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Lepton masses in an $SU(3)_L \otimes U(1)_N$ gauge model

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The $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ model of Pisano and Pleitez extends the standard model in a particularly nice way, so that, for example, the anomalies cancel only when the number of generations is divisible by 3. The original version of the model has some problems accounting for the lepton masses. We resolve this problem by modifying the details of the symmetry-breaking sector in the model.

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In Refs. [1, 2], two of us proposed a model based on the gauge symmetry

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_N . \tag{1}$$

In those original papers spontaneous symmetry breaking and fermion mass generation are assumed to arise from the vacuum expectation values (VEV's) of three scalar multiplets χ, ρ, and η, which are each triplets under $SU(3)_L$. Here we would like to point out that these scalar multiplets do not give satisfactory masses to the leptons and we resolve the problem by modifying the details of the symmetry-breaking sector. We then verify that this modification does not change the model's attractive feature or its compatibility with experiment.

We first give a brief review of the model. The three lepton generations transform under the gauge symmetry, Eq. (1), as

$$f^a = \begin{pmatrix} {\nu_L}^a \\ {e_L}^a \\ {e^c_R}^a \end{pmatrix}_L \sim (1,3,0) \ , \tag{2}$$

where $a = 1, 2, 3$ is the generation index.

Two of the three quark generations transform identically and one generation, it does not matter which, transforms in a different representation of $SU(3)_L \otimes U(1)_N$. Thus we give the quarks the following representation under Eq. (1):

$$Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ J_1 \end{pmatrix}_L \sim (3, 3, 2/3),$$

$$u_{1R} \sim (3, 1, 2/3), \ d_{1R} \sim (3, 1, -1/3), \ J_{1R} \sim (3, 1, 5/3),$$

$$Q_{2L} = \begin{pmatrix} d_2 \\ u_2 \\ J_2 \end{pmatrix}_L \sim (3, 3^*, -1/3), \tag{3}$$

$$u_{2R} \sim (3, 1, 2/3), \ d_{2R} \sim (3, 1, -1/3), J_{2R} \sim (3, 1, -4/3),$$

$$Q_{3L} = \begin{pmatrix} d_3 \\ u_3 \\ J_3 \end{pmatrix}_L \sim (3, 3^*, -1/3),$$

$$u_{3R} \sim (3, 1, 2/3), \ d_{3R} \sim (3, 1, -1/3), J_{3R} \sim (3, 1, -4/3).$$

One can easily check that all gauge anomalies cancel in this theory. However, note that each generation is anomalous. In fact this type of construction is only anomaly-free when the number of generations is divisible by 3. Thus three generations are singled out as the simplest nontrivial anomaly-free $SU(3)_L \otimes U(1)_N$ model.

We introduce the Higgs field

$$\chi \sim (1, 3, -1), \tag{4}$$

which couples via the Yukawa Lagrangian

$$\mathcal{L}^{\chi}_{\rm Yuk} = \lambda_1 \bar{Q}_{1L} J_{1R} \chi + \lambda_{ij} \bar{Q}_{iL} J_{jR} \chi^* + \text{H.c.} , \tag{5}$$

where $i, j = 2, 3$. If χ gets the VEV

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix} , \tag{6}$$

the exotic charged $5/3$ and $-4/3$ quarks ($J_{1,2,3}$) gain mass and the gauge symmetry is broken:

$$\begin{array}{c} SU(3)_c \otimes SU(3)_L \otimes U(1)_N \\ \downarrow \langle \chi \rangle \\ SU(3)_c \otimes SU(2)_L \otimes U(1)_Y . \end{array} \tag{7}$$

Even though the model has charged $5/3$ and $-4/3$ quarks there will be no fractional charged color singlet bound states, and hence no absolutely stable fractionally

*Electronic address: oscarh@physics.mcgill.ca charged particles in the model.

The usual standard model $\mathrm{U}(1)_Y$ hypercharge is given by

$$Y = 2N - \sqrt{3}\lambda_8 \ . \tag{8}$$

Here λ_8 is the Gell-Mann matrix diag$[1,1,-2]/\sqrt{3}$. The model reduces to the standard model as an effective theory at an intermediate scale.

