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Relativistic constituent quark model of electroweak properties of baryons

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We calculate the electroweak properties of nucleons and hyperons in a relativistic constituent quark model using the light-front formalism. The parameters of the model, namely, the constituent quark mass and the confinement scale, can be uniquely chosen for both the electromagnetic and weak experimental data. A consistent physical picture of the qqq system appears in this work with a symmetric nucleon wave function and an asymmetric hyperon wave function. Only for the strangeness-changing weak decays do we need nontrivial form factors of the constituent quark.

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I. INTRODUCTION

The purpose of this paper is to present the results of comprehensive calculations of electromagnetic and weak form factors of the baryon octet in a relativistic constituent quark model. This model was first formulated by Berestetskii and Terent'ev [1] and has been applied to various hadronic processes in Refs. [2,3]. Recently, new studies have been carried out by Jaus in the meson sector [4] and by Chung and Coester on the electromagnetic form factors of the nucleons [5].

In a relativistic theory the Poincaré invariance has to be respected; this means, on the quantum level, the satisfaction of the commutation relations between the generators of the Poincaré group. Dirac [6] has given a general formulation of methods to satisfy simultaneously the requirements of special relativity and Hamiltonian quantum mechanics. An extension of the Dirac classes of dynamics can be found in Ref. [7]. The light-front scheme is in particular distinguished from the other Dirac classes. Among the ten generators of the Poincaré group, there are in the light-front approach seven generators of kinematical character, and only the remaining three generators contain interactions, which is the minimal possible number. The light-front dynamics is therefore the most economical scheme for dealing with a relativistic system. If we introduce the light-front variables $p^{\pm} \equiv p^0 \pm p^3$, the Einstein mass relation $p_\mu p^\mu = m^2$ is linear in p^- and linear in p^+, in contrast with the quadratic form in p^0 and $\mathbf{p}$ in the usual dynamical scheme. A consequence is a single solution of the mass shell relation in terms of p^-, in contrast to two solutions for p^0:

$$p^- = (p_\perp^2 + m^2)/p^+ , \qquad p^0 = \pm\sqrt{\mathbf{p}^2 + m^2} .$$

The quadratic relation of p^- and $p_\perp \equiv (p^1, p^2)$ in the above equation resembles the nonrelativistic scheme [8], and the variable p^+ plays the role of "mass" in this nonrelativistic analogy. It is therefore a good idea to introduce relative variables like the Jacobi momenta when dealing with several particles. As in the nonrelativistic scheme, such variables allow us to decouple the center-of-mass motion from the internal dynamics. Hence we do not have the problems with the center-of-mass motion that occur in the bag model. The light-front scheme shows another attractive feature that it has in common with the infinite momentum technique [9]. In terms of the old-fashioned (Heitler-type, time-ordered, pre-Feynman) perturbation theory, the diagrams with quarks created out of or annihilated into the vacuum do not contribute. The usual qqq quark structure is therefore conserved as in the nonrelativistic theory. It is, however, harder to get the hadron states to be eigenfunctions of the spin operator [10].

The equation of motion of the three-quark bound state on the light-front can be reduced to a relativistic Schrödinger equation with an effective potential. Since wave functions that are solutions of the relativistic Schrödinger equation are not available, we start with two simple baryon wave functions. The constituent quark mass m_q, the length scale parameters β, and the quark form factors for the weak decay are the parameters of this model. They are fixed by fitting the relevant experimental data.

A consistent physical picture appears in this paper. The nucleon consists of a symmetric three-quark state, whereas the wave functions of the hyperons are asymmetric with a diquark forming spin-0, V-spin-0 states and spin-0, U-spin-0 states, respectively. Only for the strangeness-changing weak decay do we need nontrivial form factors. Recent works [11] have also found evidence for diquark clustering in the baryons.

In Sec. II we give a brief summary of the light-front formalism for the three-body bound state. Section III contains explicitly the asymmetric wave function on the light-front. We discuss the different choices for the ansatz of the wave function. The magnetic moments of the baryon octet are calculated in Sec. IV and the hyperon semileptonic weak decays are presented in Sec. V. We summarize our investigation in a concluding Sec. VI. In the Appendix we give the connection between the wave function and the effective potential.

II. LIGHT-FRONT FORMALISM FOR A THREE-BODY BOUND STATE

To specify the dynamics of a many-particle system one has to express the ten generators of the Poincaré group P_μ and $M_{\mu\nu}$ in terms of dynamical variables. The kinematic subgroup is the set of generators that are independent of the interaction. There are five ways to choose these subgroups [7]. Usually a physical state is defined at fixed x_0, and the corresponding hypersurface is left invariant under the kinematic subgroup.

