Temperature dependence of electric and magnetic gluon condensates

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The contribution of Lorentz nonscalar operators to finite temperature correlation functions is discussed. Using the local duality approach for the one-pion matrix element of a product of two vector currents, the temperature dependence of the average gluonic stress tensor is estimated in the chiral limit to be $\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle_T = (\pi^2/10) b T^4$. At a normalization point $\mu = 0.5$ GeV we obtain $b \approx 1.1$. Together with the known temperature dependence of the Lorentz scalar gluon condensate we are able to infer $\langle \mathbf{E}^2 \rangle_T$ and $\langle \mathbf{B}^2 \rangle_T$ separately in the low-temperature hadronic phase.

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Correlators of currents with the quantum numbers of hadrons are known to be useful to obtain information about the masses and couplings of hadrons; they are employed in the QCD sum rule approach and in lattice calculations. In both approaches the correlators are considered at large Euclidean distances or imaginary times where the dominant contribution comes from the lowest state with the corresponding quantum numbers. QCD sum rules give predictions also for form factors and structure functions of hadrons. (For a recent review of applications of QCD correlation functions see Ref. [1].)

In recent years there has been increasing interest in finite temperature QCD and hadronic physics due to the expectation that at high enough temperatures the QCD vacuum, specified by nonperturbative condensates of quark and gluon fields, will "melt" and undergo a transition to a quark-gluon plasma. Melting is usually understood in the sense that chiral symmetry restoration and deconfinement take place. The former means that with increasing temperature quark condensates evaporate, while the latter means that hadrons do not represent stable degrees of freedom. It was shown by Leutwyler and his collaborators [2] using the chiral Lagrangian approach that the quark condensate indeed decreases with rising temperature. From the usual QCD sum rules at $T = 0$ it is well known that the properties of hadrons are, to a large extent, determined by nonperturbative quark and gluon condensates [3]. Naturally, a large number of papers were devoted to the generalization of QCD sum rules to finite temperature in attempts to relate the temperature dependence of the hadronic spectrum to the temperature dependence of the condensates (see, e.g., [4–6]). In this case the vacuum average of the product of currents becomes the Gibbs average over the thermal ensemble. To calculate the Gibbs average one must choose a basis for the states. As argued in Refs. [5, 7] at temperatures which are much less than the energy scale of confinement the appropriate basis is that of hadronic states, rather than the quark-gluon basis used in early papers on the subject (see, e.g., Ref. [4]). Using this basis it was also shown [5] that at low T the thermal correlators are expressed as a mixture of zero-temperature correlators with different parity. It is also clear that if the operator product expansion (OPE) is applied to a thermal correlator then the temperature dependence appears only in the matrix elements of the operators (condensates), the coefficient functions being obtained through a perturbative calculation at $T = 0$. QCD sum rules at low temperature were recently reexamined along these lines in Ref. [8].[1] At high temperatures, corresponding to the quark-gluon plasma, the calculation of thermal correlators should be performed in a basis consisting of quark and gluon states. In this case the perturbative temperature-dependent parts of the condensates due to quarks and gluons from the thermal ensemble may be included in the coefficient functions [6].

Thus the QCD sum rule method, understood as a tool to get information about the imaginary parts of correlators via analyticity, seems to be tractable both at very low and very high temperatures, but not in the region of a phase transition where a drastic rearrangement of the spectrum takes place.

An additional feature of finite temperature sum rules is the appearance of new condensates due to Lorentz nonscalar operators; these were, of course, present in the OPE, but gave zero contribution when averaged over the vacuum. At finite temperatures Lorentz invariance is broken and these operators should contribute [1, 6]. The

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same applies to the case of finite density [9]. However, each of these new condensates is an unknown nonperturbative parameter. In principle they may be fixed from the physical spectral densities of the correlators, just as in the zero-temperature case the now well-established condensates were fixed by the hadronic spectrum.

Consider the correlator of two isovector vector currents at finite temperature T and Euclidean momentum q, where $T^2 \ll Q^2 = -q^2$ and $Q^2 \gtrsim 1$ GeV^2:

$$i\int d^4x\, e^{iqx}\sum_n \langle n|\mathcal{T} j_\mu(x) j_\nu(0) e^{(\Omega-H)/T}|n\rangle$$
$$= (g_{\mu\nu}q^2 - q_\mu q_\nu)C_1(q,T) + u^t_\mu u^t_\nu C_2(q,T)\ , \quad (1)$$