In the original papers [1, 2], electroweak symmetry breaking and fermion masses were assumed to be due to the scalar bosons

$$\rho \sim (1,3,1), \ \eta \sim (1,3,0). \tag{9}$$

These scalar bosons couple to the fermions through the Yukawa Lagrangians

$$\mathcal{L}^{\rho}_{\text{Yuk}} = \lambda_{1a}\bar{Q}_{1L}d_{aR}\rho + \lambda_{ia}\bar{Q}_{iL}u_{aR}\rho^* + \text{H.c.}, \tag{10}$$

$$\mathcal{L}^{\eta}_{\text{Yuk}} = G_{ab}\bar{f}_{aL}(f_{bL})^c\eta^* + \lambda'_{1a}\bar{Q}_{1L}u_{aR}\eta + \lambda'_{ia}\bar{Q}_{iL}d_{aR}\eta^* + \text{H.c.}, \tag{11}$$

where $a, b = 1, 2, 3$ and $i = 2, 3$. When the ρ gets the VEV

$$\langle\rho\rangle = \begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix}, \tag{12}$$

two up- and one down-type quarks gain mass. The down quark that gets its mass from the ρ is not the isospin partner of the two other up quarks.

If η gets the VEV

$$\langle\eta\rangle = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}, \tag{13}$$

then the remaining quarks get mass. However not all of the leptons get mass. This is because the first term in Eq. (11) is only nonzero when G_{ab} is antisymmetric in the generation indices (a, b). To see this note that the Lorentz contraction is antisymmetric, and the fields are Grassmannian (so that this gives an antisymmetric factor when they are interchanged) and the $\mathrm{SU}(3)_L$ contraction is antisymmetric. Explicitly writing the $\mathrm{SU}(3)_L$ indices the leptonic term in Eq. (11) we have

$$G_{ab}\bar{f}_{iaL}(f_{jbL})^c\eta^*_k\epsilon^{ijk}. \tag{14}$$

We have three antisymmetric factors; hence, only the antisymmetric part of the coupling constants G_{kl} gives a nonvanishing contribution and the mass matrix for the leptons is antisymmetric. A 3×3 antisymmetric mass matrix has eigenvalues $0, -M, M$, so that one of the leptons does not gain mass and the other two are degenerate, at least at the tree level.

The simplest way to remedy this situation is to modify the symmetry-breaking sector of the model. If the leptons are to get their masses at the tree level within the usual Higgs-boson mechanism, then we need a Higgs-boson multiplet which couples to $\bar{f}_L(f_L)^c$. Since

$$\bar{f}_L(f_L)^c \sim (1, 3 + 6^*, 0), \tag{15}$$

then the only scalars which can couple to $\bar{f}_L(f_L)^c$ must transform as a $(1, 3^*, 0)$ or $(1, 6, 0)$ (or the complex conjugate thereof). The simplest choice was the $(1, 3^*, 0)$ option which failed due to the fact that the $3 \times 3 \times 3$ SU(3) invariant is antisymmetric. However the 6 is a symmetric product of 3×3, and it can couple to $\bar{f}_L(f_L)^c$, so it seems that a Higgs-boson multiplet $S \sim (1, 6, 0)$ can give the leptons their masses.

The VEV of S must have the form

$$\langle S\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{v'}{\sqrt{2}} \\ 0 & \frac{v'}{\sqrt{2}} & 0 \end{pmatrix}. \tag{16}$$

Note that when the VEV has this form, it gives the leptons their masses and together with $\langle\rho\rangle, \langle\eta\rangle$ breaks the electroweak gauge symmetry:

$$\begin{aligned} &\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \\ &\quad\downarrow \langle\rho\rangle, \langle\eta\rangle, \langle S\rangle \\ &\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Q. \end{aligned} \tag{17}$$

It is now no longer obvious that the Higgs-boson potential can be chosen in such a way that the all the Higgs-boson fields get their desired VEV's. We must show two things: first that there exists a range of values for the parameters in the Higgs-boson potential such that the VEV's given by Eqs. (6), (12), (13), and (16) give a local minimum; and second that the number of Goldstone bosons that arise from the symmetry breaking in the scalar field sector of the theory is exactly equal to eight. This will ensure that there are no pseudo Goldstone bosons arising from the breaking of a global symmetry in the scalar sector which is larger than the SU(3)⊗U(1) gauge symmetry.

