Vacuum decay in theories with symmetry breaking by radiative corrections

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The standard bounce formalism for calculating the decay rate of a metastable vacuum cannot be applied to theories in which the symmetry breaking is due to radiative corrections, because in such theories the tree-level action has no bounce solutions. In this paper I derive a modified formalism to deal with such cases. As in the usual case, the bubble nucleation rate may be written in the form Ae^{-B}. To leading approximation, B is the bounce action obtained by replacing the tree-level potential by the leading one-loop approximation to the effective potential, in agreement with the generally adopted *ad hoc* remedy. The next correction to B (which is proportional to an inverse power of a small coupling) is given in terms of the next-to-leading term in the effective potential and the leading correction to the two-derivative term in the effective action. The corrections beyond these (which may be included in the prefactor) do not have simple expressions in terms of the effective potential and the other functions in the effective action. In particular, the scalar-loop terms which give an imaginary part to the effective potential do not explicitly appear; the corresponding effects are included in a functional determinant which gives a manifestly real result for the nucleation rate.

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I. INTRODUCTION

When studying quantum field theories in a cosmological context one often encounters situations where the field is not at the absolute minimum of the potential (the "true vacuum"), but is instead at some local minimum (a "false vacuum") which is higher in energy. The transition to the true vacuum state proceeds by a quantum-mechanical tunneling process in which the field in a small region of space tunnels through the potential energy barrier to form a bubble of true vacuum. Once nucleated, the bubble expands, converting false vacuum to true as it does so.

The bubble nucleation rate per unit volume, Γ, can be calculated by a method, due to Coleman [1], which is based on finding a "bounce" solution to the classical Euclidean field equations. Thus, for a theory with a single scalar field, one must solve

$$\partial_\mu \partial_\mu \phi \equiv \Box \phi = \frac{\partial V}{\partial \phi} \tag{1.1}$$

subject to the boundary condition that ϕ approach its false vacuum value as any of the x_μ tend to $\pm\infty$. Γ can then be written in the form

$$\Gamma = Ae^{-B} \tag{1.2}$$

where B is the Euclidean action of the bounce, while A is given by an expression involving functional determinants. In practice, the latter generally turns out to be quite difficult to evaluate, although one can show that A is equal to a numerical factor of order unity times a dimensionful quantity determined by the characteristic mass scales of the theory.

A difficulty arises when one deals with theories where the symmetry breaking is a result of radiative corrections [2]. In such cases the true vacuum is not determined by $V(\phi)$, but instead can be found only by examining the effective potential $V_{\rm eff}(\phi)$. The bounce equation (1.1) is clearly inappropriate—in fact, if $V(\phi)$ has only a single minimum there will not be any bounce solution. An obvious alternative, which has been taken by a number of authors [3], is to modify this equation by substituting $V_{\rm eff}(\phi)$ for $V(\phi)$.

Although plausible, and clearly a step in the right direction, this procedure raises some questions. The one-loop radiative corrections generate an effective action which contains not only $V_{\rm eff}$, but also terms involving derivatives (of all orders) of the fields. Can these terms be neglected when dealing with configurations, such as the bounce solution, which are not constant in (Euclidean) space-time? Even if this can be done in a first approximation, what are the nature and magnitude of the corrections which these terms generate? There are also questions relating to $V_{\rm eff}$ itself. First, the effective potential obtained by perturbative calculations differs considerably from that defined by a Legendre transform (the latter must be convex, while the former is not). The latter clearly does not lead to an appropriate bounce, but how precisely does the formalism pick out the former? Further, the perturbative effective potential is known to be complex for certain values of the fields. How is the imaginary part of $V_{\rm eff}$ to be handled?

In order to answer these questions I develop in this paper a scheme for calculating the bubble nucleation rate in theories with symmetry breaking by radiative corrections. The general idea is to use the path integral approach of Callan and Coleman [4], but to integrate out certain fields at the outset. This leads to a modified effective action which, although it differs somewhat from the usual perturbative $S_{\rm eff}$, gives a correct description of the vacuum structure of the theory and has a bounce solution which can provide the basis for a tunneling cal-

culation. The final result for Γ may again be written in the form of Eq. (1.2), with A either of order unity or proportional to a small inverse power of the coupling. As might be expected, the leading approximation to B is obtained simply by replacing V with the dominant one-loop part of $V_{\rm eff}$ in the standard procedure. The next-to-leading terms in $V_{\rm eff}$, as well as the first corrections to the derivative terms in the effective action, then come in and give calculable and significant (i.e., larger than order unity) corrections to B. However, the corrections beyond this point do not have a simple expression in terms of $V_{\rm eff}$ and the other functions entering $S_{\rm eff}$. In particular, the potentially complex parts of the effective potential do not enter the calculation directly, but only as part of more complicated functional determinants which can easily be shown to be real.

This approach can be understood by recalling the Born-Oppenheimer approximation for the calculation of molecular energy levels. Because the natural time scale for the electrons is much shorter than that for the nuclei, the electrons can be treated as adapting almost instantaneously to changes in the positions of the nuclei. This leads to an effective action, involving only the nuclear coordinates, which may be used to describe situations in which the nuclei do in fact move slowly; thus, it can be applied to the vibrational spectrum of the molecule, but should not be used when studying the scattering of high energy particles off the nucleus.

A similar situation arises in theories where radiative corrections change the vacuum structure. For definiteness, consider the case of scalar electrodynamics [2], where radiative corrections from photon-loop diagrams lead to spontaneous symmetry breaking if the scalar field self-coupling is $O(e^4)$ and the scalar mass term is sufficiently small. This relationship between the couplings implies that the natural time scale for the variation of the electromagnetic field is much less than that for the scalar field. Consequently, if we are only interested in the long wavelength modes of the latter, as is the case when studying vacuum tunneling, we can integrate out the electromagnetic field to obtain an effective action for the scalar field.

The remainder of this paper is organized as follows. Section II reviews the method of Callan and Coleman. In Sec. III, I show how this method can be adapted to theories with radiative symmetry breaking. The treatment here is somewhat formal, so that it can be applied to a variety of theories. The concrete implementation of the method in specific models is developed in Secs. IV and V, which discuss a simple model with two scalar fields and scalar electrodynamics, respectively. Section VI contains some concluding remarks.

II. THE CALLAN-COLEMAN FORMALISM

Callan and Coleman [4] evaluated Γ by calculating the imaginary part of the energy of the false vacuum. This can be obtained from the quantity

$$G(T)=\langle\phi(x)=\phi_{\rm fv}|e^{-HT}|\phi(x)=\phi_{\rm fv}\rangle = \int[d\phi]e^{-[S(\phi)+S_{\rm ct}(\phi)]} \tag{2.1}$$

where the Euclidean action

$$S(\phi)=\int d^3x\int_{-T/2}^{T/2}dx_4\left[\tfrac{1}{2}(\partial_u\phi)^2+V(\phi)\right] \tag{2.2}$$

is expressed in terms of renormalized fields and parameters, while $S_{\rm ct}$ contains the counterterms needed to make the theory finite. Although divergent, these counterterms are of higher order in the coupling constants and, as usual, are treated as perturbations.

The path integral is over all configurations such that ϕ takes its false vacuum value $\phi_{\rm fv}$ at $\pm T/2$ and at spatial infinity. In the limit $T\to\infty$ Eq. (2.1) is dominated by the lowest energy state with a nonvanishing contribution (i.e., the false vacuum) and is of the form

$$G(T)\approx e^{-\mathcal{E}T\Omega} \tag{2.3}$$

where Ω is the volume of space and $\mathcal{E}$ may be interpreted as the energy density of the false vacuum state. Because this is an unstable state, $\mathcal{E}$ is complex, with its imaginary part giving the decay rate, which in this case is simply the bubble nucleation rate. Dividing by Ω gives the nucleation rate per unit volume,

$$\Gamma=-2\,{\rm Im}\mathcal{E}\ . \tag{2.4}$$

The path integral may be approximated as the sum of the contributions about all of the stationary (or quasistationary) points of the Euclidean action $S(\phi)$: the pure false vacuum, the bounce solution with all possible locations in Euclidean space-time, and all multibounce configurations. In each case the contribution to the path integral is obtained by expanding the field about the classical solution $\bar{\phi}(x)$:

$$\phi(x)=\bar{\phi}(x)+\eta(x) \tag{2.5}$$

and then integrating over η. To leading approximation one keeps only the terms in the action which are quadratic in η. Expanding these in terms of the normal modes of $S''(\bar{\phi})=-\Box+V''(\bar{\phi})$ gives a product of Gaussian integrals. The terms of cubic and higher order in η can then be treated as perturbations, and have the effect of multiplying the Gaussian approximation for Γ by a power series in the coupling constants, with the first term being unity. The counterterms are also treated as a perturbation, with the leading contribution being from $S_{\rm ct}(\bar{\phi})$.

Thus, the integration about the trivial solution $\phi(x)=\phi_{\rm fv}$ yields

$$\begin{aligned}G_0&=NK_0e^{-S(\phi_{\rm fv})}e^{-S_{\rm ct}(\phi_{\rm fv})}(1+\cdots)\\&=NK_0e^{-\Omega TV(\phi_{\rm fv})}e^{-S_{\rm ct}(\phi_{\rm fv})}(1+\cdots)\end{aligned} \tag{2.6}$$

where N is a normalization factor,

$$\begin{aligned}K_0&=[\det S''(\phi_{\rm fv})]^{-1/2}\\&=\{\det[-\Box+V''(\phi_{\rm fv})]\}^{-1/2}\end{aligned} \tag{2.7}$$

and the dots represent terms, due to higher order perturbative effects, that can be neglected at the order to which we are working. Although the determinant is actually

divergent, the divergences are cancelled by the factor of $e^{-S_{ct}}$.

