Effect of neutrino heating on primordial nucleosynthesis

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We modify the standard code for primordial nucleosynthesis to include the effect of the slight heating of neutrinos by $e^{\pm}$ annihilations. There is a small, systematic change in the ^{4}He yield, $\Delta Y \simeq +1.5\times 10^{-4}$, which is insensitive to the value of the baryon-to-photon ratio η for $10^{-10} \lesssim \eta \lesssim 10^{-9}$. We also find that the baryon-to-photon ratio decreases by about 0.5% less than the canonical factor of $\frac{4}{11}$ because some of the entropy in $e^{\pm}$ pairs is transferred to neutrinos. These results are in accord with recent analytical estimates.

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I. INTRODUCTION

The concordance between the predictions of primordial nucleosynthesis and the observed abundances of D, ^{3}He, ^{4}He, and ^{7}Li is one of the cornerstones of the hot big-bang cosmology, and provides its earliest test. Because of this and the great interest in the very early history of the Universe, primordial nucleosynthesis has been called "the gateway to the early Universe." Further, big-bang nucleosynthesis has been exploited to provide the most accurate determination of the baryon density [1] and to probe particle physics, e.g., the stringent limit to the number of light neutrino species [2].

Over the past decade there has been continued scrutiny of primordial nucleosynthesis, both on the theoretical side and on the observational side: Reaction rates have been updated and the effect of their uncertainties quantified [3], finite-temperature corrections have been taken into account [4], and the effect of inhomogeneities in the baryon density explored [5]; the primordial abundance of ^{7}Li has been put on a firm basis [6], the production and destruction of D and ^{3}He have been studied carefully [7], and astrophysicists now argue about the third significant figure in the primordial ^{4}He abundance [8]. The result is that the "concordance region" of parameter space has continued to shrink. The predicted and measured primordial abundances agree provided the baryon-to-photon ratio lies in the narrow interval $3\times 10^{-10} \lesssim \eta \lesssim 5\times 10^{-10}$ and the equivalent number of light neutrino species $N_\nu \lesssim 3.4$ [9]. The trend motivates the study of smaller and smaller effects, and, in particular, the present examination of the small effect of the heating of neutrinos by $e^{\pm}$ annihilations.

To place our work in perspective, let us enumerate the usual assumptions underlying primordial nucleosynthesis: (i) Friedmann-Robertson-Walker cosmology; (ii) the input of various nuclear reaction cross sections, the most important of which is the matrix element for the processes that interconvert neutrons and protons; (iii) N_ν nondegenerate neutrino species, i.e., neutrino chemical potentials $|\mu_\nu| \ll T$; and (iv) the complete decoupling of neutrinos from the electromagnetic plasma before the entropy in $e^{\pm}$ pairs is transferred to photons. It is the final assumption that our work addresses.

It has long been known that neutrino interactions with the electromagnetic plasma, e.g., $\nu+\bar{\nu}\leftrightarrow e^+ + e^-$, $\nu + e^{\pm} \leftrightarrow \nu + e^{\pm}$, and so on, become ineffective (interaction rate per particle Γ less than the expansion rate H) at a temperature of the order of a few MeV. Since $e^{\pm}$ pairs do not disappear and transfer their entropy to the plasma until a temperature of the order of $m_e/3 \sim 0.1$ MeV, one expects that neutrinos do not share in the $e^{\pm}$ entropy transfer. It then follows that long after the $e^{\pm}$ pairs disappear the ratio of the photon and neutrino temperatures should be $(\frac{11}{4})^{1/3}$ and the baryon-to-photon ratio should decrease by $\frac{4}{11}$ [10].

Neither neutrino decoupling nor the disappearance of $e^{\pm}$ pairs are instantaneous events, and so one might expect neutrinos to share slightly in the $e^{\pm}$ entropy transfer and to have a higher temperature than $(\frac{4}{11})^{1/3} T_\gamma$. Because the yields of primordial nucleosynthesis are very sensitive to the neutron fraction, which around the time of nucleosynthesis is determined by the rate of neutron-proton interconversions through the processes $n+e^+ \leftrightarrow p+\bar{\nu}_e$, $n+\nu_e \leftrightarrow p+e^-$, and $n \leftrightarrow p+e^- +\bar{\nu}_e$, they depend critically upon the neutrino temperature as the rates for these processes vary as T_ν^5. Even a slight amount of neutrino heating is potentially important for the ^{4}He since its abundance is now discussed to three significant figures.

