Test of a chromomagnetic model for hadron mass differences

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An oversimplified model consisting of the QCD color-magnetic interaction has been used previously by Silvestre-Brac and others to compare the masses of exotic and normal hadrons. We show that the model can give qualitatively wrong answers when applied to systems of normal hadrons.

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Recently Silvestre-Brac [1] has used a particularly simple Hamiltonian to consider systematically the stability of ground-state exotic mesons composed of two quarks and two antiquarks. In other papers, Silvestre-Brac and Leandri [2] and Leandri and Silvestre-Brac [3] have applied the same Hamiltonian to other exotic hadrons. These papers contain references to other work on the subject.

The Hamiltonian may be written in the form

$$H=\sum_i m_i-\sum_{i<j} a\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j/m_im_j\ , \qquad (1)$$

where a is a positive constant, the λ_i are Gell-Mann matrices, the σ_i are Pauli matrices, and the m_i are effective constituent quark masses. The terms containing the Gell-Mann and Pauli matrices are an approximation to the color-magnetic interaction of perturbative QCD. Silvestre-Brac [1] omits the sum of the quark masses in (1), but it is clear from his numerical work that he actually includes them. In a comparison of the masses of systems containing the same quarks, the sum of the quark masses cancels out.

An especially interesting question about an exotic hadron is whether it is quasistable, by which we mean stable against strong decay. An exotic is quasistable (bound) if its mass is less than the mass of the two lightest hadrons into which it would otherwise be able to decay.

Silvestre-Brac understands that the Hamiltonian (1) is oversimplified. Nevertheless, he lists a number of exotic mesons which he says are candidates for being stable against strong decay into two ordinary mesons, as he calculates those exotics are bound by at least 100 MeV. He then wonders how much his results will be changed by improved calculations.

Perhaps a more important question is how well do calculations with the simple Hamiltonian (1) agree with experiment. Unfortunately, only meager experimental information exists about exotics. However, we can already give a qualitative answer to the question of how good the model is by comparing its predictions with the known masses of normal mesons and baryons. We find that the Hamiltonian (1) leads to predictions of threshold energies which can be in error by several hundred MeV.

We see no reason why the Hamiltonian (1) should be more accurate for exotic than for normal hadrons. Therefore, calculations with an improved Hamiltonian will be necessary before we can have confidence that any candidate listed in Silvestre-Brac's paper is indeed quasistable. Despite the unreliability of Silvestre-Brac's results, he is to be commended for having undertaken a systematic treatment of all exotic mesons with a Hamiltonian which, despite its flaws, has been widely used in the literature. A number of the techniques used by Silvestre-Brac should still be applicable with an improved Hamiltonian.

We introduce the quantity δ, defined as the difference in mass between a collection of initial and final state hadrons. If the initial state is an exotic hadron and the final state is the least massive collection of hadrons with the same quantum numbers, then the exotic can decay strongly if δ is positive. If δ is negative, the exotic is bound with respect to strong decay and so is quasistable. Like Silvestre-Brac, we consider ground-state hadrons only and assume that there is no orbital angular momentum in the initial state. However, we relax Silvestre-Brac's assumption that there is no orbital angular momentum in the final state because there is no reason why spin cannot be converted into orbital angular momentum so long as total angular momentum and parity are conserved.

For illustrative purposes, we test the Hamiltonian using the same parameters found by Silvestre-Brac [1]. We do this for convenience only. We have considered other parameters as well, and have found no overall improvement. The quark masses used in Ref. [1] are (in MeV)

$$m_u=m_d=m=330,\quad m_s=550,\quad m_c=1650,\quad m_b=4715\ , \qquad (2)$$

and the constant is $A=a/m^2=20$ MeV. With these parameters Silvestre-Brac [1] finds $m_\pi=m_\eta=340$ MeV, about half way inbetween the experimental values $m_\pi=138$ MeV, $m_\eta=547$ MeV. He therefore says that, if n pions are in the final state, the model underestimates δ by about $200n$ MeV. We therefore emphasize tests which do not involve pions, but we discuss pions as well.

Our first test of the Hamiltonian (1) is to consider the masses of hadrons in the absence of the color-magnetic interaction. Because we cannot turn off this interaction, we get information about such masses from experiment by spin averaging the masses of known ground-state mesons and baryons in such a way that the color-magnetic interaction cancels out in perturbation theory [4]. In those cases in which we do not have sufficient information from experiment, we make estimates based in part on a semiempirical formula for the color-magnetic interaction [5]. This exercise has already been carried out [6] for mesons, and here we do it for baryons as well.

We let the mass of a normal meson or baryon containing quarks and/or antiquarks $q_1, q_2, \ldots$ be $M_{12\ldots}$ and introduce the notation

$$E_{12\ldots} = M_{12\ldots} - \sum m_i \ . \tag{3}$$

We show in Table I the values of M_{12} and E_{12} for mesons and in Table II the values of M_{123} and E_{123} for baryons. We also show the quark content, with q standing for a u or d quark.

It is seen from Tables I and II that it is a poor approximation to assume that $E_{12\ldots}$ is the same constant for all hadrons. But that is one of the approximations necessary to obtain the oversimplified Hamiltonian of Eq. (1). The reason that the values of $E_{12\ldots}$ in Tables I and II vary in a rather haphazard way is because Silvestre-Brac did not choose the masses of the quarks so as to satisfy inequalities suggested by the Feynman-Hellman theorem [7]. But we have verified that no choice of quark masses can make the $E_{ij\ldots}$ approximately constant.

