Influence functionals and the accelerating detector

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The influence functional is derived for a massive scalar field in the ground state, coupled to a uniformly accelerating DeWitt monopole detector in $(D+1)$-dimensional Minkowski space. This confirms the local nature of the Unruh effect, and provides an exact solution to the problem of the accelerating detector without invoking a nonstandard quantization. A directional detector is presented which is efficiently decohered by the scalar field vacuum, and which illustrates an important difference between the quantum mechanics of inertial and noninertial frames. From the results of these calculations, some comments are made regarding the possibility of establishing a quantum equivalence principle, so that the Hawking effect might be derived from the Unruh effect.

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I. INTRODUCTION

A typical state of a quantum field, such as the vacuum, contains coherences between spatially distant degrees of freedom. This is the source of the nonlocality of measurement and other nonintuitive results of quantum theory. The "decoherent histories" reformulation of the theory allows one to consider quantum-mechanical systems which do not possess long-range coherence [1], and other recent work likewise examines quantum coherence specifically [2,3]. Both of these areas of research employ influence functionals as essential tools. The influence functional for a scalar field initially in the ground state, coupled to a pointlike accelerating detector, is of interest both because it provides another example of the use of influence functionals and because it sheds light on the acceleration-induced heating of the vacuum, referred to as the Unruh effect [4].

The acceleration-induced heating of the vacuum was first discussed at length by Unruh as a toy model for Hawking radiation from an eternal black hole [5]. An earlier paper by Davies equipped flat space with a static, reflecting boundary in order to model a black hole formed by collapse; this led to a similar result [6]. The original derivation of this effect begins by quantizing a scalar field in Rindler coordinates, replacing ordinary time evolution with translation along trajectories of constant proper acceleration [7,8]. From this point of view, the interaction between a pointlike accelerating detector and the field becomes a global problem: it involves relabeling the entire Fock space. It also turns the accelerating observer's personal event horizon into an apparently special location in Minkowski space, leading to the impression that the thermal effects of acceleration are somehow global properties of spacetime, rather than local effects.

On the other hand, arguments may be constructed that obtain the thermal character of the vacuum, as seen by an accelerating detector, without using the Rindler

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quantization [4,9,10]. These discussions involve time-dependent perturbation theory and an examination of the two-point function of a scalar field in terms of the detector's proper time. They suggest that the apparent horizon of the detector plays no direct role in generating the Unruh effect. This paper extends this line of argument beyond perturbation theory, deriving an exact result in the form of an influence functional. The physics involved is all implicit in the perturbative approach, but the formalism used is more powerful and perhaps less familiar.

Proceeding from the idea that acceleration implies, in a local manner, an Unruh temperature, it has been suggested that an appeal to the local equivalence of gravitation and acceleration might allow one to derive the Hawking effect from the Unruh effect [10]. Influence functionals are also used in this paper to investigate how useful a naive quantum extension of the equivalence principle might be as a basic tool, for possible application in curved spacetime. The study of a particular model for a directional accelerating detector shows that, in addition to the Unruh temperature and the differences between Rindler and Minkowski densities of states, there is another significant difference between the local behaviors of a quantum field perceived by accelerating and inertial observers: the correspondence between directions in space and orthogonal field modes breaks down in the accelerating frame. There are such significant additional effects of acceleration in quantum field theory, beyond the appearance of a temperature, that using an equivalence principle will not necessarily be as helpful as one might hope. This and other facts suggest that further study is needed if a quantum equivalence principle is to be used to approach quantum field theory in curved spacetimes.

This paper is organized as follows. A brief summary of the method of influence functionals is presented in Sec. II. In Sec. III the Unruh effect in $D+1$ dimensions is derived using influence functionals, with some comments on the distinction between thermal population and spectral density. The role of spatial regions causally disconnected from the detector is made clear in Sec. IV. Section V then deals with a directional detector in $2+1$ dimensions.

Section VI concludes, with some comments on the extension of this analysis to the case of Hawking radiation.

II. REVIEW OF INFLUENCE FUNCTIONALS

The theory of influence functionals was presented in 1963 by Feynman and Vernon. For a full discussion of the technique, the reader is referred to their original paper [11].

In many quantum-mechanical problems, at least implicitly, one has an observed system coupled to an environment which is unobserved. Both system and environment are quantum mechanical; one assumes that the complete Hilbert space may be spanned by a basis of direct-product states of the form $|\Psi_{\mathrm{evn}}\rangle|\psi_{\mathrm{sys}}\rangle$. The probability $P_{FI} = |\langle F|\hat{U}|I\rangle|^2$ of a transition from an initial state with a product wave function $\Psi_I \psi_i$ to a final state $\Psi_F \psi_f$ is given by the path integral

$$P_{FI} = \int \mathcal{D}E\, \mathcal{D}E'\, \mathcal{D}S\, \mathcal{D}S'\, \Psi_I(E_i)\psi_i(S_i)\Psi_I^*(E_i')\psi_i^*(S_i')\Psi_F^*(E_f)\psi_f^*(S_f)\Psi_F(E_f')\psi_f(S_f')$$

$$\times \exp\left[\frac{i}{\hbar}\int_{t_i}^{t_f} dt\,[L(E,S)-L(E',S')]\right]\ . \tag{1}$$

Here E and E' variables represent the environment degrees of freedom, while S and S' stand for the system under observation. E_i stands for $E(t_i)$ and similarly for the other variables and subscript. The Lagrangian is

$$L(E,S)=L_s(S)+L_e(E)+L_{\mathrm{int}}(E,S)\ ,$$

organized into a system term, an environment term, and an interaction term.

By axiom, the transition probabilities are to be summed over all final states of the unobserved sector. This leaves a truncated quantum theory describing only the observed degrees of freedom. (The initial state of the environment must be given, of course; it modifies the parameters of the truncated theory[1].) A great virtue of Feynman's path-integral formulation of quantum

mechanics, in comparison with the canonical approach, is that one can in principle carry out this truncation before considering the evolution of the observed sector. By performing first the path integration over all the unobserved degrees of freedom, one is left with a modified path integral containing only observed degrees of freedom. This modified path integral is temporally nonlocal, and it provides nonunitary time evolution, since its construction has involved replacing pure final states with decoherent mixtures in which all states of the unobserved sector are equally probable.

The part of the truncated transition probability path integral that contains the nonlocal and nonunitary evolution is called the *influence functional* $F[S,S']$. It describes completely the influence of the unobserved environment on the observed system. One can write

$$P_{fi} = \int \mathcal{D}S\, \mathcal{D}S'\, \psi_i(S_i)\psi_i^*(S_i')\psi_f^*(S_f)\psi_f(S_f')F[S,S']\exp\left[\frac{i}{\hbar}\int_{t_i}^{t_f} dt\,[L_s(S)-L_s(S')]\right]\ , \tag{2}$$

where P_{fi} denotes the probability of a transition of the observed system. Comparing (2) and (1) and using the fact that $\sum_F |F\rangle\langle F|$ is an identity operator, it is straightforward to obtain the formula for the influence functional:

$$F[S,S'] = \int \mathcal{D}E\, \mathcal{D}E'\, \Psi_I(E_i)\Psi_I^*(E_i')\delta(E_f-E_f')\exp\left[\frac{i}{\hbar}\int_{t_i}^{t_f} dt\,[L_e(E)-L_e(E')+L_{\mathrm{int}}(E,S)-L_{\mathrm{int}}(E',S')]\right]$$

$$= \langle\Psi_I|\hat{U}_{S'}^\dagger(t_f,t_i)\hat{U}_S(t_f,t_i)|\Psi_I\rangle\ . \tag{3}$$

$\hat{U}_S$ refers to the unitary time-evolution operator for the environment, with S in $L_{\mathrm{int}}(E,S)$ treated as an external source. (Because S and S' are different, $\hat{U}_{S'}^\dagger$ is not the inverse of $\hat{U}_S$.)