where $\mathcal{T}$ denotes a time-ordered product, $j_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$, $u^t_\mu = u_\mu - (u\cdot q)q_\mu/q^2$ is the transverse part of the heat bath four-velocity u_μ and $\Omega = -T\ \ln(\sum_n \langle n|e^{-H/T}|n\rangle)$. Equation (1) is the most general expression compatible with conservation of the vector current. The Lorentz-invariance-breaking term proportional to $u^t_\mu u^t_\nu$ must be absent at $T=0$. This means that $C_2(q,T)$ goes to zero as $T\to 0$, while $C_1(q,T)$ becomes the usual zero-temperature correlator. Notice that Eq. (1) may be considered to be the amplitude for forward scattering of a virtual photon by the heat bath. Then the imaginary parts of C_1 and C_2 are the structure functions of deep inelastic scattering of leptons by the heat bath [u^T_μ is similar to the transverse component of the target momentum, $p_\mu - (p\cdot q)q_\mu/q^2$].

At low T, when the contributions from all particles except pions are exponentially suppressed in the Gibbs average, the functions C_1 and C_2 may be estimated by expanding in the density of thermal pions. In the first order of this expansion only matrix elements over one-pion states are taken into account. This approximation was made in Ref. [5] for C_1. The one-pion matrix elements were estimated via PCAC (partial conservation of axial-vector current) and current algebra. It was shown that C_1 and its counterpart from the axial channel are given by T-dependent mixtures of their zero-temperature values and, as a result, the corresponding screening lengths tend to converge with increasing temperature.

The purpose of the present article is to estimate C_2. Let us start from the one-pion matrix element in the chiral limit:

$$i\int d^4x\, e^{iqx}\langle\pi(p)|\mathcal{T} j_\mu(x) j_\nu(0)|\pi(p)\rangle\ , \quad (2)$$

where we assume $p\sim T\ll Q$, since Eq. (2) is to be integrated over p with Bose occupation probabilities. If $\hat{O}_{\mu_1\mu_2\cdots\mu_n}$ is an operator of Lorentz spin n, then the matrix element $\langle\pi(p)|\hat{O}_{\mu_1\mu_2\cdots\mu_n}|\pi(p)\rangle \propto p_{\mu_1}p_{\mu_2}\cdots p_{\mu_n}$, and cannot be reduced via PCAC to a vacuum matrix element. It is clear that at low temperatures, $T\ll Q$, the main contribution to C_2 comes from operators of lowest spin, namely spin 2. In leading twist there are two spin-2 operators which are related to the energy-momentum tensor:

$$\theta^q_{\mu_1\mu_2} = \tfrac{1}{2}i(\bar{q}\gamma_{\mu_1}D_{\mu_2}q + \bar{q}\gamma_{\mu_2}D_{\mu_1}q)\quad , \quad q = u,d,s,\ldots,$$
$$\theta^G_{\mu_1\mu_2} = G^a_{\mu_1\alpha}G^{a\alpha}{}_{\mu_2} - \tfrac{1}{4}g_{\mu_1\mu_2}G^a_{\beta\alpha}G^{a\alpha\beta}\ , \quad (3)$$

where D_μ is the covariant derivative. Graphs which correspond to the contributions of these operators to the matrix element in Eq. (2) are shown in Fig. 1. If the normalization point for the operators is taken to be $\mu^2 = Q^2$, then the operator $\theta^G_{\mu_1\mu_2}$ does not contribute to the OPE in the leading log approximation, and the contribution of twist-2, spin-2 operators to Eq. (2) involves

$$\frac{1}{Q^2}\langle\pi(p)|\theta^u_{\mu\nu} + \theta^d_{\mu\nu}|\pi(p)\rangle = \frac{1}{Q^2}\langle\pi(p)|\theta^{\rm tot}_{\mu\nu} - \theta^G_{\mu\nu}|\pi(p)\rangle\ . \quad (4)$$

Here we neglected the contributions of heavy quarks. The matrix element of the total energy-momentum tensor is $\langle\pi(p)|\theta^{\rm tot}_{\mu\nu}|\pi(p)\rangle = 2p_\mu p_\nu$ [the states are normalized such that $\langle\pi(p)|\pi(p')\rangle = (2\pi)^3 2E\delta^{(3)}(\mathbf{p}-\mathbf{p}')$], while the matrix element of the gluon energy-momentum tensor,

$$\langle\pi(p)|\theta^G_{\mu\nu}|\pi(p')\rangle = bp_\mu p_\nu, \quad (5)$$

contains an unknown constant b. This constant is related to the matrix element of the energy density of the gluon field

