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Plane domain walls when coupled with the Brans-Dicke scalar field

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The space-times of plane thin domain walls are studied in the context of the Brans-Dicke (BD) theory of gravity by using distribution theory. In particular, the BD field equations are divided into two groups: one holding in the regions outside of the wall and the other holding on the wall. It is found that the equations on the wall take a very simple form, and are given explicitly in terms of the metric coefficients and the BD scalar field. As an application of the theory developed, a class of exact solutions, which represents a plane domain wall interacting with the BD scalar field, is given and studied. It is found that the surface energy density of the wall always exponentially decreases as the time develops; this is one possible solution of the domain wall problem in cosmological models founded on general relativity. The space-time is usually singular not only at the initial point, but also at spacelike infinity. However, the proper distance from the wall to the singularities at spacelike infinity is finite but exponentially increasing (in fact, inversely proportional to the surface energy density of the wall).

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I. INTRODUCTION

There have been different attempts to study the inflationary-universe models in some other theories of gravity rather than in the one of Einstein [1,2]. For example, recently La and Steinhardt [3] proposed a model, the so-called "extended" inflation, in the context of the Brans-Dicke (BD) theory of gravity [4]. It was found that, because of the interaction of the BD scalar field and the Higgs-type sector that undergoes a strongly first-order phase transition, the exponential expansion in Guth's model [5] is slowed down to a power-law one. As a result, the phase transition can be completed via bubble nucleation. Hence the "graceful exit" problem is resolved. However, Weinberg [6] and La, Steinhardt, and Bertschinger [7] soon found that the inhomogeneity caused by the nucleation of bubbles was seriously in conflict with the observational constraints of the microwave background radiation (MBR) unless the BD parameter ω is less than 25. But the latter is not consistent with the requirement $\omega > 500$ [8]. Lately, Goldwirth and Zaglauer [9] reconsidered the above problem, and found that after gravitation was taken into account bubbles are led to recollapse during most of the extended-inflation period, and the distribution of bubble sizes gets dramatically changed. Consequently, the phase transition can be completed through tiny bubbles, the existence of which is not in conflict with both of the above requirements.

On the other hand, Linde and Lyth [10] and Basu, Guth, and Vilenkin [11] have shown that, because of quantum-mechanical tunneling, topological defects, such as domain walls, cosmic strings, and monopoles also can be formed during inflation. Once they are formed, these defects will certainly interact with the BD scalar field as well as the gravitational field produced by them. Thus, to have a better understanding of the inflationary Universe, it is very important to investigate such interactions in more details and more general terms.

In this paper we shall study the interaction of domain walls with the BD scalar field and the corresponding gravitational field. Our main assumptions here are that the walls have plane and reflection symmetry, and that the typical thickness of the wall is much smaller than any other physical sizes concerned in the problem [12,13]. These assumptions may cause some doubt about our results. However, we believe that the main properties obtained in this paper should remain valid even in more realistic models.

The structure of the paper is as follows. In Sec. II, the space-time containing plane walls coupled with the BD scalar field is studied, and all the BD field equations are given in terms of distributions. It is found that the equations on the wall take a very simple form, and are given explicitly in terms of the metric coefficients, the BD scalar field, and their derivatives. It is found that for any given solution of the BD gravitational field equations, there always exists a corresponding solution, which, subject to some restrictions, represents a plane domain wall interacting with the BD scalar field. As an illustration of the theory developed in Sec. II, in Sec. III a class of exact solutions is presented and studied. Finally, the paper is closed by Sec. IV, where our main conclusions are derived.

Before proceeding, let us note that a general description of singular hypersurfaces in the BD theory was

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worked out by Suffern [14], using Israel's method [15], and the application of it to spherically symmetric bubbles was also investigated. In this paper we shall adopt a different approach [12,13], which is essentially the combination of the Newman-Penrose (NP) formalism [16] and the distribution theory [17]. In principle, both approaches give the same results and are complementary. The main advantage of the NP formalism is that the Weyl and Ricci scalars, in the present case, have their explicit physical interpretations, and with them we can easily study the interactions of the walls with gravitational and matter fields.

II. PLANE DOMAIN WALLS INTERACTING WITH THE BRANS-DICKE SCALAR FIELD

In this section, the formal development of the theory will closely follow the one given in Refs. [12] and [13]. The only difference is that, instead of considering the problem in the context of Einstein's theory of gravity, here we consider it in the one of Brans and Dicke [4]. For more details, readers may see Refs. [12] and [13].

The action of the BD theory of gravity reads [4,18]

$$A=\int d^4x\sqrt{-g}\left\{\phi R-\omega\frac{\phi_{,\lambda}\phi^{,\lambda}}{\phi}+2L_{\text{matter}}\right\}, \tag{2.1}$$

where ϕ denotes the BD scalar field, ω the BD coupling constant, and L_{matter} the Lagrangian density of matter.

