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Relativistic condensate as a source for inflation

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We consider a universe dominated by a relativistic Bose-Einstein condensate with a conserved charge quantum number. We show that the solution of the Einstein equations with the energy-momentum tensor of the condensate consists of an inflationary (de Sitter) expansion of the Universe. The cosmological constant is zero at all times in this model. We also discuss the question of transition from the inflationary to the radiation-dominated stage of the expansion.

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I. INTRODUCTION

Over the last decade, it has been shown that an early period of rapid expansion of the Universe can explain such features as the near spatial flatness of the present Universe and the uniformity of the cosmic background radiation temperature. Various models which include an early period of exponential inflation have been suggested. These include the old and new inflationary models [1] based on grand unification, the chaotic inflationary model [2], and models based on quadratic curvature terms produced by quantum renormalization effects [3]. Various problems and possible solutions have been raised in connection with these models [4,5]. The foremost problem plaguing inflationary models based on grand unification is the fine-tuning necessary to cancel a large early effective cosmological constant, so that the present value is very small. Possible solutions for this problem may require a more complete understanding of quantum gravity [6].

In this paper we suggest a simple inflationary model based on relativistic Bose-Einstein condensation (BEC) [7]. This model differs from the standard inflationary scenarios, since a conserved "charge" is required, rather than a Higgs potential. The cosmological constant is zero in the model. For a system of bosons with temperature higher than the mass, particle-antiparticle pair production processes become important so that the particle number is no longer conserved. If the system of bosons possesses a conserved charge, then the particle number is replaced in the grand canonical ensemble by the charge. (Closely related to charged relativistic BEC are nontopological soliton [8] and Q-ball [9] models. Additional aspects of these models have been studied recently by a number of authors [10].) Building on our previous study [11] of a weakly interacting relativistic BEC, we take a highly ordered relativistic condensate as the dominant matter content in the early Universe and solve the Einstein equations as well as the dynamical equation for the condensate. We find there is an inflationary stage of the expansion, with approximately the scale factor $a(t)\sim e^{Ht}$, and that the condensate decays slowly. Thus we obtain a scenario for the early Universe in which a relativistic condensate drives the de Sitter expansion. The initial condition used in the chaotic inflationary model is assumed as the requirement on the initial density of the condensate. We consider three possible mechanisms for reheating. The radiation era begins when decay of the condensate and reheating result in relativistic matter becoming dominant.

II. INFLATION DRIVEN BY THE CONDENSATE

If a theory possesses certain internal symmetries, such as a global U(1) or SO(2), then the particle system has a conserved charge due to the associated symmetry (not necessarily the electric charge). We consider such a boson system described by the complex massive scalar field with the renormalizable Lagrangian

$$\mathcal{L}=\tfrac{1}{2}\sqrt{-g}\left[-g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi^* -m^2\phi\phi^*-\xi R\,\phi\phi^*-\frac{\lambda}{4!}(\phi\phi^*)^2\right], \quad (1)$$

where R is the scalar curvature, ξ is a dimensionless coupling constant, and λ is a small self-interaction coupling constant. The spacetime background here is taken to be a spatially flat Robertson-Walker (RW) background:

$$ds^2=-dt^2+a^2(t)d\mathbf{x}^2 . \quad (2)$$

The simple model of Eq. (1) can be extended to a family of theories possessing larger global symmetries and having multiple conserved charges. For simplicity, we neglect the ξR term in Eq. (1). In the case of the de Sitter expansion $a(t)=\exp(Ht)$, with constant scalar curvature, this term has the effect of renormalizing m, so that $m^2\rightarrow m^2+12\xi H^2$.

When the charge density q is such that $q \gg m^3$, the critical temperature T_c, below which Bose-Einstein condensation occurs, is relativistic in the sense that $T_c > m$ and is given by [7,11]

$$T_c=\left[\frac{3q}{m}\right]^{1/2} . \quad (3)$$

For a given density q, BEC occurs if the temperature is lower than T_c.

