Growth of the average multiplicity of particles in high-energy collisions

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The data on the growth of the average charged-hadron multiplicity in $p(\bar{p})$-p collisions from $\sqrt{s}=5$ to 900 GeV, and in e^+e^- annihilation from 10 to 90 GeV, are described by the same slowly increasing power of s. The power approaches an asymptotic value of the order of 0.1 only at $\sqrt{s}\sim 1000$ TeV. The ratio of the average multiplicity in e^+e^- annihilation to that in pp collisions is nearly constant at 1.5; thus, the difference between these averages increases. A small but increasing energy fraction tends to be removed from particle production in the extreme fragmentation region; this sustains relatively increasing production of high multiplicities of soft particles.

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The nature of the growth with increasing c.m. energy $\sqrt{s}$ of the average number of particles [1] $\langle n(s)\rangle$ produced in high-energy collisions of hadrons is a feature of the collisions which should provide some insight into the dynamics and, also, insight into the slow possible approach to a domain of asymptotic behavior at extremely high energies. For $p(\bar{p})$-p collisions the usual parametrizations of the data [2], which exists up to $\sqrt{s}=900$ GeV, are with the forms [2,3] $\{A+B\ln s+C(\ln s)^2\}$ and [4] $a+bs^p$, with three (supposedly) constant parameters in each form. These forms are phenomenological and are distinguished by choosing a highest power of $\ln s$ or s as controlling the high-energy growth. In fact, as successively higher domains of $\sqrt{s}$ have been opened up for laboratory measurements, the "constant" parameters in these forms have tended to change. The direction of the change has been to give an increased growth of $\langle n(s)\rangle$ at the highest $\sqrt{s}$. As a recent example of this behavior, one may observe that the value $\langle n(s)\rangle\simeq 25$ at $\sqrt{s}=546$ GeV, which is given by the $(\ln s)^2$ form fit to the original [3] data obtained at the CERN Intersecting Storage Rings (ISR) whose growth curve is shown by the current [2] Particle Data Group Compilation, lies below the UA5 measurement [5,4] at $\sqrt{s}=546$ GeV, $\langle n(s)\rangle\simeq 29\pm 1$. The difference appears to be significant, even allowing for the point that the UA5 measurement [4] refers to the average multiplicity in nonsingle diffractive processes, because comparison of the measurements [6] of this multiplicity and of the inelastic multiplicity up to 63 GeV indicates that the difference between the former and the latter is about one unit. Also, as higher domains of $\sqrt{s}$ have been studied, four specific properties have been observed in hadronic collisions, which are related to the growth of the global quantity $\langle n(s)\rangle$.

(1) The multiplicity distribution continuously broadens, in particular, at high multiplicities n. In terms of the scaled variable $z=n/\langle n(s)\rangle$, probabilities increase for large z as $\sqrt{s}$ increases. In fact, the high-multiplicity part of the distribution approaches a near-exponential behavior [7], $P_n\propto e^{-k(s)z}$, controlled by a single parameter $k(s)$, which decreases from ~ 10 at $\sqrt{s}\simeq 30$ GeV to ~ 4 at $\sqrt{s}=900$ GeV.

(2) The growth of $\langle n(s)\rangle$ deviates markedly from the simplest hypothesis [8], which has it growing like the kinematic extent of a developing rapidity plateau, i.e., $d\sigma/dy_{\rm c.m.}\sim$const, out to $y_{\rm c.m.}\sim\ln s$, where $d\sigma/dy$ is an inclusive, single-particle cross section. Because of the ISR measurements [3] and continuing [4] up to 900 GeV, one knows that the stronger growth of $\langle n(s)\rangle$ arises largely from a multitude of soft pions produced near $|y_{\rm c.m.}|\sim 0$. That is, the central particle "density"

$$\rho(y_{\rm c.m.}\sim 0,s)=[1/\sigma_{\rm inel}(s)]\int d^2\mathbf{p}_{\rm T}d^3\sigma/d^2\mathbf{p}_{\rm T}dy_{\rm c.m.}$$

grows relatively strongly; in fact, a fit to a simple power form $\rho(0,s)\simeq 0.74s^{0.105}$ has been given [4] for this measured quantity in the domain $\sqrt{s}\sim 10$–900 GeV.