Varying the above action with respect to the metric we obtain the BD gravitational field equations

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{1}{\phi}\{T^M_{\mu\nu}+T^{\text{BD}}_{\mu\nu}\}, \tag{2.2}$$

where $T^M_{\mu\nu}$ and $T^{\text{BD}}_{\mu\nu}$ denote the energy-stress tensors for the matter and the BD scalar fields, respectively, and are given by

$$\begin{aligned} T^M_{\mu\nu}&=\frac{2}{\sqrt{-g}}\frac{\partial}{\partial g^{\mu\nu}}\{\sqrt{-g}\,L_{\text{matter}}\},\\ T^{\text{BD}}_{\mu\nu}&=\frac{\omega}{\phi}(\phi_{,\mu}\phi_{,\nu}-\tfrac{1}{2}g_{\mu\nu}\phi_{,\lambda}\phi^{,\lambda})+\phi_{,\mu;\nu}-g_{\mu\nu}\phi_{,\lambda}{}^{;\lambda}. \end{aligned} \tag{2.3}$$

A semicolon denotes covariant differentiation.

On the other hand, by varying the action A with respect to ϕ, we will have the equation for the ϕ field:

$$\phi_{,\mu;\nu}g^{\mu\nu}=\frac{T^M}{3+2\omega}, \tag{2.4}$$

where $T^M\equiv T^M_{\mu\nu}g^{\mu\nu}$. In this paper we shall consider the space-time with the Szekeres metric taking the form [12,19,20]

$$ds^2=2e^{-M}du\,dv-e^{-U}\{e^V\cosh W\,dx^2-2\sinh W\,dx\,dy+e^{-V}\cosh W\,dy^2\}, \tag{2.5}$$

where the functions M, U, V, and W depend only on the null coordinates u and v, and $-\infty<u,v,x,y<+\infty$, with the coordinates being number as $\{x^\mu\}\equiv\{u,v,x,y\}$.

It is not trivial, but one can prove that each solution of the system (2.2)–(2.4) with the metric (2.5) and $T^M_{\mu\nu}$ depending on u and v only has $\phi=\phi(u,v)$. Then, we find that Eq. (2.4) reads

$$2\phi_{,uv}-\phi_{,u}U_{,v}-\phi_{,v}U_{,u}=e^{-M}\frac{T^M}{3+2\omega}. \tag{2.6}$$

To proceed further, let us first choose a null tetrad as the one given by Eqs. (2.2) and (2.3) in Ref. [21]; then we find that the nonvanishing Weyl and Ricci scalars are $\Psi_0\Psi_2, \Psi_4$, and $\Phi_{00},\Phi_{02},\Phi_{11},\Phi_{22}$, Λ, and are given by Eqs. (A1) and (A2) in Ref. [22] which, for the reader's convenience, are represented in Appendix A of this paper. In terms of these nonvanishing Ricci scalars, Eq. (2.2) can be written as

$$\begin{aligned} \Phi_{ij}&=\frac{1}{\phi}\{\Phi^{\text{BD}}_{ij}+\Phi^M_{ij}\}\quad(i,j=0,1,2),\\ \Lambda&=\frac{1}{\phi}\{\Lambda^{\text{BD}}+\Lambda^M\}, \end{aligned} \tag{2.7}$$

where Φ^M_{ij}, Λ^M, Φ^{BD}_{ij} and Λ^{BD} are given by Eqs. (B1)–(B3) in Appendix B.

Following Ref. [12], let us make the following substitutions[1]:

$$z\to|z|, \tag{2.8}$$

in the metric coefficients $g_{\mu\nu}$ as well as the BD scalar field ϕ, where the function z is given via the relations

$$u=\frac{t+z}{\sqrt{2}},\quad v=\frac{t-z}{\sqrt{2}}. \tag{2.9}$$

Then, we find that the Ricci scalars Φ_{ij}'s and Λ are given exactly by Eq. (3.10) in Ref. [13], and that the Φ^{BD}_{ij}'s and Λ^{BD} are given by

$$\begin{aligned} \Phi^{\text{BD}}_{ij}&=\Phi^{\text{BD}+}_{ij}H(z)+\Phi^{\text{BD}-}_{ij}[1-H(z)]+\phi^{im}_{ij}\delta(z),\\ \Lambda^{\text{BD}}&=\Lambda^{\text{BD}+}H(z)+\Lambda^{\text{BD}-}[1-H(z)]+\lambda^{im}\delta(z), \end{aligned} \tag{2.10}$$

where $\Phi^{\text{BD}\pm}_{ij}$ and $\Lambda^{\text{BD}\pm}$ are calculated from the expressions of Eqs. (B2) and (B3) in the regions where $z>0$ and $z<0$, respectively, and ϕ^{im}_{ij} and λ^{im} are given by

$$\begin{aligned} &\phi^{im}_{00}=\tfrac{1}{2}B^2\phi_{,z}(t,0),\quad \phi^{im}_{22}=\tfrac{1}{2}A^2\phi_{,z}(t,0),\quad \phi^{im}_{02}=0,\\ &\phi^{im}_{11}=-\lambda^{im}=-\tfrac{1}{4}AB\phi_{,z}(t,0), \end{aligned} \tag{2.11}$$

[1]This is the crucial point where we introduce the domain wall: The first derivative of the metric gets a jump discontinuity at $z=0$ thus leading to a δ-like second derivative, which is connected with a δ-like matter distribution. Of course, not each δ-like matter distribution concentrated on a hypersurface may be called domain wall. First, if it is a spacelike hypersurface then it can hardly be interpreted as matter. Second, if it is a lightlike hypersurface, then it can be interpreted as gravitational shock wave [23,24] or as light impulse [25]. Third, if it is a timelike hypersurface then it can be interpreted as incoherent matter [15] or as surface tension [26]. At least, one should require an isotropic surface pressure and a kind of surface equation of state for a δ-like matter distribution to be interpretable as domain wall. In this paper we do not consider such questions.

where

$$\phi_{,z}(t,0) \equiv \lim_{z \to 0^+} \frac{\partial \phi(t,z)}{\partial z} , \quad AB \equiv e^M , \tag{2.12}$$

and $\delta(z)$ denotes the Dirac δ distribution.