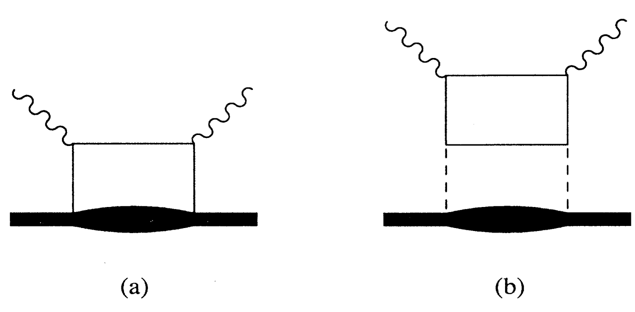
$$b = \frac{1}{2\mathbf{p}^2}\langle\pi(p)|\mathbf{E}^2 + \mathbf{B}^2|\pi(p)\rangle_{\mu=Q}\ . \quad (6)$$

Note that b depends on the normalization point, μ, in the operator product expansion. This dependence will be discussed later.

Let us try to estimate b within a quark-hadron duality approach, saturating the amplitude of Eq. (2) by hadrons, $\langle\pi|\mathcal{T} j_\mu(x)j_\nu(0)|\pi\rangle = \sum_n\langle\pi|j_\mu(x)|n\rangle\langle n|j_\nu(0)|\pi\rangle$. Focusing on spin-2 contributions to C_2, we then have

$$\frac{2-b}{Q^2} + \frac{c}{Q^4} + \cdots = \frac{1}{\pi}\int_0^\infty ds\frac{\rho(s)F_n^2(Q^2)}{s+Q^2}\ , \quad (7)$$

where $F_n(Q^2)$ is the part of the form factor $\langle\pi(p)|j_\mu|n(p+q)\rangle$ proportional to p_μ and $\rho(s)$ is the spectral density in the s channel. The states $|n\rangle$ are normalized as in Eq. (5), the n-state contribution to $\rho(s)$ being $\pi\delta(s-m_n^2)$. On the left-hand side (LHS) of Eq. (7) the term c/Q^4 denotes the contribution of three different spin-2, twist-4 operators



(a) (b)

FIG. 1. Diagrams contributing to (a) $\langle\pi|\theta^q_{\mu_1\mu_2}|\pi\rangle$ and (b) $\langle\pi|\theta^G_{\mu_1\mu_2}|\pi\rangle$. The dashed lines correspond to gluons.

[10] whose individual contributions cannot be separated. The constants b and c are considered as parameters to be fitted. The ellipsis in Eq. (7) corresponds to spin-2 terms of higher twist. Note that Eq. (7) is just the sum rule for the second moment of the deep inelastic structure function, $\int_0^1 F_2(x,Q^2)dx$, divided by Q^2. It is valid in the asymptotic region, $Q^2 \to \infty$, with all higher states in the s channel equally important in this region. Our goal here is to see whether Eq. (7) can be satisfied in a region of intermediate $Q^2 \sim 1\ \mathrm{GeV}^2$ where the RHS may be approximated by the contribution of a few low-lying states.[2]

First consider the case of charged pions. The lowest states in the s channel are the π and $a_1(1260)$ mesons. Assuming ρ dominance for the form factors (which is known to be a good approximation for the pion form factor up to $Q^2 \simeq 2\ \mathrm{GeV}^2$), $\langle\pi|j_\mu|n\rangle = -(m_\rho^2/g_\rho)\,\varepsilon_\mu^\rho\langle\pi\rho|n\rangle(Q^2+m_\rho^2)^{-1}$, where ε_μ^ρ is the ρ-meson polarization vector and $g_\rho^2/4\pi \simeq 2.9$. We obtain, for the RHS of Eq. (7),

$$\frac{8m_\rho^4}{(Q^2+m_\rho^2)^2}\left[\frac{1}{Q^2}+\frac{1}{4m_{a_1}^2 g_\rho^2(Q^2+m_{a_1}^2)}\left\{g_{a_1\rho\pi}^2+g_{a_1\rho\pi}h_{a_1\rho\pi}(Q^2-m_{a_1}^2)+\tfrac{1}{4}h_{a_1\rho\pi}^2(Q^2+m_{a_1}^2)^2\right\}\right]. \tag{8}$$

Here we used

$$\langle\pi^+(p)\rho^0(q)|\pi^+(p+q)\rangle = g_{\rho\pi\pi}\,\varepsilon^{\rho*}\cdot(2p+q) \tag{9}$$

and

$$i\langle\pi^+(p)\rho^0(q)|a_1^+(p+q)\rangle = g_{a_1\rho\pi}\,\varepsilon^{\rho*}\cdot\varepsilon^{a_1}+h_{a_1\rho\pi}\,\varepsilon^{\rho*}\cdot(p+q)\,\varepsilon^{a_1}\cdot p\,. \tag{10}$$