We shall use the light-front formalism, which is specified by the invariant hypersurface $x^+ = x^0 + x^3$ =const. The following notation is used: The four-vector is given by $x = (x^+, x^-, x_\perp)$, where $x^\pm = x^0 \pm x^3$ and $x_\perp = (x^1, x^2)$. Light-front vectors are denoted by boldface $\mathbf{x} = (x^+, x_\perp)$, and they are covariant under kinematic Lorentz transformations [12]. The three momenta $\mathbf{p}_i$ of the quarks can be transformed to the total and relative momenta to facilitate the separation of the center-of-mass motion [13]:

$$\begin{aligned} &\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3, \quad \xi = \frac{p_1^+}{p_1^+ + p_2^+}, \quad \eta = \frac{p_1^+ + p_2^+}{P^+}, \\ &q_\perp = (1-\xi)p_{1\perp} - \xi p_{2\perp}, \\ &Q_\perp = (1-\eta)(p_{1\perp} + p_{2\perp}) - \eta p_{3\perp}. \end{aligned} \tag{2.1}$$

Note that the four-vectors are not conserved, i.e., $p_1 + p_2 + p_3 \neq P$. In the light-front dynamics the Hamiltonian takes the form

$$H = \frac{P_\perp^2 + \hat{M}^2}{2P^+}, \tag{2.2}$$

where $\hat{M}$ is the mass operator with the interaction term W,

$$\begin{aligned} \hat{M} &= M + W, \\ M^2 &= \frac{Q_\perp^2}{\eta(1-\eta)} + \frac{M_3^2}{\eta} + \frac{m_3^2}{1-\eta}, \\ M_3^2 &= \frac{q_\perp^2}{\xi(1-\xi)} + \frac{m_1^2}{\xi} + \frac{m_2^2}{1-\xi}, \end{aligned} \tag{2.3}$$

with m_i being the masses of the constituent quarks. To get a clearer picture of M we transform to q_3 and Q_3 by

$$\begin{aligned} &\xi = \frac{E_1 + q_3}{E_1 + E_2}, \quad \eta = \frac{E_{12} + Q_3}{E_{12} + E_3}, \\ &E_{1/2} = (\mathbf{q}^2 + m_{1/2}^2)^{1/2}, \\ &E_3 = (\mathbf{Q}^2 + m_3^2)^{1/2}, \quad E_{12} = (\mathbf{Q}^2 + M_3^2)^{1/2}, \end{aligned} \tag{2.4}$$

where $\mathbf{q} = (q_1, q_2, q_3)$, and $\mathbf{Q} = (Q_1, Q_2, Q_3)$. The expression for the mass operator is now simply

$$M = E_{12} + E_3, \quad M_3 = E_1 + E_2. \tag{2.5}$$

The diagrammatic approach to light-front theory is well known [14,15]. It provides, in principal, a framework for a systematic treatment of higher-order gluon exchange. In this work we limit ourselves to the tree graph. Since we set $K^+ = 0$ we can preserve the correct qqq structure of the vertex. All relevant matrix elements we investigate are related to

$$\langle \mathbf{p}' | \bar{q}\gamma^+ q | \mathbf{p} \rangle \sqrt{P'^+ P^+} \equiv M^+, \tag{2.6}$$

where the state $|\mathbf{p}\rangle \equiv |p\rangle / \sqrt{p^+}$ is normalized according to

$$\langle \mathbf{p}' | \mathbf{p} \rangle = \delta(\mathbf{p}' - \mathbf{p}). \tag{2.7}$$

By writing down the tree graph for the matrix element in light-front variables for $K^+ = 0$, integrating over the negative component of the loop variables by contour methods, and replacing vertex functions by wave functions (see Appendix), we end up with the expression

$$\begin{aligned} M^+ = 3\frac{N_c}{(2\pi)^6} \int d^3q d^3Q \left(\frac{E_3' E_{12}' M}{E_3 E_{12} M'} \right)^{1/2} \\ \times \Psi^\dagger(\mathbf{q}', \mathbf{Q}', \lambda')\Psi(\mathbf{q}, \mathbf{Q}, \lambda). \end{aligned} \tag{2.8}$$

III. WAVE-FUNCTION MODELS FOR THE BARYON OCTET

In the light-front variables one can separate the center-of-mass motion from the internal motion. The wave function Ψ is therefore a function of the relative momenta $\mathbf{q}$ and $\mathbf{Q}$. The product $\Psi = \Phi\chi\phi$ with Φ = flavor, χ = spin, and ϕ = momentum distribution, is a symmetric function. We consider wave functions Ψ with spin-0, isospin-0 diquarks, with spin-0, V-spin-0 diquarks, and with spin-0, U-spin-0 diquarks, respectively. We write the proton wave function as (N_p being the normalization for the proton)

$$\begin{aligned} |p\rangle = N_p[-uud\,(\phi_1\chi^{\rho 1} + \phi_2\chi^{\rho 2}) + udu\,(\phi_1\chi^{\rho 1} - \phi_3\chi^{\rho 3}) \\ + duu\,(\phi_2\chi^{\rho 2} + \phi_3\chi^{\rho 3})]. \end{aligned} \tag{3.1}$$

The specific forms of the momentum wave functions ϕ_i and spin χ are described below. The Λ wave function is given by

$$\begin{aligned} |\Lambda\rangle = N_\Lambda[\phi_3\chi^{\rho 3}\,(uds - dus) + \phi_2\chi^{\rho 2}\,(usd - dsu) \\ + \phi_1\chi^{\rho 1}\,(sud - sdu)]. \end{aligned} \tag{3.2}$$

The wave functions for the other members of the baryon octet are obtained by changing the flavor wave function appropriately: For instance,

$$\begin{aligned} |n\rangle &= -|p\rangle\,(u \leftrightarrow d), \\ |\Sigma^+\rangle &= -|p\rangle\,(d \rightarrow s). \end{aligned} \tag{3.3}$$