The calculation of the contribution from the bounce solution $\phi_b(x)$ is similar, but must be modified to take into account the fact that $S''(\phi_b)$ has one negative eigenvalue and four zero eigenvalues; for none of the corresponding modes is the integral truly Gaussian. The zero-frequency modes are treated by introducing collective coordinates corresponding to the position of the bounce. Integrating over these gives a factor of ΩT and introduces a Jacobian factor which, for O(4)-symmetric bounces, is given by

$$J=\frac{B^2}{4\pi^2} . \tag{2.8}$$

The negative mode is handled by deforming the contour of integration. Aside from a factor of $\frac{1}{2}$, this leads to a contribution whose imaginary part is just that which would have been obtained from a naive application of the Gaussian integration formulas; i.e.,

$$\mathrm{Im}G_1=G_0G_b \tag{2.9}$$

where

$$G_b=K_1J\Omega Te^{-B}e^{-[S_{ct}(\phi_b)-S_{ct}(\phi_{fv})]}(1+\cdots) \tag{2.10}$$

with

$$B=S(\phi_b)-S(\phi_{fv}) \tag{2.11}$$

and

$$\begin{aligned}K_1&=\frac{1}{2}K_0^{-1}|\det'[-\Box+V''(\phi_b)]|^{-1/2}\\&=\frac{1}{2}\left|\frac{\det'[-\Box+V''(\phi_b)]}{\det[-\Box+V''(\phi_{fv})]}\right|^{-1/2} .\end{aligned} \tag{2.12}$$

Here det' indicates that the translational zero-frequency modes are to be omitted when evaluating the determinant. As before, the divergences in the determinant factors are cancelled by the terms containing S_{ct}.

Finally, the n-bounce quasistationary points give a contribution of the form $G_n=G_0G_b^n/n!$. The bounce contributions then exponentiate, and one finds that

$$\begin{aligned}\Gamma&=2\frac{G_b}{\Omega T}\\&=\frac{B^2}{4\pi^2}e^{-B}\left|\frac{\det'[-\Box+V''(\phi_b)]}{\det[-\Box+V''(\phi_{fv})]}\right|^{-1/2}\\&\quad\times e^{-[S_{ct}(\phi_b)-S_{ct}(\phi_{fv})]}(1+\cdots) .\end{aligned} \tag{2.13}$$

For comparison with later results we need to know the order of magnitude of the various terms in this expression. To be specific, let us assume that we can identify a small coupling λ and a dimensionful quantity σ such that the potential can be written in the form

$$V(\phi)=\lambda\sigma^4U(\phi) \tag{2.14}$$

where U involves no small couplings and the dimensionless field $\psi=\phi/\sigma$. The minima of the potential must then be located either at $\phi=0$ or at values of ϕ of order σ. (With a single scalar field the most general renormalizable potential is of the form

$$V(\phi)=\frac{m^2}{2}\phi^2+\frac{c}{3}\phi^3+\frac{\lambda}{4}\phi^4 . \tag{2.15}$$

The above assumption is then equivalent to assuming that c is of the order of $\lambda\sigma$, with $\sigma\equiv m/\sqrt{\lambda}$.)

By defining a dimensionless variable $s=\sqrt{\lambda}\sigma x$, we may write the field equations as

$$\Box_s\psi=\frac{\partial U}{\partial\psi} . \tag{2.16}$$

From the assumptions made above, this equation involves no small parameters and so has a bounce solution in which ψ is of order unity and differs from the false vacuum within a region of a spatial extent (measured in terms of s) which is also of order unity. In terms of the original variables, the bounce has ϕ of order σ and extends over a range of x of order $1/(\sqrt{\lambda}\sigma)$.

With the same change of variables, the action becomes

$$S=\frac{1}{\lambda}\int d^4s\left[\frac{1}{2}\left[\frac{\partial\psi}{\partial s^\mu}\right]^2+U(\psi)\right] . \tag{2.17}$$

Since the integrand contains no small parameters, while the volume of the bounce restricts the integration to a region of order unity, the bounce action B is of order λ^{-1}.

Similarly, the determinant factor K_1 becomes

$$\begin{aligned}K_1&=\frac{1}{2}\left|\frac{\det'\{(\lambda\sigma^2)[-\Box_s+U''(\psi_b)]\}}{\det\{(\lambda\sigma^2)[-\Box_s+U''(\psi_{fv})]\}}\right|^{-1/2}\\&=\frac{1}{2}\lambda^2\sigma^4\left|\frac{\det'[-\Box_s+U''(\psi_b)]}{\det[-\Box_s+U''(\psi_{fv})]}\right|^{-1/2}\end{aligned} \tag{2.18}$$

where the explicit factor of $\lambda^2\sigma^4$ on the second line arises because the det' factor involves four fewer modes than the det factor. With this factor extracted, the ratio of determinants is formally of order unity, although divergent.[1] Finally, the Jacobian factor J is proportional to $B^2\sim\lambda^{-2}$. Putting all of these factors together, we see that the nucleation rate is of the form

$$\Gamma=c_1\sigma^4e^{c_2/\lambda} \tag{2.19}$$

with c_1 and c_2 both of order unity.

III. A FORMALISM FOR THEORIES WITH RADIATIVE SYMMETRY BREAKING

I now describe how the methods of the previous section can be adapted to theories where radiative correc-

[1]It was asserted above that these divergences are cancelled by divergences in $S_{ct}(\bar{\phi})-S_{ct}(\phi_{fv})$. To see that these counterterms are of the correct order of magnitude, note that with our assumptions the one-loop contributions to the counterterm Lagrangian are of order $\lambda^2\sigma^4$; since the bounce has a spatial volume of order $1/(\sqrt{\lambda}\sigma)^4$, the difference in the counterterm actions is of order unity, which is what is required.

tions to the tree-level potential qualitatively change the vacuum structure. The general formalism is derived in this section, while the detailed implementation for two specific theories is described in Secs. IV and V.

In theories of this sort the fields may be divided into two sets, denoted χ_j and ϕ_j. The former give rise to the radiative corrections which effect the changes in the vacuum structure, while the various vacua are distinguished by the value of the latter. In order that the radiative corrections from the χ loops be comparable to the tree-level terms, the interactions among the ϕ_j must be weak compared to the ϕ-χ interactions. Hence, the χ_j may be viewed as "fast" degrees of freedom which can be integrated out in a Born-Oppenheimer type approximation. For simplicity of notation, I will assume here a theory with only one field of each type, both assumed to be scalars; the generalization to more complicated theories is straightforward.

As usual, the bubble nucleation rate is obtained from the quantity

$$G(T)=\int [d\phi][d\chi]e^{-S(\phi,\chi)} . \tag{3.1}$$

(To simplify the equations the counterterm action has not been explicitly written.) Because $S(\phi,\chi)$ does not display the correct vacuum structure, its classical field equations do not have a bounce solution, and so the methods of Sec. II cannot be directly applied. This difficulty can be circumvented by integrating over χ to obtain

$$G(T)=\int [d\phi]e^{-W(\phi)} \tag{3.2}$$

where

$$W(\phi)=-\ln\int [d\chi]e^{-S(\phi,\chi)} \tag{3.3}$$

may be thought of as an effective action. W has a simple graphical interpretation. The integral on the right-hand side of Eq. (3.3) is equal to the sum of all vacuum Feynman diagrams in a theory of a quantized χ field interacting with a field c-number ϕ field; i.e., the sum of all graphs with only internal χ lines and external ϕ lines. Its logarithm gives the sum of all connected vacuum graphs. In general these include one-particle reducible graphs, although in many cases these are eliminated by symmetry considerations.

$W(\phi)$ should be compared with the more familiar effective action, $S_{\rm eff}(\phi,\chi)$, which generates one-particle irreducible Green's functions. For theories in which $W(\phi)$ only receives contributions from one-particle irreducible graphs, $S_{\rm eff}(\phi,0)$ and $W(\phi)$ differ by the contribution from graphs with internal ϕ lines. Our assumptions about the size of the ϕ self-couplings imply that these graphs are suppressed. Hence, the dominant terms in $S_{\rm eff}(\phi,0)$ and $W(\phi)$ agree, and so the latter correctly reflects the vacuum structure of the theory.

One might therefore envision using the classical field equations implied by W to determine a bounce solution which would be the basis for a nucleation rate calculation. Two practical difficulties arise. First, one cannot in general obtain a closed-form expression for W, but only a perturbative series. Second, $W(\phi)$ is a nonlocal functional, so that even if a closed-form expression were available, the equations determining its stationary points would be rather unpleasant.

One way around these difficulties is to find a local action $W_0(\phi)$ which is a sufficiently close approximation to W and then use its stationary points as the basis for the nucleation rate calculation. The possibility of such an approximation arises from the fact that the nonlocality of W is significant only on distances shorter than a characteristic size set by the χ-field interactions. For ϕ fields which are slowly varying relative to these scales, $W(\phi)$ can be expanded in a "derivative expansion" of the form

$$W(\phi)=\int d^4x[\hat{V}(\phi)+\tfrac{1}{2}\hat{Z}(\phi)(\partial_\mu\phi)^2+\cdots] \tag{3.4}$$

where the dots represent terms with four or more derivatives. (A similar expansion of the effective action,

$$S_{\rm eff}(\phi,\chi)=\int d^4x[V_{\rm eff}(\phi,\chi)+\tfrac{1}{2}Z_\phi(\phi,\chi)(\partial_\mu\phi)^2+\tfrac{1}{2}Z_\chi(\phi,\chi)(\partial_\mu\chi)^2+\cdots] \tag{3.5}$$

is often made. In line with the previous remarks, $\hat{V}(\phi)$ and $\hat{Z}(\phi)$ differ from $V_{\rm eff}(\phi,0)$ and $Z_\phi(\phi,0)$ by the omission of graphs with internal ϕ lines.)

