

## Testing a stability conjecture for Cauchy horizons

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A stability conjecture previously developed to investigate quasiregular and nonscalar curvature singularities is extended here to cover the stability of Cauchy horizons. In particular, the Reissner-Nordström spacetime of charged, nonrotating black holes is considered. The conjecture predicts that the addition of infalling null dust with a power-law tail will produce a nonscalar curvature singularity at the Cauchy horizon. This prediction is verified using a Reissner-Nordström-Vaidya spacetime studied by Hiscock. The conjecture also predicts that a combination of infalling and outgoing null dust will produce a scalar curvature singularity at the Cauchy horizon. This prediction is verified using the mass inflation results of Poisson and Israel. Finally, the conjecture predicts that the addition of infalling scalar or electromagnetic waves will produce a scalar curvature singularity at the Cauchy horizon.

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### I. INTRODUCTION

In a number of papers [1–7], we have developed a stability conjecture for the investigation of quasiregular and intermediate singularities in solutions of Einstein's equations. We look at the behavior of test fields in the vicinity of the singularity, and based upon this behavior we predict what should become of the singularity if the fields are allowed to influence the geometry through back-reaction calculations using Einstein's equations. In a few cases these back-reaction calculations have actually been carried out [3,5]; in each case the results agree with the predictions of the conjecture.

In this paper we extend our conjecture to cover the stability of Cauchy horizons. We then use the conjecture to investigate the Cauchy horizon of the Reissner-Nordström geometry of charged nonrotating black holes, when various fields are added. We are able to test the conjecture for two different field configurations by comparing its predictions with the results of back-reaction calculations which have already been carried out.

In Sec. II we define quasiregular, nonscalar curvature, and scalar curvature singularities. We then state the stability conjecture we have previously applied to the first two singularity types, and tests of the conjecture. Finally, we extend the conjecture to spacetimes with Cauchy horizons. In Sec. III we begin by reviewing properties of the Reissner-Nordström spacetime, and review some previous investigations of the Cauchy horizon's stability in this geometry. Then in subsection A we derive predictions in the case of infalling null dust and compare with the result found by Hiscock [8] using the Reissner-Nordström-Vaidya spacetime, an exact solution of Einstein's equations. In subsection B we derive predictions in the case of both infalling and outgoing null dust, and compare with the corresponding back-reaction results of Poisson and Israel [9–11]. In subsection C we

use the conjecture to investigate the stability of the Cauchy horizon when minimally coupled massless scalar fields and electromagnetic fields fall into the black hole. There are no full back-reaction calculations in these cases with which our predictions can be compared. In Sec. IV we summarize our conclusions.

### II. SINGULARITY CLASSIFICATION AND THE STABILITY CONJECTURE

We use a singularity classification scheme based on one devised by Ellis and Schmidt [12]. They classified singularities in maximal spacetimes into three basic types: quasiregular, nonscalar curvature, and scalar curvature. The mildest singularity is quasiregular and the strongest is scalar curvature. At a scalar curvature singularity, physical quantities such as energy density and tidal forces diverge in the frames of all observers who approach the singularity. At a nonscalar curvature singularity, there exist curves through each point arbitrarily close to the singularity such that observers moving on these curves experience perfectly regular tidal forces [12,26]. For a quasiregular singularity, no observers see physical quantities diverge, even though their world lines end at the singularity in a finite proper time.

Our version of the Ellis and Schmidt classification scheme can be expressed mathematically. One difference between the two schemes is that Ellis and Schmidt use a  $b$ -boundary construction to define the singular points, while we simply define singular points as the end points of incomplete geodesics in maximal spacetimes. In our scheme a singular point  $q$  is a quasiregular singularity if all components of the Riemann tensor  $R_{abcd}$  evaluated in an orthonormal frame parallel propagated along an incomplete geodesic ending at  $q$  are  $C^0$  (or  $C^{0-}$ ). In other words, the Riemann tensor components tend to finite limits (or are bounded). On the other hand, a singular point  $q$  is a curvature singularity if some components are not

bounded in this way. If all scalars in  $g_{ab}$ , the antisymmetric tensor  $\eta_{abcd}$ , and  $R_{abcd}$  nevertheless tend to a finite limit (or are bounded), the singularity is nonscalar, but if any scalar is unbounded, the point  $q$  is a scalar curvature singularity.

