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Light-front quantization of the sine-Gordon model

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It is shown how to modify the canonical light-front quantization of the $(1+1)$-dimensional sine-Gordon model such that the zero-mode problem of light-front quantization is avoided. The canonical sine-Gordon Lagrangian is replaced by an effective Lagrangian which does not lead to divergences as $k^+=(k^0+k^1)/\sqrt{2}\to 0$. After canonically quantizing the effective Lagrangian, one obtains the effective light-front Hamiltonian which agrees with the naive light-front (LF) Hamiltonian, up to one additional renormalization. The spectrum of the effective LF Hamiltonian is determined using discrete light-cone quantization and agrees with results from equal-time quantization.

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I. INTRODUCTION

Recently, there has been renewed interest in the light-front quantization of the transverse lattice action of gauge theories [1–3]. The basic idea is quite simple and appealing. In Euclidean lattice gauge theory all four space-time directions are discretized [4]. In Hamiltonian lattice gauge theory, time is continuous while the three spatial directions are discretized [5]. In the transverse lattice approach one leaves the two "longitudinal" directions (x^0,x^3) continuous while discretizing the transverse directions (x^1,x^2) [1]. On the one hand, the transverse lattice thus provides a gauge-invariant ultraviolet regularization scheme and, on the other hand, it is still possible to perform canonical light-front (LF) quantization, making it a promising approach towards nonperturbative calculations of deep-inelastic structure functions.

However, there are still many unresolved renormalization issues in the context of the LF quantization of the transverse lattice action [2,3]. Those are related to similar problems appearing in the LF quantization of the sine-Gordon model [6–8]. Naturally, what one obtains for the transverse lattice action are (1 + 1)-dimensional (continuous) field theories on each of the transverse sites and links coupled through link and plaquette operators. Since the link fields are U(1) or SU(N) fields in QED and QCD, one ends up with an action that contains terms resembling gauged sine-Gordon [from $U=e^{iagA_\perp}$ in U(1)], or gauged SU(N) nonlinear σ models with many flavors in $(1+1)$ dimensions (however, we will not push the analogy too far).

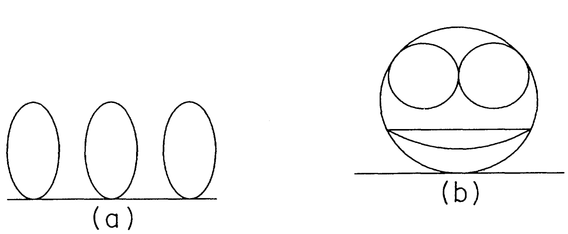

FIG. 1. (a) Tadpole diagrams in ϕ^4 theory. (b) "Generalized tadpole" diagram in ϕ^4 theory.

As a first step towards a LF quantization of these transverse lattice actions it is thus helpful to understand how to perform the LF quantization of the sine-Gordon and nonlinear σ model in $(1+1)$ dimensions.

Naively, this seems to be straightforward. However, as analyzed in detail in Ref. [6], the canonical procedure yields results that are in contradiction to known results for the sine-Gordon (SG) model [7,8].

$$\mathcal{L}_{SG}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{\alpha_0}{\beta^2}[1-\cos(\beta\phi)]\ . \tag{1}$$

For example, in equal-time quantization the Hamiltonian corresponding to $\mathcal{L}_{SG}$ becomes unbounded from below for $\beta^2\geq 8\pi$. In canonical LF quantization this occurs already for $\beta^2\geq 4\pi$ [6]. Furthermore, Green's functions computed in mass perturbation theory disagree in the two frameworks. For example, in second order in α the LF result diverges as $\beta^2\geq 4\pi$ while the correct answer (from equal-time quantization) stays finite at that point.

