PHYSICAL REVIEW D VOLUME 47, NUMBER 9 1 MAY 1993

Uncertainties in coupling constant unification

Paul Langacker and Nir Polonsky
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 16 October 1992)

The status of coupling constant unification in the standard model and its supersymmetric extension are discussed. Uncertainties associated with the input coupling constants, m_t, threshold corrections at the low and high scales, and possible nonrenormalizable operators are parametrized and estimated. A simple parametrization of a general supersymmetric new particle spectrum is given. It is shown that an effective scale $M_{\rm SUSY}$ can be defined, but for a realistic spectrum it may differ considerably from the typical new particle masses. The implications of the lower (higher) values of $\alpha_s(M_Z)$ suggested by low-energy (Z-pole) experiments are discussed.

PACS number(s): 12.10.Dm, 11.30.Pb

I. INTRODUCTION

Implications of precision Z-pole, W mass, and neutral-current data for the standard model were considered previously in Ref. [1]. Constraints on the top mass were derived, and the value of a weak angle at the Z pole, $\sin^2\theta_W(M_Z)$, was extracted from the data. It was further shown that within the supersymmetric SU(5) grand unified theory (GUT) [2]–[10] the two-loop prediction [3] of the weak-angle agrees well with the value extracted from the data, that the standard model couplings meet at a point [within the $\alpha_s(M_Z)$ uncertainty] when extrapolated to high energy, and that the scale at which they meet is high enough to prevent a too fast proton decay rate via vector-boson exchange. On the other hand, when assuming the ordinary SU(5) GUT the standard model couplings, α_1, α_2, and α_3, do not meet, and the predicted proton decay rate is much too rapid. Similar observations were made by other groups [11]. Here and below, we denote the coupling of the group G_i by α_i, where $G_i=\mathrm{U}(1)_{Y/2}$, $\mathrm{SU}(2)_L$, $\mathrm{SU}(3)_c$ for $i=1,2,3$, respectively, and α_1 is further normalized as required. All of the couplings, as well as the weak angle, are defined in this paper in the modified minimal substraction scheme ($\overline{\rm MS}$) [12,13] unless otherwise specified. The $\overline{\rm MS}$ weak angle will be denoted below by s^2.

The above observations are true for a whole class of GUT's that break to the standard model group in one step, and which predict a "grand desert" between the weak (low) and the grand unification (high) scales (one-step GUT's). In particular, they hold for larger groups such as SO(10) and E_6, which have the same relative normalization of the G_i generators, provided there are no additional matter (super)multiplets that are split into light and heavy components. However, the SU(5) model has the minimal gauge group and, in the simplest version, a minimal matter content, and is therefore useful for illustration. One should note that high-scale thresholds can modify the predictions, and thus, in principle, distinguish different one-step GUT's. If a grand desert indeed exists, and, furthermore, supersymmetry is established and characterized at future colliders, we may eventually be able to use coupling-constant unification to probe the physics near the unification and Planck scales.

We dedicate most of this paper to a more thorough discussion of one-step GUT's. Let us mention, however, that one could also fit the data to a model in which intermediate scales are introduced. In Ref. [1] left-right models [derived from nonsupersymmetric SO(10) GUT's] [4,9,14] were considered, and it was found that models with an intermediate scale $M_R\approx10^{10}$ GeV for the breaking of the right-handed $\mathrm{SU}(2)_R$ are consistent with the data. (The supersymmetric version of the model requires that M_R is close to the unification scale [1].) A more recent discussion of SO(10) models is given in Ref. [15]. Models involving *ad hoc* new matter multiplets split into light and superheavy components were also considered [16]. Such models lose most of the predictive power of the ordinary or supersymmetric grand desert theories because either the intermediate scales or the quantum numbers of the new multiplets are chosen to fit the data. We will not discuss such possibilities any further in this paper.

The better standing of the supersymmetric one-step GUT's compared to the ordinary ones has been known for some time [17–19]. However, the much more precise coupling-constant data from the CERN e^+e^- collider LEP [20] has shown this more strongly and motivated a revived interest in GUT's. As we will show below, with such precise inputs the predictions become sensitive to small correction terms (threshold corrections and others), which are often ignored. Recently, detailed calculations of the supersymmetry (SUSY) new particle (sparticle) spectrum were carried out [21–23], and constraints from proton decay via dimension-five operators [22], and from fine-tuning of the top mass [21,23], were again considered. The possible equivalence of threshold corrections at the low and high scales was pointed out in Ref.

 ©1993 The American Physical Society

[24].[1] It was also shown that in SUSY GUT's with large representations, for which sterile neutrinos can have large masses comparable to the unification scale, the light neutrino masses predicted in seesaw models [26] are smaller than those suggested by the solar neutrino problem for $\nu_e \to \nu_\mu$ oscillations [27]. The possible role of nonrenormalizable operators (NRO's) at the high scale for generating more suitable neutrino masses has been pointed out [28]. A more careful consideration of the model predictions, and in a way that consistently incorporates different correction terms that may be significant, individually or cumulatively, is now required.

Some of the possible correction terms were considered recently in Refs. [29–31]. In Ref. [31] threshold corrections at the high scale were discussed, while the sparticle ones were treated naively. In Refs. [29,30] sparticle thresholds were discussed in detail and used to constrain the high-scale gaugino mass parameter. The motivation and approach here are different. We will suggest below an alternative way to treat the sparticle thresholds. We will elaborate on an observation of Ross and Roberts [21] that a naive analysis, in which all sparticles and new Higgs particles are degenerate at a scale $M_{\rm SUSY}$ [1,11,24,25,31], can be misleading, e.g., because the average mass of the colored sparticles may be larger than that of the uncolored ones. We give a simple parametrization of the effects of an arbitrary sparticle spectrum and show that an effective $M_{\rm SUSY}$ can always be defined. However, for realistic splittings $M_{\rm SUSY}$ can differ drastically from the actual sparticle masses, and, in particular, one can have $M_{\rm SUSY} < M_Z$ [as is suggested if $\alpha_s(M_Z)$ is sufficiently large] even though the actual sparticle masses are much larger than M_Z. We will also treat the heavy t-quark threshold corrections and the m_t contribution to the input parameter uncertainties consistently, and will consider threshold and NRO correction terms at the high scale. A convenient parametrization of the high-scale threshold corrections will be suggested as well.

Below, we will use the following (updated) input values of the low-scale parameters:

$$M_Z = 91.187 \pm 0.007 \text{ GeV} . \tag{1}$$

A two-parameter fit to all Z, W, and neutral-current data yields

$$s^2(M_Z) = 0.2324 \pm 0.0006 \quad (m_t \text{ free}) , \tag{2}$$

$$m_t = 138^{+20}_{-25} \pm 5 \text{ GeV} , \tag{3}$$

where the central values assume[2] a Higgs-boson mass $m_{h^0} = M_Z$. The second error in m_t is from allowing m_{h^0} to vary from 50–150 GeV, which is a reasonable range for the light Higgs scalar in the minimal supersymmetric extension of the standard model. [The additional Higgs particles do not contribute significantly to the experimental determination of $s^2(M_Z)$. Their contributions to the running are treated as threshold corrections.] Most of the uncertainty in $s^2(M_Z)$ is due to m_t and m_{h^0}. It is convenient to use the more restrictive value

$$s_0^2(M_Z) = 0.2324 \pm 0.0003 \quad (m_t = 138 \text{ GeV}) , \tag{4}$$

which is obtained for the fixed values $m_t = 138$ GeV, $m_{h^0} = M_Z$. The uncertainties from m_t and m_{h^0} will be treated separately.

We also have[3] [32]

$$\frac{1}{\alpha(M_Z)} = 127.9 \pm 0.1 , \tag{5}$$

which is valid for $m_t = 138$ GeV. Note that the values used here for $\alpha(M_Z)$ and $s^2(M_Z)$ correspond to the definitions in [32]. This is not quite the canonical $\overline{\rm MS}$ because m_t is not decoupled; i.e., it contributes to the running even below m_t. We will correct for this and treat the uncertainty from m_t in the threshold corrections.

The largest uncertainty in the input parameters is from $\alpha_s(M_Z)$. Some of the more precise determinations are shown in Table I and Fig. 1, which are adopted from a recent review of Bethke and Catani [33] (see also [34]). It is seen that there is a tendency for the lower-energy measurements to yield smaller $\alpha_s(M_Z)$ than the Z-pole determinations.[4] However, all of the determinations except R (which still has a large statistical error) have considerable theoretical uncertainties, which could very well be underestimated, so there is no compelling evidence for a discrepancy. We will take[5]

$$\alpha_s(M_Z) = 0.120 \pm 0.010 \tag{6}$$

as a reasonable estimate, for which we have assigned a fairly conservative uncertainty.