The notation corresponds to that of Ref. [13]. Note that $g_{\rho\pi\pi}=g_\rho$ within the ρ-dominance approach. The couplings $g_{a_1\rho\pi}$ and $h_{a_1\rho\pi}$ cannot, of course, be determined from the a_1 width alone. To this end we use an effective chiral Lagrangian with spin-1 mesons [12–14]. In this approach the constants in question are expressed in terms of parameters of this Lagrangian which are fitted to reproduce masses and widths. The Lagrangian in question contains a massive Yang-Mills part and two higher derivative terms:

$$\begin{aligned}\mathcal{L}_{AV\phi} = &-\tfrac{1}{2}\mathrm{Tr}(F_{\mu\nu}^L F^{L\mu\nu}+F_{\mu\nu}^R F^{R\mu\nu})+m_0^2\mathrm{Tr}(A_\mu^L A^{L\mu}+A_\mu^R A^{R\mu})\\ &-i\xi\,\mathrm{Tr}(D_\mu U D_\nu U^\dagger F^{L\mu\nu}+D_\mu U^\dagger D_\nu U F^{R\mu\nu})+\sigma\,\mathrm{Tr}F_{\mu\nu}^L U F^{R\mu\nu}U^\dagger,\end{aligned} \tag{11}$$

where $U=\exp(2i\phi/F_\pi)$, $\phi=\phi^a\tau^a/\sqrt{2}$, $A_\mu^L=\frac{1}{2}(V_\mu+A_\mu)$, $A_\mu^R=\frac{1}{2}(V_\mu-A_\mu)$, $F_{\mu\nu}^{L,R}=\partial_\mu A_\nu^{L,R}-\partial_\nu A_\mu^{L,R}-ig\left[A_\mu^{L,R},A_\nu^{L,R}\right]$ and the covariant derivative $D_\mu U=\partial_\mu U-igA_\mu^L U+igUA_\mu^R$. The quadratic piece of this Lagrangian is nondiagonal in $\partial_\mu\phi$ and A_μ. After diagonalization the physical masses are given by

$$m_V^2=m_\rho^2=\frac{m_0^2}{1-\sigma},\quad m_A^2=m_{a_1}^2=\frac{1}{1+\sigma}\left(m_0^2+\frac{g^2F_\pi^2}{4}\right), \tag{12}$$

and F_π is related to the physical coupling $\tilde{F}_\pi = 135$ MeV through

$$\tilde{F}_\pi = ZF_\pi\,,\qquad Z^2=1-\frac{g^2\tilde{F}_\pi^2}{4m_0^2}=\frac{1-\sigma}{1+\sigma}\frac{m_V^2}{m_A^2}\,. \tag{13}$$

The couplings $g_{a_1\rho\pi}$, $h_{a_1\rho\pi}$, and $g_{\rho\pi\pi}$ are expressed through g, ξ, σ and the meson masses by

$$h_{a_1\rho\pi}=-\frac{2Z^2}{\tilde{F}_\pi}\left(\frac{2}{1-\sigma^2}\right)^{\frac{1}{2}}(\sigma+g\xi)\,, \tag{14}$$

$$g_{a_1\rho\pi}=\tfrac{1}{2}(m_V^2+m_A^2-m_\pi^2)h_{a_1\rho\pi}+\frac{m_V^2}{\tilde{F}_\pi}\left(\frac{2}{1-\sigma^2}\right)^{\frac{1}{2}}\left[(1-\sigma)(1-Z^2)+2g\xi Z^2\right]\,, \tag{15}$$

$$g_{\rho\pi\pi}=\frac{g}{\sqrt{2(1-\sigma)}}\left[1-\tfrac{1}{2}(1-Z^2)+\frac{g\xi}{(1-\sigma)}\frac{Z^4}{(1-Z^2)}\right]\,. \tag{16}$$

[2]In Ref. [11] the transverse photon structure function in the region of intermediate x was calculated starting from the $VVVV$ four-point correlation function, using the OPE in the photon virtuality p^2 and extrapolating to $p^2=0$. One could think of doing the same thing for the pion structure function, starting from the $AVVA$ correlator. It can be shown, however, that just as in the case of the longitudinal photon structure function, there are difficulties in the extrapolation to *on-shell* pions. The $AVVA$ box diagram also cannot be used, via a triple dispersion relation, to model the continuum contribution to the real part of the forward scattering amplitude in the usual manner because of the zero momentum transfer in the t channel.