Griffin has pointed out the source of the problem, which arises from an improper treatment of generalized tadpoles (Figs. 1 and 2) in the canonical LF approach [6]. The root of the problem lies in the fact that [9,10]

$$\int dp^-\frac{1}{(p^2-m^2+i\epsilon)^k}=0\quad\text{for } p^+\neq 0\ . \tag{2}$$

One can easily convince oneself that integrals such as the one in Eq. (2) appear in the diagrams in Figs. 1 and 2 as well as in many other graphs with similar topology. Therefore, unless one finds a clever way to treat these zero modes [11–14] in a canonical (and practical) framework, any field theory where such integrals appear naturally from the Feynman rules, cannot be generated from a (naive) canonical LF Hamiltonian.[1]

[1]Notice that this zero-mode problem is different from the $1/k^+$ singularities in gauge theories in the light-cone gauge. In this work we will not address the question of how to regularize gauge-field propagators.

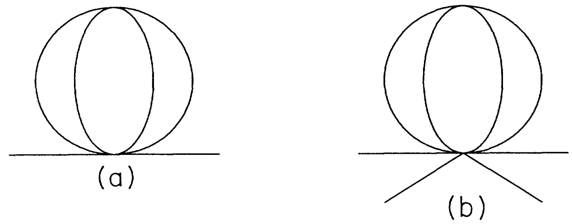

FIG. 2. Typical diagrams arising from interactions between "tadpoling" bosons in the sine-Gordon model. (a) Correction to the two-point function. (b) Similar correction to the four-point function.

So far, no practical procedure has been given that allows one to include zero modes as dynamical degrees of freedom in a LF Hamiltonian (maybe this is even impossible). The approach proposed in this article is to absorb zero-mode induced effects into renormalization constants. That this should be possible is clear from the following observation (valid for self-interacting scalar fields) in perturbation theory. Only a very distinguished class of diagrams is treated incorrectly in LF quantization (i.e., doing the p^- integrations first yields just zero). Those are diagrams such as the ones in Figs. 1 and 2 where one (or more) boson lines emanating from one vertex close back to the same vertex [they may interact with each other, as in Fig. 1(b) or 2, but not with lines from other vertices]. What those Feynman diagrams also have in common is that the loops do not depend on the external momenta. Therefore, they correspond to pointlike effects which one should be able to represent by a local counterterm. By adding the counterterm one should be able to "make up for mistakes" done in a naive approach.

In practice, we will proceed as follows. We will replace the original Lagrangian by an effective Lagrangian. In the effective Lagrangian, all generalized tadpoles and vacuum diagrams have been absorbed into a redefinition of the couplings. Therefore, those diagrams will be "forbidden" diagrams for the effective Lagrangian. This effective Lagrangian is then perfectly suited for canonical LF quantization because exactly those diagrams which are zero due to deficiencies of the canonical LF quantization are forbidden anyway.

II. CONSTRUCTING THE EFFECTIVE LAGRANGIAN

From general field theory principles it should be clear that the effective Lagrangian exists for some range of parameters. The art is to actually construct it. For reasons of clarity, we will first do this for the ϕ^4 theory[2] with

$$\mathcal{L}_{\rm int} = -\frac{\lambda}{4!}\phi^4 \; . \tag{3}$$

A very helpful observation for the derivation of $\mathcal{L}_{\rm int,eff}$ is factorization. All generalized tadpoles and vacuum diagrams factorize into a mass insertion into the external lines times a loop diagram that also occurs when one computes $\langle 0|\phi^2(0)|0\rangle$. For example, in Fig. 1(a) one obtains a mass insertion times the free field value for $\langle 0|\phi^2(0)|0\rangle$.[3] Figure 1(b) corresponds to a higher-order perturbation theory contribution to $\langle 0|\phi^2(0)|0\rangle$ times a mass insertion. Conversely, every term in the perturbative expansion for $\langle 0|\phi^2(0)|0\rangle$ corresponds to a generalized tadpole.