It must be noted that in writing Eqs. (2.10) and (2.11) we had used the assumption that ϕ is at least C^0 across the hypersurface $z=0$. This is explained by the following considerations. From Eq. (2.3) we can see that $T^{BD}_{\mu\nu}$ contains terms quadratic in the first derivatives of ϕ, and terms linear in the second derivatives. Thus, to be physically meaningful, ϕ should be at least C^0 across the surface $z=0$; otherwise, $T^{BD}_{\mu\nu}$ would contain the square of the Dirac δ function, which is both physically and mathematically unacceptable.

Since here we are mainly interested in the interaction of domain walls with the BD scalar field, in the rest of this paper we shall consider only the cases where $T^M_{\mu\nu}$ corresponds to a domain wall, namely,

$$T^M_{\mu\nu} = \sigma h_{\mu\nu} \delta(z) , \tag{2.13}$$

where σ denotes the surface energy density of the wall, $h_{\mu\nu}$ is the intrinsic three-metric of the wall and is given by

$$h_{\mu\nu} = g_{\mu\nu} - (\xi_\lambda \xi^\lambda)^{-1} \xi_\mu \xi_\nu , \quad \xi_\mu \equiv \frac{1}{\sqrt{2}} \{1,-1,0,0\} . \tag{2.14}$$

The substitution of Eqs. (2.13) and (2.14) into Eq. (B1) yields

$$\begin{aligned}
&\Phi^M_{00} = \frac{B}{4A} \sigma \delta(z) , \quad \Phi^M_{22} = \frac{A}{4B} \sigma \delta(z) , \\
&\Phi^M_{11} = -\Lambda^M = -\tfrac{1}{8} \sigma \delta(z) , \\
&\Phi^M_{01} = \Phi^M_{02} = \Phi^M_{12} = 0 .
\end{aligned} \tag{2.15}$$

Inspecting Eqs. (2.10) and (2.15), we can see that Eqs. (2.7) can be divided into two groups, one of which holds outside of the wall, and reads

$$\Phi^{\pm}_{ij} = \frac{1}{\phi} \Phi^{BD\pm}_{ij} , \quad \Lambda^{\pm} = \frac{1}{\phi} \Lambda^{BD\pm} , \tag{2.16}$$

and the other of which holds on the wall, and takes the form

$$M_{,z}(t,0) = U_{,z}(t,0) , \quad V_{,z}(t,0) = 0 = W_{,z}(t,0) , \tag{2.17}$$

$$\sigma = 2e^{M(t,0)} \{\phi(t,0) U_{,z}(t,0) - \phi_{,z}(t,0)\} \geq 0 . \tag{2.18}$$

Note that the conditions (2.17) are exactly the ones obtained for a plane domain wall in the framework of Einstein's theory of gravity [12], while the surface energy density of the wall is modified due to the presence of the ϕ field. When ϕ is constant, it will reduce to the one given in Ref. [12].

Inserting Eq. (2.17) into Eq. (3.11a) in Ref. [13], we find that the Ψ^{im}_A's vanish identically. That is, in the space-time of a plane domain wall, the gravitational field is free of an impulsive part.

The combination of Eqs. (2.13) and (2.6), on the other hand, yields

$$2\phi_{,uv} - \phi_{,u} U_{,v} - \phi_{,v} U_{,u} = e^{-M} \frac{3\sigma}{3+2\omega} \delta(z) , \tag{2.19}$$

which represents the interaction of a plane domain wall with the BD scalar field. Writing ϕ and U in the form $f = f^+ H(z) + f^- [1 - H(z)]$, where $f^+ \equiv f(t, z>0)$, and $f^- \equiv f(t, -z>0)$, we find that Eq. (2.19) is equivalent to

$$(2\phi_{,uv} - \phi_{,u} U_{,v} - \phi_{,v} U_{,u})^{\pm} = 0 \tag{2.20}$$

and

$$U_{,z}(t,0) = -\frac{2\omega}{3\phi(t,0)} \phi_{,z}(t,0) . \tag{2.21}$$

Thus, in addition to Eqs. (2.17) and (2.18), Eq. (2.21) will give another restriction on the metric coefficients and the ϕ field.

Summarizing the above results we find that the basic equations for a plane domain wall coupled with the BD scalar field consist of Eqs. (2.16)–(2.18) and (2.20) and (2.21).