Here we retain a nonzero mass for the pion for the purposes of fitting the coupling constants. The widths are expressed through these couplings as[3]

$$\Gamma_{\rho\to\pi\pi} = \frac{1}{6\pi m_\rho^2}|q_\pi|^3 g_{\rho\pi\pi}^2 , \tag{17}$$

and

$$\Gamma_{a_1\to\rho\pi} = \frac{|q_\pi|}{12\pi m_{a_1}^2}\left[2g_{a_1\rho\pi}^2 + \left(\frac{E_\rho}{m_\rho}g_{a_1\rho\pi} - \frac{m_{a_1}}{m_\rho}|q_\pi|^2 h_{a_1\rho\pi}\right)^2\right] . \tag{18}$$

With the four available parameters g, σ, ξ, and m_0 it is possible to fit both the masses and the widths of the ρ and the a_1 [14]. We have refitted these parameters using a recent value of the width, $\Gamma_{a_1} = 400$ MeV [15, 16]. There are two possible solutions:

$$\begin{aligned} &(\mathrm{A}) \quad \sigma = 0.340, \quad \xi = 0.446, \quad g = 8.37 , \\ &(\mathrm{B}) \quad \sigma = -0.291, \quad \xi = 0.0585, \quad g = 7.95 , \end{aligned} \tag{19}$$

which correspond to

$$\begin{aligned} &(\mathrm{A}) \quad g_{a_1\rho\pi} = -5.42\ \mathrm{GeV}, \quad h_{a_1\rho\pi} = -16.7\ \mathrm{GeV}^{-1}, \quad \gamma = 0.52 , \\ &(\mathrm{B}) \quad g_{a_1\rho\pi} = 4.25\ \mathrm{GeV}, \quad h_{a_1\rho\pi} = -2.05\ \mathrm{GeV}^{-1}, \quad \gamma = 0.33 . \end{aligned} \tag{20}$$

Here the quantity γ is the ratio of polar- and axial-vector contributions to radiative pion decay. Both solutions are reasonably consistent with the positive experimental value of ~ 0.4 discussed by Holstein [13]. However, it can be shown that the *opposite-sign* solution, (B), is excluded by the QCD sum rule estimates of Ioffe and Smilga [17] for the two form factors entering the nondiagonal matrix element $\langle a_1|j_\mu|\pi\rangle$. They use couplings g_1 and g_2 to parametrize these form factors in a ρ dominance approach, and the relation to $g_{a_1\rho\pi}$ and $h_{a_1\rho\pi}$ is given by

$$g_{a_1\rho\pi} = g_1 m_{a_1}, \quad h_{a_1\rho\pi} = \frac{2}{m_{a_1}}\left[g_1 + \frac{m_\rho^2}{m_{a_1}^2}g_2\right] . \tag{21}$$

While the absolute values of g_1 and g_2 obtained in Ref. [17] contain large uncertainties, they are definitely of the same sign, thus ruling out solution (B). Therefore we choose the *like-sign* solution (A).

We shall display our results for the RHS of Eq. (7) multiplied by Q^4, which according to the LHS should give the linear relation $(2-b)Q^2 + c$. The results from Eq. (8) are given by the dashed line for the charged pion case in Fig. 2. It is seen that there is a good linear dependence for $Q^2 \geq 0.9$ GeV^2 . We cannot use values of Q^2 larger than plotted in the figure since higher states, which are not accounted for, become important and ρ dominance is not applicable either. There is an excited pion state $\pi^*(1300)$ which may contribute for the values of Q^2 in question. Its coupling to $\rho\pi$ defined through $\langle\pi^*|\rho(q)\pi(p)\rangle = g^*\,\varepsilon^\rho\cdot p$ may be roughly estimated using the rather uncertain data [15] on the width, $\Gamma_{\pi^*}^{\mathrm{tot}} = 200$–$600$ MeV and $\Gamma_{\pi^*\to\pi\rho} = \frac{1}{3}\Gamma_{\pi^*}^{\mathrm{tot}}$. This gives $g^* \approx 5$. The contribution of the π^* to the RHS of Eq. (7) is then

