Possible solution to the horizon problem: Modified aging in massless scalar theories of gravity

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An early MAD (massively aged and detained) epoch during which the University becomes older than in the standard model is proposed as a possible new resolution to the horizon problem. This scenario differs from inflation in that there is no period of vacuum domination required and no entropy violation. Extensions of Einstein gravity which allow the Planck mass $m_{\rm Pl}$ to change with time as the Universe evolves may provide such a MAD resolution to the horizon problem: in a cosmology where the gravitational constant $G=m_{\rm Pl}^{2}$ is not in fact constant, the Universe may be older at a given temperature than in the standard hot big bang model. Thus, larger regions of space could have come into causal contact at that temperature. This opens the possibility that large regions became smooth without violating causality. We discuss in this paper theories of gravity in which the gravitational constant is replaced with a function of a massless scalar field. We first consider the original Brans-Dicke proposal and then address more general scalar theories. However, this resolution to the smoothness problem can more generally be a feature of any physics which allows the Planck mass to vary with time. Solutions to the equations of motion during the radiation dominated era for Brans-Dicke gravity and more general massless scalar theories of gravity are presented. In particular, we study the evolution of the field Φ which determines the Planck mass at any given time, $\Phi(t)=m_{\rm Pl}(t)^2$, in the absence of a potential for Φ. We find that, regardless of initial conditions, the Planck mass evolves towards an asymptotic value $\tilde{m}_{\rm Pl}=\tilde{\Phi}^{1/2}$. For both a Brans-Dicke cosmology and a more general scalar theory, our observable Universe could fit inside a region causally connected at some high temperature T_c prior to matter-radiation equality if there is a large disparity between the early value of the Planck mass and the Planck mass today; specifically, our causality condition is that $\tilde{m}_{\rm Pl}/m_{\rm Pl}(T_0)\gtrsim T_c/T_0$, where $m_{\rm Pl}(T_0)=M_0=10^{19}$ GeV is the Planck mass today and T_0 is the temperature of the cosmic background radiation today. Still, an additional mechanism is required to drive the Planck mass to the value M_0 before the Universe cools below a temperature of $T_0\sim 2.74°$ K. A mechanism capable of anchoring the Planck mass fast enough will necessarily accelerate the cosmological expansion and thus involves important dynamics. We suggest possible mechanisms to anchor the Planck mass and complete this MAD model.

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I. INTRODUCTION

The standard hot big bang model of the early Universe is unable to explain the smoothness of the observed Universe. In the standard cosmology, our present horizon volume would envelop many regions which were causally disconnected at earlier times. Consequently, the homogeneity and isotropy of the observed Universe is a mystery. Regions which could not have been in causal contact at earlier times seem nonetheless to be identical in temperature and other properties, as the isotropy of the cosmic background radiation attests.

The inflationary model proposed by Guth [1] addresses the horizon and flatness problems, as well as the monopole problem (if it exists). As a general class of early-universe models, inflation suggests that our Universe passes through an era of false vacuum domination during which the expansion of the Universe accelerates. The accelerated growth of the scale factor inflates a region which was initially subhorizon-sized and therefore in causal contact. If the scale factor grows sufficiently, our observable Universe fits inside one of these blown-up causally connected volumes. During inflation, the temperature of the Universe plunges, $T\propto R^{-1}$, where R is the scale factor. Therefore, the next crucial ingredient for a successful inflationary model is a period of entropy violation which reheats the Universe to some high temperature.

In this paper, we propose that a cosmology with a variable Planck mass can resolve the horizon problem without a period of vacuum domination. Further, entropy is always conserved. We call the epoch of significant variations in the Planck mass the massively aged and detained (MAD) era. (In another paper, we illustrate how our model can resolve the monopole problem while the flatness problem may persist in this model.) We have considered (in Sec. III) the Brans-Dicke proposal to replace the constant Planck mass with a scalar field,

$m_{\rm Pl} \propto \psi$, and (in Sec. IV) more general scalar theories where the Planck mass could be an arbitrary function of a massless scalar field ψ. In both cases, the energy density of the Universe begins radiation dominated and then goes over to a period of matter domination as in the standard cosmology.

We derive below the analytic solutions to the cosmological equations of motion for these alternate theories of gravity when the energy density in ordinary matter is radiation dominated. We find the scale factor, the temperature, the Hubble constant, and the horizon radius in terms of the variable Planck mass. We also find the time evolution of $m_{\rm Pl}(t)$ for early and late times during the radiation-dominated era.

Though the specifics of the cosmology depend on the initial conditions, there is a common nature to the solutions during the radiation-dominated era: whether the Planck mass initially starts out growing with time, decreasing with time, or constant, eventually it asymptotically approaches a constant value denoted by $\tilde{m}_{\rm Pl}$. Early in the radiation-dominated era, the variations in the Planck mass with time can be significant and the scale factor and the temperature evolve with the changing $m_{\rm Pl}$ in a complicated way. Thus, even though the energy density in ordinary matter is predominantly radiation, the variation in the Planck mass alters the dynamics from that of a standard radiation-dominated cosmology. Eventually, $m_{\rm Pl}$ will approach asymptotically close to a constant value $\tilde{m}_{\rm Pl}$. The Universe then evolves in a familiar way. The equations of motion reduce to those of an ordinary radiation-dominated cosmology with M_0, the usual Planck mass of 10^{19} GeV, replaced with $\tilde{m}_{\rm Pl}$. In particular, this means $R \propto \tilde{m}_{\rm Pl}^{-1/2} t^{1/2}$, $H = 1/2t$, and $d_{\rm horiz} \propto t$ where R is the scale factor, H is the Hubble constant, and $d_{\rm horiz}$ is the horizon radius. Thus, despite the underlying structure of the theory, gravity appears to be described by general relativity with a static gravitational constant.

However, in this early phase of the Universe, the strength of gravity can be much weaker than it is today, i.e., the Planck mass can be $\tilde{m}_{\rm Pl} \gg M_0$. Once the Planck mass has reached its asymptotic value $\tilde{m}_{\rm Pl}$, the age of the Universe scales as $t \propto \tilde{m}_{\rm Pl}/T^2$. A universe which has an early MAD era with large Planck mass $\tilde{m}_{\rm Pl} \gg M_0$ is therefore older at a given temperature than a universe which has today's value of the Planck mass M_0 for all time. This gives us a hint as to how such a scenario may solve the smoothness problem. If the Universe is older than in the standard model, then much larger regions of spacetime would have come into contact than we had previously supposed. This opens the possibility that large regions became smooth without violating causality. We will describe this approach to resolving the horizon problem more quantitatively later.

As in a standard cosmology, the energy density in nonrelativistic matter will eventually exceed the energy density in radiation. Thus the era of radiation domination will end as the Universe becomes matter dominated. A matter-dominated Brans-Dicke cosmology has been well studied [2], as have the constraints on such models [3]. As discussed above, in order to solve the smoothness problem, our model requires a large value of the Planck mass $\tilde{m}_{\rm Pl}$ at some time during the radiation dominated epoch. During the matter dominated era, the Planck mass will continue to evolve. However, $m_{\rm Pl}$ will not evolve enough during matter domination to reach the value of M_0 today. Furthermore, in order to avoid conflict with predicted element abundances it would be necessary to fix the strength of gravity at its standard value by the time of nucleosynthesis. In general, an additional mechanism is needed to drive the strength of gravity from its small early value to its large value observed today. We discuss possible mechanisms to anchor the Planck mass at M_0 today, including a potential for the ψ field or the consideration of other theories involving a dynamical $m_{\rm Pl}$.