Note that for any C^2 function $F(t,z)$, we have the relations

$$F_{,u}(t,-z) = F_{,\tilde{v}}(\tilde{t},\tilde{z}) , \quad F_{,v}(t,-z) = F_{,\tilde{u}}(\tilde{t},\tilde{z}) , \tag{2.22}$$

where $\tilde{t}=t$, $\tilde{z}=-z$, and $\tilde{u}$ and $\tilde{v}$ are given by Eq. (2.9) but with t, and z replaced by $\tilde{t}$ and $\tilde{z}$. Combining Eq. (A2) in Appendix A with Eq. (2.22), it is easy to show that

$$\begin{aligned}
&\Phi^-_{00}(t,z) = \Phi^+_{22}(\tilde{t},\tilde{z}) , \quad \Phi^-_{22}(t,z) = \Phi^+_{00}(\tilde{t},\tilde{z}) , \\
&\Phi^-_{02}(t,z) = \Phi^+_{02}(\tilde{t},\tilde{z}) , \\
&\Phi^-_{11}(t,z) = \Phi^+_{11}(\tilde{t},\tilde{z}) , \quad \Lambda^-(t,z) = \Lambda^+(\tilde{t},\tilde{z}) .
\end{aligned} \tag{2.23}$$

A similar expression for the Φ^{BD}_{ij}'s and Λ^{BD} can be obtained from Eq. (B2) in Appendix B. Note that in the above equation all the Ricci scalars stand for the "scale-invariant" ones, the definitions of which are given by Eq. (4.8) in Ref. [21].

Combining Eq. (2.23) with the corresponding one for the Φ^{BD}_{ij}'s and Λ^{BD}, we find that if Eq. (2.16) holds in one region (say, in the region where $z>0$), then it will also hold in the other ($z<0$), or vice versa. Therefore, we have the following conclusions.

Theorem. Assume that $\{\phi, g_{\mu\nu}\}$ is a solution of the BD gravitional field equations (2.2) of the type (2.5) with $T^M_{\mu\nu}=0$; then the substitutions (2.8) in the metric coefficients $g_{\mu\nu}$ and the BD scalar field ϕ always generate a new solution, which represents a plane domain wall interacting with the BD scalar field through Eq. (2.19), subject to the restrictions (2.17), (2.18), and (2.21).

Having completed the general description for the space-time of a plane domain wall coupled with the BD scalar field, let us consider some specific models.

III. EXACT SOLUTIONS OF PLANE DOMAIN WALLS WHEN COUPLED WITH THE BD FIELD

In this section we shall consider some exact solutions, which represent the interaction of a plane domain wall with the BD scalar field. Because of the theorem presented in the preceding section, one can see that finding such solutions essentially reduces to finding solutions of the BD field equations themselves. Thanks to the recent development carried out in this direction, several methods to generate new solutions have become available [27–29]. In this paper we shall use the method developed by Belinsky and Khalatnikov (BK) [30], which is essentially based on the relations between the BD scalar field and a common massless scalar one. The main results of BK can be summarized as follows.

If $\{\phi_0, M_0, U_0, V_0, W_0\}$ is a solution of the Einstein field equations

$$R_{0\mu\nu}=\phi_{0,\mu}\phi_{0,\nu}\ , \tag{3.1a}$$

$$2\phi_{0,uv}-\phi_{0,u}U_{0,v}-\phi_{0,v}U_{0,u}=0\ , \tag{3.1b}$$

where $R_{0\mu\nu}$ denotes the Ricci tensor built up from $g_{0\mu\nu}$, and ϕ_0 denotes the massless scalar field, then

$$\{\phi, M, U, V, W\}=\{\exp(\lambda\phi_0)\ , M_0+\lambda\phi_0, U_0+\lambda\phi_0\ , V_0\ , W_0\} \tag{3.2}$$

is a solution of the BD field equations (2.2) with $T^M_{\mu\nu}=0$. Here λ is a constant and related to the BD parameter ω via the relation

$$\lambda=\left[\frac{2}{3+2\omega}\right]^{1/2}\ . \tag{3.3}$$

Note that Eq. (3.2) essentially represents a conformal transformation of the metrics, the conformal factor is a function of the scalar field.

The usefulness of the above theorem is attributed to the facts that Eqs. (3.1) with the Szekeres metric (2.5) have been exhaustively studied recently, and a huge class of exact solutions is available now [31–33].

Inserting Eq. (3.2) into Eqs. (2.17), (2.18), and (2.21), we find

$$M_{0,z}(t,0)=U_{0,z}(t,0)\ ,\quad V_{0,z}(t,0)=0=W_{0,z}(t,0)\ , \tag{3.4}$$

$$U_{0,}(t,0)=-\frac{3+2\omega}{3}\lambda\phi_{0,z}(t,0)\ , \tag{3.5}$$

and

$$\sigma=2\exp\{M_0(t,0)+2\lambda\phi_0(t,0)\}U_{0,z}(t,0)\geq 0\ . \tag{3.6}$$

Recently, Wang [34,35] has studied a class of solutions, which represents plane domain walls interacting with the massless scalar field ϕ_0. These solutions are given by [35]

$$\begin{aligned} &M_0=2k|z|-2\beta^2\ln(\sinh 2kt)-2\alpha\beta\ln(\tanh kt)\ ,\quad W_0=0\ ,\\ &U_0=2k|z|-\ln(\sinh 2kt)\ , V_0=m\ \ln(\tanh kt)\ ,\\ &\phi_0=2\beta k|z|-\beta\ln(\sinh 2kt)-\alpha\ln(\tanh kt)\ , \end{aligned} \tag{3.7}$$

where α, β, m, and k are constant, and satisfy the conditions

$$2(\alpha^2-\beta^2)+m^2-1=0\ \text{ and }\ k>0\ . \tag{3.8}$$

Note that the above conditions are actually the direct consequence of Eqs. (3.4) and (3.6). In Ref. [35], it has been shown that regardless of the initial conditions of the wall, the corresponding surface energy density of it always exponentially decreases with the time increasing. Moreover, the space-time is singular not only at the initial point $t=0$, but also at spacelike infinity. The proper distance from the wall to these spacelike singularities is finite, and inversely proportional to the square root of the surface energy density of the wall.