After some combinatorics, one thus obtains

$$\mathcal{L}_{\rm int,eff} = -\frac{\lambda}{4!}\phi^4 - \frac{\lambda}{2!}\phi^2\frac{\langle 0|\phi^2|0\rangle}{2!} \; . \tag{4}$$

By $\langle 0|\phi^2|0\rangle$ we mean the full vacuum expectation value, computed to all orders in λ using equal-time quantization. Of course, in most cases one does not know that number. However, since it corresponds only to a mass renormalization, this parameter can be fixed by renormalizing the physical mass [15]. It is only important to keep in mind that (for the same physical masses) different bare masses will appear in equal-time quantization and in the LF approach.

If the original Lagrangian also contains higher orders in ϕ, such as a $\phi^6/6!$ term, the basic procedure is similar, although more terms appear in the effective Lagrangian. For example, for

$$\mathcal{L}_{\rm int} = -\frac{\lambda_6}{6!}\phi^6 - \frac{\lambda_4}{4!}\phi^4 \tag{5}$$

one finds

$$\mathcal{L}_{\rm int,eff} = -\lambda_6\left[\frac{\phi^6}{6!} + \frac{\phi^4}{4!}\frac{\langle 0|\phi^2|0\rangle}{2!} + \frac{\phi^2}{2!}\frac{\langle 0|\phi^4|0\rangle}{4!}\right] - \lambda_4\left[\frac{\phi^4}{4!} + \frac{\phi^2}{2!}\frac{\langle 0|\phi^2|0\rangle}{2!}\right] \tag{6}$$

and so forth. The general rule is: the higher the power of ϕ in the original Lagrangian, the more numbers [the vacuum expectation values (VEV's) of $\langle 0|\phi^k|0\rangle$] are needed to characterize the vacuum fluctuations of the field ϕ. (In general, i.e., when $\mathcal{L}_{\rm int}$ contains odd powers of ϕ, one also has to include VEV's of odd powers of ϕ, but this should be obvious.) Since there are now two extra counterterms in Eq. (6) ($\langle 0|\phi^4|0\rangle$ and $\langle 0|\phi^2|0\rangle$) one has to use two renormalization conditions when one constructs $\mathcal{L}_{\rm eff}$.[4]

In the sine-Gordon model $\mathcal{L}_{\rm int}$ contains an infinite number of powers of ϕ and it seems that one loses predictive power since one has to know the VEV's of infinitely many powers of ϕ before one can write down the effective Lagrangian. Because of some properties of exponential interactions [7] this is actually not the case. Generalizing our above "dictionary" for going from $\mathcal{L}_{\rm int}$ to $\mathcal{L}_{\rm int,eff}$ (denoted by an arrow $\rightarrow$) we note

[2]See also Refs. [15] and [16] for some related work.

[3]$\langle 0|\phi^2(0)|0\rangle$ diverges for free fields. We assume implicitly that a Pauli-Villars regulator $1/(k^2-m^2)\rightarrow 1/(k^2-m^2) - 1/(k^2-\Lambda^2)$ has been used to cut off the free field divergence.

[4]We will assume that the VEV's are not known *a priori*.

$$\frac{\phi^n}{n!} \to \sum_{k=0}^{n} \frac{\phi^{n-k}}{n!} \binom{n}{k} \langle 0|\phi^k|0\rangle = \sum_{k=0}^{n} \frac{\phi^{n-k}}{(n-k)!} \frac{\langle 0|\phi^k|0\rangle}{k!} \, . \tag{7}$$

(Here we include an irrelevant term proportional to ϕ^0 to keep the algebra simple in what follows.) Therefore

$$\begin{aligned} e^{i\beta\phi} &= \sum_{n=0}^{\infty} \frac{(i\beta)^n}{n!} \phi^n \to \sum_{n=0}^{\infty} \sum_{k=0}^{n} (i\beta)^n \frac{\phi^{n-k}}{(n-k)!} \frac{\langle 0|\phi^k|0\rangle}{k!} \\ &= \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{(i\beta)^m \phi^m}{m!} \frac{\langle 0(i\beta\phi)^k|0\rangle}{k!} \\ &= e^{i\beta\phi} \langle 0|e^{i\beta\phi}|0\rangle \end{aligned} \tag{8}$$

(we replaced $m = n - k$ in the sum).