The desired approximate action is obtained by keeping only the first two terms in the derivative expansion of $W(\phi)$, and then only the lowest order contributions to these; thus

$$W_0(\phi)=\int d^4x[\tfrac{1}{2}(\partial_\mu\phi)^2+\hat{V}_{1-\rm loop}(\phi)] . \tag{3.6}$$

Here $\hat{V}_{1-\rm loop}(\phi)$ is the sum of the tree-level $V(\phi)$ and the contributions from graphs with a single χ loop; to this order it is equal to $V_{\rm eff}(\phi,0)$. Since the loop corrections included in $\hat{V}_{1-\rm loop}(\phi)$ are precisely those responsible for altering the vacuum structure of the theory, W_0 possesses an appropriate bounce solution.

Thus, for slowly varying ϕ we may write

$$W(\phi)=W_0(\phi)+\delta W(\phi) \tag{3.7}$$

with $\delta W(\phi)$ representing subdominant terms. We can (and will) make the same decomposition for arbitrary ϕ, but δW will then not necessarily be small. In particular, because the path integral includes contributions from fields with Fourier components of arbitrarily high momentum, for which the derivative expansion is not valid, δW cannot be treated as a small perturbation within the path integral.

Nevertheless, the path integral may be approximated by expanding about the stationary points of W_0, provided that these are themselves slowly varying functions. Thus, let us suppose that the field equation implied by W_0,

$$\Box\phi=\frac{\partial\hat{V}_{1-\rm loop}}{\partial\phi} \tag{3.8}$$

has a solution $\bar{\phi}(x)$. Expanding W about this solution gives

$$W(\phi)=W(\bar{\phi})+\int d^4z\ W'(\bar{\phi};z)\eta(z)+\tfrac{1}{2}\int d^4z\,d^4z'W''(\bar{\phi};z,z')\eta(z)\eta(z')+O(\eta^3) \tag{3.9}$$

where $\eta(x)=\phi(x)-\bar{\phi}(x)$, and primes denote variational derivatives; e.g.,

$$W'(\bar{\phi};z)\equiv\left.\frac{\delta W(\phi)}{\delta\phi(z)}\right|_{\phi=\bar{\phi}}. \tag{3.10}$$

[The term linear in η is nonvanishing because $\bar{\phi}$ is not a stationary point of the full $W(\phi)$.]

We now insert Eq. (3.9) into the path integral. The cubic and higher order terms in η can be treated as small perturbations. If these are dropped, the integral becomes Gaussian. About the false vacuum, $\bar{\phi}(x)=\phi_{\rm fv}$, $W''(\bar{\phi})$ has no zero or negative eigenvalues and the integration gives

$$G_0=e^{-W(\phi_{\rm fv})}e^{W'(\phi_{\rm fv})[W''(\phi_{\rm fv})]^{-1}W'(\phi_{\rm fv})}\times[\det W''(\phi_{\rm fv})]^{-1/2}. \tag{3.11}$$

(For the sake of compactness, the spatial arguments of the variational derivatives and the associated spatial integrations have not been explicitly displayed.) As in the standard case, the bounce solution has zero-frequency modes which must be handled separately.[2] Introducing collective coordinates and proceeding as usual leads to

$$\mathrm{Im}G_1=\tfrac{1}{2}\Omega T e^{-W(\phi_b)}e^{W'(\phi_b)[W''(\phi_b)]^{-1}W'(\phi_b)}\times|\det' W''(\phi_b)|^{-1/2}J \tag{3.12}$$

where now det′ indicates that the determinant is restricted to the subspace orthogonal to the zero modes of W_0''. Similarly, $(W'')^{-1}$ is to be evaluated in this subspace, while the Jacobian J is given by

$$J=\frac{[W_0(\phi_b)-W_0(\phi_{\rm fv})]^2}{4\pi^2}. \tag{3.13}$$

The path from these results to the nucleation rate is just as before, and gives

$$\Gamma=e^{-C_1}e^{C_2}\left|\frac{\det' W''(\phi_b)}{\det W''(\phi_{\rm fv})}\right|^{-1/2}J(1+\cdots), \tag{3.14}$$

where

$$C_1=W(\phi_b)-W(\phi_{\rm fv}) \tag{3.15}$$

and

$$C_2=W'(\phi_b)[W''(\phi_b)]^{-1}W'(\phi_b)-(\phi_b\to\phi_{\rm fv}). \tag{3.16}$$

The next step is to expand the various terms in Eq. (3.14) in powers of the coupling constants. The details of this depend on the structure of the theory and the magnitudes of the various couplings. The general features described below hold for the specific theories considered in the next two sections and are likely to be true for all theories with radiative symmetry breaking. As in Sec. II, the expansion of Γ is only carried out to terms of order unity; terms which do not contribute to that order are represented by dots.

First, the bounce solution has a characteristic length scale which is large compared to those characterizing the χ-χ and χ-ϕ interactions. It therefore varies slowly enough to allow a derivative expansion of $W(\phi_b)$. (We will see that in some situations this expansion must be terminated after the first few terms, leaving a nonlocal remainder. This does not materially affect the picture outlined here.) For both $\bar{\phi}=\phi_b$ and $\bar{\phi}=\phi_{\rm fv}$, it is then possible to write $W(\bar{\phi})$ as a sum of terms of successively higher order in the couplings:

$$W(\bar{\phi})=W_0(\bar{\phi})+W_1(\bar{\phi})+W_2(\bar{\phi})+\cdots \tag{3.17}$$

with W_0 given by Eq. (3.6) and the remaining terms containing higher order contributions to the derivative expansion. For example, in the theories considered below W_1 contains two-loop contributions to $\hat{V}$ and one-loop contributions to $\hat{Z}$.

A similar expansion can be used for $W'(\bar{\phi};z)$. The zeroth order term vanishes because $\bar{\phi}$ is a stationary point of W_0, and we have

$$W'(\bar{\phi};z)=W_1'(\bar{\phi};z)+\cdots. \tag{3.18}$$

Matters are less simple for $W''(\bar{\phi};z,z')$. Although the derivative expansion can be used when $|z-z'|$ is large, the behavior for small $|z-z'|$ is sensitive to the high-momentum modes and so the derivative expansion fails even for constant $\bar{\phi}$. However, the relation

$$W''=W_0''[1+(W_0'')^{-1}\delta W''] \tag{3.19}$$

can be used to obtain formal expansions for $(W'')^{-1}$ and $\det W''$ as power series in $(W_0'')^{-1}\delta W''$. The actual utility of these expansions depends on the size of the contribution from the region of small $|z-z'|$. In the calculation of C_2 this contribution is subdominant and

$$C_2=W_1'(\phi_b)[W_0''(\phi_b)]^{-1}W_1'(\phi_b)-(\phi_b\to\phi_{\rm fv})+\cdots. \tag{3.20}$$

For the determinant factor, on the other hand, more terms must be retained:

$$\begin{aligned}\det'[W'']&=\det'[W_0'']\det[I+(W_0'')^{-1}\delta W'']\\&=\det'[W_0'']\exp\{\mathrm{tr}\ln[I+(W_0'')^{-1}\delta W'']\}\\&=\det'[W_0''[\exp\{\mathrm{tr}(W_0'')^{-1}\delta W''\\&\qquad+\tfrac{1}{2}\mathrm{tr}[(W_0'')^{-1}\delta W'']^2+\cdots\}.\end{aligned} \tag{3.21}$$

Inserting these expansions into Eq. (3.14) leads to

$$\Gamma=e^{-B_0}e^{-B_1}e^{-B_2}\left|\frac{\det' W_0''(\phi_b)}{\det W_0''(\phi_{\rm fv})}\right|^{-1/2}\frac{B_0^2}{4\pi^2}(1+\cdots) \tag{3.22}$$

[2]Because the bounce is obtained from the truncated action W_0, the usual translational modes, proportional to $\partial_\mu\phi_b$, differ slightly from the zero modes of W''; however, it can be shown that the effects due to the difference are suppressed by a power of the coupling constant and hence can be neglected at the order to which we will be working.

where

$$B_0 = W_0(\phi_b) - W_0(\phi_{\rm fv}) , \tag{3.23}$$

$$B_1 = W_1(\phi_b) + \tfrac{1}{2}{\rm tr}[W_0''(\phi_b)]^{-1}\delta W''(\phi_b) - (\phi_b \to \phi_{\rm fv}) , \tag{3.24}$$

and

$$B_2 = W_2(\phi_b) - W_0'(\phi_b)[W_0''(\phi_b)]^{-1}W_0'(\phi_b) + \tfrac{1}{4}{\rm tr}\{[W_0''(\phi_b)]^{-1}\delta W''(\phi_b)\}^2 - (\phi_b \to \phi_{\rm fv}) . \tag{3.25}$$

We thus have the naive expression for Γ, obtained by the standard procedure with $V(\phi)$ replaced by the one-loop effective potential, multiplied by a correction factor $e^{-(B_1+B_2)}$. We will find that B_1 is proportional to an inverse power of the couplings, while B_2 is of order unity. Thus, although the B_2 correction is of the same order of magnitude as the determinant factor, which in practice cannot be evaluated precisely, the B_1 factor modifies the naive result in a significant and (at least numerically) calculable manner.

The form of these correction factors can be understood. B_0 is obtained by evaluating the approximate action W_0 at its stationary points, whereas what we really want is the exact action W evaluated at its own stationary points. The difference between these two quantities can be obtained by treating $\delta W = W_1 + W_2 + \cdots$ as a small perturbation; up to second order, this gives the first term in B_1 and the first two terms in B_2. As we will see below, the remaining terms correct for the fact that W is not the full effective action and take into account contributions to the full effective action which would otherwise be neglected.

IV. EXAMPLE 1: A SCALAR FIELD THEORY

As a first example, consider a theory with two scalar fields governed by the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}(\partial_\mu\chi)^2 - V(\phi,\chi) + \mathcal{L}_{\rm ct} \tag{4.1}$$

with

$$V = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{2}m^2\chi^2 + \frac{f}{4!}\chi^4 + \frac{1}{2}g^2\phi^2\chi^2 . \tag{4.2}$$

Both μ^2 and m^2 are positive. In order that the χ-loop corrections to the effective potential be able to generate a symmetry-breaking minimum at a value $\phi=\sigma\neq 0$, λ must be $O(g^4)$, while f should be $O(g^2)$ to ensure that the $\chi\to-\chi$ symmetry remains unbroken. There is also an upper bound of order $g^2\sigma$ on μ, which will be displayed in detail below. Finally, it will be convenient to assume that m is $O(g\sigma)$.

