COMMENTS

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Comments on the metastable states of bosonized QED around a large-Z nucleus

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The bosonization technique has been recently applied by Hirata and Minakata to the study of supercritical QED around a large-Z nucleus. New charge-neutral metastable states emerge from the spectrum of the theory and their existence represents a possible explanation of the e^+e^- peaks observed in heavy-ion collisions. Actually, we suspect that such metastable states are a mere product of the approximations introduced in the treatment of the bosonized Hamiltonian. In order to support our conjecture about the origin of the new states we show that they are not suppressed by an external field approximation where both the fermion-fermion interaction and the quantum fluctuations of the electromagnetic field are removed from the bosonized formulation of QED.

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I. INTRODUCTION

In a series of interesting papers [1–5] Hirata and Minakata have proposed a stimulating explanation of the e^+e^- peaks observed in heavy-ion collisions [6–11]. It all lies within QED and it is obtained in the framework of a partial-wave bosonized QED. In such a scheme it is possible to go beyond the external field approximation and to take into account, at least partially, the quantum fluctuations of the electromagnetic field. The form of the bosonized Hamiltonian is quite involved and its spectrum can be found only at the cost of many severe approximations. Once simplified the theory predicts the existence of new neutral metastable states which are interpreted as arising from the nonperturbative aspects of QED. The energy and the width of these states suggest that they might be the cause of the narrow e^+e^- peaks observed in heavy-ion collisions. Actually, we suspect that such states would be ruled out by an improved analysis of the bosonized Hamiltonian. In other words we think that they originate from the various approximations introduced in Refs. [4,5]. In order to show this we shall bosonize the QED Hamiltonian in the background or external field approximation, where both the electron-electron interaction and the quantum fluctuations of the electromagnetic field are omitted; since this problem can be solved by means of more conventional techniques [12,13] it will be straightforward to check whether the approximations adopted in Refs. [4,5] do introduce wrong states in the spectrum of the system.

II. BOSONIZED QED IN THE EXTERNAL FIELD APPROXIMATION

QED with an external charge density $Ze\rho(\mathbf{r},t)$ is described by the Lagrangian density

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\bar{\psi}(i\gamma^\mu\partial_\mu+e\gamma^\mu A_\mu-m_e)\psi-Ze\rho(\mathbf{r},t)A_0\ , \tag{1}$$

with $\rho(\mathbf{r},t)$ normalized to unity: $\int d^3\mathbf{r}\,\rho(\mathbf{r},t)=1$. In Refs. [1,4,5] a spherically symmetric source is considered, the higher partial waves of the fields are omitted, and only the s-wave electromagnetic field and the ($j=\frac{1}{2}$)-wave spinor field are retained. As a result the theory is cast into the form of an effective two-dimensional fermionic theory. The bosonization technique [14–16] can then be used to obtain the corresponding two-dimensional boson theory which is described by the Hamiltonian

$$\mathcal{H}=\int_0^\infty dr\,(\mathcal{H}_0+\mathcal{H}_{\rm int})\ , \tag{2a}$$

with

$$\mathcal{H}_0=\sum_m \frac{1}{2}(\Pi_m^2+P_m^2+\Phi_m'^2+Q_m'^2)+\sum_{m\delta}\frac{1}{2\pi r^2}\left[1-\cos\sqrt{\pi}\left[\Phi_m+Q_m-\delta\int_r^\infty ds[\Pi_m(s)-P_m(s)]\right]\right]$$
$$+\sum_m\frac{\pi}{4}m_e^2[2-\cos(2\sqrt{\pi}\Phi_m)-\cos(2\sqrt{\pi}Q_m)]+\frac{e^2}{4\pi\sqrt{\pi}r^2}C(r)\sum_m(\Phi_m+Q_m)\ , \tag{2b}$$

$$\mathcal{H}_{\rm int}=\frac{e^2}{8\pi r^2}\left[\frac{1}{\sqrt{\pi}}\sum_m(\Phi_m+Q_m)\right]^2 . \tag{2c}$$

The fields Φ_m and Q_m are boson fields living in a (t,r) universe with $r\geq 0$. Π_m and P_m denote their canonical momenta. The index m $(=\pm 1)$ represents the z component of the angular momentum, δ $(=\pm 1)$ corresponds to the chirality, and $C(r,t)$ is defined as

$$C(r,t)=4\pi Z\int_0^r r'^2\rho(r',t)dr' . \tag{3}$$

The splitting [Eqs. (2)] of the bosonized Hamiltonian density has been introduced for later convenience. Its physical meaning will be clarified in the following.