The angular momentum $\mathbf{j}$ can be expressed as a sum of orbital and spin contributions,

$$\mathbf{j} = i\nabla_{\mathbf{p}} \times \mathbf{p} + \sum_{j=1}^{3} \mathcal{R}_{Mj}\mathbf{s}_j, \tag{3.4}$$

where $\mathcal{R}_M$ is a Melosh rotation acting on the quark spins $\mathbf{s}_j$, which has the matrix representation (for two particles)

$$\langle\lambda'|\mathcal{R}_M(\xi, q_\perp, m, M)|\lambda\rangle = \left[\frac{m + \xi M - i\boldsymbol{\sigma}\cdot(\mathbf{n}\times\mathbf{q})}{\sqrt{(m+\xi M)^2 + q_\perp^2}}\right]_{\lambda'\lambda} \tag{3.5}$$

with $\mathbf{n} = (0,0,1)$. In previous works [16] this rotation has been approximated by putting $M = M_B$. This corresponds to a weak-binding limit, which cannot be justified for a bound state in QCD. In this limit our model has a close connection to many other relativistic quark models as shown by Koerner, Hussain, and Thompson [17].

The operator $\mathbf{j}$ commutes with the mass operator $\hat{M}$; this is necessary and sufficient for Poincaré invariance of the bound state.

In terms of the relative momenta the angular momentum takes the form

$$\begin{aligned} \mathbf{j} &= i\nabla_{\mathbf{Q}} \times \mathbf{Q} + \mathcal{R}_M(\eta, Q_\perp, M_3, M)\mathbf{j}_{12} + \mathcal{R}_M(1-\eta, -Q_\perp, m_3, M)\mathbf{s}_3 , \\ \mathbf{j}_{12} &= i\nabla_{\mathbf{q}} \times \mathbf{q} + \mathcal{R}_M(\xi, q_\perp, m_1, M_3)\mathbf{s}_1 + \mathcal{R}_M(1-\xi, -q_\perp, m_2, M_3)\mathbf{s}_2 . \end{aligned} \tag{3.6}$$

We can drop the orbital contribution to obtain

$$\begin{aligned} \mathbf{j} &= \sum \mathcal{R}_i \mathbf{s}_i , \\ \mathcal{R}_1 &= \frac{1}{\sqrt{a^2+Q_\perp^2}\sqrt{c^2+q_\perp^2}} \begin{pmatrix} ac - q_R Q_L & -a q_L - c Q_L \\ c Q_R + a q_R & ac - q_L Q_R \end{pmatrix} , \\ \mathcal{R}_2 &= \frac{1}{\sqrt{a^2+Q_\perp^2}\sqrt{d^2+q_\perp^2}} \begin{pmatrix} ad + q_R Q_L & a q_L - d Q_L \\ d Q_R - a q_R & ad + q_L Q_R \end{pmatrix} , \\ \mathcal{R}_3 &= \frac{1}{\sqrt{b^2+Q_\perp^2}} \begin{pmatrix} b & Q_L \\ -Q_R & b \end{pmatrix} , \end{aligned} \tag{3.7}$$

with

$$\begin{aligned} a &= M_3 + \eta M , \quad b = m_3 + (1-\eta)M , \\ c &= m_1 + \xi M_3 , \quad d = m_2 + (1-\xi)M_3 , \\ q_R &= q_1 + iq_2 , \quad q_L = q_1 - iq_2 , \\ Q_R &= Q_1 + iQ_2 , \quad Q_L = Q_1 - iQ_2 . \end{aligned} \tag{3.8}$$

The spin-wave functions in Eqs. (3.1) and (3.2) are

$$\begin{aligned} \chi_\uparrow^{\rho 3} &= \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) , \\ \chi_\downarrow^{\rho 3} &= \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow) \end{aligned} \tag{3.9}$$

with $\chi^{\rho 2}$ and $\chi^{\rho 1}$ being the appropriate permutations of $\chi^{\rho 3}$. The spin-wave function of the ith quark is given by

$$\uparrow = \mathcal{R}_i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \downarrow = \mathcal{R}_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} . \tag{3.10}$$

The functions ϕ_i in Eqs. (3.1) and (3.2) are the momentum wave functions symmetric in the quarks different from the ith quark. We choose a harmonic oscillator and a pole-type wave function,

$$\phi_i^H = e^{-X_i}, \tag{3.11}$$

$$\phi_i^P = (1 + X_i)^{-3.5}, \tag{3.12}$$

where the X_i are the generalized forms of $M^2/2\beta^2$:

$$
\begin{aligned}
X_3 &= \frac{Q_\perp^2}{2\eta(1-\eta)\beta_Q^2} + \frac{q_\perp^2}{2\eta\xi(1-\xi)\beta_q^2} + \frac{m_1^2}{2\eta\xi\beta_q^2} + \frac{m_2^2}{2\eta(1-\xi)\beta_q^2} + \frac{m_3^2}{2(1-\eta)\beta_Q^2}\,, \\
X_2 &= q_\perp^2 \frac{(1-\eta)(1-\xi)\beta_Q^2 + \xi\beta_q^2}{2\beta_Q^2\beta_q^2\eta\xi(1-\xi)(1-\eta+\xi\eta)} + Q_\perp^2 \frac{(1-\xi)(1-\eta)\beta_q^2 + \xi\beta_Q^2}{2\beta_Q^2\beta_q^2\eta(1-\eta)(1-\eta+\xi\eta)} \\
&\quad + q_\perp Q_\perp \frac{\beta_Q^2 - \beta_q^2}{\beta_Q^2\beta_q^2\eta(1-\eta+\xi\eta)} + \frac{m_1^2}{2\eta\xi\beta_q^2} + \frac{m_2^2}{2\eta(1-\xi)\beta_Q^2} + \frac{m_3^2}{2(1-\eta)\beta_q^2}\,, \\
X_1 &= q_\perp^2 \frac{(1-\xi)\beta_q^2 + \xi(1-\eta)\beta_Q^2}{2\beta_Q^2\beta_q^2\eta\xi(1-\xi)(1-\xi\eta)} + Q_\perp^2 \frac{(1-\xi)\beta_Q^2 + \xi(1-\eta)\beta_q^2}{2\beta_Q^2\beta_q^2\eta(1-\eta)(1-\xi\eta)} \\
&\quad - q_\perp Q_\perp \frac{\beta_Q^2 - \beta_q^2}{\beta_Q^2\beta_q^2\eta(1-\xi\eta)} + \frac{m_1^2}{2\eta\xi\beta_Q^2} + \frac{m_2^2}{2\eta(1-\xi)\beta_q^2} + \frac{m_3^2}{2(1-\eta)\beta_q^2}\,.
\end{aligned}
\tag{3.13}
$$