We have previously used a stability conjecture [3] to test the stability of quasiregular and nonscalar curvature singularities. Our conjecture states the following.

*If a test field stress-energy tensor evaluated in a parallel-propagated orthonormal (PPON) frame mimics the behavior of the Riemann tensor components which indicate a particular type of singularity, then a complete nonlinear back-reaction calculation would show that this type of singularity occurs.*

We have used the conjecture to investigate the stability of quasiregular singularities in  $R \times T^3$  and  $R^3 \times S$  flat Kasner [3], Taub-Newman-Unti-Tamborino (Taub-NUT) [3], Moncrief [3], Khan-Penrose [5], and Bell-Szekeres [6] spacetimes. We have also studied the stability of the nonscalar curvature singularity in a dust-filled, Bianchi type-V locally rotationally symmetric spacetime [7]. In a few cases we have been able to test the conjecture [3,5] by comparing its predictions with exact solutions of Einstein's equations: Taub-NUT spacetime with Brill and Batakis-Cohen spacetimes. Moncrief spacetime with other Einstein-Rosen-Gowdy spacetimes, and Khan-Penrose spacetime with a Chandrasekhar-Xanthopoulos spacetime. In each case, the predictions have been verified.

Here we propose the following similar conjecture for the stability of Cauchy horizons: For all maximally extended spacetimes with Cauchy horizons, the back-reaction due to a field (whose test-field stress-energy tensor is  $T_{\mu\nu}$ ) will affect the horizon in the following manner: (1) If both  $T_{\mu}^{\mu}$  and  $T_{\mu\nu}T^{\mu\nu}$  are finite and if the stress-energy tensor  $T_{(\alpha\beta)}$  in all PPON frames is finite, then the Cauchy horizon will remain nonsingular; (2) if both  $T_{\mu}^{\mu}$  and  $T_{\mu\nu}T^{\mu\nu}$  are finite but  $T_{(\alpha\beta)}$  diverges in some PPON frame, then a nonscalar curvature singularity will be formed at the Cauchy horizon; (3) if either  $T_{\mu}^{\mu}$  or  $T_{\mu\nu}T^{\mu\nu}$  diverges, then a scalar curvature singularity will be formed at the Cauchy horizon.

### III. STABILITY TESTS OF THE REISSNER-NORDSTRÖM CAUCHY HORIZON

The extended Reissner-Nordström (RN) geometry of a nonrotating charged black hole has an outer event horizon at  $r=r_+$ , and an inner (Cauchy) horizon at  $r=r_-$ , as shown in the conformal diagram of Fig. 1. Observers falling into the black hole through  $r_+$  see light from the entire future history of the region outside the black hole, as they approach  $r_-$ . If they are then able to cross through this Cauchy horizon (CH), they will see a naked singularity, the timelike singularity at  $r=0$  shown in Fig. 1, in violation of strong cosmic censorship.

However, evidence has accumulated [13–22] showing that the CH is unstable, so observers may hit a brick wall at  $r_-$ . Fields falling in the vicinity of  $r_-$  from outside the black hole or from scattering within the black hole

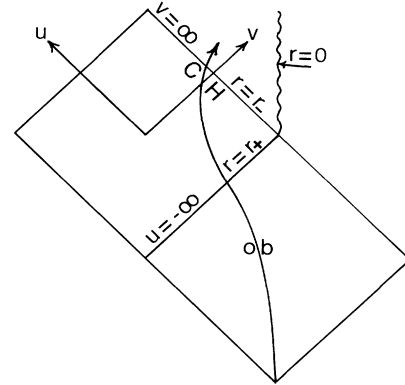


FIG. 1. Conformal diagram of a portion of the Reissner-Nordström spacetime. An observer (ob) is shown falling through the event horizon  $r=r_+$  into the interior region, and then through the Cauchy horizon (CH) at  $r=r_-$ . Just before reaching the CH, the observer can see light from the entire future of the exterior region; after passing the CH, the observer can view the  $r=0$  singularity.