The influence functional can also be used to obtain the time evolution of the observed system density matrix $\rho(S,S';t)$:

$$\rho(S_f,S_f';t_f) = \int \mathcal{D}S\, \mathcal{D}S'\, \rho(S_i,S_i';t_i)F[S,S']\exp\left[\frac{i}{\hbar}\int_{t_i}^{t_f} dt\,[L_s(S)-L_s(S')]\right]\ . \tag{4}$$

Because $F[S,S']$ is a nonunitary kernel, it provides a mechanism for decoherence in the evolution of ρ; one can inter-

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[1]It is assumed throughout this paper that the system and the environment are initially uncorrelated, so that the initial state of the Universe may be written as a product of system and environment states. For systems strongly coupled to their environment, this assumption is not realistic [3]. In the present case, a hypothetical detector will be used to probe the unperturbed state of a field, and so the uncorrelated initial state is appropriate.

pret this effect as representing the loss of information from the observed system into the unobserved environment. It is in this decohering context that influence functionals have recently found application [1,2].

A pertinent example of an influence functional is that for a collection of harmonic oscillators, distributed over all frequencies ω with spectral density $G(\omega)$, linearly coupled to an observed degree of freedom $Q(t)$. The Lagrangian for the unobserved sector, represented by the variables X_ω, is of the form

$$L = \int d\omega\, G(\omega) \int dt (\tfrac{1}{2}\dot{X}_\omega^2 - \tfrac{1}{2}X_\omega^2 + QX_\omega) \ . \tag{5}$$

From this Lagrangian and choosing the initial state of the collection of oscillators to be thermal, one may obtain the influence functional [11]

$$\ln F[Q,Q'] = -\frac{1}{2\hbar} \int \frac{d\omega}{\omega} G(\omega) \int_{t_i}^{t_f} dt \int_{t_i}^{t} dt' [Q(t) - Q'(t)] \left[\coth\frac{\hbar\omega}{2kT}[Q(t') - Q'(t')]\cos\omega(t - t') \right.$$

$$\left. - i[Q(t') + Q'(t')]\sin\omega(t - t') \right] \ , \tag{6}$$

where T is the temperature of the initial state of the ensemble of oscillators and k is the Boltzmann constant. Note that the t' integral is over the range $t' < t$.

Another useful instance of an influence functional is the one describing a free massive scalar field in $1+1$ dimensions, initially in the vacuum state, linearly coupled to an observed degree of freedom Q. The Lagrangian of the scalar field may be written

$$L = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \frac{1}{2} \left[\frac{1}{c^2}\dot{\Phi}_k\dot{\Phi}_k^* - \frac{\omega_k^2}{c^2}\Phi_k\Phi_k^* + Q(t)[A_k^*(t)\Phi_k + A_k(t)\Phi_k^*] \right] \ . \tag{7}$$

Here $\Phi_k(t)$ is the kth Fourier mode of the field and $\omega_k = c(k^2 + m^2)^{1/2}$ for $\hbar m / c$, the field mass. A_k is a coupling strength which may depend both on k and on time t. For this Lagrangian one obtains the influence functional

$$F[Q,Q'] = \exp\left[-\frac{c}{4\pi\hbar} \int_{-\infty}^{\infty} \frac{dk}{(k^2 + m^2)^{1/2}} V_k[Q,Q'] \right] \ , \tag{8}$$

where

$$V_k[Q,Q'] = \int_{t_i}^{t_f} dt \int_{t_i}^{t} dt'\, A_k(t)A_k^*(t')[Q(t) - Q'(t)]\{[Q(t') - Q'(t')]\cos\omega(t - t') - i[Q(t') + Q'(t')]\sin\omega(t - t')\} \ .$$

Equations (6)–(8) will all be used below.

III. ACCELERATING DETECTOR IN THE MINKOWSKI VACUUM

A. Unruh effect in 1+1 dimensions

A pointlike quantum-mechanical detector may be idealized as interacting with its environment only through the DeWitt monopole moment $Q(t)$ [12]. If it is linearly coupled to an otherwise free massive scalar field ϕ in $(1+1)$-dimensional Minkowski space, then it serves as a localized probe of the field. If the detector is constrained to move along a trajectory $x = x_0(\tau)$, $t = t_0(\tau)$, where x and t are Cartesian coordinates and τ is the detector's proper time, then one has the interaction Lagrangian

$$L_{int}(\phi, Q, t) = \int d\tau\, Q(\tau)\delta(t - t_0(\tau))$$

$$\times \int dk\, \Phi_k(t_0(\tau))e^{ikx_0(\tau)} \ . \tag{9}$$

The δ function will clearly serve to transform inertial time integrals into proper time integrals, in such formulas as (6) and (8). When this is done, it can be seen that (9)

gives the simple action term

$$\mathcal{S}_{int} = \int d\tau\, Q(\tau)\phi(x_0(\tau), t_0(\tau)) \ .$$

If the detector's trajectory is one of constant acceleration a, then

$$x_0(\tau) = \frac{c^2}{a}\cosh\frac{a\tau}{c} \ ,$$

$$t_0(\tau) = \frac{c}{a}\sinh\frac{a\tau}{c} \ . \tag{10}$$

Inserting this into (9) and comparing with (7), one obtains the time-dependent, k-dependent coupling of the detector to the field:

$$A_k(\tau) = \exp\left[i\frac{kc^2}{a}\cosh\frac{a\tau}{c} \right] \ . \tag{11}$$

Applying this in turn in (8) and using the δ functions to replace t and t' everywhere with the functions of proper times, $t_0(\tau)$ and $t_0(\tau')$, one derives the influence functional of the free massive scalar field on the pointlike accelerating detector:

$$F[Q,Q'] = \exp\left[-\frac{1}{2\hbar} \int_{\tau_i}^{\tau_f} d\tau \int_{\tau_i}^{\tau} d\tau' [Q(\tau) - Q'(\tau)][Q(\tau')U(\tau,\tau') - Q'(\tau')U^*(\tau,\tau')] \right] \ , \tag{12}$$

where the kernel $U(\tau,\tau')$ is defined as

$$U(\tau,\tau') \equiv \frac{c}{2\pi} \int_{-\infty}^{\infty} \frac{dk}{(k^2+m^2)^{1/2}} \exp\left[-i\frac{kc^2}{a}\left[\cosh\frac{a\tau}{c}-\cosh\frac{a\tau'}{c}\right]-i\frac{\omega c}{a}\left[\sinh\frac{a\tau}{c}-\sinh\frac{a\tau'}{c}\right]\right] .$$
(13)

Equation (12) is an implicit answer to the question of how the ground state of the scalar field appears to an accelerating observer. One need only proceed with the path integral (2), using some appropriate Lagrangian $L_s(Q)$ for the detector itself, to have an exact solution for the detector's time evolution. Rather than doing this, however, it will be more instructive to derive the Fourier transform of the apparently complicated function $U(\tau,\tau')$. The result will be of the same form as (6), providing a clear interpretation of the influence functional (12) as representing the effect of a heat bath at the Unruh temperature $kT = \hbar a/2\pi c$.