A number of authors have tried to quantify neutrino heating and its effect on the ^{4}He yield [4,11–15]. With the exception of the most recent work, Refs. [13,14], previous estimates were "one-zone" calculations; i.e., the integrated perturbation to the neutrino energy density $\delta\rho_\nu$ was calculated rather than the perturbations to the neutrino phase-space distribution functions. In the most detailed treatment [14], the Boltzmann equations for all three neutrino phase-space distributions were solved numerically, incorporating all standard neutrino interactions with the electromagnetic plasma, i.e., $\nu+\bar{\nu}\leftrightarrow e^++e^-$, $\nu+e^\pm\leftrightarrow\nu+e^\pm$, $\nu+\bar{\nu}\leftrightarrow\nu+\bar{\nu}$, and $\nu+\nu\leftrightarrow\nu+\nu$.

All authors agree that neutrino heating of the electron neutrinos increases their energy density by about 1% (slightly less for ν_μ and ν_τ, as they only have neutral-current interactions), and with one exception, all estimate the change in the mass fraction of ^{4}He synthesized to be of the order of $\Delta Y\sim 10^{-4}$, though there is no consensus as to the sign of this small change; the authors of Ref. [12] estimate the change to be 30 times larger, $\Delta Y\simeq -0.003$.

Here we incorporate the results of the most detailed treatment of neutrino heating [14] into the standard big-bang nucleosynthesis code [16]. For the interesting range of the baryon-to-photon ratio we find a systematic increase in the ^{4}He abundance of $\Delta Y=+1.5\times10^{-4}$; we find similar fractional changes for the abundances of the other light elements. By integrating the first law of thermodynamics we find that due to neutrino heating the baryon-to-photon ratio decreases by about 0.5% less than the canonical factor of $\frac{4}{11}$. All of these results are in very good agreement with the estimates made in Ref. [14].

We trace the discrepancy in the sign of the change in ^{4}He yield to other authors not considering all of the effects of neutrino heating on the ^{4}He yield. To check the claim that $\Delta Y=-0.003$ we have also modified the nucleosynthesis code to take into account the effect of neutrino heating as computed in Ref. [12]; we find, however, that the change in ^{4}He is only $\Delta Y=+1.1\times10^{-4}$, which is consistent with the more detailed treatment of neutrino heating. Since the authors of Ref. [12] give few details concerning the changes they made in the nucleosynthesis code, it is not possible to explain this discrepancy, though we are very confident that the change is not as large as they state.

Our paper is organized as follows. In Sec. II we discuss the changes that must be made in the nucleosynthesis code when neutrino heating is taken into account and how we implemented them. In the final Section, we discuss our numerical results, compare them to previous estimates for the change in ^{4}He production, and finish with some concluding remarks.

II. MODIFICATIONS TO THE STANDARD CODE

A. Role of neutrinos

The slight heating of neutrinos by $e^\pm$ annihilations causes (i) small perturbations to the neutrino phase-space distributions, and (ii) small decrease in the temperature of the electromagnetic plasma [at fixed value of the cosmic scale factor $R(t)$] since neutrinos take energy away from the electromagnetic plasma. To understand how these changes affect the outcome of nucleosynthesis, let us first review how neutrinos "participate" in primordial nucleosynthesis.

Neutrinos play several roles. First, they play an important role in determining the neutron-to-proton ratio. Specifically, the electron neutrino and antineutrino phase-space distributions affect the rates (per nucleon) for the reactions that interconvert neutrons to protons and vice versa, λ_{np} and λ_{pn}. In the standard treatment these rates are computed by integrating the well-known tree-level matrix element squared over the appropriate (thermal) Fermi-Dirac distributions (see, e.g., Refs. [4,10]). Because of neutrino heating, the electron-neutrino distribution is given by the usual thermal part plus a small perturbation, which results in small changes to the weak rates, $\delta\lambda_{pn}$ and $\delta\lambda_{np}$.