Extended inflaton [4] and hyperextended inflation [5] were both developed in the context of scalar theories of gravity. In addition to the scalar field which couples to gravity, these models require another scalar field, the inflaton field, and a potential for the inflaton. The horizon problem is resolved in the usual inflationary way as the growth of the scale factor accelerates during an era of false vacuum domination and then the Universe is reheated during a period of entropy violation. It is interesting to note that these models also need an additional mechanism, such as a potential for the Brans-Dicke field, to drive the Planck mass down to the value M_0 by today.

In Sec. II we present the action and equations of motion for the alternate theories of gravity that we are considering. Section III focuses on Brans-Dicke gravity: III A presents solutions to the equations of motion during the radiation dominated era, with solutions parametrized in terms of the Brans-Dicke field Φ; III B relates these solutions to time evolution; III C illustrates the causality condition required to solve the horizon problem; and III D discusses problems with and constraints on the scenario. Section IV presents a general discussion of the MAD era in the context of more general alternate theories of gravity in which the Brans-Dicke parameter ω is not constant. We summarize our conclusions in Sec. V.

II. ACTION

Brans and Dicke proposed an extension of Einstein gravity in which a scalar field usurps the role assumed by the gravitational constant in the Einstein action; that is, in Brans-Dicke gravity, the gravitational constant G is not a fundamental constant but is instead inversely proportional to a scalar field. More generally, G may be some more complicated function of a scalar field ψ: $G^{-1} = m_{\rm Pl}^2 = \Phi(\psi)$. The most general scalar-tensor theories [3] were originally studied by Bergmann [6] and by Wagoner [7]. Regardless of the specific form of Φ, the action for such an extension of general relativity is

$$A = \int d^4x \sqrt{-g} \left[-\frac{\Phi(\psi)}{16\pi} \mathcal{R} - \frac{\omega}{\Phi} \frac{\partial_\mu \Phi \partial^\mu \Phi}{16\pi} - V(\Phi(\psi)) + \mathcal{L}_{\rm matter} \right] , \quad (1)$$

where we have used the metric convention $(-,+,+,+)$, $\mathcal{R}$ is the scalar curvature, $\mathcal{L}_{\rm matter}$ is the Lagrangian density for all the matter fields excluding the field ψ, and $V(\Phi(\psi))$ is the potential for the field ψ. The parameter ω is defined by

$$\omega = 8\pi \frac{\Phi}{(\partial\Phi/\partial\psi)^2} .$$

Using the principle of stationary action gives the equations of motion for the scale factor of the Universe $R(t)$ and for $\Phi(t)$. In a Robertson-Walker metric these become

$$H^2 + \frac{\kappa}{R^2} = \frac{8\pi[\rho + V(\psi)]}{3\Phi} - \frac{\dot\Phi}{\Phi}H + \frac{\omega}{6}\left[\frac{\dot\Phi}{\Phi}\right]^2 , \tag{2}$$

$$\ddot\Phi + 3H\dot\Phi = \frac{8\pi}{3+2\omega}(\rho - 3p) - \frac{\partial U}{\partial\Phi} - \frac{1}{3+2\omega}\frac{d\omega}{d\Phi}\dot\Phi^2 , \tag{3}$$

where

$$\frac{\partial U}{\partial\Phi} = \frac{16\pi}{(3+2\omega)}\left[\Phi\frac{\partial V}{\partial\Phi} - 2V\right] ; \tag{4}$$

U effectively acts as a potential term in the equation of motion for Φ. $H=\dot R/R$ is the Hubble constant, while ρ is the energy density and p is the pressure in all fields excluding the ψ field.

The energy-momentum tensor of matter, $T^{\mu\nu}_{\rm matter}$, is conserved independently of the energy-momentum tensor for the scalar field, $T^{\mu\nu}_{\Phi}$. The conservation equations are

$$T^{\mu\nu}_{\rm matter\,;\nu} = 0 \tag{5}$$

and

$$-8\pi T^{\mu\nu}_{\Phi\,;\mu} = (\mathcal{R}^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}\mathcal{R})\Phi_{;\mu} . \tag{6}$$

Equation (6) returns the equation of motion (3). It can be shown that in an isotropic and homogeneous universe, the $\mu=0$ component of Eq. (5) gives

$$\dot\rho = -(\rho + p)3H . \tag{7}$$

Consider the radiation dominated era where $\rho = (\pi^2/30)g_* T^4$, $p=\rho/3$, and $g_*(t)$ is the number of relativistic degrees of freedom in equilibrium at time t. Since conservation of energy-momentum in ordinary matter does not involve Φ, we can deduce from Eq. (7) that the entropy per comoving volume in ordinary matter, $S=(\rho+p)V/T$, is conserved. For convenience we define

$$\bar S = R^3 T^3 , \tag{8}$$

where $S = \bar S \frac{4}{3}(\pi^2/30)g_S$ and g_S is the number of relativistic degrees of freedom contributing to the entropy. For practical purposes we can take $g_S = g_*$.

Once the equation of state, $p(\rho)$, and the forms of $\Phi(\psi)$ and $V(\psi)$ are specified, these equations describe the evolution of the scale factor, the energy density, and the Planck mass. In specific, the equations of motion (2) and (3) and the conservation equation (7) determine $\Phi(t)$, $\rho(t)$, and $R(t)$ up to four constants of integration. Notice that $\bar S$ is the constant of integration from integrating the energy equation (7). We take the other three constants to be the initial value of Φ, the initial value of $\dot\Phi$ [equivalently the constant C defined in Eq. (10)], and the constant of integration $\tilde\Phi$ given in Eq. (15) (the asymptotic value of Φ in the radiation-dominated era). Given these four initial conditions, the entire cosmology is specified; the equations of motion uniquely determine $R(t)$, $\Phi(t)$, and $\rho(t)$ for all time. In contrast, the standard cosmology with a constant Planck mass requires only that the value of the Planck mass and two boundary values be specified. The two values needed could, for example, be the entropy and the initial value of the scale factor.

To illustrate how the underlying structure of the Planck mass can alleviate the horizon problem, we consider different forms of $\Phi(\psi)$ in the radiation bath of the early Universe where $p(\rho)=\rho/3$. We first treat the original Brans-Dicke proposal of $\Phi=(2\pi/\omega)\psi^2$ with ω, defined above, constant. The second case we consider is general $\Phi(\psi)$ for which ω is not constant. In both cases we take $V(\psi)=0$, although in another paper we treat the model with a nonzero potential.

III. CASE a: ω=const

As a first example, take the original Brans-Dicke model where ω is constant and

$$m_{\rm Pl}^2 = \Phi = \frac{2\pi}{\omega}\psi^2 . \tag{9}$$

Also, we take $V(\psi)=0$. In the limit $\omega=\infty$, Brans-Dicke gravity is the same as Einstein gravity; here we consider arbitrary ω and comment on experimental bounds on ω below. We are interested in the behavior of the solutions during the hot radiation-dominated era of the early Universe. We assume that nonrelativistic matter energy density is negligible. Then $p=\rho/3$ and $\rho-3p=0$.