Was the temperature low enough in the early Universe to permit relativistic BEC to occur? The answer to this question depends on the mass m and the free parameter q/n_γ, where q is the total charge number density asymmetry, and n_γ is the photon number density [11]. In units of $\hbar=c=k_B=1$, the photon density n_γ is roughly of the order of the entropy density $s\sim T^3$. If we assume that the charge number asymmetry is the same as the baryon asymmetry, $q\sim 10^{-9}n_\gamma$ [12], then, for example, at temperature $T\sim 10^{16}$ GeV, Eq. (3) gives $T_c\sim 10^{19}$ GeV $\gg T$ for $m\sim 10$ GeV and $T_c\sim 10^{16}$ GeV for $m\sim 10^7$ GeV. Thus, if the mass of the boson involved is sufficiently small, then T_c will be very large, and it is reasonable to assume that in the very early Universe the bosons are in the condensed phase. The condition that $q\gg m^3$, which is necessary for relativistic BEC to occur, is met in the above examples. Even if the total charge is exactly zero, there may be situations in which particles carrying one type of charge are dominant in local regions, so that in these regions condensation may take place.

The idea that the Universe started with a "cold" dense state, with the vacuum expectation value of the scalar field large with respect to the Planck mass M_P, was suggested by Linde [13]. In his scenario spontaneous symmetry breaking took place in the early Universe in the presence of a large fermion charge density.

We follow the suggestion of Linde in taking the initial condition that the condensate order parameter σ, which is defined below, satisfies $\sigma\gg M_P$. We will see below that $\sigma\sim(n/m)^{1/2}$. Therefore, if the initial temperature is of order 10^{16} GeV and $n\sim T^3$, as in the previous examples, then the initial condition $\sigma\gg M_P$ holds if $m\leq 10^8$ GeV. If the initial temperature is larger, the upper limit on m increases as T^3.

The relativistic charged boson system we consider has a temperature T lower than T_c, so that the ground state is macroscopically occupied. We introduce a condensate wave function $\Sigma(t)$ as the $\mathbf{k}=0$ mode of the ϕ field to describe the dominant homogeneous condensate. As in the case of a nonrelativistic condensate wave function [14], we regard $\Sigma(t)$ as a classical macroscopic quantity. This c-number order parameter has the form [11]

$$\Sigma(t)=\sigma(t)e^{-i\alpha(t)} \tag{4}$$

and is large when the system is highly ordered (in momentum space). The quantity $\sigma(t)$ is related to the condensate number density $n(t)$ by $\sigma(t)\sim(n(t)/m)^{1/2}$, and $\dot\alpha$ is related to the condensate charge density q by

$$q=-i(\dot\Sigma^*\Sigma-\dot\Sigma\Sigma^*)=2\sigma^2\dot\alpha\ . \tag{5}$$

The full scalar field is written as

$$\phi(x)=\Sigma(t)+\varphi(x)\ , \tag{6}$$

where the fluctuation $\varphi(x)$ is the $|\mathbf{k}|\neq 0$ part (excited modes) of the quantum field, much smaller in magnitude that $\Sigma(t)$. The fluctuation φ, when quantized, gives rise to an ensemble of quasiparticles having an energy density proportional to T^4, much smaller, as we assumed, than that of the condensate. Thus the energy density and pressure of the boson system are dominated by the condensate contributions

$$\rho=\tfrac{1}{2}\left[\dot\Sigma\dot\Sigma^*+m^2\Sigma\Sigma^*+\frac{\lambda}{4!}(\Sigma\Sigma^*)^2\right]=\tfrac{1}{2}\left[\dot\sigma^2+\dot\alpha^2\sigma^2+m^2\sigma^2+\frac{\lambda}{4!}\sigma^4\right]\ , \tag{7}$$

$$p=\tfrac{1}{2}\left[\dot\Sigma\dot\Sigma^*-m^2\Sigma\Sigma^*-\frac{\lambda}{4!}(\Sigma\Sigma^*)^2\right]=\tfrac{1}{2}\left[\dot\sigma^2+\dot\alpha^2\sigma^2-m^2\sigma^2-\frac{\lambda}{4!}\sigma^4\right]\ . \tag{8}$$

This kind of relativistic condensate stress tensor violates the strong energy condition [11]. Here we investigate the dynamics for both the condensate and spacetime background in the situation when the spacetime background has a large expansion rate $H(t)=\dot a/a$. We will see that there is a solution in which the condensate is slowly decaying, so that the term involving $\dot\sigma^2$ in ρ and p is small. When the $\dot\alpha^2$ term is also small with respect to the remaining terms in ρ and p, then we will have $p\approx-\rho$, which gives rise to inflation. We next show that this situation can occur.