The other role neutrinos play in nucleosynthesis is to contribute to the energy density of the Universe. The total energy density determines the expansion rate of the Universe:

$$H^2=\left[\frac{\dot{R}}{R}\right]^2=\frac{8\pi G\rho_{\rm tot}}{3}\ , \tag{1}$$

where $\rho_{\rm tot}=\rho_\gamma+\rho_e+\rho_\nu+\rho_B$. Because of rapid electromagnetic interactions the electromagnetic plasma is always in thermal equilibrium so that $\rho_{\rm EM}(T_\gamma)\equiv\rho_\gamma+\rho_e$ is only a function of the photon temperature T_γ. And, of course, the baryonic contribution to the energy density, ρ_B, is very tiny, as the Universe at this early time is radiation dominated.

In the absence of neutrino heating by $e^\pm$ annihilations the neutrino temperature just redshifts with the expansion, $T_{0\nu}\propto R^{-1}$, and the neutrino energy density $\rho_{0\nu}\propto 1/R^4$. When neutrino heating is taken into account,

$$\rho_\nu=\rho_{0\nu}+\delta\rho_\nu\ , \tag{2}$$

where $\rho_{0\nu}$ is the energy density in all three neutrino species in the absence of neutrino heating, and $\delta\rho_\nu$ is the sum over all three species of the additional energy density due to neutrino heating. In Ref. [14] the evolution of the perturbation to the phase-space distribution of each species is computed; for electron neutrinos $\delta\rho/\rho$ approaches about 1.2%, and for μ or τ neutrinos about 0.6%; thus, $\delta\rho_\nu/\rho_\nu$, the average over the three species, is about 0.7%.

The neutrino energy density also appears in the first law of thermodynamics, which governs the rate at which the photon temperature decreases with time:

$$d[\rho_{\rm tot}V]=-p_{\rm tot}dV\ , \tag{3}$$

where the total pressure $p_{\rm tot}=p_{\rm EM}+p_\nu+p_B$, $p_{\rm EM}(T_\gamma)=p_\gamma+p_e$, and $V=R^3$. Since we shall assume that the three neutrino species are very light, $m_\nu\ll 1$ MeV, the neutrino pressure $p_\nu=\rho_\nu/3$. In the absence of neutrino heating the neutrino energy density drops out of Eq. (3) since $\rho_{0\nu}\propto R^{-4}$. When neutrino heating is taken into account this is no longer true; as we shall see, the additional term in this equation involving $\delta\rho_\nu$ leads to a "back reaction" resulting in a slight cooling of the electromagnetic plasma.

B. Alterations

The integral expressions for the unperturbed weak rates λ_{np} and λ_{pn} cannot be calculated in closed form, but the standard code [16] allows for either numerical integration at each temperature step, or for the use of a series approximation (in $1/T_\gamma$). We have opted for the numerical routine. The perturbations to the weak rates are implemented very simply: the numerical solutions for $\delta\lambda_{np}$ and $\delta\lambda_{pn}$ calculated in Ref. [14] are added to the unperturbed rates by means of a table.

The effect of the back reaction of neutrino heating on the electromagnetic plasma is more complicated. To begin, it is useful to describe how the evolution of the photon temperature is computed. At each time step all the energy densities and their derivatives are computed, and then stepped forward in time by a Runge-Kutta integrator. The time rate of change of the photon temperature can be written as

$$\frac{dT_\gamma}{dt}=\frac{d\ln V}{dt}\frac{dT_\gamma}{d\ln V}=3H\frac{dT_\gamma}{d\ln V} \ . \tag{4}$$

The first law can be used to calculate $dT_\gamma/d\ln V$:

$$\frac{dT_\gamma}{d\ln V}=-\frac{\rho_{\rm EM}+p_{\rm EM}+4\delta\rho_\nu/3}{d\rho_{\rm EM}/dT_\gamma+d\delta\rho_\nu/dT_\gamma} \ . \tag{5}$$

Once the evolution of the photon temperature is known, the evolution of all other quantities (light-element abundances and so on) follows as in the standard case. For example, the evolution of the baryon-to-photon ratio η is governed by

$$\frac{d\ln\eta}{dt}=-3\frac{d\ln(RT_\gamma)}{dt} \ . \tag{6}$$

Because of the $e^\pm$ annihilations, RT_γ is not constant, and η decreases with time.