are blueshifted; the energy density of these fields diverges as measured by observers falling toward the CH. Simpson and Penrose [13], for example, have shown that instabilities in an electromagnetic test field arise at the CH. Using a two-dimensional model, Hiscock [23] has shown that the stress-energy tensor of thermal Hawking radiation diverges there as well. McNamara [11,12] has examined scalar wave perturbations, showing that while the field itself remains finite at  $r_-$ , the derivatives of the field and hence the stress-energy tensor diverge. Gürsel and co-workers [16,17] have shown that the energy densities of scalar, electromagnetic, and gravitational test fields diverge in the frame of an observer falling toward the CH. Work by Matzner, Zamorano, and Sandberg [15] and by Zamorano [19] arrives at a similar conclusion. A complete first-order perturbation of the RN metric by Chandrasekhar and Hartle [18] and by Chandrasekhar [14] confirms that the flux of electromagnetic and gravitational energy diverges from the point of view of an observer falling toward  $r_-$ . While none of these results proves that the CH is unstable, because none of them includes a complete back-reaction calculation of the field effects on the geometry, they do point to a likelihood of instability.

We now explore the stability of the RN Cauchy horizon using our conjecture. In subsections A and B we use the RN metric in the form

$$ds^2 = -f(r)dv^2 + 2dv dr + r^2 d\Omega^2, \quad (1)$$

where  $f(r) = 1 - 2M/r + Q^2/r^2$  with  $Q^2 < M^2$ , and  $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ . Radial geodesics in the  $\theta = \pi/2$  plane obey

$$\dot{v} = (-\alpha \pm \sqrt{\alpha^2 + \beta f})/f, \quad (2a)$$

$$\dot{r} = \pm \sqrt{\alpha^2 + \beta f}, \quad (2b)$$

$$\dot{\phi} = 0, \quad (2c)$$

where  $\alpha$  is a constant and  $\beta=0$ ,  $-1$  for null and timelike geodesics, respectively. PPON frame vectors  $E_{(a)}^\mu$  for infalling timelike geodesics must satisfy  $E_{(0);v}^\mu E_{(a)}^\nu = 0$  and  $E_{(a)\mu} E_{(b)}^\mu = \delta_{(ab)}$ ; these vectors are

$$E_{(0)\mu} = (\alpha, -(\alpha + \sqrt{\alpha^2 - f})/f, 0, 0), \quad (3a)$$

$$E_{(1)\mu} = (-\sqrt{\alpha^2 - f}, (\alpha + \sqrt{\alpha^2 - f})/f, 0, 0), \quad (3b)$$

$$E_{(2)\mu} = (0, 0, r, 0), \quad (3c)$$

$$E_{(3)\mu} = (0, 0, 0, r) \quad (3d)$$

in the order  $v, r, \theta, \phi$ , where  $\alpha > 0$ .

### A. Infalling null dust

A test field of spherically symmetric infalling null dust has four-velocity  $u^\mu = (0, -1, 0, 0)$ , satisfying  $u_\mu u^\mu = 0$  and  $u^\mu_{;v} u^\nu = 0$ . Note that this choice of  $u^\mu$  is consistent with Eqs. (2) for  $\beta=0$ , choosing  $\alpha=1$ . The stress-energy tensor of the null dust therefore becomes

$$T^{\mu\nu} = \rho(v, r) u^\mu u^\nu = \text{diag}(0, \rho, 0, 0). \quad (4)$$

The continuity equation  $T^{\mu\nu}_{;\mu} = 0$  furthermore shows that  $\rho(v, r) = F(v)/r^2$ , where  $F(v)$  is an arbitrary function.