Substituting Eq. (3.7) into (3.2) we find the solutions

$$\begin{aligned} M&=2(1+\lambda\beta)k|z|-\beta(2\beta+\lambda)\ln(\sinh 2kt)\\ &\quad-\alpha(2\beta+\lambda)\ln(\tanh kt)\ ,\\ U&=2(1+\lambda\beta)k|z|-(1+\lambda\beta)\ln(\sinh 2kt)\\ &\quad-\alpha\lambda\ln(\tanh kt)\ ,\\ V&=m\ \ln(\tanh kt)\ ,\quad W=0\ , \end{aligned} \tag{3.9}$$

and

$$\phi=e^{2\lambda\beta k|z|}\frac{(\coth kt)^{\lambda\alpha}}{(\sinh 2kt)^{\lambda\beta}}\ . \tag{3.10}$$

To have the solutions that represent plane domain walls coupled with BD scalar field, in addition to the restrictions (3.8), the above solutions must also satisfy the restrictions given by Eqs. (2.21) or (3.5), which now read

$$\beta=-\tfrac{3}{2}\lambda\ . \tag{3.11}$$

Thus, we conclude that subject to the restrictions (3.8) and (3.11) the solutions given by Eqs. (3.9) and (3.10) represent a plane domain wall interacting with the BD scalar field.

Combining Eqs. (3.3), (3.8), and (3.11) we find

$$-\alpha_c\leq\alpha\leq\alpha_c\ , \tag{3.12}$$

where

$$\alpha_c\equiv\left[\frac{\omega+6}{2\omega+3}\right]^{1/2}\ . \tag{3.13}$$

Inserting Eqs. (3.7) and (3.11) into Eq. (3.6), on the other hand, we obtain

$$\sigma=4k\frac{(\tanh kt)^{\lambda\alpha}}{(\sinh 2kt)^{3\lambda^2/2}}\ . \tag{3.14}$$

Obviously, when $\lambda=0$, i.e., $\omega\rightarrow\infty$, the surface energy density becomes constant, and the solutions reduce to the ones discussed in Ref. [34], whereby it has been shown that the reduced solutions represent Vilenkin's planar domain wall [36] coupled with a massless scalar field. Because of the presence of the ϕ_0 field, the horizons appearing in Vilenkin's vacuum solution are replaced by spacetime singularities.

In the following we shall consider only the cases where $\lambda\neq 0$. From Eq. (3.14) it is easy to show that σ has the following asymptotic behavior

$$\sigma\sim t^{\lambda(\alpha+\beta)}\rightarrow\begin{cases}0, & -\beta<\alpha\leq\alpha_c\ ,\\ \text{const}, & \alpha=-\beta,\\ \infty, & -\alpha_c\leq\alpha<-\beta,\end{cases} \tag{3.15}$$

as $t \to 0^+$, and that

$$\sigma \sim \exp\{-3\lambda^2 kt\} \to 0 , \tag{3.16}$$

as $t \to +\infty$. Thus, similar to the case for a massless scalar field [35], the surface energy density of the wall always exponentially decreases as the time increases, independently of its initial conditions. The rate depends on the choice of the parameter λ (which is essentially the BD parameter) and the integration constant k. Thus, by properly arranging the integration constant k, the walls could disappear as fast as wanted. This is in contrast with the vacuum walls formed in grand unified theories [37]. There it was found that the walls formed by the spontaneous violation of CP invariance are so heavy that their existence would cause strong conflict to the observational constraints of MBR. Therefore, it was concluded that either model with discrete symmetry breaking is ruled out by cosmology, or there must exist some mechanisms that could make the walls disappear in a very early time of the Universe. The above example obviously shows the possibility of introducing the BD scalar field into the models. Of course, to see exactly how the BD field works, we need, among other things, to fix these parameters by some physical considerations, such as the initial conditions. However, these conditions are quite model dependent [37], and further investigations of them are quite involved and need specific models of the Universe; this is beyond the scope of this paper.