Since VEV's of odd powers of ϕ vanish in the SG model we thus obtain

$$\mathcal{L}_{\mathrm{eff}}^{\mathrm{SG}} = \frac{\alpha_0}{\beta^2} \cos(\beta\phi) \langle 0|\cos(\beta\phi)|0\rangle \, . \tag{9}$$

Note that Coleman's normal ordering formula (which eliminates only the tadpoles) is a special case of Eq. (9) where the VEV is computed in free field theory. Because of the special properties of the exponential function all generalized tadpoles factorize from the interaction term and can be absorbed into one single renormalization constant. The structure of the effective Lagrangian is the same as the original Lagrangian except for a redefinition of α. In the renormalization procedure one has to fit α_0 anyway such that one obtains the physical mass of the soliton for example.

Intuitively, one can understand the factorization of the VEV as follows. Consider, for example, the basketball diagrams (Fig. 2). Obviously, up to combinatorical factors, the same counterterm is induced for ϕ^2 as well as for ϕ^4 and all higher powers of ϕ.

Although this should be obvious from the derivation; it should be emphasized that $\mathcal{L}_{\mathrm{eff}}$ is exact (provided one knows the number $\langle 0|\cos(\beta\phi)|0\rangle$). Furthermore, the approach chosen here should not be confused with expanding ϕ around some "classical value" [16]. There one would approximate

$$\langle 0|\cos(\beta\phi)|0\rangle \approx \cos(\beta\langle 0|\phi|0\rangle)$$

which is not what is done here. Finally, it is not necessary to specify a "normal ordering" procedure in Eq. (9) since tadpoles, normal order terms, etc., are "forbidden," i.e., zero by construction (they are already in the VEV and one must not double count).

III. LIGHT-FRONT QUANTIZATION OF THE EFFECTIVE LAGRANGIAN

We have constructed an effective Lagrangian for the sine-Gordon model:

$$\mathcal{L}_{\mathrm{int}}^{\mathrm{SG}} = \frac{\alpha_0}{\beta^2} \cos(\beta\phi) \, , \tag{10}$$

$$\mathcal{L}_{\mathrm{int,eff}}^{\mathrm{SG}} = \frac{\alpha_0}{\beta^2} \cos(\beta\phi) \langle 0|\cos(\beta\phi)|0\rangle \, . \tag{11}$$

The VEV in Eq. (11) has to be computed in the full interacting theory (all orders in α and β) or determined by renormalization. By construction, $\mathcal{L}_{\mathrm{int,eff}}^{\mathrm{SG}}$ has Feynman rules different from $\mathcal{L}_{\mathrm{int}}^{\mathrm{SG}}$ in the sense that all generalized tadpoles and vacuum graphs are "forbidden."

Having constructed the effective Lagrangian it is now easy to proceed to the light-front Hamiltonian. Since all "trouble makers" (the generalized tadpoles) are absorbed into a definition of α and thus forbidden anyway one can apply the naive canonical procedure. The result is

$$P^- = \int dx^- - \frac{\alpha_{\mathrm{LF}}}{\beta^2} [:\cos(\beta\phi): - 1] \, . \tag{12}$$