The ability of GUT's to predict s^2 at the unification point ($s^2 = \frac{3}{8}$ in SU(5) and similar models [2,3]) historically led to using the prediction for $s^2(M_Z)$ [$s_0^2(M_Z)$ in our case] from $\alpha(M_Z)$ and $\alpha_s(M_Z)$ as a test of the models. However, the large uncertainty in $\alpha_s(M_Z)$ leads to a large uncertainty in the predictions, and the different input values assumed by various authors have led to some confusion. We therefore find it more instructive to use

[1]Constraints from proton decay were ignored in Ref. [24], as discussed in Ref. [25]. The conclusion of Barbieri and Hall in Ref. [24] is, however, a qualitative one, and still holds. In both Refs. [24,25] the naive effective parameter $M_{\rm SUSY}$ was used. Below, we show that $M_{\rm SUSY} < M_Z$ is allowed when sparticle mass splittings are included.

[2]For m_{h^0} varying from 50–1000 GeV with a central value of 250 GeV, as is reasonable for the nonsupersymmetric standard model, one obtains $s^2(M_Z) = 0.2325 \pm 0.0007$, $m_t = 150^{+17}_{-23}{}^{+15}_{-17}$ GeV.

[3]There is a weak correlation between the $\alpha(M_Z)$ and $s_0^2(M_Z)$ error bars, associated with the hadronic contribution to the running of α. The effect is numerically insignificant to the discussion.

[4]This has even prompted the suggestion that there may be a light gluino, which modifies the extrapolation [36].

[5]This is higher than the value 0.1134 ± 0.0035 given by the Particle Data Group [35] due to the use of resummed QCD [33] and a more conservative estimate of theoretical uncertainties.

TABLE I. Values of $\alpha_s(M_Z)$, adapted from [33]. R_τ refers to the ratio of hadronic to leptonic τ decays, DIS to deep-inelastic scattering, $\Upsilon, J/\psi$ to quarkonium decays; and LEP (R) to the ratio of hadronic to leptonic Z decays. LEP (events) refers to the event topology in $Z \to$jets. This value was derived using resummed QCD [33], in which both α_s^2 and next-to-leading logarithms are used in theoretical expressions (the same data would yield 0.119 ± 0.006 using the α_s^2 expressions). We choose $\alpha_s(M_Z) = 0.12 \pm 0.01$ as a reasonable estimate of the average.

Source	$\alpha_s(M_Z)$
R_τ	0.118 ± 0.005
DIS	0.112 ± 0.005
$\Upsilon, J/\psi$	0.113 ± 0.006
LEP (R)	0.133 ± 0.012
LEP (events)	0.123 ± 0.005
Average	0.120 ± 0.010

$s_0^2(M_Z)$ as an input in order to predict $\alpha_s(M_Z)$. We will consider both alternatives below.

In this paper we discuss in detail the SU(5) grand unification of the standard model (with one Higgs doublet) (SM), and of the minimal supersymmetric standard model (with two Higgs doublets) (MSSM). In Sec. II we review the predictions of these models, where we use $\alpha_s(M_Z)$, and alternatively $s_0^2(M_Z)$, as an input. In Sec. III we discuss in detail different correction terms that may affect these predictions. We introduce three effective mass parameters that conveniently sum the threshold corrections near M_Z. In Sec. IV we collect our results and choose reasonable ranges for the different correction terms. We then obtain (in the MSSM) the predictions

$$s_0^2(M_Z) = 0.2334 \pm 0.0025 \pm 0.0014 \pm 0.0006^{+0.0013}_{-0.0005} \pm 0.0016 \,, \tag{7}$$

$$\alpha_s(M_Z) = 0.125 \pm 0.001 \pm 0.005 \pm 0.002^{+0.005}_{-0.002} \pm 0.006 \,, \tag{8}$$

where the central values are for $m_t = 138$ GeV and $M_{\rm SUSY} = m_{h^0} = M_Z$. The first uncertainty in (7) [in (8)] is due to the $\alpha_s(M_Z)$ [$s_0^2(M_Z)$] and $\alpha(M_Z)$ error bars, and the other uncertainties in both (7) and (8) are due to sparticle thresholds, m_t and m_{h^0}, thresholds at the high scale, and NRO's at the high scale, respectively. The uncertainties quoted here refer to our choice of ranges for the different correction terms, and should be taken as such (i.e., as order of magnitude estimations rather than rigorous ranges). Note that the different theoretical uncertainties are comparable to the $\alpha_s(M_Z)$ error bar in (6) and to the corresponding uncertainty in (7). The combined theoretical uncertainty is determined by an interplay among the different terms, most of which can have either sign. [If the high-scale thresholds are not constrained as in the minimal model (see below), then none of the uncertainties has a fixed sign.] When added in quadrature, the above theoretical uncertainties yield a $+0.0026-0.0023$ ($+0.010-0.008$) combined uncertainty in the $s_0^2(M_Z)$ [$\alpha_s(M_Z)$] prediction. The predicted $\alpha_s(M_Z)$ is compared with the data in Fig. 1, while the $s^2(M_Z)$ prediction is shown in Fig. 2. The extrapolated coupling constants are shown in Fig. 3. Corresponding predictions for the unification scale and the coupling at that scale are given in Sec. IV. In all cases, it is seen that the MSSM (but not the SM) is in agreement with the prediction of unification.

The prediction for $\alpha_s(M_Z)$ is in good agreement with the value observed at the Z pole, and the larger Z-pole value for $\alpha_s(M_Z)$ predicts a smaller $s_0^2(M_Z)$, in agreement with observation. The somewhat lower $\alpha_s(M_Z)$ values suggested by low-energy experiments could be accommodated by $M_{\rm SUSY} > M_Z$, $m_t < 138$ GeV, or the introduction of NRO's. Also, in the simplest SUSY SU(5) the high-scale thresholds increase the predicted $\alpha_s(M_Z)$ when constraints from proton decay are included. However, simple extensions, e.g., replacing R parity with

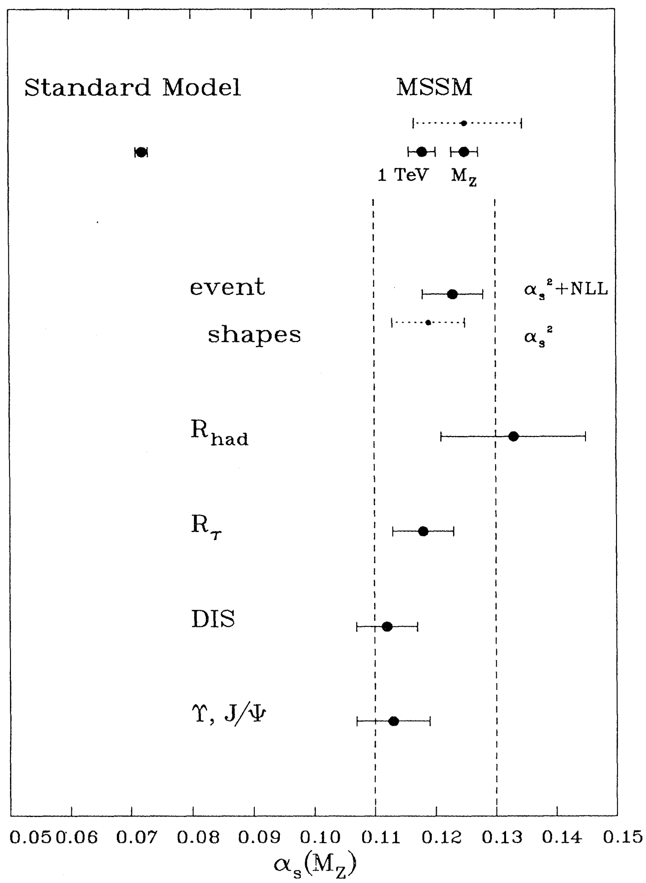

FIG. 1. Predictions for $\alpha_s(M_Z)$ from $\alpha(M_Z)$ and $s^2(M_Z)$ in ordinary (SM) and SUSY (MSSM) GUT's. In the SM case the uncertainty $\sim \pm 0.001$ includes that from $\alpha(M_Z)$ and $s^2(M_Z)$ and the negligible high-scale and NRO errors. For the MSSM the small error bars are from $\alpha(M_Z)$ and $s^2(M_Z)$ (including the m_t dependence) for $M_{\rm SUSY} = M_Z$ and $M_{\rm SUSY} = 1$ TeV. (We discuss the choice $M_{\rm SUSY} = 1$ TeV in Sec. IV.) The larger error bar includes the SUSY, high-scale, and NRO uncertainties added in quadrature. Various experimental determinations along with their nominal uncertainties are also shown. The dashed lines are the range 0.12 ± 0.01.