As anticipated in the introduction, we now consider the external or background field approximation. We also assume a time-independent external source. The Lagrangian density of QED is now given by

$$\mathcal{L}_{\rm ext}=\bar{\psi}(i\gamma^\mu\partial_\mu-m_e)\psi+eA_0\bar{\psi}\gamma^0\psi , \tag{4}$$

where A_0 is the external potential. A straightforward application of the bosonization technique gives

$$H_{\rm ext}=\int_0^\infty dr\,\mathcal{H}_0 . \tag{5}$$

We see that the effect of the external field approximation is to remove from H the term $\mathcal{H}_{\rm int}$ which so describes the quantum fluctuations of the s-wave electromagnetic field and the electron-electron interaction.

To explore the spectrum of $H_{\rm ext}$ we now follow closely the methods suggested in Refs. [1,4,5]. We first look for the configuration $(\Phi_{\rm cl},Q_{\rm cl})$ which minimizes $H_{\rm ext}$ by solving the classical equation of motion. We take the symmetric ansatz $\Phi_{\rm cl}=Q_{\rm cl}$ and we work in the approximation of vanishing canonical momenta $\Pi_{\rm cl}=P_{\rm cl}=0$; see Refs. [1,4,5] for details. As expected we find two local minima corresponding to the neutral and charged vacuum, respectively. The energies of these vacua are plotted in Fig. 1 as a function of the central charge Z. In our external field approximation the transition from the neutral-undercritical vacuum to the charged-supercritical

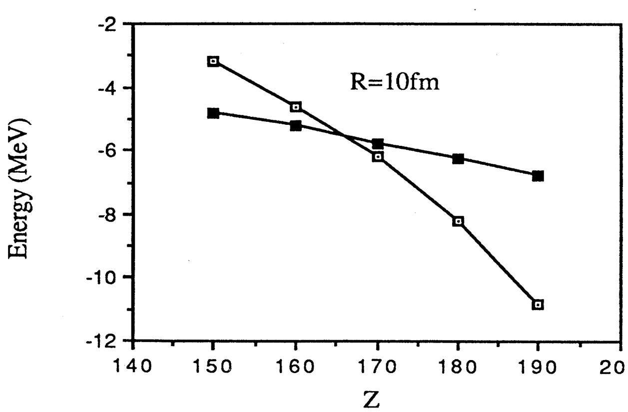

FIG. 1. The energies of the normal (solid square) and the supercritical (open square) vacua are plotted as functions of the nuclear charge Z. The external source is a uniformly charged sphere of radius $R=10$ fm.

one takes place at $Z_{\rm cr}\sim 170$. This value agrees with that of Ref. [5] and with the results obtained by more conventional tools [12,13].

In order to study the dynamics of our system, we expand the Bose fields around their background configuration:

$$\Phi_m=(\Phi_m)_{\rm cl}+\phi_m,\quad Q_m=(Q_m)_{\rm cl}+q_m ,$$
$$\Pi_m=\pi_m,\quad P_m=p_m ,$$

where the small letters represent the quantum fluctuations. Correspondingly, the Hamiltonian $H_{\rm ext}$ is expanded around its minimum up to the quadratic terms of the small fluctuations. It is useful to introduce the fields $\psi_m=\phi_m+q_m$ and $\chi_m=\phi_m-q_m$. As one can easily verify, the ψ_m and χ_m fluctuations decouple under the symmetrical ansatz $\Phi_{\rm cl}=Q_{\rm cl}$. Moreover, only ψ_m is coupled to the charge; then we focus our attention on this mode, freezing out the χ_m degrees of freedom. The effective Hamiltonian for the ψ_m fluctuations reads