(3) The entire $p(\bar{p})$-p collision system is becoming "blacker" over the impact-parameter plane [9,10]. The direct experimental evidence for this is the very slowly increasing ratio $\sigma_{\rm el}(s)/\sigma_{\rm inel}(s)$. This has reached [11] ~ 0.23 at 1800 GeV, which is still far from a fully black disk [12].

(4) A new quantitative result [13] gives the cross section for single diffractive dissociation increasing much more slowly than $\sigma_{\rm el}(s)$, leveling off in the TeV range and eventually decreasing far up in the TeV range. This is in accord with present, limited experimental indications [14,15,4], and the eventual decrease is a necessary theoretical consequence of the original [16] quantum-mechanical mechanism for diffraction dissociation of a structured projectile, when the collision system becomes increasingly black.

Given the experimental facts (1)–(4), a natural question which arises concerns the behavior of the particle density near [17] to $|x|=1$, i.e., the validity of Feynman's scaling hypothesis [8] near the boundary of the fragmentation region. On this issue, present experimental data are less clear. However, a recent experiment [18] indicates that if one considers the normalized quantity $(1/\sigma_{\rm inel})d\sigma/dy$ near the kinematic boundary $y\sim y_{\rm beam}$, there is a decrease with increasing $\sqrt{s}$, this being due, at the least, to the 30% growth in $\sigma_{\rm inel}(s)$ in the big jump from $\sqrt{s}=63$ to 630 GeV. In addition, there are long-standing and recently emphasized [19] indications from cosmic-ray interactions in the domain of $\sqrt{s}$ greater than several TeV

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for a marked decrease of the particle density in the extreme forward region from the collisions. From points (2)–(4) above, one might expect that a little energy fraction is gradually being removed from the particle density in the region near $|x|=1$ and is being utilized for a markedly increased production of soft particles from the blackening collision system. Our purpose here is to utilize this physical idea as the basis for obtaining a new form for the growth of $\langle n(s)\rangle$. Apart from a factor of $(\ln s)$ due to the kinematic extent of an approximate rapidity plateau, this form involves simply a power of s with the power slowly but continuously increasing from zero up to an asymptotic value of about 0.1. With a single fixed parameter which controls the growth of this power, we are able to represent the data on $\langle n(s)\rangle$ from $\sqrt{s}=5$ up to 900 GeV. The form then gives new predictions for the TeV energy range. We also apply these ideas to the average multiplicity from e^+e^- annihilation, describing the data from $\sqrt{s}\simeq 10$ to 90 GeV with the same parameter. We give predictions for the energy to be reached by the CERN e^+e^- collider LEP II, $\sqrt{s}\sim 200$ GeV, and for 500 GeV, while comparing the growth with that from a phenomenological form which contains an exponential in $\sqrt{\ln s}$.

The average multiplicity and inelastic cross section are formally related to the (invariant) inclusive cross section via

$$\langle n(s)\rangle \sigma_{\text{inel}}(s)=\int_{\text{all phase space}}(E\, d^3\sigma/d^3\mathbf{p})d^3\mathbf{p}/E \ . \qquad \text{(1a)}$$

Thus

$$\langle n(s)\rangle = C(s)\int_{-1}^{1} dx \Big/ \left[x^2+\left[\frac{4\lambda}{s}\right]\right]^{1/2} = C(s)\ln(s/4\lambda)\ , \qquad \text{(1b)}$$

with

$$4\lambda \simeq \langle 4(m_{\pi,K}^2+\langle p_T^2\rangle_{\pi,K})\rangle \sim (1\ \text{GeV}/c)^2$$

and

$$C(s)=(1/\sigma_{\text{inel}})\int (d^3\sigma/dy\, d^2\mathbf{p}_\text{T})d^2\mathbf{p}_\text{T}\ .$$