The Einstein equations governing the evolution of a spatially flat RW spacetime background are

$$\left[\frac{\dot a}{a}\right]^2=\frac{8\pi G}{3}\rho\ , \tag{9}$$

$$\frac{\ddot a}{a}=-\frac{4\pi G}{3}(\rho+3p)\ . \tag{10}$$

The equations of motion of the condensate are

$$\ddot\sigma+3H\dot\sigma+m^2\sigma+\frac{\lambda}{12}\sigma^3-\dot\alpha^2\sigma=0\ , \tag{11}$$

$$\ddot\alpha+2\frac{\dot\sigma}{\sigma}\dot\alpha+3H\dot\alpha=0\ . \tag{12}$$

This set of equations determines the development of $a(t)$, $\sigma(t)$, and $\alpha(t)$. Equation (12) can be integrated to give an integral constraint

$$\sigma^2\dot\alpha a^3=\text{const}\ , \tag{13}$$

which is proportional to the charge Q of the condensate in a comoving volume.

A. Case of $q/n\ll 1$

Now assume $q/n\ll 1$. On the other hand, we must require that $q\gg m^3$ for the condensate to exist. Thus q must satisfy $n\gg q\gg m^3$. Our initial condition, that $\sigma\gg M_P$, together with $\sigma\sim(n/m)^{1/2}$, implies that $n\gg mM_P^2$. Thus, if $M_P\gg m$, one has $n\gg m^3$, so that there does exist a range of q satisfying $n\gg q\gg m^3$, for which $q/n\ll 1$.

The coupling term $\dot\alpha^2\sigma$ in Eq. (11) can then be neglected, since $\sigma\sim(n/m)^{1/2}$ and $\sigma^2\dot\alpha=q/2$; so $\dot\alpha^2\sigma\sim m^{3/2}q^{1/2}(q/n)^{3/2}$. Hence this coupling term due to the charge density is much smaller than the particle mass term $m^2\sigma\sim m^{3/2}n^{1/2}$ in Eq. (11), which reduces to

$$\ddot{\sigma}+3H\dot{\sigma}+m^2\sigma+\frac{\lambda}{12}\sigma^3=0 \ . \tag{14}$$

In the same approximation, the charge term $\dot{\alpha}^2\sigma^2 \sim mq(q/n)$ in the energy and pressure of Eqs. (7) and (8) can also be neglected in comparison with the rest-mass density term $m^2\sigma^2$. Now Eqs. (7) and (8) reduce to

$$\rho=\frac{1}{2}\left[\dot{\sigma}+m^2\sigma^2+\frac{\lambda}{4!}\sigma^4\right] , \tag{15}$$

$$p=\frac{1}{2}\left[\dot{\sigma}^2-m^2\sigma^2-\frac{\lambda}{4!}\sigma^4\right] . \tag{16}$$

We can solve the set of equations (9)–(16) through two limiting cases.

1. Rest-mass dominant

First, consider the case when the self-interaction energy density is small with respect to the rest-mass energy density, $m^2 \gg (\lambda/4!)\sigma^2$. Then the self-interaction is sufficiently weak that the λ term in (14)–(16) can be neglected, and Eq. (14) is approximated by

$$\ddot{\sigma}+3H\dot{\sigma}+m^2\sigma=0 \ . \tag{17}$$

Assume that $|\dot{H}| \ll H^2$. Then, on the time scale of spacetime expansion $1/H$, H is treated as a constant, and Eq. (17) has two decaying solutions $\sigma_1(t)=\sigma_{10}e^{-\Gamma_1 t}$ and $\sigma_2(t)=\sigma_{20}e^{-\Gamma_2 t}$, where $\Gamma_1=\frac{3}{2}H-(\frac{9}{4}H^2-m^2)^{1/2}$ and $\Gamma_2=\frac{3}{2}H+(\frac{9}{4}H^2-m^2)^{1/2}$.