The normalization factors N_B in Eqs. (3.1) and (3.2) are determined from $\langle B|B\rangle = 1$. For the proton and Λ we get

$$
\begin{aligned}
\langle p|p\rangle &= N_p^2\left[(\phi_1^2+\phi_2^2+\phi_1\phi_2)_{uud} + (\phi_1^2+\phi_3^2+\phi_1\phi_3)_{udu} + (\phi_2^2+\phi_3^2+\phi_2\phi_3)_{duu}\right]\,, \\
\langle\Lambda|\Lambda\rangle &= N_\Lambda^2\left[(\phi_1^2)_{sud} + (\phi_2^2)_{usd} + (\phi_3^2)_{uds}\right]\,.
\end{aligned}
$$

Our wave functions ϕ^H and ϕ^P only differ in their high-energy behavior. The exponent of the pole-type wave function can be chosen by fitting the electromagnetic form factors of the nucleons [18]. In the limit of vanishing quark masses the corresponding quark distribution amplitudes both converge to the asymptotic form of Ref. [14]:

$$
\phi^H, \phi^P \propto \xi\eta^2(1-\eta)(1-\xi) = x_1x_2x_3, \tag{3.14}
$$

with the light-front fractions $x_i \equiv p_i^+/P^+$.

IV. MAGNETIC MOMENTS

The electromagnetic current matrix element for the transition $B \to B'\gamma$ can be written in terms of two form factors taking into account current and parity conservation:

$$
\langle B', \lambda' p' \,|J^\mu|\, B, \lambda p\rangle = \bar{u}_{\lambda'}(p')\left[F_1(K^2)\gamma^\mu + \frac{F_2(K^2)}{2M_N} i\sigma^{\mu\nu}K_\nu\right]u_\lambda(p) \tag{4.1}
$$

with momentum transfer $K = p' - p$, and the current $J^\mu = \bar{q}e\gamma^\mu q$. In order to use Eq. (2.8) we express the form factors in terms of the positive component of the current:

$$
\begin{aligned}
F_1(K^2) &= \langle B', \uparrow |J^+|\, B, \uparrow\rangle\,, \\
K_\perp F_2(K^2) &= -2M_N\,\langle B', \uparrow |J^+|\, B, \downarrow\rangle\,.
\end{aligned}
\tag{4.2}
$$

For $K^2 = 0$ the form factors F_1 and F_2 are, respectively, equal to the charge and the anomalous magnetic moment in units e and e/M_N, and the magnetic moment is $\mu = F_1(0) + F_2(0)$.

The anomalous magnetic moment for the Λ is given by

$$
K_\perp F_2(0) = -2M_\Lambda N_\Lambda^2 \frac{N_c}{(2\pi)^6} \times \int d^3q\,d^3Q \sum_{i=1}^{3} e_i|\phi_i|^2 \left\langle \chi_\uparrow^{\rho i}|\chi_\downarrow^{\rho i}\right\rangle\,, \tag{4.3}
$$

with e_i being the charge of the ith quark. The formulas for the other members of the baryon octet are analogous. The calculation of the spin matrix elements is tedious but straightforward. The explicit expressions are given in Ref. [19]. The numerical results are summarized in Table II for the four different parameter sets given in Table I. Parameter sets 1 and 2 are given as a reference for symmetric wave functions ($\beta_q = \beta_Q$), which are usually

TABLE I. The parameters of the constituent quark model: quark masses m (GeV), scale parameter β (GeV) and quark form factors f_1 and g_1 for the quark transition $s \to u$. Note that sets 1–3 are used for the harmonic oscillator wave function, whereas set 4 is used for the pole type wave function.