The scalars  $T^\mu_\mu$  and  $T^{\mu\nu} T_{\mu\nu}$  both vanish everywhere, so according to our conjecture, such null dust should not create a scalar curvature singularity (SCS) at the CH. To see whether null dust should create a nonscalar curvature singularity (NSCS) instead, we need to calculate  $T_{(ab)}$  in a PPON frame and see how it behaves as the frame approaches the CH. In an infalling PPON frame,

$$T_{(ab)} = E_{(a)}^\mu E_{(b)}^\nu T_{\mu\nu} = \begin{pmatrix} \sigma & 0 \\ 0 & 0 \end{pmatrix} \frac{(\alpha + \sqrt{\alpha^2 - f}) F(v)}{f^2 r^2}, \quad (5)$$

where  $\sigma = (\frac{1}{r_+} - \frac{1}{r_-})^{-1}$ . Therefore, as  $v \rightarrow \infty$  the behavior of  $T_{(ab)}$  is governed by that of  $F(v)/f^2$ . If this ratio diverges, a NSCS should be formed at the CH; otherwise the CH should remain nonsingular according to the conjecture. Define  $\epsilon = r - r_-$ ; then

$$f(r) = (r - r_+)(r - r_-)/r^2 = -2k_- \epsilon \quad (6)$$

to first order in  $\epsilon$ , where  $k_- = (r_+ - r_-)/2r_-^2$  is the gravity at  $r_-$ . Along an infalling timelike geodesic

$$\frac{dv}{d\epsilon} = \frac{dv}{dr} = \frac{\alpha + \sqrt{\alpha^2 - f}}{f\sqrt{\alpha^2 - f}} = -(k_- \epsilon)^{-1} \quad (7)$$

to leading order, so near the CH,  $\epsilon = \epsilon_0 e^{-k_- v}$ , where  $\epsilon_0$  is a constant. Therefore,

$$F(v)/f^2 \sim F(v) e^{2k_- v} \quad (8)$$

for large  $v$ , which diverges unless  $F(v)$  falls off fast enough. That is, according to the conjecture a NSCS should be formed at the CH by infalling null dust, unless  $F(v)$  falls off at least as fast as  $F(v) \sim e^{-2k_- v}$ , in which case the CH remains nonsingular.

There is an exact solution of Einstein's equations we

can use to test the conjecture. The Reissner-Nordström-Vaidya (RNV) [24] geometry, as considered by Hiscock [8], corresponds to spherically symmetric null dust falling into a charged black hole. The metric retains the form of Eq. (1), but now

$$f = f(v, r) = 1 - 2M(v)/r + Q^2/r^2.$$

The stress-energy tensor corresponding to this metric is

$$8\pi T^\mu_\nu = (Q^2/r^4) \text{diag}(-1, -1, 1, 1) + (2M'/r^2) \delta_\mu^1 \delta_0^\nu, \quad (9)$$

where  $M' = dM(v)/dv$ . The diagonal elements are due to the static electric field and the off-diagonal element is due to the null dust. This null dust contribution has the same form as we found for the test field on the RN geometry if we identify  $F(v) = M'(v)/4\pi$ .

Hiscock chooses infalling null dust with a power-law tail,  $M(v) = m - \delta(v_0/v)^n$  for large  $v$ , where  $m, \delta, v_0, n$  are positive constants. He then shows that scalars constructed from the Riemann tensor converge at the CH, but that there are Riemann tensor components which diverge in a PPON frame at the CH, so the CH is converted into a NSCS in the RNV spacetime, with his choice of  $M(v)$ .

Equation (8) shows that the null dust test-field stress tensor  $T_{(ab)}$  in a PPON frame in RN spacetime diverges at the CH unless  $F(v)$  falls off at least as fast as  $\exp[-2k_- v]$ . Using Hiscock's  $M(v)$ , then

$$F(v) = M'(v)/4\pi = (\delta n v_0^n / 4\pi) v^{-(n+1)},$$

which does not fall off fast enough to avoid causing a NSCS. Our conjecture therefore agrees with Hiscock's result: for purely infalling null dust with a power-law tail, the CH is converted into a NSCS in RNV spacetime.