Redefining $k = m \sinh[\eta - a(\tau+\tau')/2c]$, (13) becomes

$$U(\tau,\tau') = \frac{c}{2\pi} \int_{-\infty}^{\infty} d\eta \exp\left\{-i\frac{mc^2}{a}\left[\sinh\left[\eta+a\frac{\tau-\tau'}{2c}\right]-\sinh\left[\eta-a\frac{\tau-\tau'}{2c}\right]\right]\right\} ,$$

$$= \frac{c}{2\pi} \int_{-\infty}^{\infty} d\eta \exp\left[-2i\frac{mc^2}{a}\cosh\eta \sinh\left[\frac{a}{c}\frac{\tau-\tau'}{2}\right]\right] .$$
(14)

One now invokes a crucial identity from the theory of modified Bessel functions of imaginary order[2] (denoted $K_{i\nu}$) [13]:

$$e^{-i\alpha\sinh(x/2)} = \frac{4}{\pi}\int_0^\infty d\nu\, K_{2i\nu}(\alpha)[\cosh(\pi\nu)\cos(\nu x)-i\sinh(\pi\nu)\sin(\nu x)] .$$
(15)

Substituting $\nu \to c\omega/a$, this allows us to reexpress (13) as

$$U(\tau,\tau') = \int \frac{d\omega}{\omega} G(a,\omega)\left[\coth\frac{c\pi\omega}{a}\cos\omega(\tau-\tau')-i\sin\omega(\tau-\tau')\right] ,$$
(16)

where

$$G(a,\omega) \equiv \frac{2c^2\omega}{a\pi^2}\int_{-\infty}^{\infty} d\eta\, K_{2ic\omega/a}\left[2\frac{mc^2}{a}\cosh\eta\right]\sinh\frac{c\pi\omega}{a}$$

$$= \frac{2c^2\omega}{a\pi^2}\sinh\frac{c\pi\omega}{a}\left[K_{ic\omega/a}\left[\frac{mc^2}{a}\right]\right]^2 ,$$
(17)

using another modified Bessel function identity [14].

Inserting (16) into (12) and comparing the result with (6), one observes that the effect of the scalar field vacuum on an accelerating detector is exactly that of a heat bath at temperature $kT = \hbar a/2\pi c$, with spectral density $G(a,\omega)$.

In order to verify that these results coincide with the standard conclusion that an accelerating observer sees the Minkowski vacuum as a thermal ensemble of Rindler modes [5,8], it is worth determining what $G(a,\omega)$ would appear in a thermal influence functional derived using Rindler quantization [7]. Begin with the action for a scalar field $\phi(\rho,\xi)$ in Rindler coordinates [15],

$$\mathcal{S} = \int d\xi\left[\int_0^\infty \frac{\rho\, d\rho}{2c}(\rho^{-2}\partial_\tau\phi^2 - \partial_\rho\phi^2 - m^2\phi^2)\right.$$

$$\left.+\frac{c}{a}Q\left[\frac{c}{a}\xi\right]\phi\left[\frac{c^2}{a},\xi\right]\right] ,$$
(18)

where ρ and ξ are related to the Cartesian coordinates

used above by the definitions

$$x = \rho\cosh\xi ,$$

$$t = \rho\sinh\xi .$$
(19)

These coordinates label only the Rindler wedge of Minkowski space $x > 0$, $|t| < x$. The Minkowski line element is given by

$$ds^2 = d\rho^2 - \rho^2 d\xi^2 .$$

Following Fulling [7], one now expands the scalar field φ in Rindler modes $\varphi_\nu(\xi)$ instead of Fourier modes $\Phi_k(t)$. The φ_ν are the coefficients in an expansion of the field ϕ in solutions of the scalar wave equation

$$\rho^2\frac{d^2\phi}{d\rho^2}+\rho\frac{d\phi}{d\rho}-m^2\rho^2\Phi = \frac{d^2\phi}{d\xi^2} = -\nu^2\phi$$

separated in the ρ,ξ variables. As mentioned above, these solutions are modified Bessel functions of imaginary order, and so the Rindler modes are defined by

$$\phi(\rho,\xi) = \frac{1}{\pi}\int_0^\infty d\nu\sqrt{2\nu\sinh\pi\nu}K_{i\nu}(m\rho)\varphi_\nu(\xi) .$$
(20)

One can then use the orthogonality relation [7]

$$\frac{1}{\pi^2}\int_0^\infty \frac{d\rho}{\rho}K_{i\mu}(m\rho)K_{i\nu}(m\rho) = \frac{\delta(\mu-\nu)}{2\nu\sinh\pi\nu}$$
(21)

and the modified Bessel equation satisfied by $K_{i\nu}(m\rho)$ to rewrite (18) as

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[2]These functions appear naturally in solutions to the scalar wave equation in Rindler coordinates and therefore in wave functions for a scalar field quantized in an accelerating frame [7].

$$\mathscr{S}=\int\frac{d\xi}{c}\int_0^\infty dv\left[\tfrac{1}{2}\partial_\xi\varphi_v^2-\tfrac{1}{2}v^2\varphi_v^2+\frac{c^2}{a\pi}Q\left(\frac{c}{a}\xi\right)\sqrt{2v\sinh\pi v}K_{iv}\left(\frac{mc^2}{a}\right)\varphi_v\right].\tag{22}$$

The fastest way to obtain the influence functional for a thermal ensemble of Rindler modes with this action is to put (22) into the same form as the action derived from (5). This requires that the time variable be the proper time of the Q system and that the interaction term have the same weight as the kinetic term. Both conditions may be arranged by rescaling $\xi=(a/c)\tau$,

$$\varphi_v=\frac{a}{\pi}X_v\sqrt{2v\sinh\pi v}K_{iv}\left(\frac{mc^2}{a}\right),$$

and $v=(c/a)\omega$. One obtains $\mathscr{S}=\int d\tau L(\tau)$, where

$$L(\tau)=\int_0^\infty d\omega\frac{2c^2\omega}{a\pi^2}\sinh\frac{c\pi\omega}{a}\left[K_{ic\omega/a}\left(\frac{mc^2}{a}\right)\right]^2[\tfrac{1}{2}\dot{X}_\omega^2-\tfrac{1}{2}\omega^2X_\omega^2+QX_\omega].\tag{23}$$

The measure on ω appearing here is exactly $G(a,\omega)$ from (17). Comparison with (5) and (6) then shows that a heat bath of Rindler modes at the Unruh temperature indeed gives the same influence functional as was derived using ordinary Fourier modes populated at zero temperature.

B. Temperature versus spectral density

The result derived in Sec. III A may be expressed as the fact that an accelerating observer in the Minkowski vacuum will detect thermal radiation. In this context, however, thermal radiation must not be construed as a thermal population of Fourier modes. Such an interpretation of the Unruh effect would be inaccurate, because a heat bath of Rindler modes does not give the same influence functional as a heat bath of Fourier modes at the same temperature.