The difference between P^- and P^-_{naive} is a renormalization of α; i.e., compared to the equal-time (ET) result one finds

$$\alpha_{\mathrm{LF}} = \alpha_0 \langle 0|\cos(\beta\phi)|0\rangle \, , \tag{13}$$

while $\alpha_{\mathrm{ET}} = \alpha_0 \langle 0|\cos(\beta\phi)|0\rangle_{\mathrm{free}}$. The canonical LF quantization of $\mathcal{L}_{\mathrm{int,eff}}^{\mathrm{SG}}$ is straightforward yielding the "naive" light-front Hamiltonian except that α_{LF} differs from $\alpha_{\mathrm{naive}} = \alpha_{\mathrm{ET}}$. Since

$$\langle 0|\cos(\beta\phi)|0\rangle / \langle 0|\cos(\beta\phi)|0\rangle_{\mathrm{free}}$$

diverges as $\beta^2 \to 4\pi$ in the SG model [17] this also explains the divergences found in the naive LF approach ($\alpha_{\mathrm{LF}} = \alpha_{\mathrm{ET}}$) investigated in Ref. [6]. Also, because of these divergences, P^- [Eq. (12)] should not be used for $\beta^2 \geq 4\pi$. The eigenstates of P^- in Eq. (12) can now be obtained using standard methods, such as discrete light-cone quantization (DLCQ) [18] or the Tamm-Dancoff expansion [19]. Since all the tadpoles are already absorbed in α_{LF}, the $\cos(\beta\phi)$ in P^- [Eq. (12)] is to be understood as already normal ordered. No additional "self-induced inertias" should be added, because this would mean double counting.

A final remark concerns the value of the VEV's. The VEV's are not calculable in the light-front theory. They have to be determined through renormalization[5] (or possibly through some self-consistency conditions). For example, in an accurate numerical calculation (e.g., DLCQ with many Fock states, such that the Fock-space expansion has converged numerically) one should proceed as follows: input is β and the physical mass of the lightest boson (or any other particle). One then tunes[6] α_{LC} until one reproduces that physical mass in the spectrum of P^- [Eq. (12)]. Calculating $\langle 0|\cos(\beta\phi)|0\rangle$ becomes totally irrelevant in this kind of approach.

We will now construct and analyze the LF Hamiltonian for the sine-Gordon model (12). On the one hand, this

[5]In a renormalizable theory this is the most natural way to proceed. Only in super-renormalizable theory it is common that people take bare couplings too seriously.

[6]For P^- in Eq. (12), the "tuning" is trivial, since the VEV multiplies P^-. However, if additional terms enter, one really has to tune.

demonstrates that the procedure outlined in this article is practical. On the other hand, the explicit numerical solutions can then by used to study the "parton content" of the various physical states of the SG model. The numerical tool we will use is DLCQ [18].[7] The scalar field ϕ is put into a "light-cone box" of length $2L$ using antiperiodic boundary conditions

$$\phi(x^+,x^-)=\frac{1}{\sqrt{4\pi}}\sum_{n=1}^{\infty}\frac{1}{\sqrt{n-1/2}}\times\left[a_n\exp\left[\frac{-ik_n^+x^-}{2}\right]+a_n^\dagger\exp\left[\frac{-ik_n^+x^-}{2}\right]\right], \tag{14}$$

where

$$k_n^+=\frac{2\pi}{L}\left[n-\frac{1}{2}\right] \tag{15}$$

and the $a,a^\dagger$ satisfy canonical boson commutation relations: for example,

$$[a_k,a_q^\dagger]=\delta_{k,q}\ . \tag{16}$$

The choice of antiperiodic bc leaves out the zero modes automatically. As discussed in Sec. II, this is not a problem, as long as one uses $\alpha_{\rm LF}$ instead of $\alpha_{\rm ET}$. The expansion for ϕ (14) is inserted into P^- (12), yielding

$$P^-=\frac{L}{2\pi}H\ , \tag{17}$$

where

$$H=\frac{2\pi\alpha_{\rm LF}}{\beta^2}\left[2-\exp\left[i\tilde{\beta}\sum_{n=1}^{\infty}\frac{a_n^\dagger}{\sqrt{n-1/2}}\right]\exp\left[i\tilde{\beta}\sum_{m=1}^{\infty}\frac{a_m}{\sqrt{m-1/2}}\right]\delta_{P_f,P_i}-\exp\left[-i\tilde{\beta}\sum_{n=1}^{\infty}\frac{a_n^\dagger}{\sqrt{n-1/2}}\right]\exp\left[-i\tilde{\beta}\sum_{m=1}^{\infty}\frac{a_m}{\sqrt{m-1/2}}\right]\delta_{P_f,P_i}\right] \tag{18}$$

and $\tilde{\beta}=\beta/\sqrt{4\pi}$. Here δ_{P_f,P_i} stands symbolic for the Kronecker δ's which ensure conservation of momentum. An explicit expression would be quite lengthy and will not be given here.