Renormalization conditions must be specified in order to fix the counterterms in $\mathcal{L}_{\rm ct}$. For the lowest order approximation to the effective potential only the definitions of μ^2 and λ are needed. These can be given by choosing an arbitrary renormalization scale $\tilde\sigma$ and requiring that

$$\mu^2 = \left.\frac{\partial^2 V_{\rm eff}}{\partial\phi^2}\right|_{\phi=\chi=0} \tag{4.3}$$

and

$$\lambda = \left.\frac{\partial^4 V_{\rm eff}}{\partial\phi^4}\right|_{\phi=\tilde\sigma,\chi=0} . \tag{4.4}$$

For $\chi=0$ the effective potential is given by the sum of graphs with only external ϕ lines, all carrying zero four-momentum. The leading contributions are from tree graphs and graphs with one χ loop, as well as the corresponding counterterms. Let us denote[3] the sum of these by $V_{\text{1-loop}}(\phi)$, although it should be kept in mind that this does not include the graphs with one ϕ loop. We then have, up to an additive constant,

$$V_{\text{1-loop}}(\phi) = \frac{1}{64\pi^2}(m^2+g^2\phi^2)^2\left[\ln\left[\frac{m^2+g^2\phi^2}{m^2+g^2\tilde\sigma^2}\right]-\frac{1}{2}\right] - \frac{1}{4}\left[\frac{m^2}{\tilde\sigma^2}+y_1\right](\phi^2-\tilde\sigma^2)^2 + \left[\frac{1}{4!}(\lambda+z_1)+\frac{1}{4}\left[\frac{\mu^2}{\tilde\sigma^2}+y_1\right]\right]\phi^4 \tag{4.5}$$

where

$$y_1 = \frac{g^2m^2}{16\pi^2}\ln\left[\frac{m^2+g^2\tilde\sigma^2}{m^2}\right] \tag{4.6}$$

and

$$z_1 = \frac{g^4}{8\pi^2}\left[\frac{4g^4\tilde\sigma^4}{(m^2+g^2\tilde\sigma^2)^2} - \frac{12g^2\tilde\sigma^2}{m^2+g^2\tilde\sigma^2} - 3\right] . \tag{4.7}$$

It is most convenient to choose the renormalization point $\tilde\sigma$ to be σ, the location of the minimum of $V_{\text{1-loop}}$. The requirement $V'_{\text{1-loop}}(\sigma)=0$ then relates λ to the other parameters via

$$\lambda = -3\left[\frac{\mu^2}{\sigma^2}+y_1\right]-z_1 \tag{4.8}$$

and yields

$$V_{\text{1-loop}}(\phi) = \frac{1}{64\pi^2}(m^2+g^2\phi^2)^2\left[\ln\left[\frac{m^2+g^2\phi^2}{m^2+g^2\sigma^2}\right]-\frac{1}{2}\right] - \frac{1}{4}\left[\frac{\mu^2}{\sigma^2}+y_1\right](\phi^2-\sigma^2)^2 . \tag{4.9}$$

For σ to be a minimum, $V''_{\text{1-loop}}(\sigma)$ must be positive, which leads to the bound

$$\mu^2 < \frac{g^4\sigma^2}{16\pi^2}\left[4-\frac{m^2}{g^2\sigma^2}\ln\left[\frac{m^2+g^2\sigma^2}{m^2}\right]\right] . \tag{4.10}$$

A slightly stronger bound on μ is obtained if one requires, as we will, that the minimum at $\phi=\sigma$ be deeper than that at $\phi=0$, so that the latter corresponds to a metastable

[3]Since $\chi=0$ is to be understood for the remainder of this discussion, $V_{\rm eff}$ and related quantities will be written as functions of a single variable.

false vacuum.

The higher order contributions to the effective potential can be obtained from Feynman graphs with "dressed" propagators that take into account the background ϕ field. Thus, the effect of summing over all numbers of interactions with a constant external ϕ field is to replace the standard χ propagator $(k^2-m^2)^{-1}$ by

$$[k^2-m^2-g^2\phi^2]^{-1}\equiv[k^2-M^2(\phi)]^{-1} . \tag{4.11}$$

Because the χ-loop contributions to $V_{\rm eff}$ are of the same magnitude as the tree terms, the dressed ϕ propagator is somewhat more complicated. In order that the perturbative order of a graph increase with the number of loops, these one-loop terms must be included in the dressed propagator, which then takes the form

$$[k^2-V''_{\text{1-loop}}(\phi)]^{-1}\equiv[k^2-\mathcal{M}^2(\phi)]^{-1} . \tag{4.12}$$

This inclusion of loops in the propagator means that the loop counting for a graph with dressed propagators will not necessarily agree with that for the corresponding graphs with elementary propagators. Thus, the one ϕ-loop contribution to the effective potential will include graphs which are multiloop when drawn with elementary propagators. Some examples of these are shown in Fig. 1. In the last of these the momentum assignments have been shown explicitly in order to emphasize that they are not the ones usually associated with this graph. It should be clear from this example that graphs with both χ and ϕ internal lines must be analyzed carefully to avoid double counting.

The next contributions to $V_{\rm eff}$, of order fg^4 and g^6, arise from the two-loop graphs in Fig. 2, the one-loop counterterm graphs in Fig. 3, and the $O(fg^4)$ and $O(g^6)$ counterterms. The double-counting issue arises with the second graph of Fig. 2, which has the same topology as the graph of Fig. 1(c), but a different (i.e., the standard) assignment of momenta. However, since the latter graph is $O(g^8)$ (after renormalization), the issue can be ignored at this point.

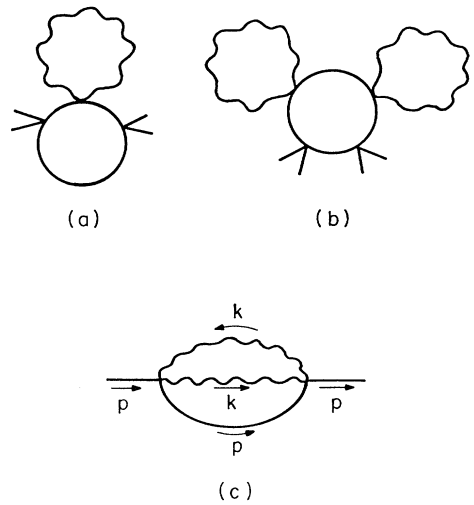

FIG. 1. Examples of graphs which, although multiloop when drawn in terms of elementary propagators, are included in the one ϕ-loop contribution to the effective potential in the model of Sec. IV. Solid and wiggly lines represent ϕ and χ propagators, respectively.

The sum of these graphs gives a contribution which, in terms of an arbitrary renormalization scale $\tilde{\sigma}$, can be written in the form

$$\begin{aligned}V_2(\phi)=&\frac{1}{2048\pi^4}[f(m^2+g^2\phi^2)+32g^4\phi^2](m^2+g^2\phi^2)\ln^2\left[\frac{m^2+g^2\phi^2}{m^2+g^2\tilde{\sigma}^2}\right]\\&+\frac{1}{64\pi^2}(c_1f+c_2g^2)(m^2+g^2\phi^2)^2\left[\ln\left[\frac{m^2+g^2\phi^2}{m^2+g^2\tilde{\sigma}^2}\right]-\frac{1}{2}\right]\\&+(c_3f+c_4g^2)m^2(m^2+g^2\phi^2)\left[\ln\left[\frac{m^2+g^2\phi^2}{m^2+g^2\tilde{\sigma}^2}\right]-1\right]-\frac{y_2}{4}(\phi^2-\tilde{\sigma}^2)^2+\left[\frac{z_2}{4!}+\frac{y_2}{4}\right]\phi^4 .\end{aligned} \tag{4.13}$$

Here the c_k are numbers of order unity which depend on the precise choice of the renormalization conditions fixing m^2, g^2, and the χ-field wave-function renormalization, while y_2 and z_2 are to be chosen so as to satisfy Eqs. (4.3) and (4.4). If $\tilde{\sigma}$ is again chosen to be σ, with the latter now understood to be the minimum of the two-loop approximation to $V_{\rm eff}$, this result combines with $V_{\text{1-loop}}$ to give

$$\begin{aligned}V_{\text{2-loop}}(\phi)=&\frac{1}{64\pi^2}(1+c_1f+c_2g^2)(m^2+g^2\phi^2)^2\left[\ln\left[\frac{m^2+g^2\phi^2}{m^2+g^2\sigma^2}\right]-\frac{1}{2}\right]\\&+\frac{1}{2048\pi^4}[f(m^2+g^2\phi^2)+32g^4\phi^2](m^2+g^2\phi^2)\ln^2\left[\frac{m^2+g^2\phi^2}{m^2+g^2\sigma^2}\right]\\&+(c_3f+c_4g^2)m^2(m^2+g^2\phi^2)\left[\ln\left[\frac{m^2+g^2\phi^2}{m^2+g^2\sigma^2}\right]-1\right]-\frac{1}{4}\left[\frac{\mu^2}{\sigma^2}+y\right](\phi^2-\sigma^2)^2 ,\end{aligned} \tag{4.14}$$

where $y=y_1+y_2$.

Among the contributions at $O(g^8)$ are those from the graphs with a single ϕ loop. These include a term proportional to $\ln V''_{\text{1-loop}}(\phi)$ which is complex whenever $V''_{\text{1-loop}}$ is negative. While such complex effective potentials are not unfamiliar, and their physical interpretation is understood [5], their implications for bubble nucleation calculations have remained somewhat unclear. We will see below how the potential problems associated with these are avoided.