We first obtain solutions to the equations of motion. Instead of finding $\Phi(t)$, $R(t)$, and $T(t)$, it is more tractable to parametrize R, T, and hence H and the horizon radius, $d_{\rm horiz}$, by Φ. We then find approximate solutions for Φ as a function of time in different regimes. Before moving on to the second case of $\omega\neq$const, we address the horizon problem in the context of our solutions.

A. Solutions to the equations of motion

With ω constant and $V(\psi)=0$, the Φ equation of motion during the radiation dominated era reduces to $\ddot\Phi + 3H\dot\Phi = 0$, so that

$$\dot\Phi R^3 = -C \quad \text{and} \quad H = -\frac{\ddot\Phi}{3\dot\Phi} , \tag{10}$$

where C is a constant of integration which can be positive, negative, or zero. If $C=0$, then $\dot\Phi=0$, the Planck mass has a constant value which we call $\tilde m_{\rm Pl}$, and the Universe evolves in the usual radiation dominated fashion, but with $G=1/\tilde m_{\rm Pl}^2$. Note that if $C>0$, then

$\dot{\Phi}<0$, while if $C<0$, then $\dot{\Phi}>0$. Here we take $\kappa=0$ to illustrate the behavior of the solutions. (In the Appendix, we present the solutions for $\kappa=\pm1$.) First we solve Eq. (2) for H:

$$H=-\frac{\dot{\Phi}}{2\Phi}[1\pm\sqrt{1+2\omega/3+(4\bar{S}^{4/3}\gamma/R^4)(\Phi/\dot{\Phi}^2)}]\ , \tag{11}$$

where $\gamma(t)=(8\pi/3)(\pi^2/30)g_*(t)$. Note that all three terms inside the square root are positive quantities. We choose the sign in front of the square root in such a way as to obtain an expanding universe with $H>0$. Thus, for $C>0$ we take the plus sign in Eq. (11), whereas for $C<0$, we take the minus sign in Eq. (11). Throughout the rest of the paper, the upper sign in equations will refer to the case $C>0$ and the lower sign to the case $C<0$. Substituting $\dot{\Phi}$ from Eq. (10) into the square root in Eq. (11) and using $H=\dot{R}/R$, we have

$$\frac{dR}{R}=-\frac{d\Phi}{2\Phi}\left[1\pm\left[1+\frac{2\omega}{3}+4\bar{S}^{4/3}\gamma\Phi C^{-2}R^2\right]^{1/2}\right]\ . \tag{12}$$

We define

$$\chi(\Phi)=4\bar{S}^{4/3}\gamma C^{-2}\left[\frac{1}{1+2\omega/3}\right]\Phi R^2 \tag{13}$$

and note that χ is always a real positive quantity. The integral of Eq. (12) becomes

$$\int_{\chi_i}^{\chi}\frac{d\chi'}{\chi'\sqrt{1+\chi'}}=\mp\int_{\Phi_i}^{\Phi}\left[1+\frac{2\omega'}{3}\right]^{1/2}\frac{d\Phi'}{\Phi'}\ , \tag{14}$$

where subscript i refers to initial values. We find the (positive) solution

$$\chi^{-1/2}=\sinh\left\{\epsilon\ln\left[\frac{\Phi}{\tilde{\Phi}}\right]\right\}\ . \tag{15}$$

Here, $\tilde{\Phi}=\Phi_i\exp[-(1/\epsilon)\mathrm{arcsinh}(\chi_i^{-1/2})]$; i.e. we have absorbed the constants of integration (which depend on the initial values of Φ and χ) into $\tilde{\Phi}$. Here, $\epsilon\equiv\pm(1+2\omega/3)^{1/2}/2$ (as noted above, an expanding universe corresponds to the plus sign in ϵ if $C>0$, or the minus sign in ϵ if $C<0$). For convenience, we define

$$\Theta\equiv\epsilon\ln\left[\frac{\Phi}{\tilde{\Phi}}\right]\ . \tag{16}$$

In many of the equations below we express the field in terms of Θ rather than Φ. As we will show below, Θ is always positive semidefinite for any value of C: it ranges from $\Theta=0$ for $\Phi=\tilde{\Phi}$ to $\Theta=\infty$ for Φ far from $\tilde{\Phi}$. From Eqs. (13), (15), and (16), we obtain an expression for the scale factor,

$$R(\Phi)=\frac{C}{\bar{S}^{2/3}}\frac{\epsilon}{\gamma^{1/2}}\tilde{\Phi}^{-1/2}\exp\left[-\frac{\Theta}{2\epsilon}\right]\frac{1}{\sinh\Theta}\ . \tag{17}$$

Note that the product $(C\epsilon)$ is always positive semidefinite.

It follows from adiabaticity that

$$T(\Phi)=\frac{\bar{S}^{1/3}}{R(\Phi)}=\frac{\bar{S}}{C}\frac{\gamma^{1/2}}{\epsilon}\tilde{\Phi}^{1/2}\exp\left[\frac{\Theta}{2\epsilon}\right]\sinh\Theta\ . \tag{18}$$

The Hubble constant $H(\Phi)$ is obtained from

$$H(\Phi)=\frac{1}{R}\frac{dR}{d\Phi}\frac{d\Phi}{dt}=-\frac{C}{R^4}\frac{dR}{d\Phi}\ , \tag{19}$$

where in the last step we used Eq. (10). We find

$$H(\Phi)=\left[\frac{\bar{S}}{C}\right]^2\frac{\gamma^{3/2}}{\epsilon^3}\frac{\tilde{\Phi}^{1/2}}{2}\exp\left[\frac{\Theta}{2\epsilon}\right]\sinh^2\Theta \times\{\sinh\Theta+2\epsilon\cosh\Theta\}\ . \tag{20}$$

The comoving horizon size is

$$\frac{d_{\text{horiz}}}{R(\Phi)}=\int_0^t\frac{dt'}{d\Phi'}\frac{d\Phi'}{R(\Phi')}=\frac{-1}{C}\int_{\Phi(t=0)}^{\Phi}R(\Phi')^2d\Phi'\ , \tag{21}$$

where we have used $\dot{\Phi}=-C/R(\Phi)^3$ in the last equality. We can integrate this to find

$$\frac{d_{\text{horiz}}}{R(\Phi)}=\left[\frac{C}{\bar{S}^{4/3}}\right]\frac{\epsilon}{\gamma}\left[\frac{1}{\tanh\Theta}-\frac{1}{\tanh\Theta(t=0)}\right]\ . \tag{22}$$

If $\Phi(t=0)$ starts out far from $\tilde{\Phi}$, i.e., $\Theta(t=0)\gg1$, Eq. (22) becomes

$$\frac{d_{\text{horiz}}}{R(\Phi)}=\left[\frac{C}{\bar{S}^{4/3}}\right]\frac{\epsilon}{\gamma}\frac{\exp[-\Theta]}{\sinh\Theta}\ . \tag{23}$$

As discussed above, $R(\Phi)$, $T(\Phi)$, $H(\Phi)$, and $d_{\text{horiz}}(\Phi)$ are determined only up to the arbitrary constants $\tilde{\Phi}$, C, and $\bar{S}$. For instance, Eq. (18) shows that one can choose the temperature at a given value of Φ by choosing $C/\bar{S}$ and $\tilde{\Phi}$ appropriately. The fourth and last constant of integration, $\Phi(t=0)$, is determined when $t(\Phi)$ is found in Sec. III B.