### B. Both outgoing and infalling null dust

Now we add outgoing spherically symmetric null dust in the region  $r_- < r < r_+$  to the infalling null dust described in subsection A, as shown in Fig. 2. We would expect to have outgoing radiation in realistic situations, coming from the surface of the collapsing star or from scattering of infalling radiation. We assume the two

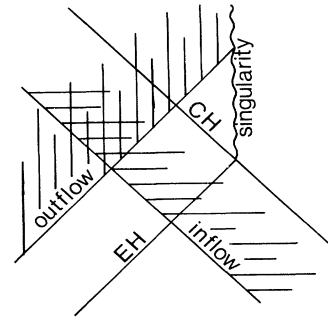


FIG. 2. Inflowing and outflowing radiation in the Reissner-Nordström geometry. Inflowing radiation moves from lower right to upper left, falling through the event horizon (EH) into the black hole. Outflowing radiation, originating at the surface of a collapsing star or from back-scattering of inflowing radiation, moves from lower left to upper right, passing through the Cauchy horizon (CH).

beams do not interact, so each is separately conserved. We will show that the addition of outgoing radiation, no matter how weak, is sufficient to cause the formation of a SCS at the CH.

The infalling, leftward-moving beam in Fig. 2 has stress-energy

$$T_L^{\mu\nu} = \rho_L(v, r) u_L^\mu u_L^\nu$$

where  $\rho_L(v, r) = F(v)/r^2$  and  $u_L^\mu = (0, -1, 0, 0)$  as in the preceding section, using the same  $(v, r, \theta, \phi)$  coordinates. The outgoing, rightward-moving beam has stress-energy

$$T_R^{\mu\nu} = \rho_R(v, r) u_R^\mu u_R^\nu,$$

where  $\rho_R(v, r) = \tilde{F}(v)/r^2$ . The four-velocity of the outgoing null dust is  $u_R^\mu = (-2/f(r), -1, 0, 0)$ , the appropriate solution of  $u_R^\mu u_{R\mu} = 0$  and  $u_R^\mu{}_{;\nu} u_R^\nu = 0$ . An initial surface for the outgoing radiation may be taken to be a three-surface with  $r = \text{const}$  within the interior region  $r_- < r < r_+$ , except that it crosses through the CH and ends at the  $r = 0$  singularity; a similar surface is shown in Fig. 2 of Ref. [7]. Most of the CH is within the Cauchy development of this initial surface. From the definition of  $T_R^{\mu\nu}$  and from  $u_R^\mu$  it follows that finite initial data leads to a divergent  $T_R^{\mu\nu}$  at the CH in  $(v, r, \theta, \phi)$  coordinates, since  $f(r) \rightarrow 0$ . Scalars constructed from  $T_R^{\mu\nu}$  are zero, however. One may be concerned that the divergence of  $T_R^{\mu\nu}$  at the CH arises from the use of infinite data at the initial surface where it crosses the CH. This concern is dispelled by calculating  $T_R^{\mu\nu}$  in the vicinity of the CH using the coordinates  $g = -(k_-)^{-1} \exp(-k_- v)$  and  $h = (r - r_-) \exp(k_- v)$ , which are nonsingular at the CH and extendable through it. In these nonsingular coordinates we find that  $T_R^{\mu\nu}$  is finite, so that a finite initial energy of outgoing null dust leads to a finite energy of outgoing dust crossing through the CH.