An inertial detector at $x=0$ coupled to a scalar field in $1+1$ dimensions may be described by the Lagrangian (7) with $A_k=0$. With an initial state at temperature T, one obtains the influence functional

$$F[Q,Q']=\exp\left[-\frac{1}{2\hbar}\int_{t_i}^{t_f}dt\int_{t_i}^t dt'[Q(t)-Q'(t)]\right.$$

$$\left.\times[Q(t')U_{\text{in}}(t,t')-Q'(t')U_{\text{in}}^*(t,t')]\right],\tag{24}$$

where the inertial kernel analogous to (13) is

$$U_{\text{in}}(t,t')=-\int\frac{d\omega}{\omega}F(\omega)\left[\coth\frac{\hbar\omega}{2kT}\cos\omega(t-t')\right.$$

$$\left.-i\sin\omega(t-t')\right],\tag{25}$$

for

$$F(\omega)=\frac{c\omega}{\pi(\omega^2-m^2c^2)^{1/2}}=\lim_{a\to0}G(a,\omega).\tag{26}$$

[One may verify that $F(\omega)$ is indeed equal to $G(0,\omega)$ by reversing the derivation of (17) from (9).]

Since, in general, $G\neq F$, the influence functionals (12) and (24) are not equal. Nevertheless, (12) does indeed represent a heat bath, because it includes the factor $\coth(\hbar\omega/2kT)$, for $kT=\hbar a/2\pi c$. It is this factor which reflects the thermal population of each mode of the bath; the spectral density only describes the distribution of the modes over frequency [16]. It is also this factor that is measured by a thermometer: In the limit where its coupling to the environment is weak, a test system acted on by an influence functional of the form of (16) or (25) will be driven toward the canonical ensemble, regardless of the spectral density [17]. This may be understood if one examines the role of the influence functional in the path integral (4) governing the observed detector system. One finds that the real parts of (25) and (13) both describe fluctuations induced by the environment, while the imaginary parts represent dissipation. The relative strength of these two effects is the true signature of a thermal environment.

C. Unruh effect in $D+1$ dimensions

The point of Sec. III B must be borne in mind when interpreting the Unruh effect beyond one dimension of space. The accelerating detector in $D>1$ spatial dimensions may be described by amending (13) so that

$$\frac{dk}{2\pi}\to\frac{d^Dk}{(2\pi)^D},$$

$$(k^2+m^2)^{1/2}\to(|\mathbf{k}|^2+m^2)^{1/2}.$$

By expressing the $(D-1)$-dimensional vector $(k_2,k_3,\ldots,k_D)$ in polar form, so that

$$k_2\equiv r\cos\alpha_1,$$

$$k_3\equiv r\sin\alpha_1\cos\alpha_2,\text{ etc.,}$$

one obtains for the D-dimensional kernel $U_D(\tau,\tau')$ an integral over k_1, r, and the angles α_n. The angular integration is trivial and contributes only a factor equal to the area of the unit $(D-2)$-sphere. The k_1 integral may be treated exactly as the k integral in Sec. IV, except that r modifies the mass term by changing $m\to(m^2+r^2)^{1/2}$. One thus obtains generalized versions of (12) and (13):

$$F_D[Q,Q']=\exp\left[-\frac{1}{2\hbar}\int_{\tau_i}^{\tau_f}d\tau\int_{\tau_i}^{\tau}d\tau'[Q(\tau)-Q'(\tau)][Q(\tau')U_D(\tau,\tau')-Q'(\tau')U_D^*(\tau,\tau')]\right] \tag{27}$$

and

$$U_D(\tau,\tau')=\int\frac{d\omega}{\omega}G_D(a,\omega)\left[\coth\frac{c\pi\omega}{a}\cos\omega(\tau-\tau')-i\sin\omega(\tau-\tau')\right]. \tag{28}$$

For $D>1$, however, one now has

$$G_D(a,\omega)=\frac{c^2\omega\sinh(c\pi\omega/a)}{2^{D-3}a\pi^{(D+3)/2}\Gamma((D-1)/2)}\int_0^{\infty}dr\,r^{D-2}\left[K_{ic\omega/a}\left[\frac{c^2}{a}(m^2+r^2)\right]^{1/2}\right]^2. \tag{29}$$

For the case $m=0$, the r integral may be obtained in a closed form. This case is analyzed in Ref. [9], where a response function related to $U_D(\tau,\tau')$ is derived which is proportional, for odd D, to the Planck distribution function. For even D, however, the Planck factor is replaced by the Fermi-Dirac distribution function, even though the problem involves a bosonic field. It is instructive to see how this comes about, for the massless scalar field does not in fact seem fermionic to an accelerating observer in any number of spatial dimensions.

The equations needed to evaluate (29) in the massless limit are [18]

$$\int_0^{\infty}dr\,r^{D-2}[K_{i\nu}(\lambda r)]^2=\frac{2^{D-4}\lambda^{1-D}}{\Gamma(D-1)}\left|\Gamma\left[\frac{D-1}{2}+i\nu\right]\right|^2\left|\Gamma\left[\frac{D-1}{2}\right]\right|^2$$

$$=\frac{\pi^{3/2}\lambda^{1-D}}{e^{\pi\nu}\pm e^{-\pi\nu}}\frac{\Gamma((D-1)/2)}{2\Gamma(D/2)}P_D(\nu), \tag{30}$$

where the plus (minus) sign applies for D even (odd). $P_D(\nu)$ is the polynomial

$$P_2=1,\quad P_{2d}(\nu)=\prod_{n=1}^{d-1}[(n-\tfrac{1}{2})^2+\nu^2],\quad d>1,\quad P_{2d+1}(\nu)=\frac{1}{\nu}\prod_{n=0}^{d-1}[n^2+\nu^2]. \tag{31}$$

From these facts one obtains

$$G_D(a,\omega)|_{m=0}=\frac{\omega P_D[(c/a)\omega]}{2^{D-1}\pi^{D/2}\Gamma(D/2)}\left[\frac{a}{c^2}\right]^{D-2}\frac{\exp\left[\frac{2\pi c}{a}\omega\right]-1}{\exp\left[\frac{2\pi c}{a}\omega\right]+(-1)^D}$$

$$\equiv\frac{\omega R_D(\omega)}{\pi}\frac{\exp\left[\frac{2\pi c}{a}\omega\right]-1}{\exp\left[\frac{2\pi c}{a}\omega\right]+(-1)^D}, \tag{32}$$

where R_D is simply a rescaled version of the polynomial P_D.

The response function $F_D(\omega)$ (equal to F_n of Ref. [9] for $n=D+1$) is defined for $\omega\in(-\infty,\infty)$ to be

$$F_D(\omega)=\lim_{L\to\infty}\frac{1}{2L}\int_{-L}^{L}d\tau\int_{-L}^{L}d\tau'\,e^{-i\omega(\tau-\tau')}U(\tau,\tau')$$

$$=\frac{\pi}{|\omega|}G_D(a,|\omega|)\frac{\mathrm{sgn}(\omega)}{\exp\left[\frac{2\pi c}{a}\omega\right]-1}$$

$$=\frac{\pi}{\omega}G_D(a,\omega)\frac{1}{\exp\left[\frac{2\pi c}{a}\omega\right]-1}, \tag{33}$$

using the fact that R_D has the same parity as D.