Even before we determine $\Phi(t)$, we can understand the general sketch of the Universe's evolution. We will find that, in all cases, the field Φ asymptotically approaches the value $\tilde{\Phi}$: for $C>0$, Φ approaches $\tilde{\Phi}$ from above; whereas for $C<0$, Φ approaches $\tilde{\Phi}$ from below. For $C=0$, $\Phi=\tilde{\Phi}$ for all time.

Let us first consider the case of $C>0$. As mentioned previously, this corresponds to $\dot{\Phi}<0$ and $\epsilon>0$. In order for the scale factor to satisfy $R\geq0$, from (17) we can see that we need $\Theta\geq0$, i.e., $\Phi\geq\tilde{\Phi}$. In addition, we need the scale factor to grow in time; again, this requires $\dot{\Phi}<0$. In short, for $C>0$, Φ starts larger than $\tilde{\Phi}$ and decreases towards the asymptotic value $\tilde{\Phi}$.

For the case of $C<0$, we have $\dot{\Phi}>0$ from (10) and $\epsilon<0$. To obtain $R\geq0$, we need $\Theta\geq0$, which in this case corresponds to the opposite limit of $\Phi\leq\tilde{\Phi}$. One can show that as Φ grows towards its asymptotic value (i.e., Θ drops), as long as $|\epsilon|>\frac{1}{2}$ (i.e., $\omega>0$), $dR/d\Theta<0$; the scale factor grows in time. In short, for $C<0$, Φ starts smaller than $\tilde{\Phi}$ and grows towards the asymptotic value $\tilde{\Phi}$.

As we have seen, as Φ approaches $\tilde{\Phi}$, for $|\epsilon|>\frac{1}{2}$

($\omega > 0$), $R(\Phi)$ grows and thus the temperature drops adiabatically. In addition, one can show (again for $\omega > 0$) that the comoving horizon size grows, as does $H^{-1}R^{-1}$. We will see below that the size of a causally connected region can grow great enough to resolve the horizon problem. As the Universe cools below the temperature of matter radiation equality ($T_{\rm eq} \approx 5.5\Omega_M h^2$ eV, where Ω_M is the fraction of the critical density contributed by matter), the reign of radiation yields to that of matter and the nature of the solutions changes. During matter domination, the equation of state is $p(\rho)=0$, and $\rho - 3p = \rho$. This alters the dynamics considerably. Thus there is a built-in off switch to end the radiation-dominated behavior of $R(\Phi)$, $T(\Phi)$, $H(\Phi)$, and $d_{\rm horiz}(\Phi)$.

We will quantify these statements below and find constraints on some of the constants of integration needed to resolve the horizon problem. In the next section, we find approximate descriptions of the behavior of Φ as a function of t. Actually, the resolution of the smoothness problem and the evolution of the cosmology can be understood without knowing Φ as a function of t. The Universe will pass through the familiar stages of baryogenesis, nucleosynthesis, matter domination, etc., as the temperature passes through the relevant energy scale. To follow the evolution of the Universe all one needs to know is the temperature as a function of Φ. One need not know the actual age of the Universe. Still, to ground the solutions in a slightly more familiar setting we will indicate below how the Universe evolves in time.

B. The age of the universe

We will determine the time evolution of the Brans-Dicke field Φ in two different limits: Φ far from $\tilde{\Phi}$ ($\Theta \gg 1$), and $\Phi \simeq \tilde{\Phi}$ ($\Theta \ll 1$). As we have seen, initially Φ may be large for $\dot{\Phi} < 0$ ($C > 0$) or small for $\dot{\Phi} > 0$ ($C > 0$). While Φ is far from its asymptotic value $\tilde{\Phi}$, the term $\dot{\Phi}/\Phi$ contributes significantly to the equations of motion [see Eq. (2)], and the evolution of the Universe is modified relative to that of an Einstein universe in a complicated way as Eqs. (17)–(23) show. Once $\Phi \approx \tilde{\Phi}$ however, the Universe evolves with time as an ordinary radiation-dominated cosmology with the Planck mass M_0 replaced with $\tilde{m}_{\rm Pl} = \tilde{\Phi}^{1/2}$.

To uncover $\Phi(t)$, return to $\dot{\Phi} = -C/R^3$ [cf. Eq. (10)]. We integrate this equation to find

$$\int_{\Phi(t=0)}^{\Phi} d\Phi' R^3(\Phi') = -Ct \; , \tag{24}$$

where $R(\Phi)$ is given in Eq. (17). To get a rough feeling for how Φ changes with t we find approximate solutions to this integral for two regimes: (1) Φ far from the asymptotic value $\tilde{\Phi}$ and (2) $\Phi \approx \tilde{\Phi}$.

1. Φ *far from* $\tilde{\Phi}$

First we consider the early regime where Φ is far from $\tilde{\Phi}$, i.e., $\Theta \gg 1$. The integral on the left-hand side of Eq. (24) is easiest to evaluate if rewritten in terms of Θ rather than Φ. Then we can approximate $\sinh\Theta \simeq e^{\Theta}/2$ in evaluating the integral. The lower limit of the Θ integral is determined by the boundary condition: $R(t=0) \to 0$. For all values of C, this initial value of the scale factor requires $\Theta(t=0) \to \infty$; i.e., $\Phi(t=0) \to \infty$ for $C > 0$ while $\Phi(t=0) \to 0$ for $C < 0$. Thus, our fourth and last integration constant is determined. [Given C, S, and $\tilde{\Phi}$, $R(t=0)$ and $\Phi(t=0)$ contain the same information via Eq. (17)]. We can now evaluate Eq. (24) to find

$$\Phi \approx \tilde{\Phi} \left[\left[\frac{\bar{S}}{C} \right]^2 \frac{\gamma^{3/2}}{8\epsilon^3} (1/2 + 3\epsilon) \right]^{-1/(1/2+3\epsilon)} \times (\tilde{\Phi}^{1/2} t)^{-1/(1/2+3\epsilon)} \; . \tag{25}$$

We see from Eqs. (25) and (17) that initially

$$R(t) \propto t^{(1+2\epsilon)/(1+6\epsilon)} \; ; \tag{26}$$

remember that $\epsilon = \pm 1/2(1+2\omega/3)^{1/2}$. It is interesting to consider the nature of these solutions for large deviations from Einstein gravity (i.e., small ω). Take $C < 0$ for $\omega \to 0$, and thus $\epsilon \to -\frac{1}{2}$. In this limit, $\Phi \to t$ while $R \to$constant. Note that this behavior of the scale factor could also be seen directly from Eq. (17). On the other hand, for $C > 0$ with $\omega \to 0$, $\epsilon \to +\frac{1}{2}$ and $\Phi \to t^{-1/2}$ while $R \to t^{1/2}$. Again, the behavior $R \propto \Phi^{-1}$ could be seen directly from Eqs. (17) and (18).