A technical note for the experts: In the nucleosynthesis code the first-law expression for $dT_\gamma/d\ln V$ is actually somewhat more complicated because it also takes into account the slight excess of electrons over positrons (electron chemical potential μ_e of order $10^{-10}T$), the tiny energy density and pressure associated with baryons, and the bookkeeping associated with nuclear-binding energies. Since these effects are small and unaffected by neutrino heating, we have left them out of our discussion here.

III. RESULTS AND CONCLUSIONS

The "input data" to the nucleosynthesis code needed to compute the effect of neutrino heating on the primordial nucleosynthesis are $\delta\rho_\nu/\rho_\nu$, $\delta\lambda_{pn}$, and $\delta\lambda_{np}$. We consider two approaches to computing these quantities: (I) the detailed Boltzmann treatment where the perturbations to the neutrino phase-space distributions are computed [14]; and (II) the bulk-heating approach, where it is assumed that the distortions to the neutrino distributions are thermal and only the bulk transfer of energy from $e^\pm$ annihilations is computed [11,12]. In the bulk-heating approach the effect of neutrino heating is a slight increase in the neutrino temperature; we use the results of Ref. [12] for δT_{ν_e} to compute $\delta\lambda_i$. While we feel that the first approach is more accurate, we have also considered the bulk-heating approach because in Ref. [12] a very large change in the ^{4}He abundance is claimed, $\Delta Y=-0.003$. We refer the reader to Refs. [12,14] for details about the two approaches.

The evolution of the energy transfer from the electromagnetic plasma to the neutrinos is shown in Fig. 1 for the two methods of computing neutrino heating; asymptotically $\delta\rho_\nu/\rho_{0\nu}$ approaches 7×10^{-3}. It is heartening that these two different treatments agree within 15% or so on the integrated magnitude of the distortion to the neutrino distributions. One consequence of the energy transfer is that there are more electron neutrinos and they have higher energies, and so the rates for the processes that interconvert neutrons and protons *increase.* However, there is no free lunch: The temperature of the electromagnetic plasma drops since it loses energy to the neutrinos [17]. Thus a second consequence of the energy transfer is a *decrease* in the neutron-proton interconversion rates due to the drop in the temperature of the electrons and positrons. This is a straightforward, but very important, implication of energy conservation.

The third consequence of the neutrino heating is also related to the drop in the temperature of the electromagnetic plasma. At a fixed time, the photon temperature is slightly lower than in the absence of heating; equivalently, at a fixed photon temperature the Universe is slightly younger than in the absence of neutrino heating. As is well appreciated, the ^{4}He abundance is determined by the neutron fraction at the onset of nucleosynthesis ($T_{\rm nuc}\sim0.07$ MeV); which, in part, is determined by the number of neutrons that have decayed by this time. Since the Universe is slightly younger, fewer neutrons

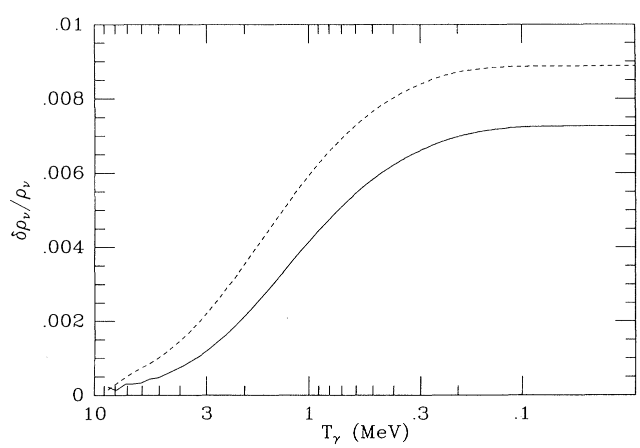

FIG. 1. The evolution of the total perturbation to the neutrino density due to heating by $e^\pm$ annihilations as a function of the photon temperature. The solid curves are the results from Ref. [14]; the dashed curves are those from Ref. [12].

will have decayed. We dub this the "clock effect."