For fixed $P^+=(2\pi/L)K$ one now diagonalizes H (eigenvalues E_n) and the invariant masses are given by

$$M_n^2=P^+P_n^-=KE_n\ . \tag{19}$$

As discussed above, we cannot determine $\alpha_{\rm LF}$ from a LF calculation. Therefore, we restrict ourselves to calculating mass ratios. Typical results, as a function of $1/K$ (to demonstrate numerical convergence as $K\to\infty$), are shown in Fig. 3. Without doing fancy fits, it is quite obvious that the DLCQ calculations reproduce (within numerical errors) the exact result for the meson spectrum as obtained in Ref. [20]:

$$M_n=2M_{\rm sol}\sin\left[\frac{n\gamma'}{16}\right], \tag{20}$$

where

$$\gamma'=\frac{\beta^2}{1-\beta^2/8\pi}\ , \tag{21}$$

i.e.,

$$\frac{M_n}{M_1}=\frac{\sin\left[\frac{n\gamma'}{16}\right]}{\sin\left[\frac{\gamma'}{16}\right]}\ . \tag{22}$$

The above formula is valid only for $n\gamma'/16<\pi/2$. When

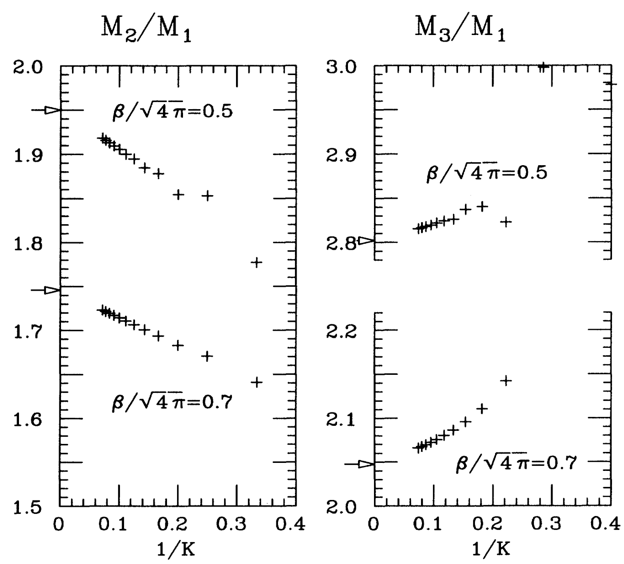

FIG. 3. Convergence of the mass ratios M_2/M_1 and M_3/M_1 for $\beta/\sqrt{4\pi}=0.5$ and $\beta/\sqrt{4\pi}=0.7$ with increasing resolution K (crosses). The analytic predictions [Eq. (22)] are indicated by arrows on the y axis.

[7]For details concerning the application of DLCQ to scalar fields see also Ref. [15].

$\gamma' > 8\pi/n$ [or $\beta^2/4\pi > 2/(n+1)$] the meson breaks up into a soliton-antisoliton pair (below we will discuss what this means in terms of a LF calculation). Other values of β and higher excited mesons (not shown here) show similarly good agreement with the analytical result (22).