2. $\Phi \approx \tilde{\Phi}$

The previous approximation breaks down for $\Phi \approx \tilde{\Phi}$. Again, it is easiest to work with $\Theta = \epsilon \ln(\Phi/\tilde{\Phi})$. When $\Phi/\tilde{\Phi}$ is near 1, then $\Theta \ll 1$ and $R[\Theta(\Phi)] \propto \Theta^{-1}$. The lowest-order contribution to the integral yields

$$\Theta = \left[\frac{(\epsilon C)^2 \tilde{\Phi}^{-1/2}}{2\bar{S}^2 \gamma^{3/2}} \frac{1}{t} \right]^{1/2} , \tag{27}$$

or, equivalently,

$$\Phi = \tilde{\Phi} \exp\left[\pm \left[\frac{C^2}{2\bar{S}^2 \gamma^{3/2} \tilde{\Phi}^{1/2} t} \right]^{1/2} \right] , \tag{28}$$

where the plus sign refers to $C > 0$ and the minus sign to $C < 0$. (By assumption, we are working near $\Phi \approx \tilde{\Phi}$ so that the exponent must be small for this approximation to be valid.)

Since Eq. (28) implies that $\Theta(t) \propto t^{-1/2}$ (times a positive constant), we have $R(t) \propto \Theta^{-1} \propto t^{1/2}$. So, as Φ approaches $\tilde{\Phi}$, the Universe evolves as an ordinary radiation-dominated universe with one modification; the Planck mass M_0 is replaced by $\tilde{\Phi}^{1/2}$.

In the standard hot big band model described by Einstein gravity, the age of the Universe as a function of temperature is given by

$$t_{\rm Einst} = \frac{M_0}{2\gamma^{1/2} T^2} \; .$$

As Φ approaches $\tilde{\Phi}$, we can see from Eqs. (28) and (18) that the age of the Universe as a function of $T(\Phi)$ mimics this form,

$$t(\Phi) = \frac{\tilde{m}_{\rm Pl}}{2\gamma^{1/2} T(\Phi)^2} \; , \tag{29}$$

where $\tilde{m}_{\rm Pl} = \tilde{\Phi}^{1/2}$. Incidentally, this is exactly the result one obtains for $C=0$ ($\dot{\Phi}=0$) and $\Phi=\tilde{\Phi}$. As discussed in the introduction, if $\tilde{m}_{\rm Pl}>M_0$, the Universe is older at a given temperature than an Einstein universe with Planck mass M_0. We will show below that if the comoving horizon volume is to become smooth at high temperatures, then $\tilde{m}_{\rm Pl}$ must greatly exceed M_0.

C. Horizon condition and discussion

To explain the smoothness of our present Universe, a region causally connected at some early time must grow big enough by today to encompass our observable Universe. Since we can see back to the time of decoupling, or perhaps nucleosynthesis, the size of the observable Universe is roughly the distance light could have traveled since that time, $\Delta t_0 \sim H_0^{-1}$, where H_0 is the Hubble constant today. Thus we can take the present comoving Hubble radius, $1/(H_0 R_0)$, as a measure of the comoving radius of the observable Universe; here R_0 is the scale factor today. Then the smoothness of the observable Universe can be explained if a comoving region of radius at least as large as $1/H_0 R_0$ is in causal contact at some time t_c before nucleosynthesis, i.e.,

$$\frac{1}{H_c R_c} > \frac{1}{R_0 H_0} , \tag{30}$$

where subscript c denotes values at the time causality is satisfied.

We can express both $H(\Phi)$ and H_0 in terms of the temperature and the Brans-Dicke field Φ. Substitution of expression (18) for $T(\Phi)$ into Eq. (20) for $H(\Phi)$ gives

$$H(\Phi)=\gamma^{1/2}\frac{T(\Phi)^2}{\Phi^{1/2}}\frac{1}{2\epsilon}\{\sinh\Theta+2\epsilon\cosh\Theta\} . \tag{31}$$

The Hubble constant today can be written as

$$H_0=\alpha_0^{1/2}\frac{T_0^2}{M_0} \tag{32}$$

where $T_0=2.6\times10^{-13}$ GeV, M_0 is the value of the Planck mass today, and $\alpha_0=\gamma(t_0)\eta_0 =(8\pi/3)(\pi^2/30)g_*(t_0)\eta_0$, where $\eta_0\sim 10^4-10^5$ is the ratio today of the energy density in matter to that in radiation. Also, we use adiabaticity, $RT=\bar{S}^{\,1/3}\propto(S/g_*)^{1/3}$, to write the causality condition as

$$\frac{\Phi_c^{1/2}}{T_c}\frac{2\epsilon}{\sinh\Theta_c+2\epsilon\cosh\Theta_c}\gtrsim\beta\frac{M_0}{T_0} , \tag{33}$$

where $\beta=[\gamma(t_c)/\alpha_0]^{1/2}[g_*(t_c)/g_*(t_0)]^{-1/3}$. To resolve the horizon problem, this constraint must be satisfied prior to matter/radiation equality.

Although it is possible for the causality condition (33) to be satisfied while Φ is still far from $\tilde{\Phi}$, we find that, for $\omega\gtrsim 1$, the solution to the horizon problem that deviates the least from Einstein gravity is obtained for $\Phi\simeq\tilde{\Phi}$ in Eq. (33). In other words, for $\omega\gtrsim1$, the lowest possible value of $\Phi_c^{1/2}=m_{\rm Pl}(T_c)$ that solves causality is given by $\Phi_c\simeq\tilde{\Phi}$. [For $C<0$, Φ is always less than $\tilde{\Phi}$, yet the previous statement still holds, e.g., if $\omega\sim500$, to better than 1%; the lowest value of Φ_c g, again for the example of $\omega\sim500$, is obtained for $\Phi_c\sim0.997\tilde{\Phi}$. If $\Phi\ll\tilde{\Phi}$, both Φ and $\tilde{\Phi}$ are driven to higher values than if they are equal to each other; this is to be expected because for large Φ_c the large factor of $\exp(\Theta_c)$ in the denominator makes the causality condition harder to satisfy]. From now on, we will examine the causality constraint for Φ near its asymptotic value $\tilde{\Phi}$.

For $\Phi\simeq\tilde{\Phi}$, $\Theta\simeq0$, $\sinh\Theta\simeq0$, $\cosh\Theta\simeq1$, and the causality condition becomes simply

$$\frac{\tilde{m}_{\rm Pl}}{M_0}\gtrsim\beta\frac{T(\tilde{\Phi})}{T_0} , \tag{34}$$

where $m_{\rm Pl}(t_c)\approx\tilde{m}_{\rm Pl}=\tilde{\Phi}^{\,1/2}$. We can specify the temperature at which we would like to resolve the causality dilemma. We are free to choose the temperature at which $\Phi=\tilde{\Phi}$ since this is equivalent to making an appropriate choice for the ratio of the arbitrary constants $\bar{S}/C$ [see Eq. (18)]. [Since $\bar{S}/C\propto T^3/\dot{\Phi}$, this amounts to making a choice for $\dot{\Phi}(t_c)$; in principle one should check that this choice is consistent with measurements of $\dot{G}/G$ today. However, since $\dot{\Phi}\propto R^{-3}$, in many cases the time derivative may be quite small and therefore unobservable by the present epoch.]