On the other hand, using Eqs. (C2) and (C3) given in Appendix C it can be shown that the nonvanishing Weyl and Ricci scalars in the regions outside of the wall are given, respectively by

$$\Psi_0 = \frac{B^2}{A^2}\Psi^4 = -2\beta m k^2 B^2 \frac{\alpha+\beta\cosh 2kt}{\sinh^2 2kt} ,$$
$$\Psi_2 = \frac{2ABk^2}{3}\frac{(2\beta^2-\alpha^2)+\alpha\beta\cosh 2kt}{\sinh^2 2kt} , \tag{3.17}$$

and

$$\begin{aligned}
\Phi_{00} = \frac{k^2B^2}{2\sinh^2 2kt}(&2\{\alpha+\beta\cosh 2kt+\beta\sinh 2kt[2H(z)-1]\}^2 \\
&+\lambda\{2\beta+(\lambda+4\beta)(\alpha^2+\beta^2)+2\alpha(1+4\beta^2+\lambda\beta)\cosh 2kt \\
&+2\beta(1+2\beta^2+\lambda\beta)\sinh^2 2kt+2(1+2\beta^2+\lambda\beta)(\alpha+\beta\cosh 2kt)\sinh 2kt[2H(z)-1]\}) , \\
\Phi_{22} = \frac{k^2A^2}{2\sinh^2 2kt}(&2\{\alpha+\beta\cosh 2kt-\beta\sinh 2kt[2H(z)-1]\}^2 \\
&+\lambda\{2\beta+(\lambda+4\beta)(\alpha^2+\beta^2)+2\alpha(1+4\beta^2+\lambda\beta)\cosh 2kt \\
&+2\beta(1+2\beta^2+\lambda\beta)\sinh^2 2kt-2(1+2\beta^2+\lambda\beta)(\alpha+\beta\cosh 2kt)\sinh 2kt[2H(z)-1]\}) , \\
\Phi_{02} = \frac{\lambda m k^2 AB}{2\sinh^2 2kt}&\{\alpha+\beta\cosh 2kt\} , \\
\Phi_{11} = \frac{k^2AB}{4\sinh^2 2kt}&\{2[\alpha^2+\beta^2+2\alpha\beta\cosh 2kt]+\lambda[4\beta+\lambda(\alpha^2+\beta^2)+2\alpha(2+\lambda\beta)\cosh 2kt]\} , \\
\Lambda = -\frac{k^2(2-3\lambda^2)AB}{12\sinh^2 2kt}&\{\alpha^2+\beta^2+2\alpha\beta\cosh 2kt\} .
\end{aligned} \tag{3.18}$$

From Eq. (3.17) one can see that the gravitational field is continuous across the wall without reflecting and absorbing [12,13]. However, for the BD scalar field it is different. In particular, the components Φ_{00} and Φ_{22} are discontinuous across the wall, although the components Φ_{02}, Φ_{11}, and Λ are continuous.

In terms of these scalars, the invariants $I \equiv R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}$, $J \equiv C_{\mu\nu\lambda\rho}C^{\mu\nu\lambda\rho}$, and $K \equiv R^{\mu\nu}R_{\mu\nu}$ are given by

$$\begin{aligned}
I &= 8\{2(3\Psi_2^2+\Psi_0\Psi_4)+\Phi_{00}\Phi_{22}+2\Phi_{02}^2+4\Phi_{11}^2+12\Lambda^2\} , \\
J &= 16(3\Psi_2^2+\Psi_0\Psi_4) , \\
K &= 8(\Phi_{00}\Phi_{22}+\Phi_{02}^2+2\Phi_{11}^2+18\Lambda^2) ,
\end{aligned} \tag{3.19}$$

where $R_{\mu\nu\lambda\rho}$ and $C_{\mu\nu\lambda\rho}$ denote the Riemann and Weyl tensors, respectively. $C_{\mu\nu\lambda\rho}$ is usually thought of as representing the gravitational field, and $R_{\mu\nu}$ the matter field. The interaction between $C_{\mu\nu\lambda\rho}$ and $R_{\mu\nu}$ is through the Bianchi identities $R^{\mu}_{\upsilon[\lambda\rho;\sigma]}=0$. To study the above invariants, let us consider the following asymptotic regions.

(i) The initial region ($|z| \ll t \to 0^+$). In this region, we find that I, J, and K have the asymptotic behavior

$$I,J,K \to \begin{cases} \text{const.} , & \alpha=-\beta , \\ \infty , & \alpha\neq-\beta . \end{cases} \tag{3.20}$$

Thus, all the solutions given by Eq. (3.9) have big bang-like singularities, except for the ones with $\alpha=-\beta$. In the latter case it can be shown from Eq. (3.17) that the corresponding solutions are Petrov-type D, while in the general case they are Petrov-type I.

(ii) The spacelike region ($t \ll |z| \to +\infty$). Then, we find that

$$I,J,K \sim \exp\left[\frac{8\omega k|z|}{2\omega+3}\right] \to \infty \ , \tag{3.21}$$

which means that the solutions are singular at spacelike infinity, too. It is this "bad" behavior that usually renders such solutions as cosmological models doubtful [38]. However, as we will show below, the proper distance from the wall perpendicular to these singularities is exponentially increasing as the time develops. Therefore, similar to the case for the massless scalar field [35], we argue that these singularities may be well extended beyond our observational horizon, before they cause any significant effects.

At a moment, say $t=t_1$, the proper distance from the wall to the singularities is given by

$$l=\int_0^{\infty} e^{-M/2}dz=\frac{2(2\omega+3)}{\omega}\sigma^{-1}(t_1) \ , \tag{3.22}$$

where σ is the surface energy density of the wall, given by Eq. (3.14).

Note that in the case of the massless scalar field, the proper distance is inversely proportional to the square root of σ [35]. Thus, in the present case the proper distance even exponentially increases faster than it does in the case of the massless scalar field.

