and L. D. Faddeev, Teor. Mat. Fiz. **4**, 153 (1970) [Theor. Math. Phys. **4**, 745 (1970)].
[13] D. Zwanziger, Phys. Rev. D **7**, 1082 (1973); **11**, 3481 (1975); **11**, 3504 (1975).
[14] H. Dahmenn, Nucl. Phys. **B202**, 365 (1981).
[15] M. S. Swanson, Phys. Rev. D **25**, 2086 (1982).
[16] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons,* 2nd ed. (Springer-Verlag, New York, 1976), Suppl. 4; see also R. Kubo, Prog. Theor. Phys. **73**, 1235 (1985); N. Papanicolaou, Phys. Rep. **24**, 229 (1976).
[17] See, for example, G. Giavarini and G. Marchesini, Nucl. Phys. **B296**, 546 (1988).
[18] B. Binegar, J. Math. Phys. **23**, 1511 (1982).
[19] The sign of energy is always taken to be positive for spinors. As a subscript, $-p$ means $(\epsilon_p, -\mathbf{p})$.
[20] S. N. Gupta, Proc. Phys. Soc. London **63**, 681 (1950); K. Bleuler, Helv. Phys. Acta **23**, 567 (1950).
[21] W. Magnus, Commun. Pure Appl. Math. **7**, 649 (1954).
[22] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento **1**, 205 (1955); **2**, 425 (1955); **6**, 319 (1957); V. Glaser, H. Lehmann, and W. Zimmermann, *ibid.* **6**, 1122 (1957).
[23] K. Haller, Acta Phys. Austriaca **42**, 163 (1975).
[24] The implementation of the subsidiary condition for the QED_4 case has been the subject of some disagreement. See D. Zwanziger, Phys. Rev. D **14**, 2570 (1976); **18**, 3051 (1978); K. Haller, *ibid.* **18**, 3045 (1978).
[25] Specifically, this allows one to set $t_0 = -t_0$ in the manipulations leading to (48).