As an example, we consider $T_c\sim3\times10^{16}$ GeV, roughly the scale of grand unification. Then the causality requirement becomes $\tilde{m}_{\rm Pl}\gtrsim10^{27}M_0$. If, instead, we take $T(\Phi_c\approx\tilde{\Phi})\simeq1$ MeV, roughly the temperature of primordial nucleosynthesis, then condition (34) requires $\tilde{m}_{\rm Pl}\gtrsim10^7M_0$.

We can verify that the Universe is old in this model. We showed with our approximate expressions for $m_{\rm Pl}$ as a function of time, that when $\Phi\approx\tilde{\Phi}$, the Universe evolves as an ordinary radiation-dominated universe with one modification; M_0 is replaced by $\tilde{m}_{\rm Pl}$. In this limit

$$t(\tilde{\Phi})=\frac{\tilde{m}_{\rm Pl}}{2\gamma^{1/2}T(\tilde{\Phi})^2} . \tag{35}$$

Since $t_c\propto1/H_c$ and $t_0\propto1/H_0$, Eq. (30) is equivalent to the statement that

$$\frac{t(\tilde{\Phi})}{R(\tilde{\Phi})}\gtrsim\frac{t_0}{R_0} . \tag{36}$$

Since $\tilde{m}_{\rm Pl}$ does in fact exceed M_0 we see from Eq. (35) that the Universe is older at a given temperature than in the standard cosmology. Writing Eq. (35) in terms of $t_{\rm Einst}$ the age of a cosmology described by Einstein gravity, gives

$$t(\tilde{\Phi})=t_{\rm Einst}\left[\frac{\tilde{m}_{\rm Pl}}{M_0}\right] \tag{37}$$

at a given temperature. In a standard cosmology, $t_{\rm Einst}\sim({\rm MeV}/T)^2$ sec. For $T_c\sim3\times10^{16}$ GeV, then $t_{\rm Einst}\sim10^{-40}$ sec while $t(\tilde{\Phi})\sim10^{-11}$ sec. For $T_c\sim1$ MeV, $t_{\rm Einst}\sim1$ sec and $t(\tilde{\Phi})\sim10^7$ sec ~3 yr.

If, as an extreme case, we take $T(\Phi_c\approx\tilde{\Phi})$ to be the temperature of matter/radiation equality, about $5.5\Omega_M h^2$ eV, then the causality condition requires

$$\tilde{m}_{\rm Pl}\gtrsim10^2M_0 . \tag{38}$$

At $T_c \sim 1$ eV, $t_{\rm Einst} \sim 10^{12}$ sec $\sim 10^5$ yr, and $t(\Phi) \sim 10^2 t_{\rm Einst} \sim 10^7$ yr. Our solution to the causality problem indeed makes the Universe at T_c older than in the standard model. However, we see that even if we push T_c to this ridiculously low value of 1 eV, the total age of the Universe today need not be changed; i.e., one can still have $t_0 \sim 10^{10}$ yr. In any case, all observable measurements of the "age of the Universe" measure the amount of time subsequent to matter domination. An arbitrarily large amount of time could have elapsed prior to decoupling without any conflict with experimental measurements.

D. Problems and constraints

The obvious difficulty with this resolution to the horizon problem is fixing the value of the Planck mass to be M_0 by today. In the Brans-Dicke model studied here without a potential, the Planck mass will be hard pressed to make it to the value M_0 today. During the matter-dominated era, Φ will initially continue to decrease with time for $C>0$ and increase with time for $C<0$ [8]. For $C<0$ then, the Planck mass will only grow larger. For $C>0$, it is conceivable that Φ will approach the value of M_0^2 during the matter-dominated era. However, observations constrain the parameter ω to be $\gtrsim 500$ for a massless Brans-Dicke theory. The rate at which Φ changes depends on ω and is very suppressed for large ω. Thus a large ω would confine Φ to near its value at the time of matter/radiation equality, which, as we have seen, may be large. For example, with $\omega=500$ and $\tilde{\Phi}^{1/2}=10^2 M_0$ at $T_c \sim 1$ eV, then today $\Phi_0^{1/2} \geq 80 M_0$. This limit can be avoided if there is a potential for Φ, e.g. a mass term such as $V(\psi)=(m_\psi^2/2)\psi^2$. The interactions measured in the time-delay experiments fall off rapidly outside the range over which the Φ field acts. If Φ has an associated mass m_ψ, then the range over which Φ acts $\lambda \sim 1/m_\psi$ could be smaller than the distances over which the observations are sensitive. Therefore a massive Brans-Dicke model could elude observation even if ω is small [9].

There are also observations of the rate of change of the gravitational constant. These observations impose a much weaker constraint than the time delay experiments. They suggest $\omega \gtrsim 5$. For comparison, if $\omega=5$ and $\tilde{\Phi}^{1/2}=10^2 M_0$, then one could have $\Phi_0^{1/2} \approx M_0$ up to order 1.

Another issue of concern is the value of the Planck mass, and thus the Hubble constant, during nucleosynthesis. If $m_{\rm Pl} \neq M_0$ during nucleosynthesis, then the predicted elemental abundances will be affected. To resolve the horizon problem, we found that the asymptotic value $\tilde{m}_{\rm Pl}=\tilde{\Phi}^{1/2}$ had to be much larger than M_0. If, for example, $m_{\rm Pl} \simeq \tilde{m}_{\rm Pl} \gg M_0$ during nucleosynthesis, then $H(\Phi) \propto T^2(\Phi)/\tilde{m}_{\rm Pl}^{1/2}$ and the large Planck mass slows the expansion of the Universe. Consequently, the temperature at which the weak interactions freeze out is lowered, the n/p ratio is maintained at its equilibrium value longer, and the value of the n/p ratio during nucleosynthesis is smaller. This works to decrease the production of ^{4}He. Compatibility with observations would then force Ω_b, the fraction of critical density in baryonic matter, to be larger. Actually, since the Hubble constant can be so much smaller than in the standard model, a situation may arise where the weak interactions are still in equilibrium during the time of ^{4}He synthesis; then the nucleosynthesis calculations would have to be redone. We have not investigated the consequences for abundances of other elements, such as deuterium, lithium, and ^{3}He. We suspect that, unless the Planck mass has returned to its present value by the time of nucleosynthesis, matching observations on all elements simultaneously will be impossible.

One could insist that the causality condition is solved for temperatures greater than an MeV and then invoke a potential to drive $m_{\rm Pl}$ to M_0 by the time of nucleosynthesis. This would also accommodate the $C=0$ scenario where $m_{\rm Pl}$ is constant at the value $\tilde{m}_{\rm Pl}$ needed to solve causality. For the previous results to hold, the potential would have to remain inconspicuous during the early evolution. The potential suggestion will be studied elsewhere.