The total null dust stress energy for both infalling and outgoing null dust is

$$T^{\mu\nu} = \rho_L(v, r) u_L^\mu u_L^\nu + \rho_R(v, r) u_R^\mu u_R^\nu \quad (10)$$

for which  $T^\mu{}_\mu = 0$ . Because of the cross-product terms, however,

$$\begin{aligned} T^{\mu\nu} T_{\mu\nu} &= \rho_L(v, r) \rho_R(v, r) [(u_L^\mu u_{R\mu})^2 + (u_R^\mu u_{L\mu})^2] \\ &= 8\rho_L(v, r) \rho_R(v, r) / f^2 \\ &= 8F(v) \tilde{F}(v) / (r^4 f^2). \end{aligned} \quad (11)$$

This scalar constructed from  $T^{\mu\nu}$  diverges as  $r \rightarrow r_-$  if (i)  $\tilde{F}(v)$  is nonzero as  $v \rightarrow \infty$  and (ii)  $F(v)/f^2$  diverges. Therefore, if there is at least some outgoing null dust and the infalling null dust has a power-law tail as used in the preceding section, a SCS is formed at the CH according to our conjecture. It is very interesting that the addition of even a tiny amount of outgoing dust is sufficient to convert the NSCS of the preceding section into a SCS. This is related, according to our method, to the non-linearity of the scalar quantity  $T^{\mu\nu} T_{\mu\nu}$ , which is able to combine properties of the two noninteracting beams.

Poisson and Israel [9–11] have constructed solutions of Einstein's equations in which the inner region  $r < r_+$  of

the RN geometry contains both infalling and outgoing spherically symmetric null dust. They show that the presence of both beams leads to the phenomenon of mass inflation, in which the effective internal gravitational-mass parameter becomes large in the absolute future of the infalling and outgoing beams. They find an exact solution in the case that the inflow and outflow are each modeled by thin null shells. The spacetime is separated into four regions, each of which is a RN solution, but with differing mass parameters. The region to the absolute future of the shell crossing has a mass which increases without bound as the infalling shell is moved closer to the CH. Poisson and Israel also show that the mass inflates along the Cauchy horizon in the more realistic case of a power-law tail for infalling null dust as long as there is some outflowing null dust. The presence of the outflow is essential for mass inflation and for the divergence of curvature at the CH, but the strength and shape of the outflow is unimportant. Divergence of the mass and curvature in the classical limit requires that the infalling radiation have a power-law tail all the way to the horizon, just as required for formation of a singularity according to the conjecture. The papers of Poisson and Israel provide a detailed explanation of mass inflation and its relation to a separation between the Cauchy horizon and an apparent horizon within it.

The stability conjecture prediction once again agrees with the results of a complete back-reaction calculation using classical general relativity. When outgoing null dust is added to null dust with a power-law tail falling into a charged black hole, a scalar curvature singularity is formed at the CH.

### C. Scalar and electromagnetic fields

Many authors have investigated the behavior of scalar and electromagnetic test fields near the CH of the RN spacetime and have shown that the flux of radiation diverges in the frame of an observer crossing the horizon. Here we show that according to our conjecture, these fields cause a SCS to form at the CH.

Using double-null coordinates  $u = r^* - t$  and  $v = r^* + t$ , where  $r^*$  is the tortoise coordinate

$$r^* = r + (2k_+)^{-1} \ln(r_+ - r) - (2k_-)^{-1} \ln(r - r_-), \quad (12)$$

with  $r_- < r < r_+$  and  $k_\pm = (r_+ - r_-)/2r_\pm^2$ , the RN metric is

$$ds^2 = f(r) du dv + r^2 d\Omega^2. \quad (13)$$

Gürsel and co-workers [16,17] show that solutions of the massless minimally coupled scalar wave equation  $\square\phi = 0$  take on the form

$$\phi(t, r, \theta, \phi) = \sum_{lm} Y_{lm}(\theta, \phi) \int_{-\infty}^{\infty} dk e^{-ikt} \frac{1}{r} \psi_{lmk}(r) \quad (14)$$

in this background spacetime, where  $\psi_{lmk}(r)$  satisfies an evolution equation. They show that the dominant contribution near the CH comes from modes in a neighborhood of  $k = 0$ . For purely infalling waves at  $r_+$ , there is scattering in the interior so outgoing waves approach the