Combining Eqs. (3.15) and (3.22) we find that when $-\beta<\alpha<\alpha_c$, l becomes unbounded as $t\to 0^+$, when $\alpha=-\beta$ it is finite, and when $-\alpha_c<\alpha<-\beta$ it becomes zero, which means that in the last case the space-time collapses into a singular point at $t=0$. On the other hand, from Eqs. (3.16) and (3.22) we can see that with the time developing l always increases exponentially, and the rate of it depends both on ω and k.

(iii) The light-cone region ($|z|\sim t\to+\infty$). In this region, it can be shown that

$$I,J,K\sim\exp\{-18\lambda^2 kt\}\to 0 \ . \tag{3.23}$$

Thus, in the light-cone region the space-time becomes locally flat.

(iv) The causal region ($|z| \ll t\to+\infty$). Then, one can find

$$I,J,K\sim\exp\{-\tfrac{4}{3}(3+9\lambda^2)kt\}\to 0 \ , \tag{3.24}$$

which means that the space-time becomes asymptotically flat, too, but the rate of it in this region is larger than the one in the light-cone region. This is because of the fact that the gravitational and the BD scalar waves propagate along the light-cone region.

Comparing the above results with the ones obtained from the "seed" solutions, we can see that the main properties of the solutions do not change under the conformal transformations (3.2). Thus, by studying one of them, we can know the other.

IV. CONCLUSIONS

In this paper, plane thin domain walls interacting with the BD scalar field have been studied. Using the distribution theory, the BD field equations have been divided into two groups, one of which is defined in the regions outside of the wall and the other of which is defined on the wall. The equations on the wall take a very simple form; they are given explicitly in terms of the metric coefficients, the ϕ field and their first derivatives. A method for generating exact solutions representing plane domain walls coupled with the BD scalar field have been developed. In fact, for any given solution of the BD field equations, the substitution (2.8) always generates such a solution (subject to several restrictions).

As an illustration of the method developed in Sec. II, in Sec. III a class of exact solutions has been presented, and the main properties have been studied. In particular, it has been shown that all the solutions (except for the ones with $\alpha=-\beta$) have big-bang-like singularities. The walls start to expand at the initial point $t=0$, and with the time developing, the surface energy density of it always exponentially decreases, independently of its initial conditions. Thus, the introduction of the BD scalar field to the inflationary models may provide a possible mechanism to resolve the domain-wall problems encountered in the GUT. In addition to the big-bang singularities, the space-time is also singular at spacelike infinity. However, since the proper distance from the wall to these singularities is exponentially increasing with the time developing, the existence of them may not cause any additional harm at all. All the above properties can be also deduced from the seed solutions. Therefore, by studying one of them, one can be able to know the main properties of the other.

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APPENDIX A: THE WEYL AND RICCI SCALARS

Corresponding to the choice of the null tetrad given by Eqs. (2.2) and (2.3) in Ref. [21], the Weyl and Ricci scalars are given by [22]

$$\begin{aligned}
\Psi_0=&-\tfrac{1}{2}B^2\{V_{,vv}\cosh W+(M_{,v}-U_{,v})V_{,v}\cosh W\\
&+2\sinh W V_{,v}W_{,v}\\
&-i[W_{,vv}+(M_{,v}-U_{,v})W_{,v}\\
&-\sinh W\cosh W V_{,v}^2]\} \ ,\\
\Psi_2=&\tfrac{1}{12}AB\{2(M_{,uv}-U_{,uv}+W_{,u}W_{,v}+\cosh^2 W V_{,u}V_{,v})\\
&+i3\cosh W[V_{,u}W_{,v}-V_{,v}W_{,u}]\} \ ,\\
\Psi_4=&-\tfrac{1}{2}A^2\{V_{,uu}\cosh W+(M_{,u}-U_{,u})V_{,u}\cosh W\\
&+2\sinh W V_{,u}W_{,u}\\
&+i[W_{,uu}+(M_{,u}-U_{,u})W_{,u}\\
&-\sinh W\cosh W V_{,u}^2]\} \ ,
\end{aligned} \tag{A1}$$

$$\Phi_{00}=\tfrac{1}{4}B^2\{2U_{,vv}-U_{,v}^2+2U_{,v}M_{,v}-W_{,v}^2-\cosh^2 W V_{,v}^2\} ,$$
$$\Phi_{11}=\tfrac{1}{8}AB\{2M_{,uv}+U_{,u}U_{,v}-W_{,u}W_{,v} -\cosh^2 W V_{,u}V_{,v}\} ,$$
$$\Phi_{02}=\tfrac{1}{4}AB\{2\cosh W V_{,uv}-\cosh W(U_{,u}V_{,v}+V_{,u}U_{,v}) +2\sinh W(V_{,u}W_{,v}+V_{,v}W_{,u}) -i[2W_{,uv}-(U_{,u}W_{,v}+W_{,u}U_{,v}) -2\sinh W\cosh W V_{,u}V_{,v}]\} , \quad \text{(A2)}$$
$$\Phi_{22}=\tfrac{1}{4}A^2\{2U_{,uu}-U_{,u}^2+2U_{,u}M_{,u}-W_{,u}^2-\cosh^2 W V_{,u}^2\} ,$$
$$\Lambda=-\tfrac{1}{24}AB\{2M_{,uv}+4U_{,uv}-3U_{,u}U_{,v}-W_{,u}W_{,v} -\cosh^2 W V_{,u}V_{,v}\} ,$$