Let us now turn to the bubble nucleation problem. The first step is to calculate $W(\phi)$. Because the Lagrangian only contains even powers of χ, only one-particle irreducible graphs contribute. One obtains

$$W(\phi)=\int d^4x\left[\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{2}\mu^2\phi^2+\frac{\lambda}{4}\phi^4\right]+\frac{1}{2}\int d^4x\,\langle x|\ln[-\Box+M^2(\phi)]|x\rangle+\text{counterterms}+\cdots \tag{4.15}$$

where the contributions from tree and one-loop graphs have been shown explicitly. The second term, arising from the loop graphs, is sensitive to the value of $\phi(y)$ over a region of size $\sim M^{-1}(\phi)\sim(g\sigma)^{-1}$. Hence, for our purposes a slowly varying ϕ will be one which varies slowly over this distance. Expanding such a field about its value at a point x leads to the derivative expansion for W.

The first term in the expansion, $\hat{V}$, is obtained by treating ϕ as a constant. To order g^4 this is the same as the standard calculation of the effective potential, and gives the result displayed in Eq. (4.9). However, the two quantities differ at order g^6, since the two-loop graph of Fig. 2(b) contributes to V_{eff} but not to $\hat{V}$. Because this graph is divergent, while the counterterm contributions to the two potentials are the same, $\hat{V}$, unlike V_{eff}, cannot be finite. This divergence is acceptable because $\hat{V}$ does not directly correspond to any physical quantity. Indeed, it combines with other divergent quantities to give a finite result for Γ.

The calculation of the next term in the derivative expansion is equivalent to extracting the term of order p^2 from the sum of graphs with two external ϕ lines carrying nonzero momentum p and any number of zero-momentum external ϕ lines. Using dressed propagators, the leading part of this sum can be represented by the single graph of Fig. 4, whose evaluation gives

$$f(p^2)=f(0)+\frac{g^4}{8\pi^2}\int_0^1 dx\,\ln\left[1+\frac{p^2x(1-x)}{M^2(\phi)}\right]. \tag{4.16}$$

For $p^2\lesssim M^2(\phi)$ this can be expanded in powers of p^2:

$$f(p^2)=f(0)+\frac{g^4}{48\pi^2}\left[\frac{p^2}{M^2(\phi)}+\frac{p^4}{5M^4(\phi)}+\cdots\right]. \tag{4.17}$$

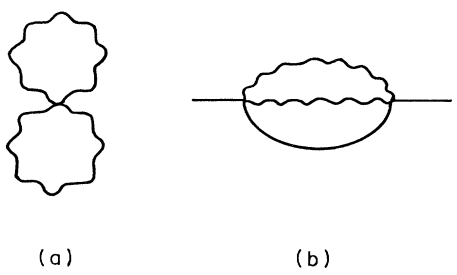

FIG. 2. The two-loop graphs which contribute to the effective potential in the model of Sec. IV at order fg^4 and g^6.

[The failure of this expansion when $p^2\gtrsim M^2(\phi)$ reflects the failure of the derivative expansion when the background ϕ field is not slowly varying.] The first term, $f(0)$, has been included in the calculation of $\hat{V}$. The $O(p^2)$ term leads to

$$\hat{Z}(\phi)=1+\frac{g^2}{48\pi^2}\left[\frac{g^2\phi^2}{m^2+g^2\phi^2}+k\right]+O(g^4), \tag{4.18}$$

where k is a constant of order unity which is chosen to enforce the ϕ wave function renormalization condition. [Note that to this order $\hat{Z}(\phi)=Z_\phi(\phi,0)$.] The $O(p^4)$ term gives a contribution to the derivative expansion of the form $H(\phi)(\Box\phi)^2$. In addition, there are other four-derivative terms, proportional to $(\partial_\mu\phi)^2\Box\phi$ and $(\partial_\mu\phi)^2(\partial_\nu\phi)^2$, which can be obtained from graphs with three or four external lines carrying nonzero momentum.

The bounce solution is determined by the approximate action of Eq. (3.6) with $\hat{V}_{\text{1-loop}}(\phi)$ being just the $V_{\text{1-loop}}(\phi)$ of Eq. (4.9). Scaling arguments similar to those of Sec. II show that it has a spatial extent of order $(g^2\sigma)^{-1}$ and thus varies slowly enough to allow a derivative expansion. By contrast, in a theory with couplings of "normal" magnitude ($\lambda\sim g^2$ and $\mu^2\sim g^2\sigma^2$) the bounce would have a size $\sim(g\sigma)^{-1}$ and would not be slowly varying. Of course, in that case the loop corrections to the effective potential would not change the vacuum structure, so the standard calculation of Γ could be used and there would be no need for a derivative expansion.

We need to know the magnitude of the various terms in the derivative expansion of $W(\phi_b)$. Since $\phi_b\sim\sigma$, we see that $\hat{V}(\phi_b)\sim g^4\sigma^4$, while our estimate of the bounce size implies that $(\partial_\mu\phi_b)^2\sim g^4\sigma^4$. More generally, consider a term containing j derivatives and k explicit factors of ϕ_b, which might be written schematically as

$$w_{jk}=H_{jk}(\phi_b)\partial^j\phi_b^k. \tag{4.19}$$

$H_{jk}(\phi)$ must have dimensions of $(\text{mass})^{4-j-k}$; since it is

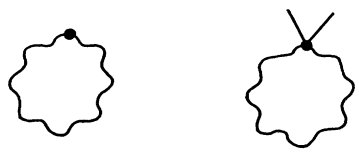

FIG. 3. The one-loop counterterm graphs which contribute to the effective potential in the model of Sec. IV at order fg^4 and g^6; the heavy dots denote counterterm insertions.

FIG. 4. A one-loop graph from which the leading contributions to $\hat{Z}(\phi)$ can be computed.

derived from graphs with only internal χ lines, the mass appearing here must be the effective χ mass, $M(\phi)$. Further, the structure of the graphs implies that there is a factor of g for each explicit factor of ϕ, except in the tree-level contribution to $\hat{Z}$. Finally, each derivative brings in a factor of $g^2\sigma$. Thus, apart from the terms contributing to $\hat{Z}$ (i.e., the case $j=k=2$),

$$w_{jk} \sim \left[\frac{g^2\sigma}{M(\phi)}\right]^j \left[\frac{g\phi}{M(\phi)}\right]^k M^4(\phi) \sim g^{4+j}\left[\frac{\phi}{\sigma}\right]^k \sigma^4 \tag{4.20}$$

where the fact that $M^2(\phi) \geq m^2 \sim g^2\sigma^2$ has been used and it has been assumed that $\phi \lesssim \sigma$.

We can now estimate the various terms entering the formula for Γ, beginning with the quantities W_j in Eq. (3.17). Because Γ involves the differences between these quantities evaluated at $\phi=\phi_b$ and at $\phi=\phi_{\rm fv}$, the integrals are effectively restricted to a region of volume $\sim(1/g^2\sigma)^4$. Using the above estimates for the integrands, we then find that the first two terms are

$$W_0 = \int d^4x\,[\hat{V}_{g^4} + (\partial\phi)^2] \sim g^{-4} \tag{4.21}$$

and

$$W_1 = \int d^4x\,[\hat{V}_{g^6} + \hat{Z}_{g^2}(\partial\phi)^2] \sim g^{-2}\ . \tag{4.22}$$

Here the subscripts indicate the order of the contributions to $\hat{V}$ and $\hat{Z}$, with $\hat{V}_{g^4}$ being the same as $\hat{V}_{\text{1-loop}}$, and $\hat{V}_{g^6}$ including both g^6 and fg^4 terms. The next term, W_2, is of order unity and contains $\hat{V}_{g^8}$ (arising from graphs with up to three loops), $\hat{Z}_{g^4}$ (arising from one- and two-loop graphs), and the one-loop contributions to the four-derivative terms.

We also need expansions of the first two variational derivatives of W. In principle, the variations should be performed before the expansion is carried out, although this turns out not to matter for the first variation. Thus, to one-loop order

$$W'(\phi;z) = -\Box\phi(z) + V'(\phi(z)) + g^2\phi(z)\langle z|[-\Box+M^2(\phi)]^{-1}|z\rangle + \text{counterterms}\ . \tag{4.23}$$

The leading contribution to the third, nonlocal, term is obtained by taking ϕ to be constant. This combines with V' to give $\hat{V}'_{\text{1-loop}}$, so that the leading approximation to W' is indeed the variation of the leading approximation to W. One can check that analogous statements hold at the next order, thus verifying Eq. (3.18).

Up to one-loop order the second variational derivative is

$$\begin{aligned} W''(\phi;z,z') = {} & \delta^{(4)}(z-z')[-\Box+\hat{V}''(\phi)] + g^2\delta^{(4)}(z-z')\langle z|[-\Box+M^2(\phi)]^{-1}|z\rangle \\ & -2g^4\phi(z)\phi(z')\langle z|[-\Box+M^2(\phi)]^{-1}|z'\rangle^2 + \text{counterterms}\ . \end{aligned} \tag{4.24}$$

For $|z-z'| \lesssim M(\phi)$ the nonlocality of the third term cannot be ignored and the derivative expansion fails, even in a spatially constant background. To see this, let us suppose that $\phi(x)$ is constant, so that we can work in momentum space with p being the momentum conjugate to $z-z'$. The last term in Eq. (4.24) is then given by the graph of Fig. 4. If p were set equal to 0 in the evaluation of the loop, then the one-loop terms shown here would give W_0'', which in momentum space is simply $W_0''(p) = p^2 + \hat{V}''_{\text{1-loop}}(\phi) = p^2 + \mathcal{M}^2(\phi)$. Subtracting this and recalling Eq. (4.16), we see that $\delta W''(p) \sim g^2p^2\ln[1+p^2/M^2(\phi)]$. If $p^2/M^2(\phi)$ is small, this can be treated as a small perturbation, but this is not the case when $p^2/M^2(\phi)$ is so large that the logarithm overcomes the factor of g^2. Correspondingly, in position space $\langle z|[W_0'']^{-1}\delta W''|z'\rangle$ is small except in a region $|z-z'| \ll M^{-1}(\phi)$. Similar estimates clearly apply with a slowly varying background field.