To summarize, there are three effects: (i) an increase in neutron-proton interconversion rates due to neutrino heating; (ii) a decrease in neutron-proton interconversion rates due to the drop in the temperature of the electromagnetic plasma; and (iii) the clock effect. The discrepancy over the sign of the change in the ^{4}He abundance traces to the fact that with the exception of Ref. [14], all other authors have only considered the first of these three effects.

First, consider the change due to the distorted electron-neutrino distribution. For simplicity let us begin by assuming that the perturbation is thermal (method II), characterized by a change in the electron-neutrino temperature δT_{ν_e}. The change in the neutron fraction X_n at the onset of nucleosynthesis due to a change in the temperature of either the neutrinos or the electromagnetic plasma is found numerically to be [18]

$$\delta X_n = -0.1\frac{\delta T}{T} \ . \qquad (7)$$

It is easy to understand the sign in Eq. (7): when the temperature rises, the rates for neutron-proton interconversions increase and the neutron fraction tracks its equilibrium abundance,

$$\frac{X_n}{1-X_n} = \exp\left[\frac{-\Delta m}{T}\right] ,$$

longer, which leads to a lower neutron abundance when nucleosynthesis commences.

What is δT_{ν_e}? Since the electron neutrinos have both charged- and neutral-current weak interactions, they get more than their share of the energy transferred to the neutrinos, about as much as μ and τ neutrinos combined. Therefore,

$$\frac{\delta T_{\nu_e}}{T_{\nu_e}} = \frac{1}{4}\frac{\delta\rho_{\nu_e}}{\rho_{\nu_e}} \simeq \frac{3}{8}\frac{\delta\rho_\nu}{\rho_\nu} \ . \qquad (8)$$

Figure 1 shows that $\delta\rho_\nu/\rho_\nu \simeq 7\times 10^{-3}$, and thus it follows that the change in the neutron fraction due to the fact that electron neutrinos are hotter is $\delta X_n^\nu = -2.6\times 10^{-4}$.

This is not the whole story. There is a change in the neutron fraction of opposite sign due to the slight decrease in the temperature of electrons and positrons, which we also estimate by Eq. (7). If we assume that electrons and positrons are relativistic (a good approximation since the neutron fraction freezes out at a temperature of about 0.7 MeV) and ignore the small differences between $e^\pm$'s and γ's due to statistics, then electrons, positrons, and photons each lose the same amount of energy due to neutrino heating. Remembering $\delta\rho_{\rm EM} = -\delta\rho_\nu$, it follows that

$$\frac{\delta T_\gamma}{T_\gamma} \simeq -\frac{1}{4}\frac{\delta\rho_\nu}{\rho_\nu} \ . \qquad (9)$$

Comparing Eqs. (8) and (9) we see that the fractional change in the electron temperature is $-\frac{2}{3}$ that of the electron-neutrino temperature, leading to an *increase* in the neutron fraction that is only $\frac{2}{3}$ as large, $\delta X_n^\gamma \simeq 1.7\times 10^{-4}$. The predicted net change in the neutron fraction is thus

$$\delta X_n \equiv \delta X_n^\nu + \delta X_n^\gamma \simeq -0.1\left[\frac{3}{8}-\frac{1}{4}\right]\frac{\delta\rho_\nu}{\rho_\nu} \simeq -9\times 10^{-5} . \qquad (10)$$

Figure 2 shows δX_n as a function of temperature; the numerical results agree well with this simple analytical prediction.

Figure 2 also shows the result of incorporating neutrino heating into the code via method I, where the distor-