For $\beta^2 \geq 4\pi$ one obtains negative invariant masses and the results become nonsensical. This should not come as a surprise since $\alpha_{\rm LF}$ becomes infinite for $\beta^2 = 4\pi$. Extending the DLCQ calculation beyond this point will most likely not work, as we will see below. The point is that the Fock-space expansion (in terms of elementary bosons, created and destroyed by $a^\dagger$ and a) for solitons diverges. This is quite obvious since the asymptotic values of ϕ at $\pm\infty$ are different from each other in the presence of a soliton. Different asymptotic values can only be accomplished by coherent states with elementary bosons. For $\beta^2 \geq 4\pi$ all mesons break up into soliton-antisoliton pairs. Therefore, mass eigenstates will have a divergent Fock-space expansion beyond this point (no convergence in K). It is conceivable that one can develop techniques to overcome this difficulty. However, this would go beyond the scope of this article and we will restrict ourselves to $\beta^2 < 4\pi$.

Let us now analyze the structure of the mesons for various couplings of β. The mass ratios (Fig. 3) indicate already stronger binding (between the bosons) as one increases β. This expectation is confirmed when one looks at structure functions

$$f_q = \langle \psi_p | a_q^\dagger a_q | \psi_p \rangle \tag{23}$$

and the obvious continuum generalization $f(x)$ where momenta are measured in units of the total momentum.

The numerical results for the β evolution of the ground state and the first excited state meson structure functions are shown in Fig. 4. For small β, the nth meson is a weakly bound state of n elementary bosons. The structure functions are peaked at $1/n$. As one increases β a sea of bosons develops and the peak depletes. Finally, at $\beta^2/4\pi = 2/(n+1)$ the sea develops a life of its own and the meson decays. The caterpillar (meson) has turned into a butterfly (soliton-antisoliton) pair. At that point the corresponding structure function starts diverging as