	Harmonic oscillator			Pole type
Parameters	Set 1	Set 2	Set 3	Set 4
$m_u = m_d$	0.33	0.267	0.26	0.263
m_s	0.55	0.40	0.38	0.38
β_{qN}	0.16	0.56	0.55	0.607
β_{QN}	0.16	0.56	0.55	0.607
$\beta_{q\Lambda}$	1.00	0.60	0.55	0.607
$\beta_{Q\Lambda}$	1.00	0.60	0.80	0.90
$\beta_{q\Sigma}$	1.00	0.60	0.80	0.90
$\beta_{Q\Sigma}$	1.00	0.60	0.40	0.45
$\beta_{q\Xi}$	1.08	0.62	0.80	0.90
$\beta_{Q\Xi}$	1.08	0.62	0.36	0.40
f_{1us}	1.00	1.00	1.19	1.28
g_{1us}	1.00	1.00	1.19	1.28

used in the literature [5,20]. Set 1 uses relatively large quark masses normally found in nonrelativistic models ($m_u = m_d \approx M_{\rm nucleon}/3$). The magnetic moments can be reproduced very well, but the semileptonic weak decay data deviate by more than an order of magnitude. This is due to the special choice of the β parameter for the nucleon and the hyperons:

$$\beta_N \ll \beta_\Sigma \approx \beta_\Lambda \approx \beta_\Xi \ , \tag{4.4}$$

which results in a too large suppression for the $\Delta S = 1$ transitions, since the wave-function overlap is small. Parameter set 2, on the other hand, gives good values for the semileptonic decays but is bad at fitting the magnetic moments. Within the symmetric wave-function model do we find, that either the magnetic moments can be fitted and the weak decay data are poorly fitted or vice versa. The opposite statement in Ref. [3] has to be questioned because their numerical results for the magnetic moments are wrong. Our results agree with Ref. [20] on this point. The inconsistency just described between the electromagnetic and the weak sector can be resolved by using asymmetric wave functions (parameter sets 3 and 4). All electroweak properties in Table II can be fitted with this wave function. Set 3 uses the harmonic oscillator wave function in Eq. (3.11), and set 4 uses the pole-type wave function in Eq. (3.12). The only essential difference between these two types of wave functions is the high-energy behavior [18]. The twelve parameters in Table I are overcounted because $\beta_{qN} = \beta_{q\Lambda} = \beta_{ud}$ and $\beta_{q\Sigma} = \beta_{q\Xi} = \beta_{us}$ being the scale parameters for the diquarks ud and us, respectively. After fitting the mass $m_u = m_d$, we fix the strange quark mass to be $m_s/m_u \sim 1.4 - 1.6$ [21]. Therefore, we have only nine degrees of freedom, but it is not obvious that a reasonable fit is possible, since the relations are nonlinear. We get, however, an excellent agreement with data for the asymmetric wave functions (sets 3 and 4). The neutron magnetic moment could be improved by introducing electromagnetic quark form factors [5].

V. HYPERON SEMILEPTONIC BETA DECAY

In the low-energy limit the standard model for semileptonic weak decays reduces to an effective current-current interaction Hamiltonian

$$H_{\rm int} = \frac{G}{\sqrt{2}} J_\mu L^\mu + \text{H.c.} \ , \tag{5.1}$$

where $G \simeq 10^{-5}/M_p^2$ is the weak-coupling constant,

$$L^\mu = \bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_\nu + \bar{\psi}_\mu \gamma^\mu (1-\gamma_5)\psi_\nu \tag{5.2}$$

is the lepton current, and

TABLE II. Electroweak properties of the baryon octet. The calculations with symmetric wave functions (sets 1 and 2) and asymmetric wave functions (sets 3 and 4) are compared. Note that set 1 is only able to fit the magnetic moments, whereas set 2 is best at fitting the weak decays. Sets 3 and 4 reproduce all electroweak data in an excellent way. The magnetic moments are given in units of the nuclear magneton; the decay rates are given in units of $10^6\,{\rm s}^{-1}$ (except the nucleon decay is in units of $10^{-3}\,{\rm s}^{-1}$). Experimental data are from Ref. [26].

Quantity	Expt.	Set 1	Set 2	Set 3	Set 4
$\mu(p)$	2.79 ± 10^{-7}	2.85	2.78	2.82	2.81
$\mu(n)$	-1.91 ± 10^{-6}	−1.83	−1.62	−1.66	−1.66
$\mu(\Sigma^+)$	2.42 ± 0.05	2.59	3.23	2.63	2.61
$\mu(\Sigma^-)$	-1.160 ± 0.025	−1.30	−1.36	−1.14	−1.13
$\mu(\Lambda)$	-0.613 ± 0.004	−0.48	−0.72	−0.69	−0.69
$\mu(\Xi^0)$	-1.250 ± 0.014	−1.25	−1.87	−1.25	−1.24
$\mu(\Xi^-)$	-0.6507 ± 0.0025	−0.99	−0.96	−0.67	−0.76
$g_1/f_1(n \to p e^- \bar{\nu}_e)$	1.2573 ± 0.0028	1.63	1.252	1.248	1.260
$\sqrt{3/2}g_1(\Sigma^\pm \to \Lambda e^\pm \nu_e)$	0.742 ± 0.018	0.80	0.736	0.759	0.704
$g_1/f_1(\Lambda \to p e^- \bar{\nu}_e)$	0.718 ± 0.015	0.957	0.826	0.759	0.745
$g_1/f_1(\Sigma^- \to n e^- \bar{\nu}_e)$	-0.340 ± 0.017	−0.319	−0.275	−0.255	−0.255
$g_1/f_1(\Xi^- \to \Sigma^0 e^- \bar{\nu}_e)$	1.287 ± 0.158	1.594	1.362	1.212	1.192
$g_1/f_1(\Xi^- \to \Lambda e^- \bar{\nu}_e)$	0.25 ± 0.05	0.319	0.272	0.270	0.255
$\Gamma(n \to p e^- \bar{\nu}_e)$	1.125 ± 0.003	1.76	1.152	1.113	1.13
$\Gamma(\Sigma^+ \to \Lambda e^+ \nu_e)$	0.25 ± 0.06	0.29	0.24	0.25	0.21
$\Gamma(\Sigma^- \to \Lambda e^- \bar{\nu}_e)$	0.387 ± 0.018	0.47	0.389	0.41	0.36
$\Gamma(\Lambda \to p e^- \bar{\nu}_e)$	3.169 ± 0.053	0.14	3.51	3.37	3.22
$\Gamma(\Sigma^- \to n e^- \bar{\nu}_e)$	6.88 ± 0.23	0.16	5.74	6.13	6.47
$\Gamma(\Xi^- \to \Lambda e^- \bar{\nu}_e)$	3.36 ± 0.18	0.10	2.96	2.35	2.76
$\Gamma(\Xi^- \to \Sigma^0 e^- \bar{\nu}_e)$	0.53 ± 0.10	0.02	0.55	0.66	0.76
$\Gamma(\Lambda \to p \mu^- \bar{\nu}_\mu)$	0.60 ± 0.13	0.02	0.58	0.56	0.53
$\Gamma(\Sigma^- \to n \mu^- \bar{\nu}_\mu)$	3.04 ± 0.27	0.07	2.54	2.77	2.93
$\Gamma(\Xi^- \to \Lambda \mu^- \bar{\nu}_\mu)$	2.1 ± 2.1	0.03	0.80	0.65	0.76