W'' enters the formula for the nucleation rate both through the quantity C_2 defined by Eq. (3.16) and through the determinant factor. In both cases Eq. (3.19) leads to a formal expansion in which the higher order terms are suppressed if $\delta W''$ is indeed a small perturbation; i.e., if the contribution of the short-distance, high-momentum region is suppressed. Roughly speaking, C_2 can be viewed as corresponding to a tree graph in which two factors of $\delta W'$ are connected by a ϕ propagator. The magnitude of the momentum running through this propagator is set by the spatial extent of the bounce solution, and is indeed small enough for $\delta W''$ to be treated as a perturbation. (This can be checked by a detailed examination of the contribution from the small $z-z'$ region.) We may therefore write

$$\begin{aligned} C_2 = {} & \int d^2z\, d^4z'\, W_1'(\phi_b;z) W_1'(\phi_b;z') \\ & \times \langle z|[W_0''(\phi_b)]^{-1}|z'\rangle - (\phi_b \to \phi_{\rm fv}) + \cdots\ . \end{aligned} \tag{4.25}$$

The size of the bounce restricts the z integration to a volume of size $(1/g^2\sigma)^4$, while the falloff of $[W_0'']^{-1}$ restricts $z-z'$ to a similar volume. The two factors of W_1' are each of order $g^6\sigma^3$, while $[W_0'']^{-1}$ gives a factor of roughly $(z-z')^2$. Combining these facts, one finds that the terms shown explicitly in Eq. (4.25) are of order unity.

For the determinant factor, we recall from Eq. (3.21) that $\det W''$ can be written as a product of $\det W_0''$ and the

exponential of a power series in $(W_0'')^{-1}\delta W''$. The former combines with the Jacobian to give σ^4 times a factor of order unity, as in the standard calculation. The latter corresponds to a sum of one-loop graphs in which a number of insertions of $\delta W''$ are connected by an equal number of ϕ propagators. This sum has an ultraviolet divergence which can be attributed to the graphs with one and two insertions of $\delta W''$; even after the cancellation of these divergences by counterterm contributions, we should expect the high-momentum region to be enhanced, and these terms to be anomalously large. This is in fact what happens. Working in momentum space, and ignoring the spatial variation of ϕ for the sake of clarity, we may write the nth term in the expansion as

$$\mathrm{tr}[(W_0'')^{-1}\delta W'']^n = \int d^4x\, d^4p \{[W_0'']^{-1}(p)\delta W''(p)\}^n . \tag{4.26}$$

The momentum integration is quartically divergent; after cancellation of infinities by counterterms a finite term proportional to the fourth power of $M(\phi)$, the largest relevant mass, remains. There is a factor of g^2 for each $\delta W''$. Finally, because we must eventually take the difference between the contributions of the bounce and of the pure false vacuum, the x integration is effectively restricted to a volume $\sim(g^2\sigma)^{-4}$. The net result is a contribution of order g^{2n-4}, implying that both the $n=1$ and the $n=2$ terms are of order unity or larger, as indicated in Eq. (3.21).

The dominant, $O(g^{-2})$, contribution from the $n=1$ term is obtained by treating ϕ as a constant. This term then essentially reproduces the two-loop graph of Fig. 2(b), which contributes to $V_{\rm eff}$ but not to $\hat{V}$. [The definition of W_0'' actually requires a subtraction corresponding to the graph of Fig. 1(c), but this is simply the correction for double counting noted previously.] To this order, then, the net effect is to simply replace $\hat{V}$ by the full effective potential. Furthermore, since $\hat{Z}$ and Z_ϕ are equal at $O(e^2)$, it might seem that the derivative expansion of $W(\phi)$ is simply being replaced by the derivative expansion of $S_{\rm eff}(\phi,\chi=0)$.

However, when we go to the next higher order it becomes clear that this is not the case. At $O(g^8)$, $V_{\rm eff}$ differs from $\hat{V}$ by the contribution from two- and three-loop graphs with both χ and ϕ internal lines plus the contribution from graphs with a single ϕ loop. The former set of graphs is reproduced by the $n=2$ term in the expansion of the determinant and by the next-to-leading contribution of the $n=1$ term. The contribution of the latter set is contained in the factors of $\det W_0''$; indeed, for a constant background ϕ the logarithm of this determinant is just the spatial integral of this part of the effective potential. Matters are more complicated when this determinant is evaluated in the background of the bounce. Since the effective ϕ mass is a factor of g smaller than the effective χ mass, the derivative expansion of $\det W_0''$ requires a more slowly varying background field than is needed for the expansion of the terms in $W(\phi)$ arising from χ loops. Precisely because the bounce is determined by the field equations of W_0, its characteristic size is incompatible with a derivative expansion of $\det W_0''(\phi_b)$, and so the ϕ-loop contribution to $V_{\rm eff}$ cannot be simply isolated.

This resolves a puzzle associated with the complexity of the perturbative effective potential. As noted above, the ϕ-loop contribution to $V_{\rm eff}(\phi)$ is complex for values of ϕ such that $V''_{\text{1-loop}}(\phi)$ is negative. Since the bounce solution $\phi_b(x)$ lies in this range for some values of x, a manifestly complex result for the nucleation rate would have been obtained if the calculation had explicitly involved the full effective potential.[4]

Of course, the fact that this effective potential contribution cannot be isolated does not rule out the possibility that a complexity problem could arise with the full determinant, where it would be manifested by the existence of a negative eigenvalue of $W_0''(\phi_b)$ in addition to the one which occurs for any bounce. However, from the fact that the bounce corresponds to the solution of a minimization problem (that of finding the path through configuration space with the smallest WKB tunneling exponent), it is easy to show [6] that the bounce solution of minimum action can never have additional negative modes. Hence, $|\det' W_0''(\phi_b)|^{1/2}$ is real and the imaginary part of the effective potential has no effect on the bubble nucleation calculation.

To summarize these results, we may write

$$\Gamma = \tilde{A} e^{-(\bar{B}_0+\bar{B}_1)} , \tag{4.27}$$

where

$$\bar{B}_0 = \int d^4x \{[V^g_{\rm eff}(\phi_b)+\tfrac{1}{2}(\partial_\mu\phi_b)^2] - (\phi_b\to\phi_{\rm fv})\} = O(g^{-4}) , \tag{4.28}$$

$$\bar{B}_1 = \int d^4x \{[V^g_{\rm eff}(\phi_b)+\tfrac{1}{2}Z^g_\phi(\phi_b)(\partial_\mu\phi_b)^2] - (\phi_b\to\phi_{\rm fv})\} = O(g^{-2}) , \tag{4.29}$$

and the preexponential factor $\tilde{A}$, now understood to include the contribution from the $O(1)$ part of the exponent in Eq. (3.22), is equal to σ^4 times a dimensionless factor of order unity.

V. EXAMPLE 2: SCALAR ELECTRODYNAMICS

A second example is provided by scalar electrodynamics [2]. In terms of the real and imaginary parts, ϕ_1 and ϕ_2 of the scalar field, the Lagrangian takes the form

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\phi_1 + eA_\mu\phi_2)^2 + \frac{1}{2}(\partial_\mu\phi_2 - eA_\mu\phi_1)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 , \tag{5.1}$$

where $\phi^2\equiv\phi_1^2+\phi_2^2$. In order that the one-loop radiative corrections give rise to the desired vacuum structure with both a symmetric minimum at $\phi=0$ and a symmetry-breaking minimum at $\phi=\sigma$, we require that λ be $O(e^4)$ and that μ^2 be positive and $O(e^4\sigma^2)$. As with the previ-

[4]Although $\det W_0''(\phi_{\rm fv})$ can be expanded in a derivative expansion, no complexity problem arises because $V_{\rm eff}(\phi_{\rm fv})$ is real.

ous example, μ and λ are to be fixed by Eqs. (4.3) and (4.4), while the other renormalization conditions will not be explicitly given.

It is most convenient to calculate the effective potential in Landau gauge. In this gauge all graphs with a zero-momentum external ϕ line entering a ϕ-ϕ-A_μ vertex vanish; this leads to a considerable reduction in the number of graphs to be calculated.[5] A further simplification follows from the observation that $V_{\rm eff}$ can only depend on ϕ^2, so that we may calculate it with $\phi_2=0$ and $\phi_1=\phi$. To $O(e^4)$ the result is

$$V_{\text{1-loop}}(\phi)=\frac{1}{2}\mu^2\phi^2+\frac{\lambda}{4!}\phi^4 + \frac{3e^4}{64\pi^2}\phi^2\left[\ln\left[\frac{\phi^2}{\bar{\sigma}^2}\right]-\frac{25}{6}\right]. \tag{5.2}$$

If, as before, we choose $\bar{\sigma}=\sigma$, the minimum of $V_{\text{1-loop}}$, we can eliminate λ and, after adding a constant, write

$$V_{\text{1-loop}}(\phi)=\frac{3e^4}{64\pi^2}\phi^4\left[\ln\left[\frac{\phi^2}{\sigma^2}\right]-\frac{1}{2}\right] -\frac{1}{4}\frac{\mu^2}{\sigma^2}(\phi^2-\sigma^2)^2 . \tag{5.3}$$

Requiring that $V''_{\text{1-loop}}(\sigma)>0$ implies that $\mu^2<3e^4\sigma^2/16\pi^2$; for this asymmetric minimum to be the true vacuum, the stronger condition $\mu^2<3e^4\sigma^2/32\pi^2$ must hold.