CH. In the vicinity of the CH ( $v \rightarrow \infty, u < 0$ ) Gürsel and co-workers show that the  $lm$  mode approaches

$$\phi_{lm}(u, v, \theta, \phi) \sim \left[ \frac{A_{lm}}{v^{2l+2}} + \frac{B_{lm}}{u^{2l+2}} \right] Y_{lm}(\theta, \phi), \quad (15)$$

where  $A_{lm}$  and  $B_{lm}$  are constants. The stress-energy tensor is

$$T_{\mu\nu} = \frac{1}{4\pi} \left[ \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}S \right], \quad (16)$$

where  $S = g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}$ . The behavior of the stress-energy scalars is decided by the behavior of  $S$ , since  $T^\mu{}_\mu = -S$  and  $T^{\mu\nu}T_{\mu\nu} = S^2$ . The metric elements  $g^{uv} = g^{vu} = 2/f$  diverge quickly at the CH, so the dominant term of  $S_{lm}$  is

$$\begin{aligned} S_{lm} &\sim (4/f)\phi_{lm,u}\phi_{lm,v} \\ &\sim C_{lm} Y_{lm}^2(\theta, \phi) \frac{e^{k(u+v)/2}}{(uv)^{2l+3}} \end{aligned} \quad (17)$$

where  $C_{lm}$  is constant and we have used

$$f(r) \rightarrow -(2k_- r_-) \exp[-k_-(u+v)/2]$$

as  $r \rightarrow r_-$ . Expression (17) for  $S_{lm}$  diverges as  $v \rightarrow \infty$ , so the stress-energy scalars  $T^\mu{}_\mu$  and  $T^{\mu\nu}T_{\mu\nu}$  both diverge. According to our conjecture, these scalar fields will therefore cause a scalar curvature singularity to form at the CH.

Gürsel and co-workers show that electromagnetic waves behave near the CH in a manner similar to that of  $\phi_{lm}$ . Therefore although  $T^\mu{}_\mu = 0$  for any electromagnetic wave, the scalar  $T^{\mu\nu}T_{\mu\nu}$  diverges as  $v \rightarrow \infty$ , so again our conjecture predicts that these waves also form a SCS at the CH.

It is important to emphasize that the solutions of Gürsel and co-workers and other authors take account of the power-law tail of infalling radiation, and also back-scattering for  $r < r_+$ , which produces outgoing radiation qualitatively similar to the outgoing null dust used in the preceding section. Therefore it is not surprising that these scalar and electromagnetic waves lead to the same conclusion as infalling plus outgoing null dust.

#### IV. CONCLUSION AND DISCUSSION

We have proposed a stability conjecture for Cauchy horizons in which the behavior of test fields near the horizon can be used to predict whether the Cauchy horizon will remain regular, or will be converted instead into a nonscalar or scalar curvature singularity when a full nonlinear back-reaction calculation is carried out. We have used the conjecture to predict the fate of the Reissner-Nordström-Cauchy horizon under the influence of infalling null dust, of the combination of infalling and outgoing null dust, and of infalling and outgoing scalar fields and electromagnetic fields. With infalling null dust a nonscalar curvature singularity should be formed, while in the other cases a scalar curvature singularity should be formed.

Full back-reaction calculations have been made by Hiscock for purely infalling null dust and by Poisson and Israel for the combination of infalling and outgoing null dust, using the Reissner-Nordström-Vaidya spacetime. The type of singularity formed at the Cauchy horizon in these solutions is in agreement with the predictions of our conjecture.

It seems clear from these results that when realistic fields are allowed to perturb the idealized Reissner-Nordström geometry of a charged nonrotating black hole, a scalar curvature singularity is formed at the Cauchy horizon, although a quantum theory of gravity would have more to say about the structure of the singularity within dimensions of the Planck length. The singularity may serve as a brick wall, closing off the need to contemplate the extended Reissner-Nordström solution past the Cauchy surface and the consequent violation of strong cosmic censorship, or it may be sufficiently weak to permit penetration [25], in which case what happens beyond the horizon would remain an open question.

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