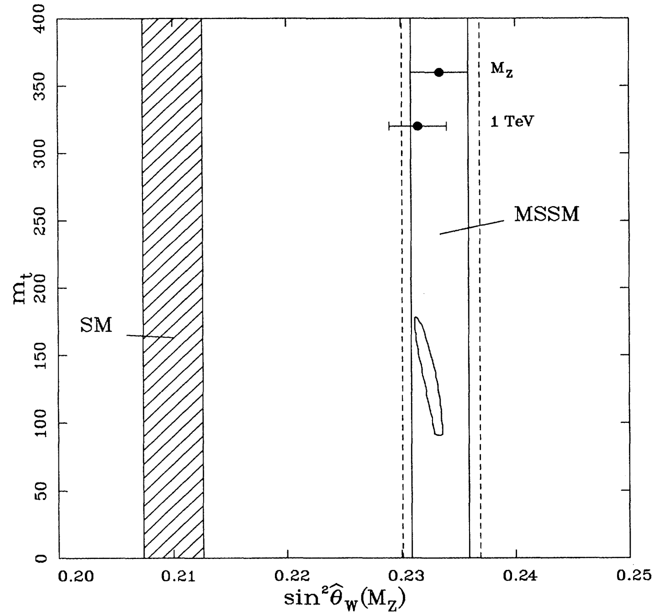

FIG. 2. Predictions for $s^2(M_Z)$ from $\alpha(M_Z)$ and $\alpha_s(M_Z)$ in ordinary (SM) and SUSY (MSSM) GUT's, compared with the region allowed by the data at 90% C.L. The smaller ranges of uncertainties are from $\alpha_s(M_Z)$ and $\alpha(M_Z)$ only, while the larger ones include the various low and high scale uncertainties added in quadrature. The predictions for $M_{\text{SUSY}}=M_Z$ and 1 TeV (see discussion in Sec. IV) are shown for comparison.

baryon parity [37], or the introduction of additional matter supermultiplets at the high scale, would allow smaller $\alpha_s(M_Z)$. We discuss the high-scale thresholds in more general terms in Sec. V, where we introduce effective parameters, similar to those introduced for the sparticles in Sec. III. Throughout this paper we display the various expressions in a transparent form, which enables one to generalize our discussion and to use the results elsewhere. We summarize our conclusions in Sec. VI.

II. ONE- AND TWO-LOOP PREDICTIONS

When solving for the running of the couplings in any GUT scenario with no intermediate scale, we can reduce the problem to one of a grand desert and account for all thresholds near the desert boundaries by properly defining correction terms. If one uses a two-loop β function for the running, then one-loop threshold corrections [38–40] usually suffice. The normalized couplings are then [39,18]

$$\frac{1}{\alpha_i(M_Z)}=\frac{1}{\alpha_G}+b_i t+\theta_i-\Delta_i \quad \text{for } i=1,2,3 \,, \tag{9}$$

where $t\equiv(1/2\pi)\ln(M_G/M_Z)$, M_G is the grand unification scale (which serves as the high-scale boundary of the desert), and α_G is the coupling at that point.

$$\theta_i\equiv\frac{1}{4\pi}\sum_{j=1}^{3}\frac{b_{ij}}{b_j}\ln\left[\frac{\alpha_j(M_G)}{\alpha_j(M_Z)}\right]$$

are the two-loop terms, and b_i (b_{ij}) are the one-(two-)loop β function coefficients,

$$\mu\frac{d\alpha_i}{d\mu}=\frac{b_i}{2\pi}\alpha_i^2+\sum_{j=1}^{3}\frac{b_{ij}}{8\pi^2}\alpha_i^2\alpha_j \,, \tag{10}$$

which can be calculated using, for example, Refs. [41,42].

Δ_i are threshold and other corrections, which should be calculated to a precision consistent with the θ_i. Our ignorance of their exact values suggests that they should be reasonably parametrized and estimated within a given model, and then translated into theoretical uncertainties on any predictions. This will be carried out in the following sections. We will also show that for reasonable masses for the sparticles, the MSSM can be treated as a two-scale model with all mass effects included in the threshold corrections.

At the Z threshold (which serves as the low-scale boundary of the desert), we have

$$\frac{1}{\alpha_i(M_Z)}=\frac{3}{5}\frac{1-s^2(M_Z)}{\alpha(M_Z)},\frac{s^2(M_Z)}{\alpha(M_Z)},\frac{1}{\alpha_s(M_Z)} \quad \text{for } i=1,2,3 \,,$$

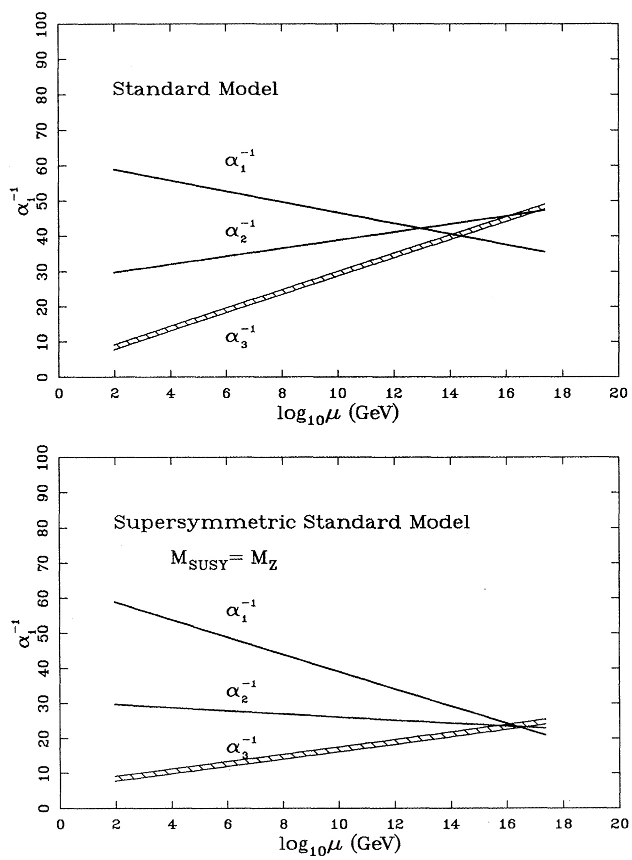

FIG. 3. The running coupling in ordinary (SM) and SUSY (MSSM) GUT's, assuming $s^2(M_Z)=0.2324\pm0.0006$, $1/\alpha(M_Z)=127.9\pm0.2$ (the larger uncertainty compared to (5) is due to m_t), and $\alpha_s(M_Z)=0.120\pm0.010$, for $M_{\text{SUSY}}=M_Z$. The uncertainties from threshold effects are best seen in Figs. 1 and 2.

respectively. $s^2(M_Z)$ [which we replace with $s_0^2(M_Z)$], $\alpha(M_Z)$, and $\alpha_s(M_Z)$ are the three low-scale ($\overline{\rm MS}$) parameters defined previously, evaluated at the Z pole. By taking linear combinations of (9) one obtains explicit expressions for the two high-scale parameters t and α_G, and for one low-scale parameter, in terms of the other two, the β function one-loop coefficients and the two-loop and correction terms.

The two-loop terms can be rewritten using the lowest-order solution for the couplings [39,18]:

$$\theta_i = \frac{1}{4\pi}\sum_{j=1}^{3}\frac{b_{ij}}{b_j}\ln(1+b_j\alpha_G t)\ , \tag{11}$$

where the one-loop expressions for α_G and t are to be substituted.[6] For a given model one can then predict t, α_G, and either $s_0^2(M_Z)$, which we will refer to as case (a), or $\alpha_s(M_Z)$, which we will refer to as case (b), in terms of $\alpha(M_Z)$ and either $\alpha_s(M_Z)$ or $s_0^2(M_Z)$.

We list in Table II the general expressions for t, α_G, and $s^2(M_Z)$ [$\alpha_s(M_Z)$],[7] where we define a linear combination of the one-loop β function coefficients, $D\equiv 5b_1+3b_2-8b_3$. The correction term for each expression is of the same form as the one-loop term, only with θ_i replaced by $-\Delta_i$. Note that we have exactly the same expressions when replacing $s^2(M_Z)$ with $s_0^2(M_Z)$, except that the t quark and the Higgs particle contributions to the correction terms Δ_i are different. For the two models studied in this paper, the SM and the MSSM, the β functions can be found in Ref. [18], where the dependence on the number of fermion families and Higgs doublets is explicitly given. For completeness we give b_i, b_{ij}, and D for the SM (MSSM) with our choice of three families and one (two) Higgs doublet(s) in Table III. Then, using Tables II and III and the input parameters, we can calculate the two-loop terms for each case. These are listed in Table IV, where we also compare the θ_i values calculated using the one-loop t and α_G and those calculated iteratively. For different values of the low-scale input parameters the two-loop terms should be recalculated, though for a small change the difference is negligible.