As in the previous example, the higher order contributions to the effective potential are most easily obtained from Feynman graphs with modified propagators. In Landau gauge with $\phi_2=0$ these are the dressed photon propagator with an effective photon mass

$$M(\phi)=e\phi \tag{5.4}$$

and the dressed scalar propagators with masses given by

$$\mathcal{M}_1^2(\phi)=\frac{\partial^2 V_{\text{1-loop}}}{\partial\phi_1^2}\bigg|_{\phi_1=\phi,\phi_2=0}=V''_{\text{1-loop}}(\phi) \tag{5.5}$$

and

$$\mathcal{M}_2^2(\phi)=\frac{\partial^2 V_{\text{1-loop}}}{\partial\phi_2^2}\bigg|_{\phi_1=\phi,\phi_2=0}=\phi^{-1}V'_{\text{1-loop}}(\phi) . \tag{5.6}$$

In gauges other than Landau gauge there would also be mixed ϕ_2-A_μ propagators. As before, the inclusion of loops in the dressed scalar propagators implies that caution must be used to avoid double counting.

[5]The issue of gauge dependence is an important one, with some issues still to be settled. It is known that the effective potential is gauge dependent beyond lowest order; this is acceptable because it is not in general a directly measurable quantity [7]. Physically measurable quantities, on the other hand, should not depend on the choice of gauge. However, an investigation [8] of the gauge dependence of the bubble nucleation rate in scalar electrodynamics found an apparent dependence on the gauge choice.

The $O(e^6)$ contributions to $V_{\rm eff}$ come from the two-loop graphs of Fig. 5, one-loop counterterm graphs identical to those of Fig. 3, and the $O(e^6)$ counterterms. Combining these with the $O(e^4)$ terms, with the renormalization point now chosen to be the minimum of the two-loop approximation to the effective potential, gives [9]

$$V_{\text{2-loop}}(\phi)=\frac{3e^4}{64\pi^2}(1+ce^2)\phi^4\left[\ln\left[\frac{\phi^2}{\sigma^2}\right]-\frac{1}{2}\right] +\frac{5e^6}{512\pi^4}\phi^4\ln^2\left[\frac{\phi^2}{\sigma^2}\right]-\frac{1}{4}\frac{\mu^2}{\sigma^2}(\phi^2-\sigma^2)^2 \tag{5.7}$$

where c, a number of order unity, depends on the precise specification of the renormalization conditions.

The contributions from purely scalar loops enter at $O(e^8)$. These make the effective potential complex, since $\mathcal{M}_1^2$ is negative in the region where $V''_{\text{1-loop}}<0$, while $\mathcal{M}_2^2<0$ when $|\phi|<\sigma$.

As in the previous example, the function $Z_\phi(\phi)$ will also enter the calculation of Γ. At $O(e^2)$ this receives contributions from the graphs of Fig. 6, and is given by

$$Z_\phi(\phi)=\frac{3e^2}{16\pi^2}\left[\ln\left[\frac{\phi^2}{\sigma^2}\right]+k\right] \tag{5.8}$$

where k, of order unity, enforces the wave-function renormalization condition.

Let us now turn to the calculation of the bubble nucleation rate. Before going into the details, we can use the example of the previous section to anticipate a number of the results. The bounce is presumably determined by field equations involving $V_{\text{1-loop}}$. It is then easy to see that it must have a constant phase, and so by a global gauge rotation can be made entirely real. Furthermore, the spatial extent of the bounce must be of order $(e^2\sigma)^{-1}$, thus making it slowly varying enough to justify a derivative expansion of the photon-loop terms, but not of those due to graphs with purely scalar loops. This will make it possible for the $O(e^6)$ part of the effective potential to enter the calculation of Γ in a straightforward fashion, while still avoiding the troublesome $O(e^8)$ terms.

There are at least two possible approaches to the calculation. Since the symmetry breaking arises from radiative corrections involving photon loops, the most straightforward procedure would be to integrate over the gauge field to obtain an action $W(\phi_1,\phi_2)$. Since A_μ enters the Lagrangian at most quadratically, this integration can be done exactly, yielding

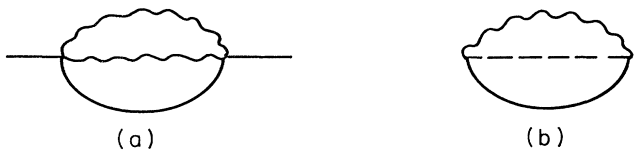

FIG. 5. The two-loop graphs which give $O(e^6)$ contributions to the effective potential of scalar electrodynamics. Solid, dashed, and wiggly lines refer to ϕ_1, ϕ_2, and photon propagators, respectively.

$$W(\phi_1,\phi_2)=\int d^4x\left[\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{2}\mu^2\phi^2+\frac{\lambda}{4}\phi^4\right]+\frac{1}{2}\mathrm{tr}\int d^4x\,\langle x|\ln[-\delta_{\mu\nu}\Box+\partial_\mu\partial_\nu+\delta_{\mu\nu}M^2(\phi)]|x\rangle$$
$$-\frac{1}{2}\int d^4x\,d^4x' j^\mu(x)j^\nu(x')\langle x|[-\delta_{\mu\nu}\Box+\partial_\mu\partial_\nu+\delta_{\mu\nu}M^2(\phi)]^{-1}|x'\rangle+\text{counterterms} \tag{5.9}$$

where $j_\mu=e(\phi_2\partial_\mu\phi_1-\phi_1\partial_\mu\phi_2)$ and the factors of $\delta_{\mu\nu}$ arise from the Euclidean metric. The second term on the right-hand side corresponds to graphs with a single photon loop, and is analogous to the one χ-loop term in the model of Sec. IV. The last term corresponds to a graph with a photon line connecting two currents. While it vanishes for both the constant false vacuum solution and the bounce, it does not vanish in general and, indeed, it is clearly needed to bring the $O(e^6)$ part of the effective potential into the exponent of Γ. However, because it cannot be expanded as a sum of local terms, the analysis of this term introduces complications not encountered in the previous example.

These complications are avoided by an alternative approach. Since the bounce can be chosen to be entirely real, it should be possible to integrate out ϕ_2, as well as A_μ, from the very beginning to obtain an effective action

$$W(\phi)=\int d^4x\left[\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{2}\mu^2\phi^2+\frac{\lambda}{4}\phi^4\right]+\frac{1}{2}\mathrm{tr}\ln G+\text{counterterms}+\cdots\,. \tag{5.10}$$

Here the dots represent multiloop contributions while G is the matrix

$$G=\left.\begin{bmatrix}\dfrac{\delta^2S}{\delta A_\mu\delta A_\nu} & \dfrac{\delta^2S}{\delta A_\mu\delta\phi_2}\\ \dfrac{\delta^2S}{\delta\phi_2\delta A_\nu} & \dfrac{\delta^2S}{\delta\phi_2^2}\end{bmatrix}\right|_{\phi_1=\phi,\phi_2=A_\mu=0}\equiv\begin{bmatrix}G_{\mu\nu}(\phi) & G_{\mu2}(\phi)\\ G_{2\nu}(\phi) & G_{22}(\phi)\end{bmatrix}. \tag{5.11}$$

In Landau gauge with a constant background $\phi_1(x)$ the off-diagonal entries of this matrix vanish. For a nonconstant but slowly varying background one can write

$$G=\begin{bmatrix}G_{\mu\nu}(\phi) & 0\\ 0 & G_{22}(\phi)\end{bmatrix}\left[I+\begin{bmatrix}0 & G_{\nu\mu}^{-1}(\phi)G_{\mu2}(\phi)\\ G_{22}^{-1}(\phi)G_{2\nu}(\phi) & 0\end{bmatrix}\right] \tag{5.12}$$

and then expand W as

$$W=\int d^4x\left[\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{2}\mu^2\phi^2+\frac{\lambda}{4}\phi^4\right]+\frac{1}{2}\mathrm{tr}\ln G_{\mu\nu}+\ln\det G_{22}+\frac{1}{2}\mathrm{tr}G_{\nu\mu}^{-1}G_{\mu2}G_{22}^{-1}G_{2\nu}+\text{counterterms}+\cdots\,. \tag{5.13}$$

The first (tree-level) term and the second term, corresponding to one-photon-loop effects, dominate. Aside from a minor point noted below, these can be treated just like the analogous terms for the previous model. Let us write these as $W_0+\delta_1W$, with W_0, given by Eq. (3.6), again being the approximate action which determines the bounce solution $\phi_b(x)$. The spatial extent of the bounce is such that $\delta_1W(\phi_b)$ can be expanded in a derivative expansion. This expansion does not give any further contribution to $\hat{V}$, but does give the contribution to $\hat{Z}$ corresponding to the graph of Fig. 6(a).

Let us consider next the fourth term, which may be denoted δ_2W. This corresponds to one-loop graphs with both a photon propagator and a ϕ_2 propagator. Because G_{22} is obtained from the fundamental action S, the latter propagator will have a mass $\hat{\mathcal{M}}_2(\phi)$ given by the tree-level potential rather than by $V_{1\text{-loop}}$, as is the mass $\mathcal{M}_2(\phi)$ of Eq. (5.6). The replacement $\hat{\mathcal{M}}_2\to\mathcal{M}_2$ is obtained by summing over multiloop graphs with arbitrary numbers of photon loops attached to the ϕ_2 line, as in Fig. 7; let us denote the result of this replacement by $\hat{\delta}_2W(\phi)$. [Since both $\hat{\mathcal{M}}_2$ and $\mathcal{M}_2$ are a factor e smaller than the effective photon mass, $\hat{\delta}_2W(\phi)$ and $\delta_2W(\phi)$ actually differ only in subdominant terms.] About the bounce solution $\hat{\delta}_2W(\phi)$ can also be expanded in a derivative expansion, with the leading term being the contribution of graph 6(b) to $\hat{Z}(\phi)$.

The third term, $\ln\det G_{22}(\phi)$, is the contribution from

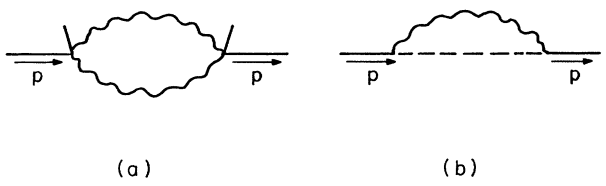

FIG. 6. The one-loop graphs which give $O(e^2)$ contributions to $Z_\phi(\phi)$ in scalar electrodynamics.