While the Planck mass drops and the strength of gravity grows, the Universe will cool. Notice from (34) that the ratio $\tilde{m}_{\rm Pl}/M_0$ must be greater than the ratio $T(\tilde{\Phi})/T_0$. Thus, the causality condition in (34) requires that, subsequent to $T(\tilde{\Phi})$, the Planck mass must drop faster than the temperature does. Since $H=\dot{R}/R$, the causality condition is equivalent to the statement that $\dot{R}_0 \gtrsim \dot{R}_c$, i.e., $\ddot{R}>0$ (for $R \propto t^p$, this implies $p>1$) [10]. Thus, this period where $m_{\rm Pl}$ drops faster than T is associated with accelerated growth of the scale factor (however, note that MAD does not require the 60 e-foldings of expansion that inflation does). It may be that during this stage persistent anisotropies and inhomogeneities which could not be smoothed out by causal microphysics are diluted by the rapid growth of the scale factor. Further, it is during this stage that density perturbations responsible for the formation of large-scale structure could be imprinted.

IV. CASE b: $\omega \neq$const

A. Solution to the equations of motion [11]

Here we extend the analysis to the more general case of ω not constant, again with no potential for the Brans-Dicke field. We will find the solutions to $R(\Phi)$, $H(\Phi)$, and $T(\Phi)$ here. The solutions during the radiation-dominated era are very similar in spirit to the previous solutions for $\omega=$const. However, the case of ω not constant does allow the possibility of a small value of ω at early times which matches onto $\omega \geq 500$ today. (Actually, the observational constraints need to be reinterpreted if $\omega \neq$const.)

If Φ has any functional form other than the minimal $\propto \psi^2$, then $\omega = 8\pi\Phi/(\partial\Phi/\partial\psi)^2$ will also be a function of ψ. The equation of motion (3) in the radiation-dominated era for the case of no potential is

$$\ddot{\Phi}+3H\dot{\Phi}=-\frac{d\omega}{d\Phi}\frac{\dot{\Phi}^2}{3+2\omega}=-\dot{\omega}\frac{\dot{\Phi}}{3+2\omega} , \tag{39}$$

so that

$$\dot{\Phi}R^3=-\frac{C}{(1+2\omega/3)^{1/2}},\quad H=-\frac{\ddot{\Phi}}{3\dot{\Phi}}-\frac{2\dot{\omega}/3}{6(1+2\omega/3)}. \tag{40}$$

We can use results (39) and (40) in Eq. (2) as we did in the previous section. This time, we define

$$\chi=4\bar{S}^{4/3}\gamma C^{-2}\Phi R^2 \tag{41}$$

[this time, there is no factor of $(1+2\omega/3)^{-1}$ in our definition of χ]. Again, we obtain Eq. (14). Since $\omega(\Phi)$ is as yet unspecified here, we define

$$\Sigma(\Phi)=\pm\int_{\tilde{\Phi}}^{\Phi}\frac{d\Phi'}{2\Phi'}[1+2\omega(\Phi')/3]^{1/2}, \tag{42}$$

where the upper (lower) sign refers to $C>0$ $(C<0)$ and $\Sigma(\Phi)>0$. We have absorbed all constants of integration into the constant $\tilde{\Phi}$. If one prefers to work in terms of initial values, then one can relate them through the integral

$$\int_{\Phi_i}^{\tilde{\Phi}}\frac{d\Phi}{2\Phi}(1+2\omega/3)^{1/2}=\mp\operatorname{arcsinh}(\chi_i^{-1/2}).$$

The solution to (14) in this case becomes

$$\chi^{-1/2}=\sinh\Sigma(\Phi). \tag{43}$$

From Eqs. (41) and (43), we can find

$$R(\Phi)=\frac{|C|}{2\bar{S}^{2/3}\gamma^{1/2}}\frac{1}{\Phi^{1/2}\sinh\Sigma(\Phi)}. \tag{44}$$

It follows that

$$T(\Phi)=\frac{2\bar{S}\gamma^{1/2}}{|C|}\Phi^{1/2}\sinh\Sigma(\Phi). \tag{45}$$

For use in the constraint (30), we find $H(\Phi)$ as before,

$$H(\Phi)=4\gamma^{3/2}\left[\frac{\bar{S}}{C}\right]^2\frac{\sinh^2\Sigma\Phi^{1/2}}{(1+2\omega/3)^{1/2}}\times\{(1+2\omega/3)^{1/2}\cosh\Sigma\pm\sinh\Sigma\}. \tag{46}$$

$H(\Phi)$ can be expressed in terms of the temperature and Σ using Eq. (45) in Eq. (46).

$$H(\Phi)=\gamma^{1/2}\frac{T^2(\Phi)}{\Phi^{1/2}}\frac{1}{(1+2\omega/3)^{1/2}}\times\{(1+2\omega/3)^{1/2}\cosh\Sigma\pm\sinh\Sigma\}. \tag{47}$$

The comoving horizon size is

$$\frac{d_{\text{horiz}}}{R(\Phi)}=\left[\frac{|C|}{2\gamma\bar{S}^{4/3}}\right]\left[\frac{1}{\tanh\Sigma}-\frac{1}{\tanh\Sigma(\Phi_i)}\right]. \tag{48}$$

In order to solve the smoothness problem, we need

$$\frac{1}{H(\Phi_c)R(\Phi_c)}\gtrsim\frac{1}{H_0R_0}$$

as before. This gives a constraint similar to the ω=constant scenario. Written in terms of the temperature, the constraint is

$$\frac{\Phi_c^{1/2}}{T(\Phi_c)}\frac{(1+2\omega_c/3)^{1/2}}{\{(1+2\omega_c/3)^{1/2}\cosh\Sigma_c\pm\sinh\Sigma_c\}}\gtrsim\beta\frac{M_0}{T_0}, \tag{49}$$

where β is defined below Eq. (33) and again the subscript c indicates the values at the time causality is solved. Again, the causality condition can be satisfied with small Σ_c (i.e., $\sinh\Sigma_c\sim 0$ and $\cosh\Sigma_c\sim 1$); then Eq. (49) becomes

$$\frac{\Phi_c^{1/2}}{M_0}\gtrsim\beta\frac{T_c}{T_0}. \tag{50}$$

We see that a large early value of the Planck mass can solve the causality condition at high temperatures.

Again, as in the case of pure Brans-Dicke gravity, Φ approaches an asymptotic value $\tilde{\Phi}$ from either above or below. We can see this by looking at Eq. (40). For $\omega>0$, we can see that $\dot{\Phi}\le C/R^3\to 0$ as $R\to\infty$, and Φ approaches the constant value $\tilde{\Phi}$. Notice that Eq. (44) is consistent with this behavior: as Φ approaches $\tilde{\Phi}$, $\Sigma(\tilde{\Phi})\to 0$ and the scale factor grows very large.

As an example we take $\Phi(\psi)=\hat{\Phi}\exp(\psi/\hat{\psi})$ where $\hat{\psi}$ is a constant mass scale and $\hat{\Phi}$ is a constant with units of mass squared. Then

$$\omega=8\pi\frac{\hat{\psi}^2}{\Phi}. \tag{51}$$

Since $\omega\propto\Phi^{-1}$, ω is small for large values of Φ. As Φ decreases, ω increases and automatically turns off the change in Φ. With this form of ω we find $\Sigma(\Phi)$ to be

$$\Sigma(\Phi)=\pm\left[\sinh^{-1}\left[\frac{3}{2\omega(\Phi)}\right]^{1/2}-\sinh^{-1}\left[\frac{3}{2\omega(\tilde{\Phi})}\right]^{1/2}\right]\mp\{[1+2\omega(\Phi)/3]^{1/2}-[1+2\omega(\tilde{\Phi})/3]^{1/2}\}. \tag{52}$$

The description of this cosmology is very similar to the ω=const scenario. Initially, $\dot{\Phi}$ is either positive, negative, or zero. If $\dot{\Phi}$ is positive, then the Planck mass starts small and increases toward the asymptotic value $\tilde{m}_{\rm Pl}=\tilde{\Phi}^{1/2}$. If $\dot{\Phi}$ is negative, then the Planck mass begins large and decreases toward $\tilde{m}_{\rm Pl}$. To satisfy the causality condition when the Planck mass is at the asymptotic value $\tilde{m}_{\rm Pl}$, we need $\tilde{m}_{\rm Pl}/M_0\gtrsim\beta\tilde{T}/T_0$.