Consider formally separating the "kinematic" factor in Eq. (1b) into two contributions:

$$2\left\{\int_0^{(1-\delta a)} dx \Big/ \left[x^2+\frac{4\lambda}{s}\right]^{1/2}\right\} + 2\int_{(1-\delta a)}^{1} dx \Big/ \left[x^2+\frac{4\lambda}{s}\right]^{1/2} \sim \{\ln(s/4\lambda)-2\ln|1-\delta a|\}+2\ln|1-\delta a|\ , \qquad (2)$$

with $\delta a \ll 1$ initially. The last term in Eq. (2) is to be considered (to within a factor C) as the average multiplicity arising from the extreme fragmentation region, i.e., within δa of $|x|=1$. The energy going into these fast fragments [20] is

$$\sim\sqrt{s}\int_{(1-\delta a)}^{1} x\, dx \Big/ \left[x^2+\frac{4\lambda}{s}\right]^{1/2} \cong \sqrt{s}\ \delta a\ . \qquad (3)$$

We suggest that the probability for these fragments (i.e., the density in this region of rapidity) is going away as $\sqrt{s}$ increases and that the domain δa in which this occurs (away from $|x|=1$) is slowly but continuously opening up. We replace δa by a parametrized differential element $da=d(s_0/s)^\epsilon$ with $\epsilon>0$ and obtain, at any s, an effective $a(s)$ from

$$a(s)=\int_{s_0}^{s} da=(s_0)^\epsilon \epsilon\int_{s_0}^{s} du/u^{1+\epsilon}=1-(s_0/s)^\epsilon\ . \qquad (4)$$

Thus $a(s_0)=0$, and $a(s\to\infty)\to 1$.

We choose an initial [21] energy for the evolution of $\langle n(s)\rangle$ as $\sqrt{s_0}=5$ GeV, which is the first data point on the relevant graph [2] of the Particle Data Group. There is a fixed parameter ϵ. We hypothesize that the energy removed from the fragmentation region sustains a power growth of $\langle n(s)\rangle$, with a power $p(\epsilon,s)$ slowly increasing from zero, to a limiting value as $s\to\infty$. The structure of $\langle n(s)\rangle$ is then taken as

$$\langle n(s)\rangle = A\{\ln s-2\epsilon\ln(s/25)\}\{[1-(25/s)^\epsilon]\sqrt{s}\}^{p(\epsilon,s)}\ , \qquad (5)$$

with

$$\tfrac{1}{2}p(\epsilon,s)=(25)^\epsilon\epsilon\int_{25}^{s} du/u^{1+\epsilon}(\ln u)\ .$$

Thus $p(5\ \text{GeV})=0$.

An overall normalization parameter is A; *a priori* we expect it to be essentially unity and to thus disappear as a parameter, leaving the single parameter ϵ which controls the evolution of the power growth of $\langle n(s)\rangle$. The first bracketed quantity involving $(\ln s)$ simply reflects the developing kinematic extent of a central rapidity plateau; there is, of course, a slight reduction from the curtailment as one approaches the fragmentation region, as measured by $\epsilon\ll 1$. The physical meaning of $A\simeq 1$ is that "initially" (i.e., at $\sqrt{s_0}=5$ GeV) there is about one particle per unit of rapidity. The specific structure for the evolving power is evaluated from the following heuristic argument. Consider an imaginary statistical situation where particle production is simply proportional to $\sqrt{s}$. Remove a fraction δa. The changed effective power is then $p=1+\delta p$:

$$\langle n(s)\rangle \propto \sqrt{s}\,(1-\delta a)=(\sqrt{s}\,)^p$$

$$\Longrightarrow \ln\sqrt{s}+\ln(1-\delta a)=(1+\delta p)\ln\sqrt{s}$$

$$\Longrightarrow \delta p\simeq -\frac{2\delta a}{\ln s},\quad \delta a\ll 1\ . \qquad (6)$$