$$H_\psi=\frac{1}{2}\sum_m\left[\frac{\partial\psi_m}{\partial t}\right]^2+\left[\frac{\partial\psi_m}{\partial r}\right]^2 + \left[2\frac{f}{r^2}+\pi^2m_e^2\right]\cos[2\sqrt{\pi}\Phi_{\rm cl}(r)]\psi_m^2 \tag{6}$$

and the fields equations are

$$\frac{\partial^2\psi_m}{\partial t^2}-\frac{\partial^2\psi_m}{\partial r^2}+\left[2\frac{f}{r^2}+\pi^2m_e^2\right]\cos[2\sqrt{\pi}\Phi_{\rm cl}(r)]\psi_m=0 . \tag{7}$$

In Eqs. (6) and (7) the constant factor $f=\sqrt{e}$ $(e=2.718\ldots)$ has a nontrivial origin. It is obtained by renormal ordering the bosonized Hamiltonian with respect to the "physical" masses of the problem. On this crucial point the reader should consult Refs. [4,5]. Setting $\psi_m=e^{\pm i\omega t}\phi_m(r)$ we obtain a Schrödinger-type equation for $\phi_m(r)$:

$$-\frac{d^2}{dr^2}\phi_m+V(r)\phi_m=\omega^2\phi_m , \tag{8}$$

where $V(r)$ is given by

$$V(r)=\left[2\frac{f}{r^2}+\pi^2m_e^2\right]\cos[2\sqrt{\pi}\Phi_{\rm cl}(r)] . \tag{9}$$

This potential is very similar to that obtained in Refs. [4,5] and only an additive term $\Delta V\sim e^2/\pi r^2$ has been removed by the external field approximation. In Fig. 2 we plot the potential $V(r)$ for a uniformly charged sphere of radius $R=10$ fm and $Z=170$. Again we found a good agreement with the results of Refs. [4,5]. In particular, even in the external field approximation, the potential develops the pocket structure responsible for the trapping of the boson excitation. If we employ a WKB approximation to solve Eq. (8) we explicitly verify that the first excitation is trapped in the potential well. Both the energy and the width of this state agree with the values found in Refs. [4,5].

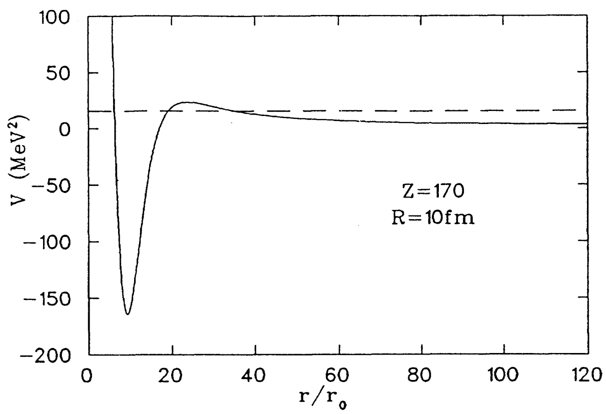

FIG. 2. The potential (9) felt by the small fluctuations is plotted for a uniformly charged sphere of radius $R=10$ fm and $Z=170$. The radial coordinate is measured in r_0 units, r_0 being the classical electron radius. The dashed line corresponds to the energy squared of the state trapped in the potential well.

III. DRAWBACKS IN THE APPROXIMATED TREATMENT OF BOSONIZED QED

We are now in a position to draw some conclusions from the foregoing results. The crucial point is that the metastable states of Refs. [4,5] are still present in our external field approximation, obtained by neglecting the quantum fluctuations of the electromagnetic field and the interaction between the electrons. Within this approximation, the spectrum of QED around a large-Z nucleus has been extensively treated by Greiner, Müller, and Rafelski in Refs. [12,13]. In particular, they have shown that no e^+e^- resonance can appear in the considered system whereas only positron resonances are present in the spectrum of supercritical QED. If we assume their analysis to be correct, we are forced to conclude that the metastable states found in Refs. [4,5] are merely a result of the several approximations introduced there. In order to avoid misunderstandings about this point, we find the following warning appropriate. Some authors have suggested that QED in strong external fields undergoes a transition to a new confining phase [17,18]. In the simplified context we are dealing with there is no room for such a phase since, as we have stressed many times, the fermion-fermion interaction and the quantum aspects of the electromagnetic field have been omitted from the very beginning. Our previous conclusion then remains unaffected, even if a new confining phase of QED actually exists in nature.