Now we assume that during the early period the condensate density is extremely high so that the condensate satisfies $\sigma \gg M_P$. Thus Eq. (9) gives $H \gg m$, which yields

$$\Gamma_1 \simeq \frac{1}{3}\left[\frac{m^2}{H}\right] \ll m \ , \tag{18}$$

and $\Gamma_2 \simeq 3H - m^2/3H \simeq 3H$. Thus the first characteristic decay time $1/\Gamma_1$ is much greater than the RW spacetime expansion scale time $1/H$, giving a slow-decay mode. The second characteristic decay time $1/\Gamma_2$ is smaller than $1/H$, giving a fast-decay mode.

Because of the Einstein equation, a solution for σ which is dominated by the fast-decay mode is not consistent with the above assumption that $|\dot{H}| \ll H^2$. Therefore a consistent solution exists only if $\sigma_{10} \gg \sigma_{20}$. After a relatively short time, only the slowly decaying solution $\sigma_1(t)$ will remain. Then Eq. (15) reduces to $\rho \simeq \frac{1}{2}m^2\sigma^2=\frac{1}{2}m^2\sigma_0^2 e^{-2\Gamma_1 t}$. This slowly decaying energy density determines, through Eq. (9), a slowly decaying Hubble's constant $H(t)=H_0 e^{-\Gamma_1 t}$ with $H_0=(\sqrt{4\pi/3})m\sigma_0/M_P$. This solution is consistent with the assumption $|\dot{H}| \ll H^2$ because $\sigma \gg M_P$. Then the scale factor is found from Eq. (9):

$$a(t)=a_0\exp\left[\int_0^t H(t')dt'\right] =a_0\exp\left[\frac{H_0}{\Gamma_1}(1-e^{-\Gamma_1 t})\right] ;$$

since we have $\Gamma_1 \ll m \ll H$, this reduces to the de Sitter expansion $a(t)=a_0 e^{H_0 t}$ in the early expanding Universe.

2. Self-interaction dominant

In the second case, the self-interaction potential-energy density dominates over the rest-mass energy density $(\lambda/4!)\sigma^2 \gg m^2$. Equation (14) reduces to

$$\ddot{\sigma}+3H\dot{\sigma}+\frac{\lambda}{12}\sigma^3=0 \ . \tag{19}$$

This equation is analogous to that in the chaotic inflationary model [2]. From Eq. (9) follows $H=(\sqrt{\pi\lambda/18})\sigma^2/M_P$, which is substituted into Eq. (19). Using the so-called "slow rolling" approximation [2], one neglects the $\ddot{\sigma}$ term of Eq. (18) and obtains the equation

$$\dot{\sigma}+\gamma\sigma=0 \ ,$$

yielding a decaying condensate solution

$$\sigma=\sigma_0 e^{-\gamma t} \ ,$$

where

$$\gamma=\frac{\lambda^{1/2}M_P}{6\sqrt{2\pi}} \ . \tag{20}$$

Then Eq. (9) with $\rho \sim \lambda\sigma^4/2\times 4!$ gives $H=H_0' e^{-2\gamma t}$, where $H_0'=(\sqrt{\pi/18})\lambda^{1/2}\sigma_0^2/M_P$. And the scale factor is obtained:

$$a(t)=a_0\exp\frac{H_0'}{2\gamma}(1-e^{-2\gamma t}) \ ,$$

where $a_0=a(0)$. For the condensate $\sigma(t)$ to decay slowly requires that $\gamma \ll H_0'$. This implies that the initial condensate must satisfy $\sigma_0 \gg M_P$, which is the same as the condition on σ given in Sec. II A 1. For t small with respect to γ^{-1}, the scale factor $a(t) \approx a_0 e^{H_0' t}$.

B. Case of $q=n$

Now consider the case $q=n$. That is, no antiparticles are present, and so the charge number density equals the particle number density. Then, by $\sigma \sim (n/2m)^{1/2}$ and $2\sigma^2\dot{\alpha}=q$, we have $\dot{\alpha} \approx m$, and if λ is not very small, then Eq. (11) becomes the same as Eq. (19). Therefore, as in the case when the self-interaction is dominant, one finds a slowly decaying condensate and an inflationary expansion.

Thus, in both cases ($q/n \ll 1$ and $q=n$), the model predicts a de Sitter expansion as well as a slowly decaying condensate. Note also that in the case when inflation occurs the strong energy condition is violated by the condensate because $\rho+3p \simeq -2\rho < 0$.