$$J_\mu = V_\mu - A_\mu \ , \quad V_\mu = V_{ud}\bar{u}\gamma_\mu d + V_{us}\bar{u}\gamma_\mu s \ , \quad A_\mu = V_{ud}\bar{u}\gamma_\mu\gamma_5 d + V_{us}\bar{u}\gamma_\mu\gamma_5 s \ , \tag{5.3}$$

is the hadronic current, and V_{ud}, V_{us} are the elements of the Kobayashi-Maskawa mixing matrix. The τ-lepton current cannot contribute, since m_τ is much too large.

The matrix elements of the hadronic current between spin-$\frac{1}{2}$ states are

$$\langle B', p' | V^\mu | B, p\rangle = V_{qq'}\bar{u}(p')\left[f_1(K^2)\gamma^\mu - \frac{f_2(K^2)}{M_i}i\sigma^{\mu\nu}K_\nu + \frac{f_3(K^2)}{M_i}K^\mu\right]u(p) \ , \tag{5.4}$$

$$\langle B', p' | A^\mu | B, p\rangle = V_{qq'}\bar{u}(p')\left[g_1(K^2)\gamma^\mu - \frac{g_2(K^2)}{M_i}i\sigma^{\mu\nu}K_\nu + \frac{g_3(K^2)}{M_i}K^\mu\right]\gamma_5 u(p) \ , \tag{5.5}$$

where $K = p - p'$ and M_i is the mass of the initial baryon. The quantities f_1 and g_1 are the vector and axial-vector form factors, f_2 and g_2 are the weak magnetism and electric form factors, and f_3 and g_3 are the induced scalar and pseudoscalar form factors, respectively. T invariance implies real form factors. We do not calculate f_3 and g_3, since we put $K^+ = 0$ and their dependence on the decay spectra is of the order

$$\left(\frac{m_l}{M_i}\right)^2 \ll 1 \ , \tag{5.6}$$

where m_l is the mass of the final charged lepton. The other form factors are

$$\begin{aligned} f_1 &= \langle B', \uparrow | V^+ | B, \uparrow\rangle \ , \quad K_\perp f_2 = M_i \langle B', \uparrow | V^+ | B, \downarrow\rangle \ , \\ g_1 &= \langle B', \uparrow | A^+ | B, \uparrow\rangle \ , \quad K_\perp g_2 = -M_i \langle B', \uparrow | A^+ | B, \downarrow\rangle \ . \end{aligned} \tag{5.7}$$

We generalize the Dirac quark current for the $s \to u$ transition by introducing constituent quark form factors f_{1us} and g_{1us}:

$$\bar{u}\gamma^\mu(1-\gamma_5)s \to \bar{u}\gamma^\mu(f_{1us} - g_{1us}\gamma_5)s.$$

We therefore have an effective $\tilde{f}_1 = f_{1us}f_1$ and an effective $\tilde{g}_1 = g_{1us}g_1$. Ignoring the lepton mass the rate Γ is given by [22]

$$\begin{aligned} \Gamma = G^2\frac{\Delta M^5|V|^2}{60\pi^3}\Bigg[&\left(1 - \frac{3}{2}\beta + \frac{6}{7}\beta^2\right)\tilde{f}_1^2 + \frac{4}{7}\beta^2 f_2^2 + \left(3 - \frac{9}{2}\beta + \frac{12}{7}\beta^2\right)\tilde{g}_1^2 \\ &+ \frac{12}{7}\beta^2 g_2^2 + \frac{6}{7}\beta^2\tilde{f}_1 f_2 + (-4\beta + 6\beta^2)\tilde{g}_1 g_2 + \frac{4}{7}\beta^2(\tilde{f}_1\lambda_f + 5\tilde{g}_1\lambda_g)\Bigg] \ , \end{aligned} \tag{5.8}$$

where β is defined as $\beta = (M_i - M_f)/M_i$, and $\Delta M = M_i - M_f$, M_i, M_f being the masses of the initial and final baryon, respectively. The K^2 dependence of f_2 and g_2 is ignored and f_1 and g_1 are expanded as

$$\tilde{f}_1(K^2) = \tilde{f}_1(0) + \frac{K^2}{M_i^2}\lambda_f \ , \quad \tilde{g}_1(K^2) = \tilde{g}_1(0) + \frac{K^2}{M_i^2}\lambda_g \ . \tag{5.9}$$

We correct the rate from Eq. (5.8) to include the effect of the nonvanishing lepton mass and the effect of the radiative corrections [22,23].