$$\frac{2g^{*2}m_\rho^4}{g_\rho^2(Q^2+m_\rho^2)^2(Q^2+m_{\pi^*}^2)} . \tag{22}$$

The result of taking into account the π^* is shown in Fig. 2

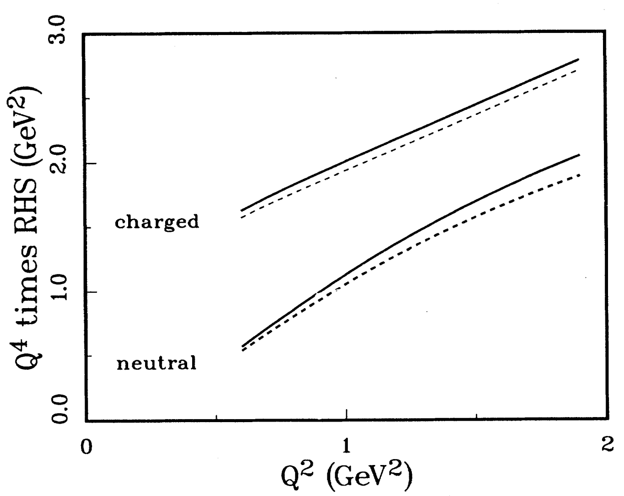


FIG. 2. The RHS of Eq. (7) multiplied by Q^4 shown as a function of Q^2. In the case of charged pions, the dashed curve is obtained with π- and a_1-meson intermediate states and the solid curve also includes the π^* meson. In the case of neutral pions, the dashed curve is obtained with an ω-meson intermediate state and the solid curve also includes the ω^* meson.

[3]We note that the *minus* sign in Eq. (18) is correct in contrast to Refs. [13, 14] which are written with an incorrect *plus* sign.

by the solid line. It is clear that the effect of the π^* is quite small. By fitting a straight line to the curve we estimate $b = 1.14$ and $c = 1.14$ GeV^2.

The matrix element of the gluon field energy density, Eq. (6), must be the same for charged and neutral pions. This may be used to check our calculation. So, let us now consider the case of neutral pions. Isotopic spin invariance forbids the π^0 and a_1^0 mesons in the s channel so the lowest allowed state is the ω meson. The $\omega\rho\pi$ vertex has the form

$$i\langle\pi(p)\rho(q)|\omega(p+q)\rangle = g_{\omega\rho\pi}\epsilon^{\alpha\beta\sigma\tau}\,\varepsilon^{\omega}_{\alpha}\varepsilon^{\rho}_{\beta}p_\sigma q_\tau\ , \qquad (23)$$

where $\varepsilon^\omega_\alpha$ and ε^ρ_β are the polarization vectors of the ω and ρ mesons. Then the contribution of the ω to the RHS of Eq. (7) is

$$\frac{2m_\rho^4 Q^2}{(Q^2+m_\rho^2)^2(Q^2+m_\omega^2)}\left(\frac{g_{\omega\rho\pi}}{g_\rho}\right)^2 . \qquad (24)$$

To be consistent we should use the value of the coupling constant $g_{\omega\rho\pi}$ obtained from the decay $\omega \to \pi\gamma$ using ρ dominance [18], $g_{\omega\rho\pi} \simeq 14.9$ GeV^{-1}. The corresponding Q^2 dependence of Eq. (24) (multiplied by Q^4) is shown in Fig. 2 by the dashed curve for the neutral case.

There is, however, an excited state, $\omega^*(1390)$, which can contribute. The dominant decay mode is to the $\rho\pi$ channel and taking this to account for the full width of 230 ± 40 MeV [15], we deduce a coupling constant $g_{\omega^*\rho\pi} = 5.29$ GeV^{-1}. The result of including both the ω and ω^* is shown by the solid line in Fig. 2. There is a noticeable curvature and a linear fit in this case results in larger uncertainties: $b =$ 1–1.2 and $c =$ 0–0.3 GeV^2. While the value for b agrees with the one obtained from charged pions, it is clear that the intercept c is different. This should have been expected since c involves the contributions of quark operators and their averages over charged and neutral pions need not be the same.

Thus, we adopt the value $b \simeq 1.14$, corresponding to a normalization point $\mu \sim Q \sim 1$ GeV. This is in good agreement with the value $b = 1.03$ deduced from the analysis of Ref. [8] in which the matrix element $\langle\pi|\theta^{u+d}_{\mu\nu}|\pi\rangle$ was extracted from a fit [19] to the quark and gluon distribution functions in the pion. In the leading log approximation the dependence on the normalization point is determined by the renormalization group. However, as is well known [20], operators of the same twist get mixed under renormalization due to radiative gluon corrections. The diagonal combinations in the case of two quark flavors are

$$\begin{aligned}
\theta^{\rm tot}_{\mu\nu} &= \theta^u_{\mu\nu} + \theta^d_{\mu\nu} + \theta^G_{\mu\nu} && [0]\ ,\\
R_{\mu\nu} &= \theta^u_{\mu\nu} + \theta^d_{\mu\nu} - \tfrac{3}{8}\theta^G_{\mu\nu} && [-\tfrac{44}{87}]\ ,\\
\Delta_{\mu\nu} &= \theta^u_{\mu\nu} - \theta^d_{\mu\nu} && [-\tfrac{32}{87}]\ .
\end{aligned} \qquad (25)$$