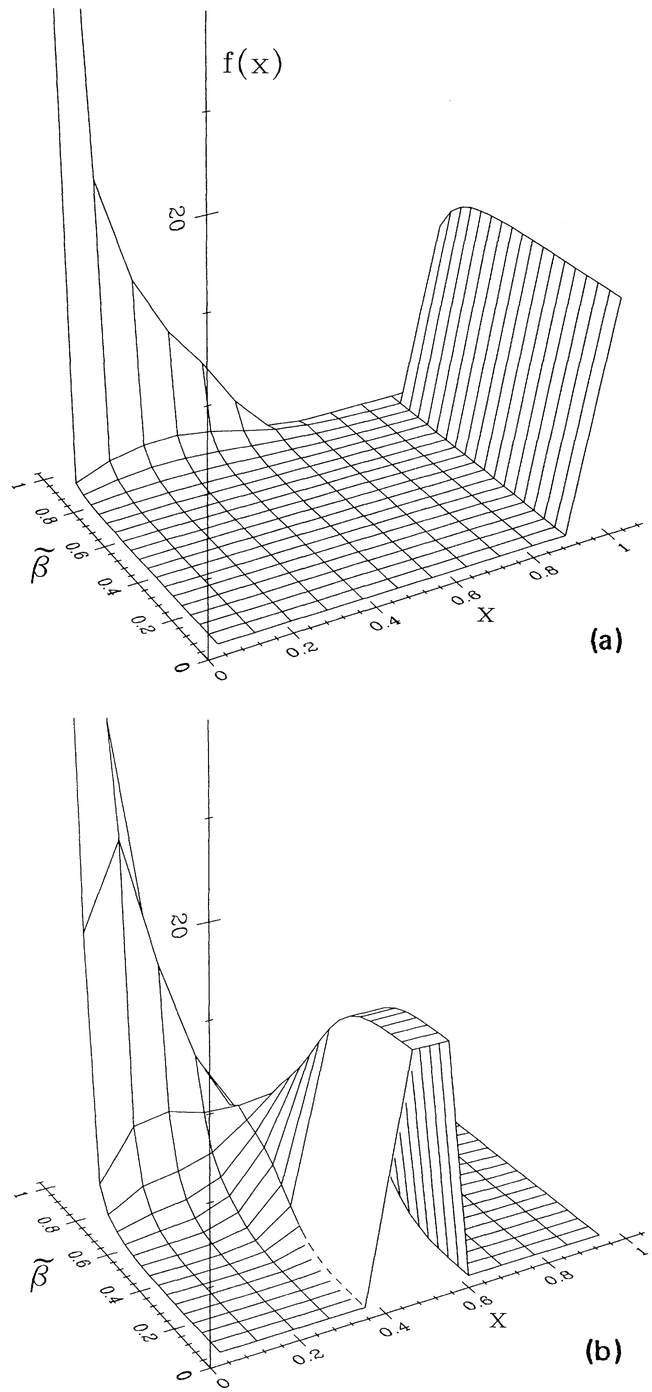

FIG. 4. "Evolution" of the structure functions with $\tilde\beta = \beta/\sqrt{4\pi}$. (a) Ground-state meson, $n=1$; (b) first excited state, $n=2$.

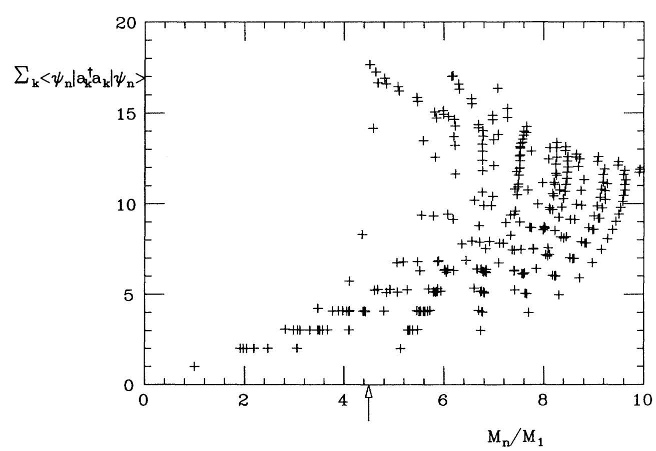

FIG. 5. Total occupation number $\sum_k \langle \psi_n | a_k^\dagger a_k | \psi_n \rangle$ for each state vs mass M_n at $\beta/\sqrt{4\pi} = 0.5$. The calculation was done with $K=25/2$ and $26/2$ (to accommodate even and odd solutions under $\phi \to -\phi$). Each cross corresponds to one state n in the spectrum. Notice the sudden increase around the mass of a soliton-antisoliton pair (20) indicated by an arrow. All masses in units of M_1.

$x \to 0$ indicating the drastic change of the internal structure of the state. The divergence reflects the impossibility to describe solitons in a Fock-space expansion. Nevertheless, this unique signature allows one to extract the soliton masses (again in units of the lightest meson mass) from a DLCQ calculation.

If one plots the number of elementary bosons in a DLCQ spectrum versus the invariant mass for a fixed value of β there is a sudden increase at the soliton-antisoliton threshold (Fig. 5). In that mass region, the occupation number for states with small occupation in Fig. 5 has already converged (with K). These states can be identified with meson-meson scattering states. In those states in Fig. 5, where the occupation number is already high, it keeps increasing with K showing no signs of convergence. These states are most likely soliton-antisoliton scattering states in which case one could determine the soliton mass by determining the mass where the edge occurs.

IV. SUMMARY

We have isolated a distinctive class of diagrams for self-interacting scalar fields (generalized tadpoles) which are incorrectly set to zero in canonical LF quantization. These diagrams have then been absorbed into a redefinition of the coupling constants in the interaction Lagrangian. For the special case of the sine-Gordon model all coupling constants acquire the same renormalization factor.

Canonical LF quantization of the effective Lagrangian is straightforward. We constructed the effective LF Hamiltonian for the sine-Gordon model and solved it using DLCQ. The numerical results for the meson spectrum agree with known results (from equal-time quantization) to better than 1%. The natural next step would be to construct the effective LF Hamiltonian for theories with fermions and gauge fields because this brings us closer to QED and QCD. Work for this is in progress.

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