The form factors in Eq. (5.8) are calculated by using Eqs. (5.7), (3.1), and (3.2). As an example we give the form factors g_1 for the decay $\Lambda \to pl^-\bar{\nu}_l$:

$$g_1 = \langle p, \uparrow | A^+ | \Lambda, \uparrow\rangle = N_p N_\Lambda\left(\phi_1\phi_3\langle\chi_\uparrow^{\rho 3}|A^+|\chi_\uparrow^{\rho 1}\rangle - \phi_2\phi_3\langle\chi_\uparrow^{\rho 3}|A^+|\chi_\uparrow^{\rho 2}\rangle - 2\phi_3^2\langle\chi_\uparrow^{\rho 3}|A^+|\chi_\uparrow^{\rho 3}\rangle\right) \ . \tag{5.10}$$

The calculations for the spin matrix elements are tedious but simple algebra. The exact formulas are given in Ref. [19]. In the limit of symmetric wave functions and $K^2 = 0$ do we get, for the above decay $\Lambda \to pl^-\bar{\nu}_l$,

$$g_1(0) = -\sqrt{\frac{3}{2}}\frac{N_c}{(2\pi)^6}\int d^3q\, d^3Q \frac{\phi^\dagger(M)\phi(M)(b'b - Q_\perp^2)(a'a + Q_\perp^2)^2}{(a'^2 + Q_\perp^2)(a^2 + Q_\perp^2)\sqrt{(b'^2 + Q_\perp^2)(b^2 + Q_\perp^2)}} \ . \tag{5.11}$$

TABLE III. Form factors f_1 and g_1 for the various semileptonic weak β decays. Parameter sets 3 and 4 of Table I are used.

	Set 3		Set 4	
Decay	f_1	g_1	f_1	g_1
$n \to p$	1.00	1.25	1.00	1.26
$\Sigma^{\pm} \to \Lambda$	–0.04	0.62	–0.05	0.58
$\Lambda \to p$	–1.04	–0.79	–0.95	–0.71
$\Sigma^- \to n$	–0.87	0.22	–0.83	0.21
$\Xi^- \to \Sigma^0$	0.71	0.86	0.72	0.86
$\Xi^- \to \Lambda$	0.91	0.25	0.92	0.24

The K^2 dependence of the form factors f_1 and g_1 is calculated by their derivatives at $K^2 = 0$. The form factor g_2 vanishes or is very small as it should be. The weak magnetism form factor f_2 agrees with the conserved vector current hypothesis within 5%. The form factors f_1 and g_1 are given in Table III for the various decays. We summarize the ratios g_1/f_1 and the rates Γ for all the measured semileptonic weak decays in Table II. Sets 3 and 4 give an excellent fit for all experimental data, except for the ratio g_1/f_1 for the decays $\Lambda \to p$ and $\Sigma^- \to n$. This is, however, a general property of every quark model due to its SU(6) flavor-spin symmetry. The ratio

$$\frac{g_1/f_1(\Lambda \to pe^-\bar{\nu}_e)}{g_1/f_1(\Sigma^- \to ne^-\bar{\nu}_e)} \tag{5.12}$$

is constrained to be -3 in the models in contrast with the experimental value -2.11 ± 0.15 for $g_2 = 0$.

VI. SUMMARY AND OUTLOOK

We have shown that there exists a relativistic quark model with diquark clustering that provides a framework, in which we have overall an excellent and consistent picture of the whole baryon octet for the magnetic moments and the semileptonic weak decays. The physical picture of the baryon octet is as follows (parameter sets 3 and 4 in Table I). There is no diquark clustering in the nucleon sector ($\beta_{qN} = \beta_{QN}$). In the strange sector we have a strong diquark clustering for the Σs ($\beta_{q\Sigma} \sim 2\beta_{Q\Sigma}$) and Ξs ($\beta_{q\Xi} \sim 2\beta_{Q\Xi}$) and a small one for the Λ ($1.5\beta_{q\Lambda} \sim \beta_{Q\Lambda}$). The diquark *us* pair ($\beta_{q\Sigma}, \beta_{q\Xi}$) is more tightly bound than the *ud* pair ($\beta_{qN}, \beta_{q\Lambda}$) as we might expect. The low-momentum properties do not depend on the two different wave functions chosen. It would, however, be illuminating to derive the momentum wave function from a potential. To complete the study of the baryons one should also include the effects of higher Fock states.

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APPENDIX: CONNECTION BETWEEN THE WAVE FUNCTION AND THE POTENTIAL

It is instructive to give some details on the derivation of the equation of motion for the wave function.

We shall assume only two-particle forces interacting in a ladder-type pattern so that the dynamics of the three-body system is governed by the Bethe-Salpeter (BS) interaction kernel for the two-body system and the relativistic Faddeev equations.