The numbers in square brackets are the anomalous dimensions γ of the corresponding diagonal operators which are renormalized multiplicatively:

$$\hat{O}_Q = \kappa^\gamma \hat{O}_\mu, \quad \kappa = \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} = \frac{\ln(Q/\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\rm QCD})}\ , \qquad (26)$$

where $\Lambda_{\rm QCD} \approx 150$ MeV. Then the evolution of b, defined by Eq. (5), under a change of the normalization point is given by

$$b(\mu) = \frac{16}{11}\left(1-\kappa^{44/87}\right) + b(Q)\kappa^{44/87}\ . \qquad (27)$$

It can be seen that according to Eq. (27) b decreases with μ and becomes zero at $\mu = 1.1\Lambda_{\rm QCD}$. At the standard normalization point used in QCD sum rules, $\mu = 0.5$ GeV, we get $b = 1.06$. Note that the small value of the normalization point for which $b = 0$ (meaning that there is no gluon component in the pion) agrees with the results of Ref. [21] where it was shown that a quark model description of deep inelastic scattering of leptons on nucleons is consistent with experimental data provided $\mu \approx m_\pi$.

Coming back to finite temperatures, the temperature dependence of the condensate $\langle\mathbf{E}^2+\mathbf{B}^2\rangle$ is determined by the integral over the thermal pion phase space:

$$\langle\mathbf{E}^2+\mathbf{B}^2\rangle_T = 3b\int\frac{d^3p}{(2\pi)^3}\frac{|\mathbf{p}|}{\exp(|\mathbf{p}|/T)-1} = \frac{b\pi^2}{10}T^4\ , \qquad (28)$$

where the factor of 3 in front of the integral accounts for the three charge states of pions. The structure function C_2 in Eq. (1) is obtained in the same way:

$$C_2(Q,T) = \frac{\pi^2T^4}{10Q^2}\left(2-b+\frac{\bar{c}}{Q^2}+\cdots\right) + O\left(\frac{T^6}{Q^4}\right)\ , \qquad (29)$$

where $\bar{c} = \frac{2}{3}c_{\rm charged} + \frac{1}{3}c_{\rm neutral} \approx \frac{2}{3}$ is the charge averaged value of the constant c.

Let us now briefly summarize what is known about behavior of condensates at low temperatures in the chiral limit. The temperature dependence of the usual (Lorentz scalar) condensates at low T was considered on the basis of chiral perturbation theory up to three-loop order [2]. The low T expansion of the quark condensate begins with a term of order T^2/F_π^2, because for pions with zero momentum the matrix element $\langle\pi|\bar{q}q|\pi\rangle$ is nonzero and proportional to $\langle0|\bar{q}q|0\rangle/F_\pi^2$. In the case of the operator $G^a_{\mu\nu}G^{a\mu\nu}$, which is a chiral singlet, the one-pion matrix elements vanish. The T dependence of the gluon condensate is related through the trace anomaly to $\langle\theta^\mu_\mu\rangle_T$. The first nonzero contribution to this matrix element appears only at the three-loop level. As a result, the T dependence of the gluon condensate begins at order T^8/F_π^4:

$$\begin{aligned}
\left\langle\frac{\alpha_s}{\pi}G^2_{\mu\nu}\right\rangle_T &= \left\langle\frac{\alpha_s}{\pi}G^2_{\mu\nu}\right\rangle_0\\
&\quad -\frac{4\pi^2}{3645}N_f^2(N_f^2-1)\frac{T^8}{F_\pi^4}\left(\ln\frac{\Lambda_p}{T}-\frac{1}{4}\right)\\
&\quad +\cdots\ ,
\end{aligned} \qquad (30)$$

where $\Lambda_p \simeq 275$ MeV is a scale encountered in the three-loop calculation of the pressure of a hot pion gas within chiral perturbation theory [2]. The sign of this contribution corresponds to the melting of the gluon conden-