The Higgs-boson potential has the form

$$\begin{aligned} V(\eta,\rho,\chi,S) = {} & \lambda_1[\eta^\dagger\eta - v^2]^2 + \lambda_2[\rho^\dagger\rho - u^2]^2 + \lambda_3[\chi^\dagger\chi - w^2]^2 + \lambda_4[\mathrm{Tr}(S^\dagger S) - v'^2]^2 + \lambda_5[2\,\mathrm{Tr}(S^\dagger S S^\dagger S) - (\mathrm{Tr}[S^\dagger S])^2] \\ & + \lambda_6[\eta^\dagger\eta - v^2 + \rho^\dagger\rho - u^2]^2 + \lambda_7[\eta^\dagger\eta - v^2 + \chi^\dagger\chi - w^2]^2 \\ & + \lambda_8[\chi^\dagger\chi - w^2 + \rho^\dagger\rho - u^2]^2 + \lambda_9[\eta^\dagger\eta - v^2 + \mathrm{Tr}(S^\dagger S) - v'^2]^2 \\ & + \lambda_{10}[\rho^\dagger\rho - u^2 + \mathrm{Tr}(S^\dagger S) - v'^2]^2 + \lambda_{11}[\chi^\dagger\chi - w^2 + Tr(S^\dagger S) - v'^2]^2 \\ & + \lambda_{12}[\rho^\dagger\eta][\eta^\dagger\rho] + \lambda_{13}[\chi^\dagger\eta][\eta^\dagger\chi] + \lambda_{14}[\rho^\dagger\chi][\chi^\dagger\rho] + f_1\epsilon_{i,j,k}\eta_i\rho_j\chi_k + f_2\rho^T S^\dagger\chi + \text{H.c.} \end{aligned} \tag{18}$$

A detailed analysis shows that for all λ's>0 there exist values of f_1 and f_2 such that the potential is minimized by the desired VEV's and such that there are no pseudo Goldstone bosons. The above potential leads exactly to 8 Goldstone bosons which are absorbed by the gauge bosons which acquire a mass. However even without a rigorous analysis one expects such a result to be true for the following reasons. If the terms λ_5, f_1, and f_2 are zero, the above potential is positive definite and zero when

$$\langle\eta\rangle=\begin{pmatrix}v\\0\\0\end{pmatrix},\ \langle\rho\rangle=\begin{pmatrix}0\\u\\0\end{pmatrix},$$
$$\langle\chi\rangle=\begin{pmatrix}0\\0\\w\end{pmatrix},\ \langle S\rangle=\begin{pmatrix}0&0&0\\0&0&\frac{v'}{\sqrt{2}}\\0&\frac{v'}{\sqrt{2}}&0\end{pmatrix}. \tag{19}$$

Hence the above VEVs are a minimum of the potential. Note that the $\lambda_{12},\lambda_{13}$, and λ_{14} terms in the Higgs-boson potential are very important for the alignment of the vacuum, as they imply that the three vectors $\langle\eta\rangle,\langle\rho\rangle$, and $\langle\chi\rangle$ are orthogonal (in the complex three-dimensional mathematical space).

Now allow λ_5 to be nonzero and positive. This term is positive in most of the parameter space of the matrix S. For it to be negative we must have large values of $\mathrm{Tr}[S^\dagger S]$ which in turn would imply large values for terms such as $\lambda_4,\lambda_{10},\lambda_{11}$. Unless we allow fine-tuning of the potential so that $\lambda_4,\lambda_{10},\lambda_{11}$ are very small, the desired S VEV minimizes the potential. However any S VEV such that $\mathrm{Tr}[S^\dagger S]=v'^2$ will minimize the potential and make it zero. Now consider nonzero f_1, f_2. These trilinear terms ensure that the largest continuum symmetry of the scalar potential is SU(3)⊗U(1). In addition to this the f_2 term is linear in S and this term induces a VEV for S proportional to the VEV of $\langle\rho\chi^T\rangle$ which has the desired form [given in Eq. (16)].

The potential in Eq. (18) is the most general SU(3)⊗U(1) gauge invariant, renormalizable Higgs-boson potential for the three triplets and the sextet, which also respects the discrete symmetry

$$\rho\to i\rho,\ \chi\to i\chi,\ \eta\to-\eta,\ S\to-S. \tag{20}$$

If the fermions transform as

$$f_L\to if_L,\ Q_{1L}\to-Q_{1L},\ Q_{iL}\to-iQ_{iL},$$
$$u_{aR}\to u_{aR},\ d_{aR}\to id_{aR}, \tag{21}$$

the entire Lagrangian is kept invariant. This symmetry is important since it prevents the trilinear terms

$$\eta^TS^\dagger\eta \ \text{ and } \ \epsilon^{ijk}\epsilon^{lmn}S_{il}S_{jm}S_{kn} \tag{22}$$

from appearing in the Higgs-boson potential. These terms make analysis of the Higgs-boson potential more complicated and lead to nonzero Majorana neutrino masses. To see that without the discrete symmetry Majorana neutrino masses would occur, define

$$S=\begin{pmatrix}\sigma_1^0&h_2^-&h_1^+\\h_2^-&H_1^{--}&\sigma_2^0\\h_1^+&\sigma_2^0&H_2^{++}\end{pmatrix}. \tag{23}$$