In Table V we give the predictions corresponding to our central values of the input parameters, but not including any correction terms. One can clearly see that the MSSM is consistent with these values (see also Figs. 1–3). Cases (a) and (b) in the MSSM are consistent with each other at the two-loop order. Also the prediction of t in that model is large enough to prevent an observed proton decay via a heavy vector boson exchange [43]. The value of t corresponds to $M_G\sim 2.5\times10^{16}$ GeV, so that $\tau_{p\to e^+\pi^0}\sim M_G^4\sim 10^{37\pm1}$ yr, much larger than the experimental lower limit [43] of 10^{33} yr. In the SM the inconsistency between cases (a) and (b) implies that SM unification is inconsistent with the present values of the input parameters (see also Figs. 1–3). Also, the SM prediction of t is inconsistent with proton decay limits[8] in either case (a) or (b). For case (a) [case (b)] one predicts the unacceptable values $M_G\sim 4.6\times10^{14}$ GeV and $\tau_{p\to e^+\pi^0}\sim 10^{31\pm1}$ yr (8.5×10^{12} GeV and $10^{24\pm1}$ yr).

The above failures of the SM cannot be resolved by adding either more light Higgs doublets or additional fermion families. As is well known, additional fermion families represent complete GUT multiplets, which affect all the b_i's equally. Hence, the $\alpha_s(M_Z)$, $s_0^2(M_Z)$, and t predictions are only modified at the two-loop level. (α_G is affected at one-loop.) On the other hand, extra Higgs families are part of partial GUT multiplets, which affect the predictions at one loop. When adding Δn_H Higgs doublets in case (a), the $s_0^2(M_Z)$ prediction increases, but t decreases, increasing the proton decay rate. For $\Delta n_H=6$ one has $M_G\sim 4\times10^{13}$ GeV and $\tau_p\sim 6\times10^{26}$ yr. In case (b), $\alpha_s(M_Z)$ increases with n_H, but eventually changes sign, and adding enough Higgs doublets so that t has an acceptable value drives $\alpha_s(M_Z)$ negative. In the MSSM, extra Higgs supermultiplets will destroy the successful predictions for $s_0^2(M_Z)$ and $\alpha_s(M_Z)$. For completeness we display the changes in the predictions for additional fermion family and Higgs (super)multiplets in Table VI.

III. A FORMAL DISCUSSION OF THE CORRECTION TERMS

This section will be devoted to the correction terms Δ_i:

$$\Delta_i = \Delta_i^{\rm conversion} + \sum_{\rm boundary}\sum_{\zeta}\frac{b_i^{\zeta}}{(2\pi)}\left[\ln\left[\frac{M_\zeta}{M_{\rm boundary}}\right]-C^{J_\zeta}\right] + \Delta_i^{\rm top}+\Delta_i^{\rm Yukawa}+\Delta_i^{\rm NRO}\ . \tag{12}$$

The first term is a constant, which depends only on the gauge group G_i [44]:

$$\Delta_i^{\rm conversion}\equiv -\frac{C_2(G_i)}{12\pi}\ , \tag{13}$$

where $C_2(G_i)$ is the quadratic casimir operator for the adjoint representation, $C_2(G_i)=N$ [0] for $G_i={\rm SU}(N)$ [U(1)]. $\Delta_i^{\rm conversion}$ results from the need to use the dimensional-reduction ($\overline{\rm DR}$) scheme in the MSSM, so

[6] In practice we will use the full two-loop values for t and α_G in θ_i, solving iteratively. The difference between the two procedures is of higher order.

[7] We use a Taylor expansion to convert the prediction of $1/\alpha_s(M_Z)$ to an expression for $\alpha_s(M_Z)$. In Table II we give the zeroth- (one-loop) and first- (two-loop) order terms in the expansion. This gives $\sim 99.2\%$ accuracy. We will include the second-order term when evaluating $\alpha_s(M_Z)$.

[8] In the SM case one has approximately $\tau_{p\to e^+\pi^0}$ (yr) $\sim 10^{31\pm1}(M_G/4.6\times10^{14}\ {\rm GeV})^4$, so that $\tau>10^{33}$ yr corresponds to $M_G>10^{15}$ GeV or $t>4.8$. In the MSSM the $e^+\pi^0$ rate is suppressed by M_G^{-4} but is slightly enhanced by a factor of ~ 3 due to the larger α_G.

TABLE II. (a) t, $1/\alpha_G$, and $s^2(M_Z)$ [or alternatively $s_0^2(M_Z)$] predictions in terms of $\alpha(M_Z)$, $\alpha_s(M_Z)$, and the β-function coefficients, case (a). (b) t, $1/\alpha_G$, and $\alpha_s(M_Z)$ predictions in terms of $\alpha(M_Z)$, $s^2(M_Z)$ [or alternatively $s_0^2(M_Z)$] and the β function coefficients, case (b). The correction terms are of the same form as the two-loop terms, only with θ_i replaced by $-\Delta_i$.

	One-loop term	Two-loop term
	(a)	
t	$\frac{1}{D}\left[\frac{3}{\alpha(M_Z)}-\frac{8}{\alpha_s(M_Z)}\right]$	$-\frac{1}{D}(5\theta_1+3\theta_2-8\theta_3)$
$\frac{1}{\alpha_G}$	$\frac{1}{D}\left[\frac{-3b_3}{\alpha(M_Z)}+\frac{5b_1+3b_2}{\alpha_s(M_Z)}\right]$	$-\frac{1}{D}((5b_1+3b_2)\theta_3-b_3(5\theta_1+3\theta_2))$
$s^2(M_Z)$	$\frac{1}{D}\left[3(b_2-b_3)+5(b_1-b_2)\frac{\alpha(M_Z)}{\alpha_s(M_Z)}\right]$	$-\frac{5\alpha(M_Z)}{D}((b_2-b_3)\theta_1+(b_3-b_1)\theta_2+(b_1-b_2)\theta_3)$
	(b)	
t	$\frac{3-8s^2(M_Z)}{5(b_1-b_2)\alpha(M_Z)}$	$+\frac{\theta_2-\theta_1}{b_1-b_2}$
$\frac{1}{\alpha_G}$	$\frac{3b_2[1-s^2(M_Z)]-5b_1s^2(M_Z)}{5(b_2-b_1)\alpha(M_Z)}$	$+\frac{b_2\theta_1-b_1\theta_2}{b_1-b_2}$
$\alpha_s(M_Z)$	$\frac{5(b_1-b_2)\alpha(M_Z)}{Ds^2(M_Z)-3(b_2-b_3)}$	$-\frac{25(b_1-b_2)\alpha(M_Z)^2}{[Ds^2(M_Z)-3(b_2-b_3)]^2}[(b_2-b_3)\theta_1+(b_3-b_1)\theta_2+(b_1-b_2)\theta_3]$

that the algebra is kept in four dimensions [45]. Thus, we convert the $\overline{\rm MS}$ couplings above M_Z:

$$\frac{1}{\alpha_{i_{\overline{\rm MS}}}}=\frac{1}{\alpha_{i_{\overline{\rm DR}}}}-\Delta_i^{\rm conversion}\,. \tag{14}$$

For consistency we will also use $\overline{\rm DR}$ in the SM case, though this is not required. α_G is then given in its $\overline{\rm DR}$ definition.

The second term sums over the one-loop threshold corrections [39]. b_i^ζ is the (decoupled) contribution of a heavy field ζ to the β function coefficient b_i between M_ζ and $M_{\rm boundary}$. C^{J_ζ} is a mass-independent number, which depends on the spin J_ζ of ζ and on the regularization scheme used. In $\overline{\rm MS}$ (using dimensional regularization) one has[9] $C^1_{\overline{\rm MS}}=\frac{1}{21}$, $C^{1/2}_{\overline{\rm MS}}=C^0_{\overline{\rm MS}}=0$ [39]. These are to be used at the low-scale boundary, while at the other boundary (using dimensional reduction[10]) we have $C^J_{\overline{\rm DR}}\equiv 0$ [44]. (If one converts α_G back to its $\overline{\rm MS}$ definition, then the sum of the two conversion terms reproduces the $\overline{\rm MS}$ mass-independent term.)

The summation in (12) can account for a particle

TABLE III. The β function coefficients [18] and their linear combination $D\equiv 5b_1+3b_2-8b_3$.

	SM	MSSM
b_i	$\begin{pmatrix}\frac{41}{10}\\ -\frac{19}{6}\\ -7\end{pmatrix}$	$\begin{pmatrix}\frac{66}{10}\\ 1\\ -3\end{pmatrix}$
b_{ij}	$\begin{pmatrix}3.98 & 2.7 & 8.8\\ 0.9 & \frac{35}{6} & 12\\ 1.1 & 4.5 & -26\end{pmatrix}$	$\begin{pmatrix}7.96 & 5.4 & 17.6\\ 1.8 & 25 & 24\\ 2.2 & 9 & 14\end{pmatrix}$
D	67	60

TABLE IV. (a) Two-loop terms for the case (a) calculated using one loop values for the parameters (OL), and iteratively (TL). (b) The same as (a) except for case (b).