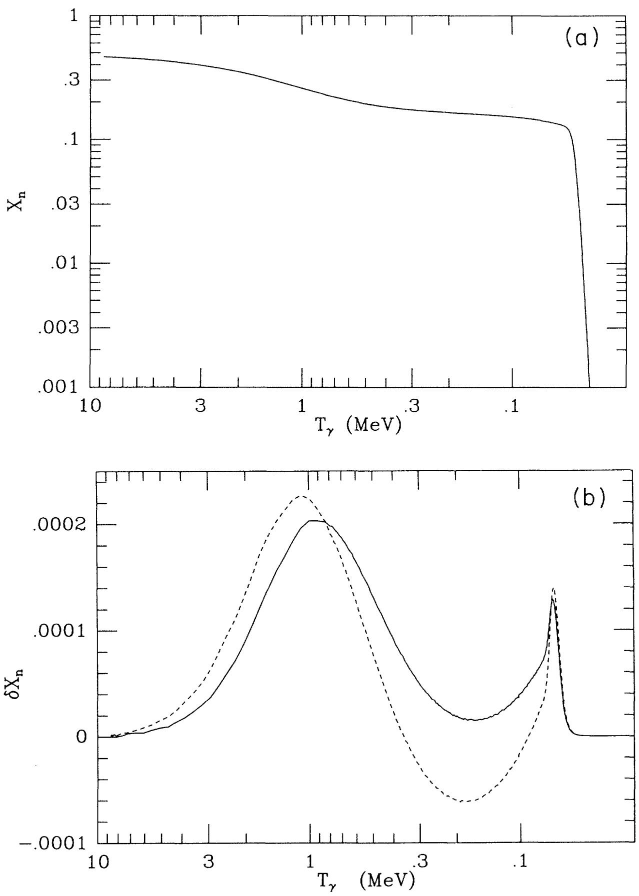

FIG. 2. (a) The evolution of the neutron fraction X_n as a function of the photon temperature: X_n tracks its equilibrium value until $T_\gamma \sim 0.3$ MeV, when it levels off because of the freeze out of the weak interactions; it then slowly decreases due to neutron decays; the precipitous drop occurs because of the onset of nucleosynthesis ($T_\gamma \sim 0.07$ MeV). (b) The change in the neutron fraction δX_n due to the effects of neutrino heating as a function of the photon temperature: δX_n begins to level off at $T_\gamma \sim 0.2$ MeV due to the freeze out of the weak interactions; it then rises because at a given value of T_γ the Universe is younger and fewer neutrons have decayed ("clock effect"); it drops to zero when nucleosynthesis commences. The solid curves are based upon the results of Ref. [14]; the dashed curves upon those of Ref. [12]; all results are for $\eta = 3\times 10^{-10}$.

tion is not assumed to be thermal. In fact, as discussed in Ref. [14], the perturbation to the neutrino spectra is highly nonthermal due to the fact that more high-energy neutrinos are produced in the process of neutrino heating since neutrino cross sections rise with energy. This excess of high-energy neutrinos further enhances the neutron-production rate, which as the temperature drops is becoming more suppressed by the neutron-proton mass difference, and therefore we expect δX_n^{ν} to be larger, which is precisely what is seen in Fig. 2. Since δX_n^{ν} is larger, the near cancellation between δX_n^{ν} and δX_n^{γ} is even more precise: For method I, $\delta X_n \simeq -2\times10^{-5}$.

For reference, the mass fraction of ^{4}He synthesized is related to the neutron fraction at freeze out by $Y\simeq1.33X_n$ (see, e.g., Ref. [14]). Thus, the predicted change in the ^{4}He mass fraction due to the first two effects is $\Delta Y_{1+2}\simeq-3\times10^{-5}$ (method I) and -1.1×10^{-4} (method II).

The clock effect involves the age of the Universe at the epoch at which nucleosynthesis commences, $T=T_{\rm nuc}\simeq0.07$ MeV. Since the Universe is slightly younger when nucleosynthesis commences when neutrino heating is taken into account, fewer neutrons decay from the time that the neutron fraction freezes out, leading to a larger ^{4}He abundance. In Ref. [14] the change in the ^{4}He abundance due to the clock effect was estimated to be $\Delta Y_{\rm clock}\simeq+1.5\times10^{-4}$. Figure 3 shows the total change in ^{4}He abundance as computed by our modified version of the standard code. For method I, ΔY is about $+1.5\times10^{-4}$, while for method II it is about $+1.1\times10^{-4}$, which indicates that the $\Delta Y_{\rm clock}\sim2\times10^{-4}$, in reasonable accord with the previous estimate.