Now one naturally asks the question why the condensate $\sigma(t) \propto \sqrt{n(t)/m}$ decays slowly (compare γ or Γ_1 with H). Multiplying Eq. (11) by $\dot{\sigma}$, making use of the law of conservation of charge in Eq. (12), we can put it in the form

$$\frac{d}{dt}\left[\frac{1}{2}\left[\dot\sigma^2+\dot\alpha^2\sigma^2+m^2\sigma^2+\frac{\lambda}{4!}\sigma^4\right]\right]=-3H(\dot\sigma+\dot\alpha^2\sigma^2)\ ,$$

which by Eqs. (7) and (8) is the conservation law of energy:

$$\frac{d}{dt}\rho=-3H(\rho+p)\ . \tag{21}$$

Here the large negative pressure of the condensate does work on the condensate as the Universe expands. This keeps ρ nearly constant.

III. EXCITATION PRODUCTION AND OTHER REHEATING MECHANISMS

We now consider the possibility of reheating and termination of the de Sitter stage. In one scenario, quasiparticles are produced by excitation of the condensate during the early stages of its decay. This excitation is produced through coupling of the condensate to the excited modes of the field as a result of the quartic self-interaction. There is also production of bosons by the change in the form of the metric as the condensate σ decays. After the condensate has decayed away, one is left with a relativistic boson gas. The model itself does not exclude the possibility of scalar bosons which make up the condensate decaying into light species of particles, such as fermions [12]. Indeed, this is another mechanism by which reheating could occur. However, we will see that if the condensate field σ remains large with respect to the Planck mass M_P during the inflationary stage, then the creation of bosonic excitations dominates over the creation of light particles as a result of the decay of the condensate bosons.

In order to investigate the excitation effects, let us write the full field operator as in Eq. (6). We also assume that the fluctuation field is small compared with the condensate field, $|\varphi| \ll \sigma(t)$. Then, up to order of φ^2, the equations of motion for the condensate and fluctuation are approximately given by

$$\ddot\sigma+3H\dot\sigma+m^2\sigma+\frac{\lambda}{12}\sigma^3-\dot\alpha^2\sigma+\frac{\lambda}{6}\langle\varphi^2\rangle\sigma=0\ , \tag{22}$$

$$\ddot\varphi+3H\dot\varphi-\nabla^2\varphi+m^2\varphi+\frac{\lambda}{6}\sigma^2\varphi=0\ . \tag{23}$$

In Eq. (22), $\langle\varphi^2\rangle$ is the thermal average of the fluctuation field.

In the following we consider the case $(1/4!)\lambda\sigma^2 \gg m^2$. Equation (22) differs from Eq. (11) by the $(\lambda/6)\langle\varphi^2\rangle\sigma$ term, which is the coupling between the condensate and fluctuation. The effects of such a term have been discussed in Ref. [15] for the Higgs field in the standard inflationary model. It has two effects. One is to modify the mass parameter by a temperature-dependent term. Another effect is to provide a dissipation term through the effect of the time dependence of σ producing bosons whose energy diffuses into lighter particles through decay, thus producing entropy and irreversibility. That is, Eq. (22) can be put in the form

$$\ddot\sigma+3H\dot\sigma+m_R^2\sigma+\frac{\lambda}{12}\sigma^3-\dot\alpha^2\sigma+F(\sigma)\dot\sigma=0\ , \tag{24}$$

where $m_R^2=m^2+(\lambda/24)T^2$, with m being understood as the renormalized mass (renormalization can also generate a curvature term, which renormalizes the mass during de Sitter stage), and $F(\sigma)$ is the dissipation coefficient, which represents the decay of condensate bosons into light particles of other species. For example, if the boson field couples to the fermion field ψ through the interaction

$$g\sigma\bar\psi\psi\ ,$$

with g being the coupling constant, then the condensate bosons can decay into fermions with the dissipation coefficient [15]

$$F(\sigma)\approx 10^{-4}\lambda^{3/2}g^2\sigma\ . \tag{25}$$

This dissipation term serves as a damping term in Eq. (22). Note that the $F(\sigma)\dot\sigma$ term is not time-reversal invariant and gives rise to dissipation, while the $3H\dot\sigma$ term in Eq. (24) is time-reversal invariant and comes from the expansion of the comoving volume.