	SM		MSSM	
Two-loop term	OL	TL	OL	TL
	(a)			
θ_1	0.22	0.21	0.67	0.69
θ_2	0.29	0.28	1.09	1.13
θ_3	-0.41	-0.40	0.56	0.58
	(b)			
θ_1	0.16	0.16	0.64	0.71
θ_2	0.21	0.21	1.05	1.16
θ_3	-0.27	-0.28	0.54	0.60

[9]Different regularization conventions give $C^{1/2}_{\overline{\rm MS}}=-\ln\sqrt{2}$ [38,39].

[10]When using dimensional reduction the loop integrals are analytically continued away from $d=4$ (as for dimensional regularization). On the other hand, the algebra of the fields is not continued and is kept in $d=4$ (i.e., $g^{\mu\nu}g_{\mu\nu}=4$). Therefore, no constants arise when taking the limit $d\rightarrow 4$ [44].

threshold as long as two-loop terms between this threshold and the boundary are negligible, i.e.,

$$\left| b_i \alpha_i (M_{\text{boundary}}) \ln \left[\frac{M_\xi}{M_{\text{boundary}}} \right] \right| \ll 2\pi \ . \tag{15}$$

This allows a split of more than 3 orders of magnitude for all relevant cases. Thus (12) can correctly account for a reasonable sparticle spectrum.

At the low-scale boundary we have to consider the top, Higgs, and sparticle thresholds. The $SU(2)_L$ symmetry is broken by the top quark mass in the range $M_Z - m_t$, questioning the validity of accounting for the top in the above threshold summation. Furthermore, the values of the input parameters and m_t are correlated in a complicated way. Similar considerations apply to the SM Higgs boson. We therefore omit these two thresholds from the summation and discuss them separately below. In the MSSM we assume a light SM Higgs boson ($m_{h^0} \approx M_Z$) and a heavy decoupled doublet, which is included with the sparticles. Using tree-level sum rules [46] one can show that in such a limit $SU(2)_L$ breaking is negligible in the Higgs sector. (This conclusion is still valid when radiative corrections to the Higgs-boson masses [47,48] are considered.) We will further assume a good symmetry in the sparticle sector (i.e., $SU(2)_L$-breaking effects are typically $<(m_t/m_{t\ \text{squark}})^2$ and are negligible for our purposes).

In the SM we can then omit the low-scale boundary from the summation in (12), while in the MSSM we are left with the sparticles and the heavy Higgs doublet. The sparticle and Higgs-boson masses can be calculated given a small number of high-scale parameters—i.e., a universal gaugino mass $M_{1/2}$; a universal scalar mass m_0; the Higgs mixing parameter μ_{mixing}; a universal trilinear coupling A; and the top Yukawa coupling h_t (we omit all other Yukawa couplings)—by solving a set of coupled renormalization-group equations (RGE's) [49,50].[11] Other mass parameters, such as the universal bilinear coupling B, are related to the parameters above by boundary conditions and the constraint setting the weak breaking scale [46]. One can then solve the one-loop RGE's for a given set of parameters, and predict a specific sparticle spectrum [21–23]. Substituting in (12) gives the desired correction. However, this is a lengthy and not very enlightening procedure for our purpose of estimating small correction terms. We use instead a parametrization in terms of three low-energy effective parameters defined by

$$\sum_\xi \frac{b_i^\xi}{2\pi} \ln \left[\frac{M_\xi}{M_Z} \right] \equiv \frac{b_i^{\text{MSSM}} - b_i^{\text{SM}}}{2\pi} \ln \left[\frac{M_i}{M_Z} \right] \quad \text{for } i = 1,2,3 \ , \tag{16}$$

TABLE V. (a) Numerical predictions of t, $1/\alpha_G$, and $s_0^2(M_Z)$ in case (a). (b) Numerical predictions of t, $1/\alpha_G$, and $\alpha_s(M_Z)$ in case (b). Input parameters are indicated by parentheses. No correction terms are included. The near equality of cases (a) and (b) for the MSSM is a reflection of the success of the coupling constant unification.

	SM		MSSM	
	One loop	Two loop	One loop	Two loop
(a)				
t	4.73	4.65	5.28	5.25
$\frac{1}{\alpha_G}$	41.46	41.32	24.18	23.49
$\frac{1}{\alpha(M_Z)}$		(127.9)		
$s_0^2(M_Z)$	0.2070	0.2100	0.2304	0.2335
$\alpha_s(M_Z)$		(0.120)		
(b)				
t	4.01	4.02	5.21	5.29
$\frac{1}{\alpha_G}$	42.44	42.45	24.51	23.28
$\frac{1}{\alpha(M_Z)}$		(127.9)		
$s_0^2(M_Z)$		(0.2324)		
$\alpha_s(M_Z)$	0.070	0.072	0.113	0.125

TABLE VI. (a) The two-loop predictions of the SM and MSSM in case (a) for $\Delta F = 1$ additional fermion family and $\Delta n_H = 1$ or 2 additional light Higgs (super)multiplets. Input parameters are indicated by brackets. No correction terms are included. (b) The same as (a) except in case (b). Note that for a negative $\alpha_s(M_Z)$ the Taylor expansion is not valid.

	SM		MSSM	
	$\Delta F = 1$	$\Delta n_H = 1$	$\Delta F = 1$	$\Delta n_H = 2$
(a)				
t	4.69	4.58	5.32	4.76
$\frac{1}{\alpha_G}$	34.83	40.83	11.52	22.09
$\frac{1}{\alpha(M_Z)}$		(127.9)		
$s_0^2(M_Z)$	0.2099	0.2141	0.2345	0.2562
$\alpha_s(M_Z)$		(0.120)		
(b)				
t	4.05	4.06	5.41	5.78
$\frac{1}{\alpha_G}$	36.70	41.67	10.69	15.82
$\frac{1}{\alpha(M_Z)}$		(127.9)		
$s_0^2(M_Z)$		(0.2324)		
$\alpha_s(M_Z)$	0.072	0.077	0.130	Negative

[11]Our notation follows that of Ref. [49], aside from self-explanatory subscripts.

TABLE VII. The MSSM low-energy parameters calculated for the spectra of Ref. [21]. An $SU(2)_L$ doublet is identified with its heavier member. Masses are in GeV.

High-scale parameters					Low-scale parameters			
$M_{1/2}$	m_0	$\mu_{\rm mixing}$	A	B	m_t	M_1	M_2	M_3
140	190	190	0	0	160	261	207	352
230	120	−120	0	0	100	282	245	527

where the summation is over the sparticles and the heavy Higgs doublet. The low-scale sparticle spectrum can be crudely parametrized in terms of the high-scale parameters[12] (with a reasonable assumption about the Higgs mass) [6,8,29], in order to learn about the relationship between the high-scale parameters and M_i. One finds that the case $M_{1/2} \gg m_0 \approx \mu_{\rm mixing} \approx M_Z$ ($m_0 \approx \mu_{\rm mixing} \gg M_{1/2} \approx M_Z$) corresponds to $M_3 \gg M_1, M_2$ ($M_1 \gg M_2, M_3$).[13] The parameters can be split by a factor of a few. As will become clear below, it is important to note that we do not expect to have $M_2 \gg M_1$ and/or $M_2 \gg M_3$. We have also calculated M_i for the realistic spectra[14] given in Ref. [21] (see Table VII). The parameters M_i can be calculated exactly in any other model using (16), and once calculated, all correction terms are given below.

The discussion so far has only assumed a SM gauge group, $S \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_{Y/2}$ (with the proper normalization of α_1) in the desert, and has been independent of the GUT gauge group. The high-scale corrections do depend on the group. For definiteness we first assume that this is SU(5), for both the SM and the MSSM. A minimal choice of massive (super)multiplets at the high scale is then (listing S quantum numbers) $(\bar{3},2,\frac{5}{6})\oplus$c.c. massive vector (super)multiplets;[15] (8,1,0), (1,3,0),(1,1,0) massive real Higgs (Majorana super)multiplets [embedded in a **24** of SU(5)]; and a $(3,1,-\frac{1}{3})$ complex Higgs (Dirac super)multiplet [embedded in a **5** of SU(5)] [4,9]. We thus introduce three mass parameters M_V, M_{24}, and M_5 for the vector, real-Higgs (Majorana), and complex-Higgs (Dirac) (super)multiplet thresholds, respectively, and we assume mass degeneracy within each of these (super)multiplet classes. (We show how to generalize this in Sec. V.) We then identify $M_G \equiv \max(M_V, M_{24}, M_5)$, so that SU(5) is complete above M_G. In the MSSM, proton decay via dimension-five operators constrains $M_5 \gtrsim 10^{16}$ GeV, and the validity of perturbation theory in the Higgs sector constrains $M_5 \lesssim 3M_V$ [22]. This suggests $M_G \equiv M_5$ in the MSSM. Though we shall not impose this (allowing other solutions to the proton decay problem [37]), one has to bear in mind the possible need to carefully adjust M_5 in the MSSM, and the general dependence between these parameters, determined by the details of the Higgs sector Lagrangian.