It is therefore easy to see that, for $m=0$, the response function is equal to a polynomial multiplied by a either a Planck or a Fermi-Dirac factor:

$$F_D(\omega)=\frac{R_D(\omega)}{\exp\left[\frac{2\pi c}{a}\omega\right]+(-1)^D}, \tag{34}$$

in agreement with Ref. [9]. This is in contrast with the response function F_D^{in} for inertial thermal radiation in $D+1$ dimensions, which always involves the Planck factor:

$$F_D^{\mathrm{in}}(\omega)=\frac{2^{1-D}\pi^{(2-D)/2}|\omega|^{D-2}\mathrm{sgn}(\omega)}{\Gamma(D/2)(e^{\omega/kT}-1)}. \tag{35}$$

The important point to realize in interpreting (34) is that if one ignores the parity of F_D, then information about the real and imaginary parts of $U_D(\tau,\tau')$ has been

discarded. As emphasized in Sec. III B, these real and imaginary parts play drastically different roles in the detector path integral (4) and so are physically distinguishable. To check the statistics of the bath oscillators, rather than simply extracting the most obvious statistical factor from the response function, one must check whether the ratio of its even and odd parts is of the form $\coth(\hbar\omega/2kT)$. This confirms that (34) actually describes a bosonic thermal environment:

$$\frac{F_D(-\omega)+F_D(\omega)}{F_D(-\omega)-F_D(\omega)}=\coth\frac{\pi c}{a}\omega \ . \tag{36}$$

So the scalar field does appear as a bath of bosonic oscillators populated thermally at the Unruh temperature, and this is true for all values of D; but $G_D(\omega)$ is such that the scalar field heat bath felt by an accelerating observer has in general an effect different from that of thermal radiation in an inertial frame. This effect is only partially described by the response function, but may be represented exactly by the influence functional.

IV. ROLE OF LONG-RANGE COHERENCE

This section examines the role played in generating the Unruh effect by those of the scalar field's degrees of freedom that lie outside the past light cone of the trajectory of the detector; this is equivalent to considering the role of vacuum-state coherences between degrees of freedom

inside and outside the light cone. For simplicity, the analysis is again restricted to $1+1$ dimensions. The past light cone of a constant-acceleration trajectory from $\tau=-\infty$ to ∞, with the past horizon as an initial data surface, of course contains the whole Rindler wedge. If one wishes to consider the evolution of the detector over all time, then the formulation of the Unruh problem in terms of Rindler modes seems correctly to attach importance to the horizon. But (16) describes a thermal environment, regardless of τ_i and τ_f: an accelerating detector "feels" a heat bath over any time interval, however short.[3] Hence one can consider the behavior of the detector between any two finite proper times τ_i and τ_f, and the Unruh effect will still appear. The past light cone of a finite trajectory does not coincide with the Rindler horizon; consequently, the role of quantum coherences between field degrees of freedom separated by spacelike intervals is distinct from the role of the horizon. The former may best be studied using the influence functional formalism.

To begin, consider the simple case of a harmonic oscillator driven by an external force $J(t)$, with the Lagrangian

$$L_{\rm HO}=\tfrac{1}{2}\dot{q}^2-\tfrac{1}{2}\Omega^2q^2+Jq \ . \tag{37}$$

The amplitude for a transition between initial and final q states is

$$\langle q_f|\hat{U}_J(t_f,t_i)|q_i\rangle=N\exp\left[\frac{i}{\hbar}\frac{(\Omega/2)}{\sin\Omega(t_f-t_i)}\left[(q_f^2+q_i^2)\cos\Omega(t_f-t_i)-2q_iq_f\right.\right.$$

$$\left.\left.+\frac{2}{\pi}\int_{t_i}^{t_f}dt' \, J(t')[q_f\sin\Omega t'+q_i\sin\Omega(t_f-t')]+\cdots\right]\right] , \tag{38}$$

where the ellipsis denotes terms independent of q_i and q_f.

The influence functional for the oscillator, on the system represented by J, is given by

$$F[J,J']=\int dq_i dq_i'\psi_i(q_i)\psi_i^*(q_i')\int dq_f\langle q_i'|\hat{U}_{J'}^{\dagger}|q_f\rangle\langle q_f|\hat{U}_J|q_i\rangle \ , \tag{39}$$

where ψ_i is the wave function for the initial state of the oscillator. One may extract and simplify the kernel in this definition, writing

$$\int dq_f\langle q_i'|\hat{U}_{J'}^{\dagger}|q_f\rangle\langle q_f|\hat{U}_J|q_i\rangle=\mathcal{A}[J,J']\delta\left[q_i-q_i'-\frac{1}{\Omega}\int_{t_i}^{t_f}dt[J(t)-J'(t)]\sin\Omega(t-t_i)\right]$$

$$\times\exp\left[\frac{i}{\hbar}\frac{q_i+q_i'}{2}\int_{t_i}^{t_f}dt[J(t)-J'(t)]\cos\Omega(t-t_i)\right] \ . \tag{40}$$

Since q_f is the same in both amplitudes in the first line, the q_f^2 term in (38) is canceled by the Hermitian conjugate term, leaving an exponent linear in q_f. The integration over final states then converts this into a δ function, which is then used to eliminate (q_i-q_i') from the expression. The prefactor $\mathcal{A}$ is independent of both q_i and q_i'.

This analysis may be generalized to the case of the massive scalar field, coupled as prescribed by (7), (9), and (11), since the Fourier modes $\Phi_i(k)\equiv\Phi_k(t_i)$ are closely analogous to the Schrödinger picture position operators of an infinite set of decoupled oscillators. One obtains, for the kernel analogous to (40),

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[3]The time necessary for the detector to reach equilibrium with the environment is a separate problem, not considered here.

$$\langle \Phi_i'(k)|\hat{U}_{Q'}^\dagger\hat{U}_Q|\Phi_i(k)\rangle=\mathcal{A}[Q(\tau),Q'(\tau)]\prod_k\delta\left[\Phi_i(k)-\Phi_i'(k)-\int_{\tau_i}^{\tau_f}d\tau\frac{Q-Q'}{\sqrt{2\pi}(k^2+m^2)^{1/2}}e^{ikx_0(\tau)}\sin\omega_k[t_0(\tau)-t_i]\right]$$

$$\times\exp\left[\frac{ic}{\sqrt{2\pi\hbar}}\int_{-\infty}^\infty dk\frac{\Phi_i(k)+\Phi_i'(k)}{2}\int_{\tau_i}^{\tau_f}d\tau(Q-Q')e^{ikx_0(\tau)}\cos\omega_k[t_0(\tau)-t_i]\right]. \qquad (41)$$

Fourier transforming back to field variables, so that $\Phi_i(k)$ is replaced by $\phi_i(x)$, for x the inertial spatial coordinate mapping the initial data surface $t=t_i$, this amplitude may be rewritten

$$\langle \phi_i'(x)|\hat{U}_{Q'}^\dagger\hat{U}_Q|\phi_i(x)\rangle=\mathcal{A}[Q(\tau),Q'(\tau)]\prod_x\delta\left[\phi_i(x)-\phi_i'(x)-\int_{\tau_i}^{\tau_f}d\tau(Q-Q')G[x_0(\tau)-x,t_0(\tau)-t_i)]\right]$$

$$\times\exp\left[-\frac{i}{\hbar}\int_{-\infty}^\infty dx\frac{\phi_i(x)+\phi_i'(x)}{2}\int_{\tau_i}^{\tau_f}d\tau(Q-Q')\frac{\partial}{\partial t_i}G[x_0(\tau)-x,t_0(\tau)-t_i]\right], \qquad (42)$$

where the odd Green's function for the scalar wave equation is defined by

$$G[x,y]\equiv\frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty\frac{dk}{(k^2+m^2)^{1/2}}e^{ikx}\sin\omega_k t . \qquad (43)$$

$G[x,t]$ vanishes outside the light cone $|ct|>|x|$.