Now we consider the reheating temperature $T_{\rm RT}$, which is roughly speaking, the thermal temperature of the light particle, for example, the fermions, created in the boson decay processes. This problem has been examined in Refs. [16] and [12] for the Higgs field in the standard inflationary models. Since the mathematical structure of Eq. (24) is similar, one obtains, as in those models [12],

$$T_{\rm RT}\approx 0.5[F(\sigma)M_P]^{1/2}\ . \tag{26}$$

We next compare this $T_{\rm RT}$ with the temperature of excitations that are created at the end of the de Sitter expansion.

The particle production occurring from Eq. (23) as a result of transition from the inflationary to the radiation-dominated expansion can be obtained from the results of Ford and Panthinayake [17,18] (see also Ref. [19]). In the simple model of a sudden transition from the de Sitter to the radiation-dominated power-law expansion, the energy density of the created excited modes is of the order [17,18] of

$$\rho_{\rm ex}\approx 10^{-2}H^4\approx\lambda^2\sigma^8/M_P^4\ , \tag{27}$$

where H and σ refer to the de Sitter stage of the expansion just before transition to the power-law expansion. The corresponding equivalent temperature is of the order of [17,18]

$$T_{\rm ex}\approx\lambda^{1/2}\sigma^2/M_P\ . \tag{28}$$

This effect is due to the creation of particles by the gravitional field of the expanding Universe.

Another term creating excitations in Eq. (23) is $(\lambda/6)\sigma^2\varphi$, which represents a coupling between the zero-momentum condensate σ and the excited modes φ. A discussion of the excitation production due to this coupling has been given in Ref. [20]. It is found that because of the slow damping of the condensate, the resulting excitations have a Bose-Einstein distribution function with the effective temperature

$$T_e \approx \gamma \ , \tag{29}$$

where γ is the condensate decay constant given earlier in Eq. (20). In the present case, we have $\lambda\sigma^2 \gg m^2$, so that, by Eq. (20),

$$T_e \approx 10^{-1}\lambda^{1/2} M_P \ . \tag{30}$$

Recall that, initially we had $\sigma_0 \gg M_P$. Comparing the two excitation temperatures $T_{\rm ex}$ and T_e, we see that if σ is still sufficiently large just before transition occurs, then

$$T_{\rm ex} \gg T_e \ . \tag{31}$$

Since the condensate decay constant γ is small compared to H, σ may remain large after a long period of inflation.

Now we can compare the reheating temperature $T_{\rm RT}$ caused by the $F(\sigma)\dot{\sigma}$ term in Eq. (24) with the excitation temperature $T_{\rm ex}$. By Eqs. (25) and (26), we have

$$\begin{aligned} T_{\rm RT} &\approx 10^{-2}\lambda^{3/4} g(\sigma M_P)^{1/2} \\ &\approx 10^{-2}\lambda^{1/4} g(M_P/\sigma)^{3/2} T_{\rm ex} \ll T_{\rm ex} \ . \end{aligned} \tag{32}$$

Since $\lambda \ll 1$, and $g \approx 1$, if the condition $M_P/\sigma \leq 1$ holds just before transition, then $T_{\rm RT} \ll T_{\rm ex}$. In fact, in that case the temperature T_e produced as a result of the decay of the condensate σ, and given in Eq. (30), clearly also satisfies $T_{\rm RT} \ll T_e$.

Then the temperature of excitations created by the damping of the condensate, T_e, is higher than the reheating temperature $T_{\rm RT}$ of relativistic light particles created by the decay of condensate bosons, but is smaller than the temperature $T_{\rm ex}$ resulting from the creation of particles due to the change in the gravitational field at the end of the inflationary era. Note that one has

$$T_{\rm ex} \geq 10^{15} \ {\rm GeV} \ , \tag{33}$$

as long as the following condition is satisfied:

$$\lambda^{1/4} \left[\frac{\sigma}{M_P} \right] \geq 10^{-2} \ .$$

We have seen that there are several possible sources of reheating in this model.

It will be interesting to study cosmological density perturbations associated with this charged relativistic BEC model.

The model suggested in this paper presents a scenario for an inflationary stage of the expansion driven by relativistic BEC. Here the Universe starts from a highly ordered cold beginning (in the sense that $T < T_c$) and evolves into a radiation-dominated hot universe.

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