We now discuss the heavy top threshold. We must consider both the effect on the running and on the experimental determination of the couplings at M_Z. In the $\overline{\rm MS}$ scheme to account for $m_t > M_Z$ one can define threshold corrections [39] to $\alpha(M_Z)$ and $\alpha_s(M_Z)$, i.e., $(b_Q^{\rm top}/2\pi)\ln(m_t/M_Z)$ and $(b_3^{\rm top}/2\pi)\ln(m_t/M_Z)$, respectively, where $b_Q^{\rm top}$ and $b_3^{\rm top}$ are the top contributions to the relevant one-loop β function slope. The first of these corrections is equivalent to the slightly nonstandard $\alpha(M_Z)$ definition of Ref. [32], which we use. Thus, for our central value of $m_t = 138$ GeV, our value of $\alpha(M_Z)$ already includes the top threshold correction, and we need to further correct $\alpha(M_Z)$ only for different values of m_t. Thus

$$\Delta_\alpha^{\rm top} = \frac{8}{9\pi}\ln\left[\frac{m_t}{138\ {\rm GeV}}\right], \tag{17}$$

$$\Delta_{\alpha_s}^{\rm top} = \frac{1}{3\pi}\ln\left[\frac{138\ {\rm GeV}}{M_Z}\right] + \frac{1}{3\pi}\ln\left[\frac{m_t}{138\ {\rm GeV}}\right]. \tag{18}$$

Similarly, the m_t threshold corrections are already included in the $s^2(M_Z)$ definition of Ref. [32]. However, the input value of $s^2(M_Z)$ extracted from the data depends both quadratically and logarithmically on m_t. In particular, the value $s_0^2(M_Z) = 0.2324 \pm 0.0003$ in (4) is for the best fit value $m_t = m_{t0} = 138$ GeV. For other m_t the corresponding $s^2(M_Z)$ is

$$s^2 = s_0^2 - \frac{3G_F}{8\sqrt{2}\pi^2} s_0^2 \frac{1-s_0^2}{1-2s_0^2}[(m_t)^2 - (m_{t0})^2], \tag{19}$$

where G_F is the Fermi coupling, and we have neglected

---

[12] In the limit $h_t \to 0$ this parametrization can be made exact.

[13] Constraints derived from proton decay favor $m_0 \gg M_{1/2}$ [22]. However, we then may have $\mu_{\rm mixing} < m_0$. If so, M_3 and M_1 become closer.

[14] M_G and α_G of Ref. [21] differ slightly from ours due to different values of input parameters and a different calculational procedure. The procedure there incorporates the sparticle effects iteratively, and thus the $\alpha_s(M_Z)$ prediction is automatically corrected for sparticle thresholds. Also, $\alpha_s(M_Z) < 0.118$ and a fine-tuning constraint were imposed, and m_t was assigned so the constraint setting the weak breaking scale is satisfied. However, constraints on the spectrum parameters derived from proton decay limits [22] were not considered. (The proton decay and fine-tuning constraints do not agree.) We use the spectra given in Ref. [21] for illustration only, and ignore minor inconsistencies. When small $SU(2)_L$ breaking occurs, we identify a doublet threshold with that of the heavier member. Our results and conclusions do not depend on any specific choice of spectrum.

[15] Supermultiplets are defined as in Ref. [18]. A massive vector supermultiplet consists of a real massive vector, a Dirac spinor, and a real scalar. A Dirac (Majorana or chiral) supermultiplet consists of a Dirac (Majorana or Weyl) spinor and two (one) complex scalars.

logarithmic dependences on m_t. We then have $s^2(M_Z)(m_t)=s_0^2(M_Z)+\Delta_{s^2}^{\rm top}$, where

$$\Delta_{s^2}^{\rm top}\approx -1.03\times 10^{-7}\ {\rm GeV}^{-2}[(m_t)^2-(m_{t0})^2]\ . \quad (20)$$

We take the reference value of $s_0^2(M_Z)$ in (4) as our input value for both case (a) and case (b). That is, $s_0^2(M_Z)$ can be viewed as a convenient parametrization of the precisely known M_Z. The m_t dependence of the "true" $s^2(M_Z)$ in $\Delta_{s^2}^{\rm top}$ will be included together with the threshold corrections in $\Delta_i^{\rm top}$. Thus

$$\Delta_1^{\rm top}=\frac{8[1-s^2(M_Z)]}{15\pi}\ln\left[\frac{m_t}{138\ {\rm GeV}}\right]-\frac{3}{5}\frac{\Delta_{s^2}^{\rm top}}{\alpha(M_Z)}\ , \quad (21)$$

$$\Delta_2^{\rm top}=\frac{8s^2(M_Z)}{9\pi}\ln\left[\frac{m_t}{138\ {\rm GeV}}\right]+\frac{\Delta_{s^2}^{\rm top}}{\alpha(M_Z)}\ , \quad (22)$$

$$\Delta_3^{\rm top}=0.04+\frac{1}{3\pi}\ln\left[\frac{m_t}{138\ {\rm GeV}}\right]\ . \quad (23)$$

The SM Higgs boson has $\Delta_\alpha^{h^0}=\Delta_{\alpha_s}^{h^0}=0$ and $\Delta_{s^2}^{h^0}\ll\Delta_{s^2}^{\rm top}$. We therefore neglect possible contributions to Δ_i from the SM Higgs boson, and $s_0^2(M_Z)=0.2324$ is consistent with $m_{h^0}=M_Z$. We account then for different values of m_{h^0} as a part of the 0.0003 error bar.

When evaluating $\Delta_i^{\rm top}$ it is convenient to use $s^2(M_Z)=s_0^2(M_Z)=0.2324$ rather than the one-loop prediction (as we could choose to do in case (a) [39]). This induces a weak dependence of the $s_0^2(M_Z)$ prediction on the $s_0^2(M_Z)$ input value via the first terms in (21) and (22), but in practice the effect is negligible. As a matter of fact, all the logarithmic contributions to $\Delta_i^{\rm top}$ (and not just the ones that appear in $\Delta_{s^2}^{\rm top}$) are negligible in comparison to the quadratic ones, and will be omitted later. Table VIII lists the different contributions to $\Delta_i^{\rm top}$ (which are the same in the SM and the MSSM).

Another issue that is related to the heavy top is the contribution of the top Yukawa coupling h_t to the two-loop β function [42,51–53]. If $h_t\approx 1$, we need to reintroduce the relevant term (that was neglected above) in the β function (10), i.e.,

TABLE VIII. The top correction terms $\Delta_i^{\rm top}$. (These are common for the SM and the MSSM.)

	Constant term	Logarithmic term	Quadratic term
Δ_1	-0.15	$+0.13\ln\left[\frac{m_t}{138\ {\rm GeV}}\right]$	$+0.15\left[\frac{m_t}{138\ {\rm GeV}}\right]^2$
Δ_2	$+0.25$	$+0.065\ln\left[\frac{m_t}{138\ {\rm GeV}}\right]$	$-0.25\left[\frac{m_t}{138\ {\rm GeV}}\right]^2$
Δ_3	$+0.04$	$+0.105\ln\left[\frac{m_t}{138\ {\rm GeV}}\right]$	

$$\mu\frac{d\alpha_i}{d\mu}=\frac{b_i}{2\pi}\alpha_i^2+\sum_{j=1}^{3}\frac{b_{ij}}{8\pi^2}\alpha_i^2\alpha_j-b_{i;\rm top}\frac{h_t^2}{16\pi^2}\frac{\alpha_i^2}{2\pi}\ , \quad (24)$$

where $b_{i;\rm top}$ can be calculated using, for example, Refs. [42,52] and are of the order of magnitude of unity. In the SM, $b_{i;\rm top}=\frac{17}{10},\frac{3}{2},2$ for $i=1,2,3$, respectively [42,51]. In the MSSM there are (to this order) two additional Yukawa terms in which a Higgsino is coupled to a top squark and a top quark. One then has [53] $b_{i;\rm top}=\frac{26}{5},6,4$ for $i=1,2,3$. h_t is running and is coupled to α_i at the one-loop order. $\Delta_i^{\rm Yukawa}$ are functions of the couplings h_t and α_G at the unification point, and of the unification point parameter t, and have to be calculated numerically.

Let us consider only the MSSM, where the effect is relevant. A heavy top can then also imply a large Yukawa coupling for the b quark h_b, i.e., $m_t h_b/m_b h_t=\tan\beta$, where $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets [46]. For a large enough $\tan\beta$ one could have $h_b\approx h_t$ [54]. However, such a situation is not very likely. Proton decay via dimension-five operators constrains $\tan\beta$ (i.e., $\tan\beta\le 4.7$ for $m_t=125$ GeV, assuming $\alpha_s(M_Z)=0.113\pm0.005$) [22]. We will keep neglecting h_b. (One should note that the requirement $\sin\beta<1$ places a lower bound on h_t for a fixed m_t.) We calculate the Yukawa correction by solving numerically the coupled RGE's [53]. The results are given in Table IX in terms of the corrections to the predictions, H_{s^2}, H_{α_s}, H_t, and H_{1/α_G}, rather then in terms of $\Delta_i^{\rm Yukawa}$.