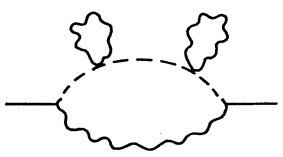

FIG. 7. A multiloop graph whose contribution must be added to $\delta_2W(\phi)$.

one-loop graphs with only internal ϕ_2 propagators. As with $\delta_2 W$, these propagators have the mass $\mathcal{M}_2$. Summing over graphs with photon loops attached to the ϕ_2 line again replaces this mass by $\mathcal{M}_2$ and replaces $G_{22}(\phi)$ by

$$\bar{G}_{22}(\phi)=-\Box+\mathcal{M}_2^2(\phi)\ . \tag{5.14}$$

Two points should be noted here. First, the effects of the change of mass are not subdominant, as they were in the previous case. Second, this term cannot be expanded in a derivative expansion about the bounce.

Doing the path integral about the bounce and the pure false vacuum and then proceeding as in the previous example leads to

$$\Gamma=e^{-B_0}e^{-B_1}e^{-B_2}\left|\frac{\det' W_0''(\phi_b)}{\det W_0''(\phi_{\rm fv})}\right|^{-1/2} \times\left[\frac{\det\bar{G}_{22}''(\phi_b)}{\det\bar{G}_{22}''(\phi_{\rm fv})}\right]^{-1/2}\frac{B_0^2}{4\pi^2}(1+\cdots) \tag{5.15}$$

where

$$B_0=W_0(\phi_b)-W_0(\phi_{\rm fv})\ , \tag{5.16}$$

$$B_1=\delta_1 W(\phi_b)+\delta_2 W(\phi_b)+{\rm tr}[W_0''(\phi_b)]^{-1}\delta_1 W''(\phi_b) +{\rm tr}[W_0''(\phi_b)]^{-1}\delta_2 W''(\phi_b)-(\phi_b\rightarrow\phi_{\rm fv})\ , \tag{5.17}$$

and B_2, containing higher order terms, is of order unity. The leading term in the exponent, B_0, contains the e^4 part of the effective potential and the tree-level kinetic term. The four terms in B_1 reproduce, in the order in which they appear, graphs 6(a) and 6(b) of the $O(e^2)$ part of Z_ϕ and graphs 5(a) and 5(b) of the $O(e^6)$ part of $V_{\rm eff}$. Although some of the $O(e^8)$ terms in $V_{\rm eff}$ are contained in B_2, the potentially complex ones, arising from scalar loops, are contained entirely within the explicit determinant factors. Because these determinants cannot be expanded in derivative expansions, the imaginary part of the effective potential does not explicitly enter the calculation. Just as before, it is necessary to look for negative eigenvalues in the spectra of the operators in these determinants. By viewing the bounce as the optimum tunneling path in the space of field configurations involving both ϕ_1 and ϕ_2, it is clear that there is still just a single negative mode, involving only ϕ_1. However, in addition to the four translational modes there is a fifth zero-frequency mode, corresponding to a phase rotation of the bounce. This must be treated by introducing a collective coordinate, replacing $\det\bar{G}_{22}(\phi_b)$ by $\det'\bar{G}_{22}(\phi_b)$, and multiplying by a Jacobian factor

$$J_{\rm phase}=\left[2\pi\int d^4x\ \phi_b^2(x)\right]^{1/2}\sim\frac{1}{e^4\sigma}\ . \tag{5.18}$$

There remains a minor point, which was noted above. Because the photon mass $M(\phi)$ is proportional to ϕ, there can be infrared problems if $\phi\ll\sigma$, as is the case far from the center of the bounce, where $\phi_b(x)$ exponentially approaches the false vacuum $\phi_{\rm fv}=0$. These can be seen most clearly by recalling the estimate (4.20) for the effective action contribution containing j derivatives and k factors of ϕ. With $M(\phi)=e\phi$, the previous estimate is replaced by

$$w_{jk}\sim g^{4+j}\left[\frac{\sigma}{\phi}\right]^{j=4}\sigma^4\ . \tag{5.19}$$

For small ϕ, the derivative expansion cannot be carried out beyond the four-derivative terms. However, if we extract from W the potential terms and the two-derivative terms, the remainder is $O(g^4)$ and gives an $O(1)$ contribution to the exponent B, which can eventually be absorbed in a redefinition of the prefactor.

To summarize, the bubble nucleation rate is again given by an expression of the form of Eq. (4.27), with

$$\bar{B}_0=\int d^4x\{[V_{\rm eff}^{e^4}(\phi_b)+\tfrac{1}{2}(\partial_\mu\phi_b)^2] -(\phi_b\rightarrow\phi_{\rm fv})\}=O(e^{-4}) \tag{5.20}$$

and

$$\bar{B}_1=\int d^4x\{[V_{\rm eff}^{e^6}(\phi_b)+\tfrac{1}{2}Z_\phi^{e^2}(\phi_b)(\partial_\mu\phi_b)^2] -(\phi_b\rightarrow\phi_{\rm fv})\}=O(e^{-2}) \tag{5.21}$$

and the preexponential factor $\bar{A}$ now being of order σ^4/e^4.

VI. CONCLUDING REMARKS

In this paper I have shown how the decay rate of a metastable vacuum can be calculated in a theory whose vacuum structure is determined by radiative corrections. As in the standard case, the result may be written as a dimensionful prefactor times the exponential of an action involving a bounce solution. To leading approximation this exponent is just the tree-level action supplemented by the dominant one-loop contribution to the effective potential; in scalar electrodynamics it is $O(1/e^4)$. The first correction to the exponent arises from the next-to-leading contributions to the effective potential and the leading correction to the tree-level kinetic part of the effective action. Although smaller than the leading terms, these give an addition to the exponent which is larger than order unity [e.g., it is $O(1/e^2)$ in scalar electrodynamics] and is thus more important than the prefactor. It does not appear that this correction need have any particular sign, but rather that it might increase the nucleation rate in some theories and reduce the rate in others. Further, the separation between the leading and next-to-leading terms in the bounce action is dependent on the precise specification of the renormalization conditions. In fact, the existence of a correction term of this magnitude follows simply from the requirement that the physical nucleation rate be independent of the renormalization scheme and of the particular definition of the coupling constants of the theory.

All further corrections may be absorbed into the prefactor. Although some of these (those corresponding to graphs with internal lines of the heavy "χ particles") can be identified with particular terms in the effective potential and the other functions entering the effective action, this is not true of all the higher corrections. Specifically,

the ϕ-loop graphs which give rise to complex terms in the effective potential cannot, when calculated in the background of the bounce, be expanded in a derivative expansion. Consequently, the imaginary part of the effective potential does not explicitly enter the bubble nucleation calculation, and the problems of interpretation which it would entail are avoided. In particular, the expression obtained for the nucleation rate can be shown to be real.

In the examples considered in this paper, it was obvious that the standard formalism for vacuum decay had to be modified, since the tree-level potential had only a single vacuum and so could not possibly lead to a bounce solution. However, one could easily construct examples in which the tree-level potential had several inequivalent minima, but where the one-loop corrections to the effective potential were comparable to the tree-level terms and changed the vacuum structure. In such cases a bounce solution to the tree-level action would exist and could be used to calculate the nucleation rate, but the result would not be current.[6] Working with the standard formalism, would one see any indication that the calculation was unreliable? The answer is yes. To see this, consider a model similar to that of Sec. IV, but with an additional ϕ^3 interaction of magnitude such that $V(\phi,\chi=0)$ has two minima. Further, assume that the ϕ self-interactions can be written in the form

$$V(\phi,\chi=0)=\lambda\sigma^4 U(\phi/\sigma) \tag{6.1}$$

with U involving no small couplings and the relation between λ and the couplings involving χ being the same as in Sec. IV. Applying the standard formalism in this case, one will obtain a bounce solution with nontrivial $\phi(x)$ but with $\chi=0$ everywhere and will be led to an expression for the nucleation rate which differs from Eq. (2.13) only by a multiplicative factor

$$K_\chi=\left[\frac{\det\{-\Box+M^2[\phi_b(x)]\}}{\det[-\Box+M^2(\phi_{\rm fv})]}\right]^{-1/2} \tag{6.2}$$

arising from the functional integration over χ.

The magnitudes of the various terms entering the expression for Γ can then be estimated by the methods of Sec. II. In particular, since the spatial extent of the bounce solution is of order $1/(\sqrt{\lambda}\sigma)$, it is useful to define the dimensionless variable $s=\sqrt{\lambda}\sigma x$, in terms of which the bounce has spatial extent of order unity. The bounce action is then seen to be of order $1/\lambda$, while the determinants arising from the functional integration over ϕ combine with the Jacobian to give a factor of order unity. However, the remaining factor, K_χ, now becomes

$$K_\chi=\left[\frac{\det\{-\Box_s+M^2[\phi_b(s)]/\lambda\sigma^2\}}{\det[-\Box_s+M^2(\phi_{\rm fv})/\lambda\sigma^2]}\right]^{-1/2}. \tag{6.3}$$

Because the χ-ϕ couplings have been assumed to be large compared to the ϕ self-interactions, $M^2(\phi)/\lambda\sigma^2$ is large (of order $1/g^2$). As a result, K_χ is not simply a factor of order unity which can be included in the prefactor. Instead, it gives potentially large corrections whose evaluation [10] is nontrivial; in other words, the standard algorithm has broken down.

One final note. This work has been concerned solely with the problem of bubble nucleation by quantum-mechanical tunneling at zero temperature. Issues similar to those encountered here also arise in connection with the problem of finite temperature bubble nucleation, even in theories where radiative corrections have little effect on the zero-temperature vacuum structure. These will be considered elsewhere.

ACKNOWLEDGMENTS

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[6] A similar situation could arise in the study of solitons. The calculation of the quantum corrections to the soliton energy has many similarities to the decay rate calculation considered in this paper. Here too, the standard calculation should break down if the one-loop corrections to the effective potential are large enough.

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