One can choose the functional form of $\omega(\Phi)$, or equivalently of $\Phi(\psi)$, so that $m_{\rm pl}$ drops faster than T does. We have found that one way to get the Planck mass to drop quickly enough is with a negative ω. Although we could certainly write down such a scheme, a negative ω corresponds to a negative kinetic energy term which may be plagued with ills. Alternatively, for $\epsilon\sim-\frac{1}{2}$ and so very small values of ω, it may be that such a large ratio of Planck masses is not needed. The value of ω could then grow to exceed 500. We are currently investigating this possibility.

V. CONCLUSIONS

In a MAD cosmology, the Universe is older at a given temperature than in a standard cosmology, old enough to explain the smoothness of our observable Universe. The comoving size of a causally connected region $1/(HR)$ is correspondingly larger. Since the observed Universe only reaches out to about recombination (or possibly back to nucleosynthesis), a rough estimate of the comoving radius of the observable Universe is the present comoving Hubble radius $H_0^{-1}/R_0 \sim \Delta t_0/R_0$, where Δt_0 is roughly the time elapsed since the Universe became matter dominated. The smoothness of the observable Universe can be explained if the Universe ages sufficiently so that the comoving horizon size at the temperature of nucleosynthesis is $H^{-1}/R \gtrsim \Delta t_0/R_0$. Observations of the age of the Universe from the Hubble diagram, nucleocosmochronology, and ages of globular clusters only place limits on the age of the Universe subsequent to the time when stars formed; thus, the Universe may in fact become much older in the radiation-dominated era than one would expect from the standard model.

We found that an early period with large Planck mass ages the Universe sufficiently so that our entire observable Universe was once causally connected. For a scalar theory of gravity without a potential during the radiation-dominated era, the Planck mass approaches an asymptotic value, $\tilde{m}_{\rm Pl}$. This asymptotic value can be chosen to satisfy the causality condition at a specified temperature T_c:

$$\tilde{m}_{\rm Pl}/M_0 \gtrsim T_c/T_0 \ . \tag{53}$$

For instance, if $T_c \sim 3\times 10^{16}$ GeV, roughly the scale of grand unification, we need the asymptotic value $\tilde{m}_{\rm Pl} \gtrsim 10^{27}M_0$. Or, for $T_c \sim 1$ MeV, the temperature of nucleosynthesis, we need $\tilde{m}_{\rm Pl} \gtrsim 10^{7}M_0$. However, it is difficult for the Planck mass to drop quickly enough after T_c so that it reaches $M_0 = 10^{19}$ GeV by today. In fact, we expect that the Planck mass must drop to today's value M_0 by the time of nucleosynthesis so that the standard predictions of element abundances will not be drastically altered. In the original Brans-Dicke model, $m_{\rm Pl}$ is unable to reach M_0 in time. We suggest that $m_{\rm Pl}$ may be driven down to almost its present value by the time of nucleosynthesis if there is a potential in the theory for $m_{\rm Pl}$ or if a negative Brans-Dicke parameter ω could be tolerated. As the Planck mass drops and the strength of gravity grows, the cosmological expansion will accelerate. During this stage a spectrum of primordial density perturbations may be produced. Quantum fluctuations in G or other particle fields may lead to significant density perturbations in the matter background. It is possible that density perturbations relevant for large-scale structure can be produced. Inevitably, any perturbations would imprint some signature on the cosmic background radiation. In a separate paper we also discuss the monopole problem and the flatness problem [12].

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APPENDIX

We present here the solutions to the equations of motion during the radiation-dominated era for a Brans-Dicke theory with $\kappa = \pm 1$. The Φ equation of motion reduces to $\ddot{\Phi} + 3H\dot{\Phi} = 0$ so that, as before,

$$\dot{\Phi}R^3 = -C \quad \text{and} \quad H = -\frac{\ddot{\Phi}}{3\dot{\Phi}} \ . \tag{A1}$$

Solving Eq. (2) for H with $C \neq 0$ and $\kappa \neq 0$ gives

$$H = -\frac{\dot{\Phi}}{2\Phi}\left[1 + 2\epsilon\sqrt{1+\chi - Q^2\chi^2}\right] \tag{A2}$$

where $\epsilon = \pm(1+2\omega/3)^{1/2}/2$, χ is defined as in (13), and

$$Q^2 = \frac{\epsilon^2 C^2}{\gamma^2 \bar{S}^{8/3}}\kappa \ . \tag{A3}$$

Using $H = \dot{R}/R$, the definition of χ, and rearranging, we are left with the integral

$$\int_{\chi_i}^{\chi} \frac{d\chi'}{\chi'\sqrt{1+\chi' - Q^2\chi'^2}} = \mp \int_{\Phi_i}^{\Phi} \left[1 + \frac{2\omega}{3}\right]^{1/2} \frac{d\Phi'}{\Phi'} \ . \tag{A4}$$

Integrating this equation and using $R = (\epsilon C/\bar{S}^{2/3}) \times(\gamma\Phi)^{-1/2}\chi^{1/2}$, from the definition of χ, we find

$$R = \frac{\epsilon C}{\bar{S}^{2/3}\gamma^{1/2}}\frac{1}{\Phi^{1/2}}\left\{\frac{1}{\sinh^2\Theta + Q^2\exp(-2\Theta)}\right\}^{1/2} , \tag{A5}$$

where, as before, $\Theta = \epsilon\ln(\Phi/\tilde{\Phi})$. The temperature of the Universe is found from adiabaticity to be

$$T = \frac{\bar{S}\gamma^{1/2}}{\epsilon C}\Phi^{1/2}\{\sinh^2\Theta + Q^2\exp(-2\Theta)\}^{1/2} \ . \tag{A6}$$

The causality condition becomes

$$\frac{\Phi_c^{1/2}}{T_c} 2\epsilon \left[\frac{(\sinh^2\Theta_c + Q^2\exp(-2\Theta_c))^{1/2}}{\sinh^2\Theta_c + 2\epsilon \sinh\Theta_c \cosh\Theta_c + Q^2(1-2\epsilon)\exp(-2\Theta_c)} \right] \gtrsim \beta \frac{M_0}{T_0} . \tag{A7}$$

Notice that as $Q^2 \rightarrow 0$ (A5), (A6), and (A7) reduce to the corresponding results for a flat Universe. Similarly, for large Θ, $e^{-2\Theta} \rightarrow 0$, and we have the same causality condition as in the case of the flat Universe. We will further discuss the case of nonzero curvature, together with the issue of flatness, in another paper.

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