Instead of the full two-loop numerical calculation one could use an approximation in which h_t is constant. Then the new term in (24) is realized as a negative correction to b_i, and

$$\Delta_i^{\rm Yukawa}\approx b_{i;\rm top}\frac{h_t^2}{16\pi^2}t\ , \quad (25)$$

or $\Delta_i^{\rm Yukawa}/h_t^2\approx 0.17,0.20,0.13$, for $i=1,2,3$, respectively. One can see from Table IX that taking $h_t\approx 1\approx h_{\rm fixed}$ is a reasonable approximation ($h_{\rm fixed}$ is the fixed point of the one-loop top Yukawa RGE [55,49,53,56]).

Lastly, we consider contributions from nonrenormalizable operators at the high scale, which may be induced by the physics between M_G and $M_{\rm Planck}\approx 1.22\times 10^{19}$ GeV [57,58]. We consider only dimension-five operators,

$$-\frac{1}{2}\frac{\eta}{M_{\rm Planck}}{\rm Tr}(F_{\mu\nu}\Phi F^{\mu\nu})\ ,$$

where η is a dimensionless parameter and $F_{\mu\nu}$ is the field strength tensor. In the SU(5) model Φ is the **24** real-Higgs (Majorana super)multiplet. (Contributions from higher-dimension operators are suppressed by powers of $M_{\rm Planck}^{-1}$.) When Φ acquires an expectation value the effect is to renormalize the gauge fields, which can be absorbed into a redefinition of the couplings. It is shown in Refs. [57,58] that the running couplings at M_G are related to the underlying gauge coupling $\alpha_G(M_G)$ by $1/\alpha_i(M_G)=(1+\epsilon_i)/\alpha_G$, where $\epsilon_i=\eta k_i\sqrt{r/\pi\alpha_G}(M_V/M_{\rm Planck})$. In the SU(5) model $r=\frac{2}{25}$ and $k_i=\frac{1}{2},\frac{3}{2},-1$ for

TABLE IX. The corrections to the predictions in the MSSM due to different values of the top Yukawa coupling h_t at the unification point. $\tan\beta$ is calculated using $m_t=138$ GeV and is not required to obey any limits. $\sin\beta<1$ gives a lower bound on h_t. The corrections are denoted by H, with self-explanatory subscripts.

			Case (a)			Case (b)		
$h_t(M_G)$	$h_t(M_Z)$	$\tan\beta$	H_{s^2}	H_t	H_{1/α_G}	H_{α_s}	H_t	H_{1/α_G}
0.300	0.794	17.13	−0.000 08	+0.002	+0.04	−0.0003	−0.001	+0.05
0.400	0.903	1.84	−0.000 12	+0.003	+0.06	−0.0004	−0.002	+0.08
0.600	1.015	1.25	−0.000 19	+0.004	+0.09	−0.0006	−0.003	+0.12
0.800	1.067	1.11	−0.000 24	+0.005	+0.12	−0.0008	−0.004	+0.16
1.000	1.095	1.05	−0.000 29	+0.006	+0.15	−0.0010	−0.004	+0.19
1.200	1.111	1.02	−0.000 33	+0.007	+0.17	−0.0012	−0.005	+0.22
1.400	1.122	1.00	−0.000 37	+0.008	+0.18	−0.0013	−0.005	+0.25
1.600	1.129	0.98	−0.000 40	+0.009	+0.20	−0.0014	−0.006	+0.27

$i=1,2,3$, respectively. We treat these operators perturbatively (i.e., for $|\eta|<10$), by defining

$$\Delta_i^{\mathrm{NRO}} \equiv -\eta k_i \left[\frac{r}{\pi\alpha_G^3}\right]^{1/2} \frac{M_G}{M_{\mathrm{Planck}}}\,, \tag{26}$$

where it is sufficient to use the one-loop expressions for α_G and $M_G=M_Z e^{2\pi t}$. (26) is valid in the MSSM as well [57], and different normalizations and scales can be absorbed in η.

Like θ_i, Δ_i^{NRO} and $\Delta_i^{\mathrm{Yukawa}}$ depend on the input parameters through t and α_G. We use the full two-loop values for t and α_G when estimating these correction terms (consistent with solving for θ_i iteratively). At the price of a minor technical inconsistency, we always use the two-loop values of t and α_G given in Table V(a).

The different contributions to Δ_i, in the SM and the MSSM, are listed in Tables VIII–X. From Tables X(b) and VIII we learn that different contributions to Δ_i in the MSSM are *a priori* comparable, and a comparison with Table X(a) suggests that they are more significant, by number and magnitude, than in the SM. These points were stressed recently in Ref. [24].

At this point one is able to write explicit expressions for $1/\alpha_G$, t, and $s_0^2(M_Z)$ [$1/\alpha_G$, t, and $\alpha_s(M_Z)$]. We give below those for $s_0^2(M_Z)$, $\alpha_s(M_Z)$, $t(\alpha,s_0^2)$, and $\alpha_G(\alpha,s_0^2)$ in the MSSM, which are the main results of this section. We hereafter neglect all logarithmic contributions to Δ_i^{top}. Constant correction terms are included in the functions δ_i, which are normalized such that the conversion term is unity. Our best guesses for the values of the functions H are $H_{s^2}=-0.0003^{+0.0002}_{-0.0001}$, $H_{\alpha_s}=-0.0010^{+0.0007}_{-0.0004}$, $H_t=-0.004^{+0.003}_{-0.002}$, $H_{1/\alpha_G}=+0.19^{+0.08}_{-0.14}$, corresponding to $h_t=1$ at the unification point and the range given in Table IX:

TABLE X. (a) The different correction terms Δ_i in the [minimal SU(5)] SM. (b) The different correction terms Δ_i in the [minimal SU(5)] MSSM.

	$\Delta_i^{\mathrm{conversion}}$	Δ_i^V	Δ_i^{24}	Δ_i^5	Δ_i^{SUSY}	$\Delta_i^{\mathrm{NR)}}$
			(a)			
Δ_1		$-\frac{35}{4\pi}\ln\left[\frac{M_V}{M_G}\right]$		$+\frac{1}{30\pi}\ln\left[\frac{M_5}{M_G}\right]$		-0.008η
Δ_2	$-\frac{1}{6\pi}$	$-\frac{21}{4\pi}\ln\left[\frac{M_V}{M_G}\right]$	$+\frac{1}{6\pi}\ln\left[\frac{M_{24}}{M_G}\right]$			-0.0024η
Δ_3	$-\frac{1}{4\pi}$	$-\frac{7}{2\pi}\ln\left[\frac{M_V}{M_G}\right]$	$+\frac{1}{4\pi}\ln\left[\frac{M_{24}}{M_G}\right]$	$+\frac{1}{12\pi}\ln\left[\frac{M_5}{M_G}\right]$		$+0.0016\eta$
			(b)			
Δ_1		$-\frac{5}{\pi}\ln\left[\frac{M_V}{M_G}\right]$		$+\frac{1}{5\pi}\ln\left[\frac{M_5}{M_G}\right]$	$+\frac{5}{4\pi}\ln\left[\frac{M_1}{M_Z}\right]$	-0.015η
Δ_2	$-\frac{1}{6\pi}$	$-\frac{3}{\pi}\ln\left[\frac{M_V}{M_G}\right]$	$+\frac{1}{\pi}\ln\left[\frac{M_{24}}{M_G}\right]$		$+\frac{25}{12\pi}\ln\left[\frac{M_2}{M_Z}\right]$	-0.042η
Δ_3	$-\frac{1}{4\pi}$	$-\frac{2}{\pi}\ln\left[\frac{M_V}{M_G}\right]$	$+\frac{3}{2\pi}\ln\left[\frac{M_{24}}{M_G}\right]$	$+\frac{1}{2\pi}\ln\left[\frac{M_5}{M_G}\right]$	$+\frac{2}{\pi}\ln\left[\frac{M_3}{M_Z}\right]$	$+0.028\eta$

$$s_0^2(M_Z)=0.2+\frac{7}{15}\frac{\alpha(M_Z)}{\alpha_s(M_Z)}(1\pm\delta_\alpha\pm\delta_{\alpha_s})$$
$$+0.0031+H_{s^2}+\frac{\alpha(M_Z)}{60\pi}(\delta_1+\delta_2)\ , \qquad (27)$$

$$\alpha_s(M_Z)=\frac{7\alpha(M_Z)}{15s_0^2(M_Z)-3}(1\pm\delta_\alpha\pm\delta_{s^2})$$
$$+0.012+H_{\alpha_s}+\frac{28\alpha(M_Z)^2}{[60s_0^2(M_Z)-12]^2\pi}(\delta_1+\delta_2)\ , \qquad (28)$$