There are a couple of fine points to be made about the baryon-to-photon ratio. In the standard scenario the baryon-to-photon ratio decreases by a factor of $\frac{4}{11}$ from its prenucleosynthesis value to its postnucleosynthesis value, due to the entropy transfer from $e^{\pm}$ pairs to the photons (see Fig. 4). When neutrino heating is taken into account the decrease is less, by about 0.5% (see Fig. 4), which means that for a *fixed value of* η *today, the value of* η *before* $e^{\pm}$ *annihilations was smaller.* We remind the reader that one always specifies the yields of primordial nucleosynthesis in terms of the present value of the baryon-to-photon ratio. This suggests a fourth effect of neutrino heating on nucleosynthesis, involving the fact that the value of η at early times is always smaller when neutrino heating is taken into account; we dub this the η effect [14]. This effect for most values of η is small because somewhat before the onset of nucleosynthesis η has reached its asymptotic (present) value; see Fig. 4. For the most interesting values, $10^{-10}\lesssim\eta\lesssim10^{-9}$, ΔY is insensitive to the value of η. For extreme values of η it becomes η dependent.

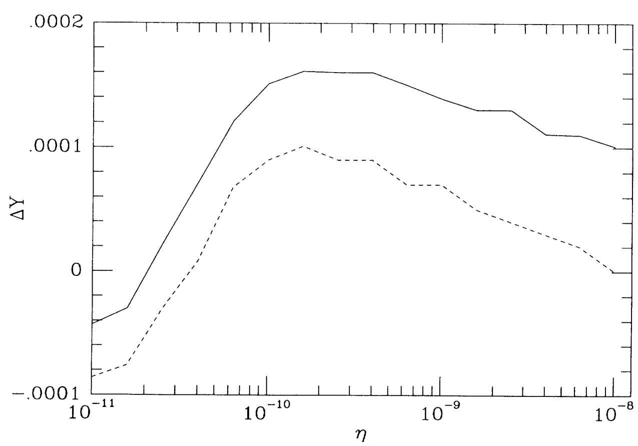

FIG. 3. The change in the predicted ^{4}He abundance due to neutrino heating as a function of the present baryon-to-photon ratio. The solid curves are based upon the results of Ref. [14], the dashed curves upon those of Ref. [12].

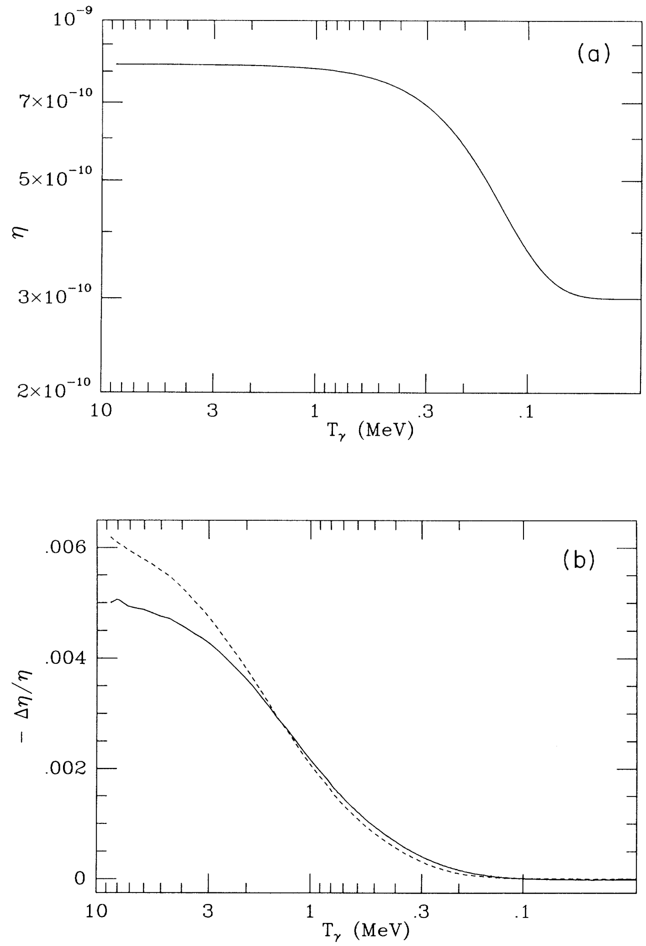

FIG. 4. (a) The evolution of the baryon-to-photon ratio as a function of the photon temperature; (b) the change in baryon-to-photon ratio, $\Delta\eta/\eta$, due to neutrino heating as a function of photon temperature. Note, we have chosen the initial value of η with and without neutrino heating so that the final value is identical. The solid curves are based upon the results of Ref. [14], the dashed curves upon those of Ref. [12].