Note that S couples with leptons via the Yukawa Lagrangian

$$2\mathcal{L}_{lS}=-\sum_l G_l[\,(\bar\nu_{lL}^c\nu_{lL}\sigma_1^0+\bar l_L^c l_L H_2^{++}+\bar l_R l_L^c H_1^{--})+(\bar\nu_{lR}^c l_L+\bar l_R^c\nu_L)h_1^+ +(\bar\nu_{lR}^c l_L^c+\bar l_R\nu_{lL})h_2^-+(\bar l_R^c l_L^c+\bar l_R l_L)\sigma_2^0]+\text{H.c.} \tag{24}$$

The neutrino gets a Majorana mass if $\langle\sigma_1^0\rangle\neq0$ and it is this VEV which the symmetry (20), (21) keeps equal to zero by preventing the terms in (22). If we did not impose the symmetry we could always fine-tune $\langle\sigma_1^0\rangle$ to zero, but this is a more unattractive option.

Note that the VEV's of ρ,η, and S break the gauge symmetry while preserving the tree-level mass relation $M_W^2=M_Z^2\cos^2\theta_W$. [Here we define $\sin\theta_W\equiv e/g$, the ratio of the electric charge coupling to the SU(2)$_L$ coupling after χ acquires a VEV.] One way to see this is to note that under the intermediate scale gauge group SU(2)$_L$⊗U(1)$_Y$, the VEV's of ρ and S transform as members of a $Y=1$, SU(2)$_L$ doublet, and the custodial SU(2)$_C$ symmetry is not broken. One can also see this explicitly by working out the vector bosons masses. The mass matrix for the neutral gauge boson is the following in the (W^3,W^8,B_N) basis:

$$M^2=\frac14 g^2\begin{pmatrix}a+b+a'&\frac{1}{\sqrt3}(a-b+a')&-2tb\\\frac{1}{\sqrt3}(a-b+a')&\frac13(a+b+4c+a')&\frac{2}{\sqrt3}t(b+2c)\\-2tb&\frac{2}{\sqrt3}t(b+2c)&4t^2(b+c)\end{pmatrix} \tag{25}$$

with the notation

$$a=2v^2,\ b=2u^2,\ c=2w^2,\ a'=2v'^2,\ t=g_N/g_{\mathrm{SU(3)}_L}\,.$$

We can verify that $\det M^2=0$.

The eigenvalues of the matrix in Eq. (25) are $0,M_Z^2$, and $M_{Z'}^2$, respectively. In the approximation that $c\gg a,b,a',M_Z^2$ and $M_{Z'}^2$ become

$$M_Z^2=\frac{g^2}{4}(a+b+a')\frac{1+4t^2}{1+3t^2}, \tag{26}$$

$$M^2_{Z'} = \frac{g^2}{3}(1+3t^2)c. \tag{27}$$

Hence we can see that $M^2_{Z'}$ is very massive since it depends only on w. In Ref. [2] a lower bound of 40 TeV have been obtained by considering the contribution to the K^0-$\bar{K}^0$ mass difference of the heavy Z'^0.

There is a charged gauge boson with mass given by

$$M^2_W = \tfrac{1}{4}g^2(a+b+a'). \tag{28}$$

Then, as in Ref. [2],

$$\frac{M^2_Z}{M^2_W} = \frac{1+4t^2}{1+3t^2}. \tag{29}$$

In order to get consistency with experimental data Eq. (29) must be numerically equal to $1/\cos^2_W$, where θ_W is the weak mixing angle in the standard electroweak model. This definition of the weak mixing angle is consistent with the previous definition (i.e., $\sin\theta_W = e/g$) at tree level since we can always define

$$t^2 = \frac{s^2_W}{1-4s^2_W} \quad .$$

This shows that at tree level the ρ parameter is equal to one as in the standard model.

We have reviewed the model proposed in [1,2] and have shown how to modify it to yield a realistic lepton mass spectrum at tree level. We have proven that a Higgs-boson potential allowing such a change exists and we have verified that such a modification does not lead to any problems such as a tree level ρ parameter different from one.

The model has many unique features. In particular it is only anomaly-free if the number of generations is a multiple of three. Models of this type deserve our attention and study.

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