Of course, empirically there is no statistical situation. Rather, particle growth starts as the extent of the rapidity domain $\sim(\ln s)$. However, we use as the (positive) power growth, which evolves from $p(5\ \text{GeV})=0$, the δp in Eq. (6). With $\delta a\Longrightarrow da$, one then obtains the effective power at any $\sqrt{s}>5$ GeV, given in Eq. (5), in terms of

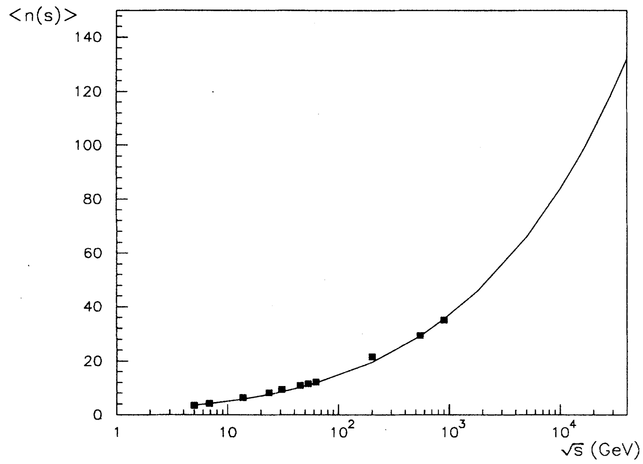

FIG. 1. Curve shows the $\langle n(s)\rangle$ calculated from Eq. (5) with $\epsilon=0.07$, $A=1.12$, compared to the existing data (from Refs. [2–4,6] and references therein) for $p(\bar{p})$-p collisions from $\sqrt{s}=5$ to 900 GeV, with predictions up to 40 TeV.

the single parameter ϵ.

In Fig. 1 we compare the average multiplicity growth produced by the relatively simple structure in Eq. (5), whose physical motivation we have sketched, with all of the data from $\sqrt{s}=5$ to 900 GeV, and we continue the curve up to 40 TeV. In Table I we give a more detailed comparison of Eq. (5) with experimental numbers, and we list the effective power of $s, p(\epsilon,s)$ at each energy. There are two parameters in this representation of the data; the essential parameter is $\epsilon=0.07$, with $A=1.12$, i.e., nearly unity. The predictions for high energies differ markedly from those of the $(\ln s)^2$ curve [2], which gives $\langle n\rangle=34$ at 1.8 TeV, for example.

A striking, simple feature emerges from applying Eq. (5) *with the same value of* $\epsilon=0.07$ to the data of $\langle n(s)\rangle$ from e^+e^- annihilation [2] from $\sqrt{s}=10$ to 90 GeV. An increase of the overall normalization to $A=1.67$, i.e., by a factor of 1.5, results in a description of the e^+e^- data. This is shown in Table II, where also predictions are given for $\sqrt{s}$ up to 500 GeV. In 1974, before any data on $\langle n(s)\rangle$ from high-energy e^+e^- annihilation was available, it was predicted [22] on rather general (geometric) grounds that the average multiplicity in e^+e^- annihilation *in ratio* to that in *pp* collisions would be $\sim\frac{3}{2}$. The difference between these average multiplicities thus grows with increasing $\sqrt{s}$. In Table II we also give the $\langle n(s)\rangle$ produced by a form which is motivated by perturbative QCD at the parton level in e^+e^- annihilation and which has been fit [23] phenomenologically to the physical charged-hadron data by the DELPHI Collaboration. The form involves the exponential of $[\ln(s/\Lambda^2)]^{1/2}$, and the fit involves two parameters, an overall normalization and a Λ in a process-dependent function $\alpha_s(\ln(s/\Lambda^2))$. This representation of the data is thus sensitive to the running of this particular α_s; the required value of α_s at each energy, from the fit parameter Λ, is also given in Table II. The extreme sensitivity to a particular value is illustrated by taking $\alpha_s=0.125$ at 91 GeV (i.e., a possible empirical value) instead of 0.11 as in Table II; for the same normalization, $\langle n(s)\rangle$ is then reduced to 14.6. Thus, with Λ increased from 138 to 330 MeV, increasing the overall normalization parameter from 0.066 to 0.094 is required to restore agreement with the data. At the highest energy, the power behavior $s^{p(\epsilon,s)}$ begins to be above the form $\sim e^{c[\ln(s/\Lambda^2)]^{1/2}}$. It is noteworthy that long ago in a fundamental paper [24,25] Polyakov derived, from field theory, a limiting growth of $\langle n(s)\rangle$ as a simple power s^p in e^+e^- annihilation, using equations of unitarity, analyticity, and a physical condition of similarity in the creation of hadrons.