Define the intersection of the light cone of the accelerating trajectory with the initial data surface to be the initial data region S. (See Fig. 1, where $t_i=0$ is chosen for convenience.) The fact that G vanishes outside the light cone may then be used to rewrite (42) as

$$\langle \phi_i'(x)|\hat{U}_{Q'}^\dagger\hat{U}_Q|\phi_i(x)\rangle=\mathcal{H}_S[\phi,\phi']\times\prod_{x\notin S}\delta(\phi_i(x)-\phi_i'(x)), \qquad (44)$$

where $\mathcal{H}_S$ is a functional of the field degrees of freedom lying inside the initial data region S.

The influence functional may formally be obtained from (44) by inserting the initial-state wave functionals $\Psi_i[\phi_i(x)]$ and $\Psi_i^*[\phi_i'(x)]$, and then integrating:

$$F[Q,Q']=\int\prod_x d\phi_i(x)d\phi_i'(x)\Psi_i[\phi_i(x)]\Psi_i^*[\phi_i'(x)]\langle\phi_i'(x)|\hat{U}_{Q'}^\dagger\hat{U}_Q|\phi_i(x)\rangle$$

$$=\int\prod_{x\in S}d\phi_i(x)d\phi_i'(x)\mathcal{H}_S[\phi,\phi']\int\prod_{x\notin S}d\phi_i(x)d\phi_i'(x)\delta(\phi_i-\phi_i')\Psi_i[\phi_i(x)]\Psi_i^*[\phi_i'(x)]$$

$$\equiv\int\prod_{x\in S}d\phi_i(x)d\phi_i'(x)\mathcal{H}_S[\phi,\phi']\rho_S[\phi_i(x),\phi_i'(x)], \qquad (45)$$

where the reduced density matrix ρ_S describing the state of the field degrees of freedom lying within the region S is defined implicitly in the last line. The product over the continuous index x is meant to describe the infinite-dimensional integrals over all initial field configurations $\phi_i(x)$ and $\phi_i'(x)$, using the ordinary measure $d\phi_i d\phi_i'$ in order to avoid confusion with path integrals over $\phi(x,t)$.

In general, initial data for a scalar field influence functional are provided by the density matrix for the field on the initial data surface, which in this case is the x axis.
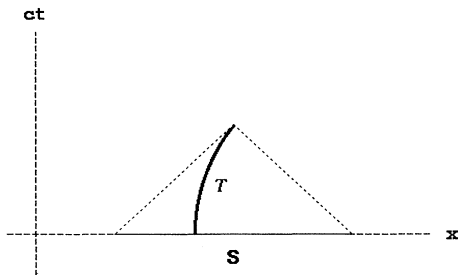

FIG. 1. Initial data region S.

Where the field is initially in a pure state Ψ_i, one has the initial density matrix $\rho_i(\phi,\phi')=\Psi_i[\phi]\Psi_i^*[\phi']$. In the particular case of a pointlike detector, however, (45) shows that the influence functional involves no more than a trace over the initial values of those field degrees of freedom lying outside the past light cone of the detector. In this case, therefore, the reduced density matrix ρ_S, formed by taking the trace of ρ_i over all the degrees of freedom outside S, provides sufficient initial data for the evolution of the detector between the given initial and final times.

Of course, S may be extended outside the past light cone as far as one likes, and it will still be true that the influence functional involves only a trace over the field modes outside S. Defining ρ_i, and then obtaining ρ_S by tracing out the modes of the field which lie outside the initial data region, is therefore a procedure which makes no reference whatever to the detector's trajectory or event horizon, and involves only ordinary, inertial quantization. Thus, even though the long-range coherences of the scalar field ground state do affect the form of the reduced density matrix, it can nevertheless clearly be seen that the Unruh effect does not arise from any interaction between

the long-range coherence of the vacuum and the geometry of the Rindler wedge. On the contrary, the Unruh effect originates in the identity (15) relating functions of inertial and accelerating time.

V. DIRECTIONAL DETECTOR
EXHIBITING DECOHERENCE

Section III showed that the thermal radiation detected by an accelerating observer has a different spectral density than that of a thermal ensemble of ordinary Fourier field components. This section illustrates another, and perhaps more profound, difference between the acceleration heat bath and inertial thermal radiation.

The accelerating detectors studied so far have been coupled to the field omnidirectionally: they do not discriminate between different directions in space. In the debate over whether or not the acceleration heat bath is isotropic, various authors have considered directional detectors, whose couplings single out preferred angles [19–21]. This section examines a model slightly different from previous ones. This model has a novel feature that is shown clearly in the influence functional formalism: if the detector's narrow line of sight is pointed away from its direction of acceleration, then the influence of the sca-

lar field vacuum on the detector is not like that of a stationary ensemble of oscillators. Instead, the field acts as a quantum measurement device that causes the detector's quantum state to evolve rapidly into a decoherent mixture of eigenstates of the monopole moment operator Q. In the limit where the angular aperture of the detector is extremely narrow, this is all that the scalar field does.

The present directional detector is similar to that of Ref. [21] in that its window of angular sensitivity is infinitesimal. The latter model uses Rindler modes to describe the scalar field, however. Since these modes do not possess a momentum quantum number in the direction of acceleration, the preferred angle of Ref. [21] is defined by a ratio of transverse momentum to energy. Such an angle is not actually a direction in space, even in the detector's rest frame. In this respect the present model is more like that of Ref. [20] in that it describes a directional detector in terms of its coupling to Fourier modes in an instantaneously comoving inertial frame.[4] The angle θ referred to in the discussion below will therefore be a true spatial direction, held constant in the frame of the accelerating detector.

Since the problem of the accelerating directional detector has azimuthal symmetry, all of its significant features are encompassed in the case $D=2$. In two spatial dimensions, then, consider the coupling

$$L_{\text{int}}(\tau) = \frac{Q(\tau)}{8\pi^2} \int \frac{d^2k}{(k_1^2 + k_2^2 + m^2)^{1/2}} W(k_1, k_2; \theta, \tau) \Phi_{k_1, k_2} e^{ik_1 x_0(\tau) - i\omega t_0(\tau)} . \tag{46}$$

$W(k_1, k_2; \theta, \tau)$ is a projection operator that vanishes outside a narrow window, of infinitesimal width ϵ, around the polar angle θ. In the rest frame of the detector at proper time τ, it is given by

$$W(k_1^\tau, k_2^\tau; \theta, \tau) = \int_0^\infty x \, dx \int_\theta^{\theta+\epsilon} d\alpha \, \delta(k_1 - x \cos\alpha) \delta(k_2 - x \sin\alpha) . \tag{47}$$

The cases $\theta = 0, \pi$ turn out to be difficult to evaluate in closed form. It will therefore be assumed that $\sin\theta \gg \epsilon > 0$.