We next test the color-magnetic piece of the interaction with mesons and baryons. We have noted that, in the simplified Hamiltonian of Eq. (1), the constant $A = a/m^2$ has the value $A = 20$ MeV. But we can use the data on the color-magnetic splitting of mesons and baryons to obtain A in selected cases [4]. We need to relax the assumption that A is a constant and write $A = A_{12}$ for mesons and $A = A_{ij,k}$ for baryons. Our notation for baryons is that $-A_{ij,k}$ is the coefficient of the term $\lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j m^2/(m_i m_j)$ in the Hamiltonian. The values of A which can be obtained from the existing data [8] are given in Table III for mesons and in Table IV for baryons.

We see from Table III that, although the values of A_{12} are approximately constant for mesons containing at least one q quark, the values change considerably for mesons containing only heavier quarks. From Table IV, we see that we have only data for baryons containing at least one q quark and no quarks heavier than the s. For these baryons, the values of $A_{ij,k}$ are approximately constant, but a different constant than for mesons containing only q and s quarks. Thus, the assumption of constant A is not borne out by experiment. We expect that the values of A will be still different for exotic mesons.

We next consider the annihilation of a proton by an anti-Ω baryon at rest in the reaction

$$\bar{\Omega} + p \to K^+ + K^+ + K^0 \ . \tag{4}$$

(We choose this reaction because it does not have any pions in the final state.) The reaction (2) has not been seen, but its observation is not necessary for the purposes of our test of the simple model. We know the masses of the particles involved. Therefore, we can compare the value of δ calculated from the experimental masses with the value calculated from the Hamiltonian (1) with the parameters used by Silvestre-Brac. We obtain

$$\begin{aligned} &\delta(\text{experiment}) = 1124 \text{ MeV} \ , \\ &\delta(\text{model}) = 474 \text{ MeV} \ . \end{aligned} \tag{5}$$

Thus, the simple model gets the threshold wrong by 650 MeV.

If we assume, like Silvestre-Brac, that spin cannot be converted into orbital angular momentum, then the reaction (4) is forbidden. Instead, the reaction with the lowest-mass final state is

$$\bar{\Omega} + p \to K^{*+} + K^+ + K^0 \ . \tag{6}$$

In this case we have

$$\begin{aligned} &\delta(\text{experiment}) = 726 \text{ MeV} \ , \\ &\delta(\text{model}) = 218 \text{ MeV} \ . \end{aligned} \tag{7}$$

TABLE I. Spin-averaged meson masses M_{12} from Ref. [6] and values of E_{12} using Eqs. (2) and (3). Masses are in MeV.

Quark content	Mass M_{12}	Energy E_{12}
$q\bar{q}$	612	−48
$q\bar{s}$	793	−87
$s\bar{s}$	936	−164
$q\bar{c}$	1974	−6
$q\bar{b}$	5313	268
$s\bar{c}$	2075	−125
$s\bar{b}$	5397	132
$c\bar{c}$	3068	−232
$b\bar{b}$	9446	16

TABLE II. Spin-averaged baryon masses M_{123} and values of E_{123} using Eqs. (2) and (3). Masses are in MeV.

Quark content	Mass M_{123}	Energy E_{123}
qqq	1086	96
qqs	1270	60
qss	1433	3
sss	1587	−63
qqc	2448	138
qqb	5810	435
qsc	2586	56
ssc	2736	−14

TABLE III. Strength A_{12} in MeV of the color-magnetic interaction in selected mesons. The results are adapted from Ref. [4].

Quark content	Vector	Pseudoscalar	Strength A_{12}
$q\bar{q}$	$\rho(770)$	$\pi(138)$	30
$q\bar{s}$	$K^*(894)$	$K(496)$	31
$q\bar{c}$	$D^*(2009)$	$D(1867)$	33
$q\bar{b}$	$B^*(5325)$	$B(5279)$	31
$s\bar{c}$	$D_s^*(2110)$	$D_s(1969)$	55
$c\bar{c}$	$J/\psi(3097)$	$\eta_c(2980)$	137

TABLE IV. Strength $A_{ij,k}$ in MeV of the color-magnetic interaction in selected baryons. The results are adapted from Ref. [4].

Quark content	Spin-$\frac{3}{2}$ baryon	Spin-$\frac{1}{2}$ baryon	Spin-$\frac{1}{2}$ baryon	$A_{12,3}$	$A_{13,2}$	$A_{23,1}$
qqq	$\Delta(1232)$	$N(939)$	[a]	18	18	18
qqs	$\Sigma^*(1385)$	$\Sigma(1193)$	$\Lambda(1116)$	19	20	20
ssq	$\Xi^*(1533)$	$\Xi(1318)$	[a]		22	22

[a]State does not exist.

In this case, the model is "only" off by about 510 MeV. But we see no reason why the reaction (4) should be inhibited by the presence of orbital angular momentum when the energy released is so high (1124 MeV).

As still another example, we consider the reaction

$$\pi+\Upsilon\rightarrow B^*+\bar{B}\ . \tag{8}$$

Again comparing the value of δ from experiment with the calculated value, we obtain

$$\delta(\text{experiment})=-1010\text{ MeV}\ ,$$
$$\delta(\text{model})=-304\text{ MeV}\ , \tag{9}$$

a difference of 706 MeV. Even if we allow an extra -200 MeV for the pion in the initial state, the calculated threshold is still off by 506 MeV.

Our last example is the simplest. Whereas experimentally the splitting between the J/ψ and η_c is 117 MeV, the model predicts it to be only 17 MeV.

We have no reason to believe that the Hamiltonian (1) is any better for exotic hadrons than it is for the normal hadrons in our examples. Quite the contrary; because so little is known about exotics we expect the uncertainties to be greater. In conclusion, in view of the fact that the model gets thresholds for normal hadron reactions wrong by as much as 650 MeV in our examples, we do not regard predictions that some exotic mesons are bound by about 100 MeV as any indication that these exotics are in fact quasistable.

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