For large values of η, ΔY depends upon η because of the η effect. As one increases η the onset of nucleosynthesis occurs earlier; for large enough η it occurs before η has reached its asymptotic value; thus, the value of η during nucleosynthesis is slightly *smaller* when neutrino heating is taken into account; since Y increases monotonically with η, the amount of ^{4}He synthesized decreases due to this effect. This is precisely the behavior seen in Fig. 3: For $\eta \gg 10^{-9}$, ΔY decreases.

To understand why ΔY also decreases for very small values of η, we must first recall why the primordial helium abundance drops so precipitously for small values of η (for $\eta \lesssim 10^{-11}$ the mass fraction of D synthesized is actually greater than that of ^{4}He). Small η means that number densities of all nuclear species are small, so that nuclear-reaction rates are correspondingly lower: $\Gamma_{\rm nuclear} \propto \eta$. For extremely low values of η, by the time nucleosynthesis commences nuclear-reaction rates have become ineffective ($\Gamma_{\rm nuclear} \lesssim H$), and the amount of ^{4}He produced depends upon the relative effectiveness of the nuclear reactions: $Y \propto \Gamma_{\rm nuclear}/H$. Neutrino heating increases the expansion rate (at fixed photon temperature), therefore the $\Gamma_{\rm nuclear}/H$ is *smaller* and less ^{4}He is synthesized. This is precisely what is seen in Fig. 3: For $\eta \ll 10^{-10}$, ΔY decreases with decreasing η.

To conclude, neutrino heating affects the synthesis of ^{4}He in four distinct ways; by incorporating the effect of the slight heating of neutrinos by $e^{\pm}$ annihilations into the standard nucleosynthesis code we have quantified its effect on nucleosynthesis and clarified previous conflicting estimates. The net result of the four effects is a slight increase in the mass fraction of ^{4}He synthesized, $\Delta Y \simeq +1.5\times 10^{-4}$, for the interesting range of η. Since an additional neutrino species (or other light particle species) also increases the ^{4}He abundance, by about $\Delta Y \simeq 0.013$, taking account of neutrino heating *improves* the limit to the equivalent number of light neutrino species by about $\Delta N_\nu \simeq 0.01$, which, like the change in the ^{4}He abundance, is a small effect.

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[17] In Ref. [14] the back reaction of neutrino heating on the electromagnetic plasma is carefully discussed; at fixed value of the cosmic scale factor, T_γ is lower when neutrino heating is taken into account by $\delta T_\gamma = -\delta\rho_\nu/(d\rho_{\rm EM}/dT_\gamma) \approx -\frac{1}{4}(\delta\rho_\nu/\rho_{0\nu})T_\gamma$, which follows by energy conservation.

[18] The empirical relation $\delta X_n \simeq -0.1\delta T_i/T_i$ can be derived. To do so, define all perturbations with respect to fixed value of the scale factor, cf. Ref. [14]. The freeze-out value of X_n is set by its equilibrium value when $\Gamma/H \sim 1$: $X_n/(1-X_n) = \exp(-\Delta m/T_f)$, where $(\Gamma/H)_{T_f} = 1$. Write $\Gamma = aR^{-5}(r_\nu^5 + r_e^5)$ and $H = bR^{-2}$, where $R^{-1} = T_{0\nu}$ (neutrino temperature in absence of $e^{\pm}$ heating), $r_\nu = T_\nu/T_{0\nu} \sim 1$, $r_e = T_\gamma/T_{0\nu} \sim 1$. The effective temperature "felt" by nucleons, which determines the equilibrium neutron-to-proton ratio, $T_{\rm eff} \sim (T_\nu + T_\gamma)/2 = R^{-1}(x_\nu + x_e)$. Since $\delta\rho_{\rm EM} = -\delta\rho_\nu$ neutrino heating does not affect the expansion rate. After some algebra it follows that $\delta X_n = -\frac{1}{3}X_n(1-X_n)(\Delta m/T_f)\delta T_i/T_i \approx -0.1\delta T_i/T_i$.