To calculate the influence functional, it will be necessary to have W at two different times τ and τ' expressed in the same inertial frame, since the kernel $U(\tau, \tau')$ will become

$$U(\tau, \tau'; \theta) = \frac{1}{4\pi^2} \int \frac{d^2k}{(k_1^2 + k_2^2 + m^2)^{1/2}} W(k_1, k_2; \theta, \tau) W(k_1, k_2; \theta, \tau')$$

$$\times \exp\{i(k_1[x_0(\tau) - x_0(\tau')] - \omega(k_1, k_2)[t_0(\tau) - t_0(\tau')])\} . \tag{48}$$

It will be convenient to choose for this frame the one in which the detector is at rest at proper time $\bar{\tau} \equiv \frac{1}{2}(\tau + \tau')$. In this frame the detector's locations at times τ and τ' are given by

$$x_0(\tau) = x_0(\tau') = \frac{c^2}{a} \cosh\Delta\tau , \quad t_0(\tau) = -t_0(\tau') = \frac{c}{a} \sinh\Delta\tau , \tag{49}$$

for $\Delta\tau = \frac{1}{2}(\tau - \tau')$. Boosting (k_1, k_2) into the $\bar{\tau}$ frame, one obtains

$$W(k_1, k_2; \theta, \tau) = \int_0^\infty x \, dx \int_\theta^{\theta+\epsilon} d\alpha \, \delta(k_2 - x \sin\alpha) \delta(k_1 \cosh\Delta\tau + (x^2 + m^2)^{1/2} \sinh\Delta\tau - x \cos\alpha) . \tag{50}$$

$W(k_1, k_2; \theta, \tau')$ is given by a similar equation, in which the plus sign in the argument of the last δ function is replaced by a minus sign.

To leading order in ϵ, the narrow limits of integration on α in (50) simply impose $\alpha = \theta$. Therefore, to leading order

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[4]The spatial as well as the temporal frequencies perceived by the accelerating observer differ from those in the instantaneously comoving inertial frame [22]. The description of the directional detector in terms of its coupling to Fourier modes in instantaneously comoving frames presumes that the detector performs its angular discrimination within a sufficiently short distance for this effect to be negligible.

in ϵ and for $\sin\theta \gg \epsilon > 0$, one can obtain

$$W(k_1,k_2;\theta,\tau)W(k_1,k_2;\theta,\tau') \simeq \frac{\epsilon^2}{\sin\theta} \int_0^\infty dx\, x^2 \delta(k_2 - x\sin\theta)\delta(2(x^2+m^2)^{1/2}\sinh\Delta\tau)$$

$$\times \delta(k_1\cosh\Delta\tau + (x^2+m^2)^{1/2}\sinh\Delta\tau - x\cos\theta)$$

$$= \delta(\tau-\tau')\frac{\epsilon^2}{\sin\theta} \int_0^\infty \frac{x^2\,dx}{(x^2+m^2)^{1/2}} \delta(k_1 - x\cos\theta)\delta(k_2 - x\sin\theta) \ . \tag{51}$$

Substituting this and (49) into (48), one obtains

$$U(\tau-\tau';\theta) = \frac{\epsilon^2}{4\pi^2\sin\theta}\delta(\tau-\tau')\int_0^\infty \frac{x^2\,dx}{x^2+m^2}$$

$$\equiv \frac{\epsilon^2 A}{\sin\theta}\delta(\tau-\tau') \ . \tag{52}$$

A is divergent as written, but it may be regulated in the manner described in the Appendix, by assuming that the field-detector coupling has a cutoff at some large Rindler energy. The regulated version of A is finite and positive.

The reason for the δ function on $(\tau-\tau')$ is that a polar angle such as θ is not Lorentz invariant. As viewed in the laboratory frame, a fixed polar angle in the accelerating frame decreases with time (if it is between 0 and π), converging toward the direction of acceleration [23]. Hence, if the detector's window is bounded by two very close angles, held constant in the accelerating frame, then this window will not overlap itself at different times. Consequently, the product of the two window projection operators at different times vanishes unless the two times are the same (to zeroth order in ϵ).

To leading order in ϵ, the influence functional implied by (52) is simply

$$F[Q,Q';\theta] = \exp\left[-\frac{\epsilon^2 A}{2\hbar\sin\theta} \int_{\tau_i}^{\tau_f} d\tau [Q(\tau)-Q'(\tau)]^2 \right] \ . \tag{53}$$

When $F[Q,Q';\theta]$ is used to obtain the time evolution of the detector density matrix $\rho(Q,Q')$, it clearly suppresses all paths $[Q(\tau),Q'(\tau)]$ except those in which $(Q-Q')$ rapidly approaches zero. The density matrix $\rho(Q,Q')$ evolves toward being diagonal: The influence functional (53) drives the detector's quantum state toward a decoherent mixture of eigenstates of the monopole mo-

ment operator Q. Such decoherence is a generic effect of influence functionals for large environments acting on small systems, but (53) is remarkable for its simplicity. It decoheres the detector in the Q-state basis; it does so in an obvious and direct manner, and it does nothing else.

The reason for this simple behavior lies in the laboratory-frame time dependence of the detector's time-independent angular window. The variation in an inertial frame of the angle of receptivity means that an accelerating detector with a narrow aperture, directed away from the axis of motion, couples at each instant to a succession of different (and orthogonal) modes of the scalar field. This is the physical interpretation of (53). It implies that the association of a fixed direction in space with a given field mode, which is a basic feature of quantum field theory in inertial frames and is an important element in our notion of quantum fields as representing particles, breaks down in noninertial frames.

It can also be shown that the forward and backward ($\theta=0$ or π) directions, which represent orthogonal field modes in inertial quantization, are associated with the same mode of the field in the Rindler quantization. So not only can a fixed angle correspond to a succession of modes, but distinct angles can correspond to a single mode.

The fact that the δ function in (52) may be related to the elementary relationship between inertial and accelerating angular directions should confirm that it is not an unrealistic artifact of some approximation that has been used. If the calculations are extended to include terms of third order in the narrow angular width ϵ, then the δ function is indeed blurred into a narrow window, but the basic features of (53) are not significantly changed. A brief sketch of the extension of (52) and (53) to third order follows.

One can obtain by straightforward means the result that

$$U(\tau,\tau';\theta) = \frac{\epsilon^3}{\sin\theta}[\,\tilde{A}\cot\theta + iB\csc\theta\,]\delta(\tau-\tau') + \frac{\epsilon}{4\pi^2\sin\theta}\int_0^\epsilon d\gamma \int_0^\infty dx \frac{x^2}{x^2+m^2}\delta\left[\tau-\tau' - \frac{x\sin\theta}{(x^2+m^2)^{1/2}}\gamma\right] + O(\epsilon^4) \ , \tag{54}$$

where $\tilde{A}$ and B are real constants defined by integrals needing to be regulated in the same manner as that in (52). The imaginary term proportional to B and the smeared δ function lead to new terms in the influence functional:

$$F[Q,Q';\theta] = \exp\left[-\frac{\epsilon^2}{2\hbar\sin\theta} \int_{\tau_i}^{\tau_f} d\tau \left[[\,A + \epsilon\cot\theta\,\tilde{A}\,][Q(\tau)-Q'(\tau)]^2 \right.\right.$$

$$\left.\left. + i\epsilon\csc\theta B[Q(\tau)^2 - Q'(\tau)^2] + \epsilon C\frac{d}{d\tau}[Q(\tau)-Q'(\tau)]^2 \right]\right] \ , \tag{55}$$

where a new real constant C is defined by the x integral in (54). The term in $F[Q,Q';\theta]$ proportional to B is simply an addition to the quadratic potential term in the Q-system Lagrangian $L(Q)$. The term arising from the smearing of the δ function also tends to decohere the system in the Q basis.