It is now useful to identify the approximation which brings the wrong states into the spectrum of QED. As far as the vacuum state is concerned, our results seem to be reasonable. As shown in Fig. 1 the transition from the neutral vacuum to the charged one is clearly reproduced. Moreover, the value of the critical charge $Z_{\rm cr}$ lies in the expected range. It is then natural to search for the bug in the "small fluctuation" approximation, that is in the expansion of the bosonized Hamiltonian up to the quadratic terms of the boson excitations. We now give a simple argument supporting this hypothesis. Let us consider, from a classical point of view, the field equations (7) and let $\xi(r,t)$ be the classical fluctuation defined as

$$\xi(r,t)=e^{-i\omega t}\xi_\omega(r)+e^{i\omega t}\xi^*_\omega(r)\ , \tag{10}$$

where ω is the energy of the trapped state and ξ_ω satisfies Eq. (8). From the effective Hamiltonian (6) we get the classical energy $E_{\rm cl}$ of the fluctuation:

$$E_{\rm cl}=2\omega^2\int_0^\infty dr|\xi_\omega(r)|^2\ . \tag{11}$$

Since $\xi_\omega(r)$ is confined in a region of length $L\sim 50$ fm (the width of the potential well), we can write

$$E_{\rm cl}\cong 2\omega^2\langle|\xi_\omega|^2\rangle L\ , \tag{12}$$

where $\langle|\xi_\omega|^2\rangle$ is the average of $|\xi_\omega|^2$ in the potential well. Consequently,

$$\langle\xi_\omega\rangle\equiv\sqrt{\langle|\xi_\omega|^2\rangle}\cong\left[\frac{E_{\rm cl}}{2\omega^2 L}\right]^{1/2} \tag{13}$$

and this relation gives us a rough estimate of the fluctuation amplitude as a function of its energy $E_{\rm cl}$. In the exact Hamiltonian the boson fields appear as argument of a cosine function. The condition $\langle\xi_\omega\rangle\ll 1$ should then be satisfied in order to rely on the small fluctuation approximation. Using Eq. (13) we obtain

$$E_{\rm cl}\ll 2m_e\left[\frac{\omega}{m_e}\right]^2\left[\frac{L}{r_0}\right]\alpha\ , \tag{14}$$

where r_0 is the classical electron radius. Inserting the numerical values of $L\sim 50$ fm and $\omega\sim 2m_e$, the last inequality boils down to $E_{\rm cl}\ll 2m_e$. Since $2m_e$ is a lower bound for the physical boson excitations, it is very hard to have $\langle\xi\rangle\ll 1$ at the quantum level.

IV. CONCLUSIONS

We have applied the methods developed in Refs. [1–5] to bosonize the lowest partial wave of QED within the external field approximation, where both the electron-electron interaction and the quantum fluctuations of the electromagnetic field are omitted. As a result we have verified that the approximation scheme adopted in Refs. [4,5] introduce nonexisting states in the spectrum of QED, namely, the metastable states assumed to be the origin of the narrow e^+e^- peaks observed in heavy-ion collisions. We have also given a hint to identify the approximation responsible for the described drawback: it is the "small fluctuation" one, that is the expansion of the bosonized Hamiltonian up to the quadratic terms of the boson excitations.

As a by-product of our analysis we have obtained a rather simple expression for $\mathcal{H}_{\rm int}$, the bosonized Hamiltonian density describing the fermion-fermion interaction and the quantum fluctuations of the electromagnetic field. The results of Sec. II suggest that $\mathcal{H}_{\rm int}$ cannot dramatically influence the properties of QED around a large-Z nucleus. This circumstance could be taken as an indication against the hypothesis that QED in strong Coulomb fields undergoes a phase transition [17,18]. Although this conjecture is an exaggerated extrapolation, it agrees with the results of Refs. [19–21] which show that static configurations of background fields are unlikely to trigger the transition to a new confining phase of QED. Obviously, the role played by time-dependent external fields is still an open question.

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