In summary, the growth of $\langle n(s)\rangle$ in hadronic collisions, and also in e^+e^- annihilation, can be described

TABLE I. $\langle n(s)\rangle$ calculated from Eq. (5) with $\epsilon=0.07$, $A=1.12$ are given from $\sqrt{s}=5$ to 10^5 GeV. The effective power of $s, p(\epsilon,s)$ is also given for each energy. The existing data for $p(\bar{p})$-p collisions from $\sqrt{s}=5$ to 900 GeV (Refs. 2–4,6) and references therein) includes a systematic error, as well as the statistical error. The data at 200, 546, and 900 GeV exclude single diffractive dissociation.

$\sqrt{s}$ (GeV)	$p(\epsilon,s)$	$\langle n(s)\rangle$	$\langle n(s)\rangle_{\text{expt}}$
5.0	1.7×10^{-4}	3.61	3.43 ± 0.5
6.85	1.2×10^{-2}	4.09	4.25 ± 0.5
13.8	3.2×10^{-2}	5.78	6.37 ± 0.9
23.6	4.3×10^{-2}	7.52	8.12 ± 0.8
30.8	4.8×10^{-2}	8.54	9.43 ± 0.6
45.2	5.3×10^{-2}	10.21	10.86 ± 0.6
53.2	5.5×10^{-2}	11.00	11.55 ± 0.6
62.8	5.7×10^{-2}	11.86	12.25 ± 0.6
200	6.9×10^{-2}	19.54	21.2 ± 1.2
546	7.5×10^{-2}	29.25	29.4 ± 1.2
900	7.8×10^{-2}	35.44	35.2 ± 1.2
1800	8.1×10^{-2}	45.86	
17 000	8.8×10^{-2}	99.97	
40 000	9.0×10^{-2}	132.05	
100 000	9.1×10^{-2}	176.25	

TABLE II. $\langle n(s)\rangle$ calculated from Eq. (5) with $\epsilon=0.07$, $A=1.67$ compared to the existing data for e^+e^- annihilation from $\sqrt{s}=10$ to 91 GeV (Refs. [2,23] and references therein). Also listed are the $\langle n(s)\rangle$ calculated from the fit formula involving $e^{c[\ln(s/\Lambda^2)]^{1/2}}$ given in Ref. [23], with the value of $\alpha_s(\ln(s/\Lambda^2))$ at each energy.

$\sqrt{s}$ (GeV)	$\langle n(s)\rangle$	$\langle n(s)\rangle_{\text{expt}}$	$\alpha_s(s)$	$\langle n(s)\rangle$
10	7.34	7.5 ± 1	0.16	7.87
14	8.68	9.30 ± 0.5	0.149	9.26
22	10.84	11.30 ± 0.5	0.137	11.42
34.8	13.49	13.59 ± 0.5	0.127	14.03
43.6	14.98	15.08 ± 0.5	0.122	15.48
91	20.84	20.7 ± 0.8	0.11	21.11
200	29.13		0.098	28.94
500	42.14		0.089	41.06

by the same slowly growing power of s. The power approaches an asymptotic limit [26] of the order of 0.1 only at much higher $\sqrt{s} \sim 1000$ TeV. The approach to a simple power behavior is consistent with an asymptotic expectation, based upon general dynamical equations [24]. It constitutes an important clue to dynamical aspects of multihadron production, which is the dominant process in high-energy collisions.

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