VI. CONCLUSION

The apparent heating of the vacuum observed along an accelerating trajectory seems to be a fundamental result in noninertial, relativistic quantum field theory. An acceleration provides a natural energy scale, but that this scale should appear precisely as a temperature is an example of rare natural simplicity. The underlying reason for this simplicity seems originally to have been sought in the behavior of certain globally preferred coordinate systems near event horizons [5,8]. This understanding of the phenomenon of acceleration temperature had distressing implications, however, in that the results of local measurements seemed to depend on global properties of spacetime.

The analysis presented above confirms that global issues arise only when they are explicitly invoked, in trying to discuss the state of a quantum field over the entire Rindler wedge. The thermal effects may be isolated from the global problem by the use of influence functional methods, which allow one to determine the properties of the field as probed by a pointlike detector. The ultimate source of the acceleration temperature is then seen to be the relation between sines and cosines of the proper times of inertial and accelerating observers. The Unruh effect may be considered a distant cousin to the inertial forces of Newtonian mechanics, in that it is a property of the acceleration itself.

The calculations that lead to this result require a reduced density matrix for the initial state of the quantum field within a region of space. In Minkowski space this information is available because the correct ground state is known. If these methods are to be extended to problems in curved spacetime, in particular the Hawking radiation of black holes, some proposal for the field ground state must be assumed. As suggested by Jacobson [10], the ground state might be constrained to possess certain properties measured by local observers; for example, a static detector "near" the horizon might be required to behave as an accelerating detector in flat space. This gravitational quantum equivalence principle[5] would provide the standard Hawking effect by means of the Unruh effect.

One might hope to construct a quantum theory in curved spacetime based on such a quantum equivalence principle, plus some mechanism for propagating thermal radiation from "near" regions out into flatter regions. (This propagation process is obviously needed to let Un-

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[5]The quantum-mechanical extension of the local equivalence of gravitation and acceleration should not be confused with the "quantum equivalence principle" of Refs. [24,25], which compares radiation reaction and vacuum fluctuations.

ruh radiation generated at an event horizon be detected far away as Hawking radiation.) Yet the propagation of the thermal radiation does not obviously have to interfere with its generation via the naive quantum equivalence principle. Unless this principle can be shown to be peculiar to the neighborhood of a horizon, it should apply equally well in the case of static detectors outside any spherically symmetric matter distribution and at any distance. These applications of the principle would not yield the standard results.

Even if one could establish the Hawking temperature by appeal to the Unruh effect, it seems unlikely that a gravitational quantum equivalence principle would provide the correct spectral density of the Hawking radiation. While two heat baths with different spectral densities are both thermal environments, they may have significantly different physical effects, which one would want to understand. Furthermore, the one-to-one association of local directions with orthogonal field modes, valid in inertial frames in flat space, has been seen to fail for a constantly accelerating frame. These difficulties do not imply that the gravitational quantum equivalence principle is wrong, but they do suggest that it will not be as easy to use as one might hope. While the Unruh temperature is a genuine property of acceleration *per se*, there is more to acceleration than temperature. The idea of formulating quantum theory in curved spacetime in terms of observations made using local detectors would seem to deserve further study. The technique of influence functionals should be of use in this regard.

ACKNOWLEDGMENTS

APPENDIX: RINDLER CUTOFF ON INERTIAL FREQUENCY

In $D > 1$ spatial dimensions, ultraviolet-divergent frequency integrals can appear in accelerating detector problems. Discarding the high-frequency limit by appeal to the properties of physical detectors is perhaps sophistical, since any linear acceleration that produces a measurable Unruh temperature will destroy any ordinary apparatus [4]. Nevertheless, it seems reasonable that the most meaningful results are to be extracted from the divergent integrals by imposing a cutoff at some high-frequency Γ, in the frame of the accelerating detector.

Given a function $f(x)$ with a Fourier transform $F(k)$, a cutoff on k may be imposed by changing

$$f(x) \rightarrow f_\Gamma(x) = \int_{-\Gamma}^{\Gamma} dk \, F(k) e^{ikx}$$

$$= \int_{-\infty}^{\infty} dz \frac{\sin\Gamma z}{\pi z} f(x+z) \, .$$

Thus one may impose a Rindler energy cutoff on the coupling of the detector to the field, by substituting in (46):

$$e^{ik_1 x_0(\tau) - i\omega t_0(\tau)} \rightarrow \int_{-\infty}^{\infty} dz \frac{\sin\Gamma z}{\pi z} e^{ik_1 x_0(\tau+z) - i\omega t_0(\tau+z)} \, .$$

$$\text{(A1)}$$

To obtain the regulated version of A in Eq. (52) of Sec. VI, note that the δ function $\delta(\tau-\tau')$ forces $\tau=\tau'=\bar{\tau}$, so that, in the $\bar{\tau}$ frame,

$$x_0(\tau+z) \to \frac{c^2}{a}\cosh\frac{az}{c} ,$$

$$t_0(\tau+z) \to \frac{c}{a}\sinh\frac{az}{x} .$$

Because the z integral is now included in the coupling, (52) is changed so that

$$\int_0^\infty \frac{x^2 dx}{(x^2+m^2)^{1/2}} \to \int_0^\infty \frac{x^2 dx}{(x^2+m^2)^{1/2}}|C_\Gamma(x)|^2 , \qquad (A2)$$

where

$$C_\Gamma(x) \equiv \int_{-\infty}^\infty dz \frac{\sin\Gamma z}{\pi z}\exp\left[i\frac{c^2}{a}\left[x\cos\theta\cosh\frac{az}{c} - (x^2+m^2)^{1/2}\sinh\frac{az}{c}\right]\right]$$

$$= \int_{-\infty}^\infty dz \frac{\sin\Gamma z}{\pi z}\exp\left[i\frac{c^2}{a}(x^2\sin^2\theta+m^2)^{1/2}\sinh\left[B(x)-\frac{az}{c}\right]\right] . \qquad (A3)$$

Here $B(x)$ is defined for convenience, such that

$$\sinh B(x) \equiv \frac{x\cos\theta}{(x^2\sin^2\theta+m^2)^{1/2}} .$$

Using the identity [26]

$$e^{i\lambda\sinh\xi} = \frac{2}{\pi}\int_0^\infty d\mu\, K_{i\mu}(\lambda)\left[\cosh\frac{\pi\mu}{2}\cos\mu\xi + i\sinh\frac{\pi\mu}{2}\sin\mu\xi\right] , \qquad (A4)$$

(A3) may be written

$$C_\Gamma(x) = \frac{2}{\pi}\int_{-\infty}^\infty dz \frac{\sin(\Gamma z)}{\pi z}\int_0^\infty d\mu\, K_{i\mu}\left[\frac{c^2}{a}(x^2\sin^2\theta+m^2)^{1/2}\right]\cos\mu\left[B(x)-\frac{az}{c}-\frac{i\pi}{2}\right]$$

$$= \frac{2}{\pi}\int_0^{(c/a)\Gamma} d\mu\, K_{i\mu}\left[\frac{c^2}{a}(x^2\sin^2\theta+m^2)^{1/2}\right]\cos\mu\left[B(x)-\frac{i\pi}{2}\right] . \qquad (A5)$$

For fixed order and large argument, the modified Bessel functions of the second kind have the asymptotic behavior [27]

$$K_{i\mu}(\lambda) \sim \left[\frac{\pi}{2\lambda}\right]^{1/2} e^{-\lambda}[1+O(\lambda^{-1})] .$$

Therefore the factor $|C_\Gamma(x)|^2$ does suppress the ultraviolet divergence in A.

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