Using the Faddeev decomposition for the vertex function $\Gamma = \Gamma^{(1)} + \Gamma^{(2)} + \Gamma^{(3)}$, we can write down a BS equation for the various components in operator notation:

$$\Gamma^{(1)} = T^{(1)} G_2 G_3 (\Gamma^{(2)} + \Gamma^{(3)}) \tag{A1}$$

with

$$G_i = \not{p}_i - m_i, \quad T^{(1)} = (1 - VG_2G_3)^{-1} V \;, \tag{A2}$$

and similarly for $\Gamma^{(2)}$ and $\Gamma^{(3)}$. V is the one gluon exchange kernel between two quarks, and T is already the ladder sum to all orders. It is useful to consider the second iteration of the vertex equation, which is given by

$$\mathbf{\Gamma} = U G_1 G_2 G_3 \mathbf{\Gamma} \;, \tag{A3}$$

where $\mathbf{\Gamma} = (\Gamma^{(1)}, \Gamma^{(2)}, \Gamma^{(3)})$ and U is the matrix:

$$U_{ij} = \begin{cases} T^{(i)} G_j T^{(k)} & \text{for } i \neq j \text{ with } k \neq i, j \;, \\ T^{(i)} (G_k T^{(l)} + G_l T^{(k)}) & \text{for } i = j \text{ with } k \neq l \neq i \;. \end{cases} \tag{A4}$$

The four-dimensional Eq. (A3) can be reduced to a three-dimensional equation

$$\mathbf{\Gamma} = W g_3 \mathbf{\Gamma} \;, \quad W = (1 - UR_3)^{-1} U \tag{A5}$$

by writing $G_1 G_2 G_3 = g_3 + R_3$, where g_3 has only three-particle singularities. We choose a g_3, which puts the quarks on their mass shells:

$$g_3 = (2\pi i)^2 \int ds \frac{1}{P^2 - s} \prod_{i=1}^{3} \delta^+(p_i^2 - m_i^2)(\not{p}_i + m_i) \;, \tag{A6}$$

where P is the total momentum of the bound state, $s = (p_1 + p_2 + p_3)^2$ and p_i are restricted by $p_i^+ \geq 0$. We get

$$g_3 = (2\pi i)^2 \delta(p_2^2 - m_2^2)\delta(p_3^2 - m_3^2)\Theta(\xi)\Theta(1-\xi)\Theta(\eta)\Theta(1-\eta) \frac{\Lambda^+(p_1)\Lambda^+(p_2)\Lambda^+(p_3)}{\xi\eta(P^2 - M^2)} \tag{A7}$$

with the spin projection operator

$$\Lambda^+(p_i) = \sum_\lambda u(p_i,\lambda)\bar{u}(p_i,\lambda) \ . \tag{A8}$$

Writing

$$\begin{aligned}
\hat{g}_3 &= \frac{1}{P^2 - M^2} \ , \\
\hat{\Gamma}^{(i)} &= \left(\frac{M_3 M}{E_1 E_2 E_3 E_{12}}\right)^{1/2} \Gamma^{(i)} u(p_1\lambda_1)u(p_2\lambda_2)u(p_3\lambda_3) \ , \\
\hat{W}_{ij} &= \left(\frac{M_3 M_3' M M'}{E_1 E_1' E_2 E_2' E_3 E_3' E_{12} E_{12}'}\right)^{1/2} u(p_1\lambda_1)u(p_2\lambda_2)u(p_3\lambda_3) W_{ij} \bar{u}(p_1'\lambda_1')\bar{u}(p_2'\lambda_2')\bar{u}(p_3'\lambda_3'),
\end{aligned} \tag{A9}$$

we are led to the integral equation

$$\begin{aligned}
\hat{\Gamma}^{(i)}(\mathbf{q},\mathbf{Q},\lambda_1,\lambda_2,\lambda_3) = \frac{1}{(2\pi)^6} \sum_{\lambda_1'\lambda_2'\lambda_3' j} \int d^3q' d^3Q' \hat{W}^{ij}(\mathbf{q},\mathbf{q}',\mathbf{Q},\mathbf{Q}',\lambda_1,\lambda_1',\lambda_2,\lambda_2',\lambda_3,\lambda_3') \\
\times \hat{g}_3(\mathbf{q}',\mathbf{Q}')\hat{\Gamma}^{(j)}(\mathbf{q}',\mathbf{Q}',\lambda_1',\lambda_2',\lambda_3') \ .
\end{aligned} \tag{A10}$$

We can write this equation in terms of the wave function Ψ. The Faddeev decomposition is $\Psi = \Psi^{(1)} + \Psi^{(2)} + \Psi^{(3)}$, the relation to the vertex function is $\Psi^{(i)} = \hat{g}_3 \hat{\Gamma}^{(i)}$, and writing $\mathbf{\Psi} = (\Psi^{(1)}, \Psi^{(2)}, \Psi^{(3)})$ we get

$$(M_B^2 - M^2)\mathbf{\Psi} = \hat{W}\mathbf{\Psi} \tag{A11}$$

with M_B being the mass of the baryon. If we put $\hat{W} = MW + WM + W^2$ we see that the wave function is an eigenfunction of the mass operator $\hat{M}^2$, given in Eq. (2.3):

$$\hat{M}^2\Psi = M_B^2\Psi, \tag{A12}$$

which is equivalent to the equation usually used in constituent quark models [24]:

$$(E_{12} + E_3 + W)\Psi = M_B\Psi \ . \tag{A13}$$

This last equation is the starting point for an explicit calculation of the wave function, which has been done for the meson sector [25].

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