and

$$\Psi_1=\Psi_3=\Psi_{01}=\Psi_{10}=\Psi_{12}=\Psi_{21}=0 . \quad \text{(A3)}$$

APPENDIX B: THE SCALARS Φ_{ij}^M, Λ^M, Φ_{ij}^{BD}, AND Λ^{BD}

In terms of $T_{\mu\nu}^M$, the Φ_{ij}^M's and Λ^M are given by

$$\Phi_{00}^M=\tfrac{1}{2}T_{\mu\nu}^m l^\mu l^\nu , \quad \Phi_{22}^M=\tfrac{1}{2}T_{\mu\nu}^M n^\mu n^\nu ,$$
$$\Phi_{01}^M=\tfrac{1}{2}T_{\mu\nu}^M l^\mu m^\nu , \quad \Phi_{02}^M=\tfrac{1}{2}T_{\mu\nu}^M m^\mu m^\nu ,$$
$$\Phi_{12}^M=\tfrac{1}{2}T_{\mu\nu}^M n^\mu m^\nu , \quad \Lambda^M=\tfrac{1}{24}T^M , \quad \text{(B1)}$$
$$\Phi_{11}^M=\tfrac{1}{4}T_{\mu\nu}^M(l^\mu n^\nu+m^\mu \overline{m}^\nu) .$$

Similarly, the Φ_{ij}^{BD}'s and Λ^{BD}, in terms of ϕ and its derivatives, are given by

$$\Phi_{00}^{\mathrm{BD}}=\frac{B^2}{2}\left\{\phi_{,vv}+\phi_{,v}M_{,v}+\frac{\omega}{\phi}\phi_{,v}^2\right\} ,$$
$$\Phi_{22}^{\mathrm{BD}}=\frac{A^2}{2}\left\{\phi_{,uu}+\phi_{,u}M_{,u}+\frac{\omega}{\phi}\phi_{,u}^2\right\} ,$$
$$\Phi_{02}^{\mathrm{BD}}=-\frac{AB}{4}\{\cosh W(\phi_{,u}V_{,v}+\phi_{,v}V_{,u}) -i(\phi_{,u}W_{,v}+\phi_{,v}W_{,u})\} , \quad \text{(B2)}$$
$$\Phi_{11}^{\mathrm{BD}}=\frac{AB}{8}\left\{2\phi_{,uv}+\phi_{,u}U_{,v}+\phi_{,v}U_{,u}+\frac{2\omega}{\phi}\phi_{,u}\phi_{,v}\right\} ,$$
$$\Lambda^{\mathrm{BD}}=-\frac{AB}{24}\left\{3(2\phi_{,uv}-\phi_{,u}U_{,v}-\phi_{,v}U_{,u})+\frac{2\omega}{\phi}\phi_{,u}\phi_{,v}\right\} ,$$

and

$$\Phi_{01}^{\mathrm{BD}}=\Phi_{12}^{\mathrm{BD}}=0 . \quad \text{(B3)}$$

APPENDIX C: THE WEYL AND RICCI SCALARS UNDER A CONFORMAL TRANSFORMATION

Assume that the interval ds_0^2 is related to ds^2 given by Eq. (2.5) by a conformal transformation

$$g_{\mu\nu}=e^{-\Sigma}g_{0\mu\nu} , \quad \text{(C1)}$$

where Σ is function of u and v only, then it can be shown that the Weyl and Ricci scalars in the two cases are related by

$$\Psi_A=\Psi_A^0 \quad (A=0,1,2) , \quad \text{(C2)}$$

and

$$\Phi_{00}=\Phi_{00}^0+\tfrac{1}{4}\{2\Sigma_{,vv}+\Sigma_{,v}(2M_{0,v}+\Sigma_{,v})\} ,$$
$$\Phi_{22}=\Phi_{22}^0+\tfrac{1}{4}\{2\Sigma_{,uu}+\Sigma_{,u}(2M_{0,u}+\Sigma_{,u})\} ,$$
$$\Phi_{02}=\Phi_{02}^0-\tfrac{1}{4}\{\cosh W_0(\Sigma_{,u}V_{0,v}+\Sigma_{,v}V_{0,u}) -i(\Sigma_{,u}W_{0,v}+\Sigma_{,v}W_{0,u})\} , \quad \text{(C3)}$$
$$\Phi_{11}=\Phi_{11}^0+\tfrac{1}{8}\{2\Sigma_{,uv}+\Sigma_{,u}\Sigma_{,v}+\Sigma_{,u}U_{0,v}+\Sigma_{,v}U_{0,u}\} ,$$
$$\Lambda=\Lambda^0-\tfrac{1}{8}\{2\Sigma_{,uv}-\Sigma_{,u}\Sigma_{,v}-\Sigma_{,u}U_{0,v}-\Sigma_{,v}U_{0,u}\} ,$$

where quantities with the index zero denote the ones calculated from $g_{0\mu\nu}$, and quantities without the index zero denote the ones calculated from $g_{\mu\nu}$. Note that all the Weyl and Ricci scalars appearing in Eqs. (C2) and (C3) stand for the "scale-invariant" ones defined by Eq. (4.8) in Ref. [21].

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