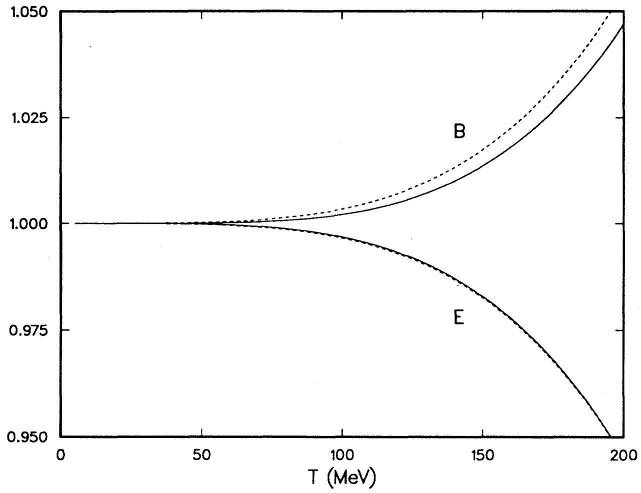


FIG. 3. The curves labeled B and E give, respectively, $\langle \mathbf{B}^2\rangle_T/\langle \mathbf{B}^2\rangle_0$ and $\langle \mathbf{E}^2\rangle_T/\langle \mathbf{E}^2\rangle_0$ as a function of temperature. The normalization point is $\mu = 0.5$ GeV. The dashed curves give the results for zero pion mass and the solid curves correspond to nonzero pion mass. In case E these two curves are indistinguishable.

sate with rising temperature. However, this melting is much slower than in the case of the quark condensate, and $\langle G^2\rangle_T$ is practically constant up to $T \sim 150$ MeV, that is, in the region of applicability of the approximation of a hadronic gas.

One-pion matrix elements of Lorentz nonscalar operators cannot be estimated using the soft pion approach, because they are proportional to the pion momentum p. Since $p \sim T$, the corresponding condensates naturally vanish as $T \to 0$. Since $\langle \mathbf{B}^2 - \mathbf{E}^2\rangle_T \simeq \langle \mathbf{B}^2 - \mathbf{E}^2\rangle_0$, we get from Eq. (28) the T dependence of the condensates of chromomagnetic and chromoelectric fields:

$$\begin{aligned} \langle \mathbf{B}^2\rangle_T &= \langle \mathbf{B}^2\rangle_0 + \frac{b\pi^2}{20}T^4 , \\ \langle \mathbf{E}^2\rangle_T &= \langle \mathbf{E}^2\rangle_0 + \frac{b\pi^2}{20}T^4 , \end{aligned} \tag{31}$$

where $\langle \mathbf{B}^2\rangle_0 = -\langle \mathbf{E}^2\rangle_0 \simeq 2\times 10^{-2}$ GeV^4, using a renormalization scale $\mu = 0.5$ GeV. We indicate the predicted ratios $\langle \mathbf{B}^2\rangle_T/\langle \mathbf{B}^2\rangle_0$ and $\langle \mathbf{E}^2\rangle_T/\langle \mathbf{E}^2\rangle_0$ by the dashed curves in Fig. 3. It is seen that the T dependence is rather weak at low T and, at $T \sim 150$ MeV, the condensates are changed from their $T = 0$ value by only about 1%. The fact that the change is small is qualitatively consistent with the results extracted from the lattice data [6]; however, we do not agree with the lattice predictions for the sign. We suggest that the lattice calculations are probably not sufficiently accurate to predict such small effects.

We notice that keeping m_π finite would not affect the values of b and c within the accuracy of our approach. The only differences would appear in the integral over the thermal distribution function, Eq. (28), and in a lower order contribution to Eq. (30). It is straightforward to perform the calculation numerically and this results in the solid curves shown in Fig. 3. We observe that Eq. (31) is a good approximation; indeed, for the electric field the results are indistinguishable. We remark that at very low T, $T \ll m_\pi$, we have, for $\mu = 0.5$ GeV,

$$\begin{aligned} \langle \mathbf{B}^2\rangle_T &= \langle \mathbf{B}^2\rangle_0 - 0.033 m_\pi^{5/2} T^{3/2} e^{-m_\pi/T} , \\ \langle \mathbf{E}^2\rangle_T &= \langle \mathbf{E}^2\rangle_0 + 0.20 m_\pi^{5/2} T^{3/2} e^{-m_\pi/T} . \end{aligned} \tag{32}$$

The numerical effect is exceedingly small, but it is interesting to observe that the magnetic condensate $\langle \mathbf{B}^2\rangle_T$ slightly decreases at very low T before increasing. The behavior of $\langle \mathbf{E}^2\rangle_T$ is, however, monotonic.

Finally we briefly comment on the effects of higher spin and twist operators. The averages of Lorentz nonsinglet operators of spin larger than 2 are necessarily proportional to higher powers of T and their contribution to thermal correlators will be suppressed by powers of T^2/Q^2. The operators of spin 2, but of higher twist, are suppressed by μ_h^2/Q^2, where μ_h is some hadronic mass scale $\sim \Lambda_{\rm QCD}$. In the case of vector currents three twist-4, spin-2 operators [10] contribute to the constant c in Eq. (7) and to disentangle individual contributions some extra information must be used. In our opinion, this problem deserves further consideration.

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