$$t=\frac{3-8s_0^2(M_Z)}{28\alpha(M_Z)}(1\pm\delta_\alpha\pm 0.2\delta_{s^2})$$
$$+0.08+H_t+\frac{5}{168\pi}(\delta_3+\delta_4+\delta_5)\ , \qquad (29)$$

$$\frac{1}{\alpha_G}=\frac{3-36s_0^2(M_Z)}{28\alpha(M_Z)}(1\pm\delta_\alpha\pm 0.2\delta_{s^2})\ ,$$
$$-1.23+H_{1/\alpha_G}-\frac{5}{168\pi}(\delta_3+\tfrac{33}{5}\delta_4+\delta_6)\ , \qquad (30)$$

where[16]

$$\delta_1=1-12\ln\left[\frac{M_V}{M_G}\right]-6\ln\left[\frac{M_{24}}{M_G}\right]+18\ln\left[\frac{M_5}{M_G}\right]$$
$$+25\ln\left[\frac{M_1}{M_Z}\right]-100\ln\left[\frac{M_2}{M_Z}\right]+56\ln\left[\frac{M_3}{M_Z}\right]\ , \qquad (31)$$

$$\delta_2=+3.9+47.4\left[\left[\frac{m_t}{138\text{ GeV}}\right]^2-1\right]+8.00\eta\ , \qquad (32)$$

$$\delta_3=-30\ln\left[\frac{M_V}{M_G}\right]+\frac{6}{5}\ln\left[\frac{M_5}{M_G}\right]+\frac{15}{2}\ln\left[\frac{M_1}{M_Z}\right]\ , \qquad (33)$$

$$\delta_4=1+18\ln\left[\frac{M_V}{M_G}\right]-6\ln\left[\frac{M_{24}}{M_G}\right]-\frac{25}{2}\ln\left[\frac{M_2}{M_Z}\right]\ , \qquad (34)$$

$$\delta_5=+7.6\left[\left[\frac{m_t}{138\text{ GeV}}\right]^2-1\right]+0.53\eta\ , \qquad (35)$$

$$\delta_6=+34.2\left[\left[\frac{m_t}{138\text{ GeV}}\right]^2-1\right]+5.01\eta \qquad (36)$$

and

$$\delta_\alpha=\alpha(M_Z)\delta\left[\frac{1}{\alpha(M_Z)}\right]\ , \qquad (37)$$

$$\delta_{\alpha_s}=\frac{\delta(\alpha_s(M_Z))}{\alpha_s(M_Z)}\ , \qquad (38)$$

$$\delta_{s^2}=\frac{\delta(s_0^2(M_Z))}{s_0^2(M_Z)-0.2}\ . \qquad (39)$$

[16]Negligible inconsistencies between (31)–(36) and Tables VIII–X may exist due to roundoff.

The third term in (27), and the second terms in (28), (29), and (30) are two-loop terms. These, as well as δ_2, δ_5, δ_6, and the functions H, depend weakly on the values of input parameters used. All the other expressions can be similarly constructed. Implications of these results are considered in the following section, where we also estimate the values of the correction terms and their uncertainties (see Table XI).

IV. THE CORRECTION TERMS IN THE MSSM

We are now equipped to discuss the correction terms in the MSSM (where their contribution is significant) more quantitatively. From (31) we can realize the meaning of the naive parameter M_{SUSY} mentioned above, i.e.,

$$25\ln\left[\frac{M_1}{M_Z}\right]-100\ln\left[\frac{M_2}{M_Z}\right]+56\ln\left[\frac{M_3}{M_Z}\right]$$
$$\equiv -19\ln\left[\frac{M_{\text{SUSY}}}{M_Z}\right]\ . \qquad (40)$$

That is, the effect of an arbitrary sparticle spectrum on the $s_0^2(M_Z)$ and $\alpha_s(M_Z)$ predictions can always be parametrized in terms of the (same) parameter M_{SUSY}. On the other hand, the $1/\alpha_G$ and t uncertainties have different dependences on the M_i. It is important to note that the coefficient on the right-hand side (RHS) of (40) is small due to cancellations, while those on the LHS are large. In the case $M_2 \gg M_1$ and/or $M_2 \gg M_3$ mentioned above, the LHS of (40) (and therefore δ_1) can grow significantly, and M_{SUSY} can then be large. However, excluding such a case implies that $M_{\text{SUSY}}\approx 1$ TeV can be achieved only by some adjustment of the parameters. It is not enough to have large M_i in order to have a large M_{SUSY}. For example, ($M_1\approx M_2\approx 1$ TeV, $M_3\approx 2$ TeV) correspond to $M_{\text{SUSY}}\approx 130$ GeV and ($M_1\approx 850$ GeV, $M_2\approx 840$ GeV, $M_3\approx 1$ TeV) correspond to $M_{\text{SUSY}}\approx 495$ GeV. On the other hand, a small M_{SUSY} does not imply a low spectrum. For example, ($M_1\approx 550$ GeV, $M_2\approx 540$ GeV, $M_3\approx 980$ GeV) or ($M_1\approx 600$ GeV, $M_2\approx M_3\approx 266$ GeV) both correspond to $M_{\text{SUSY}}\approx M_Z$. One can even have $M_{\text{SUSY}}\ll M_Z$ (a large positive contribution to δ_1). For example, for the two spectra of Ref. [21] given in Table VII we have $M_{\text{SUSY}}\approx 32$ GeV and 21 GeV, respectively. Thus, M_{SUSY} does not teach one about the actual spectrum, and the widely chosen range of $M_Z < M_{\text{SUSY}} < 1$ TeV does not represent the possible sparticle spectra properly, as was emphasized in Ref. [21].

For $M_5=M_G$, as is suggested by proton decay constraints, the high-scale threshold contribution to δ_1 is always positive. This was emphasized recently in Ref. [29]. If one combines the two observations, a positive δ_1 is likely. Such a situation is not favored in the MSSM, as the predictions for $s_0^2(M_Z)$ and $\alpha_s(M_Z)$ are already slightly higher than the central input values of these parameters. (It could even signal the model failure if the $\alpha_s(M_Z)$ value is determined to be near the lower end of the 0.120 ± 0.010 range, as was also emphasized in Ref. [29].)

TABLE XI. The different contributions to the theoretical uncertainties of the $s_0^2(M_Z)$, $\alpha_s(M_Z)$, $t(\alpha,s_0^2)$, and $(1/\alpha_G)(\alpha,s_0^2)$ predictions in the MSSM. The ranges of the parameters and the corresponding uncertainties serve as an order of magnitude estimate only, and in some cases are chosen to be smaller than those displayed in the figures.

	$s_0^2(M_Z)$	$\alpha_s(M_Z)$	$t(\alpha,s_0^2)$	$\frac{1}{\alpha_G}(\alpha,s_0^2)$
Input value	0.2324	0.120		
Error bar	± 0.0003	± 0.010		
One-loop prediction	0.2304	0.113	5.21	24.51
Two-loop correction	$+0.0031$	$+0.012$	$+0.08$	-1.23
Yukawa correction (H)	-0.0003	-0.001	-0.004	$+0.19$
Constant correction	$+0.0002$	$+0.001$	$+0.01$	-0.06
$\alpha_s(M_Z)$ error bar	± 0.0025			
$s_0^2(M_Z)$ error bar		± 0.001	± 0.01	∓ 0.04
$M_1=4M_Z$, $M_2=M_3=M_Z$	$+0.0014$	$+0.005$	$+0.10$	-0.10
$M_1=M_2=M_3=6M_Z$	-0.0014	-0.005	-0.08	$+1.27$
$m_t=159$ GeV	$+0.0006$	$+0.002$	$+0.02$	-0.10
$m_t=113$ GeV	-0.0006	-0.002	-0.02	$+0.10$
$M_V=0.3M_G$, $M_{24}=0.05M_G$, $M_5=M_G$	$+0.0013$	$+0.005$	$+0.31$	-0.11
$M_V=M_{24}=M_G$, $M_5=0.5M_G$	-0.0005	-0.002	-0.01	$+0.01$
$\eta=\pm 5$	± 0.0016	± 0.006	± 0.025	∓ 0.23

Requiring a negative δ_1-1 can then severely constrain the spectrum parameters. However, until the M_i are known in detail it is not clear to us that there is really a problem, and at the present time we do not find much point in elaborating on the δ_1-1 sign. Furthermore, the above situation can be compensated by a negative δ_2, i.e., if either $\eta<0$ or $m_t<138$ GeV, or a combination of the two. Theoretical knowledge of $\Delta_i^{\rm NRO}$ is thus important for a more quantitative discussion, especially once $\alpha_s(M_Z)$ is more accurately known and the top is found. The discussion above stresses once again a major weakness of the MSSM-proton decay via dimension-five operators. If M_G is not strongly constrained, δ_1-1 can be made negative without any constraints on the sparticle spectrum. This is the situation in a simple extension of the MSSM in which the discrete Z_2 R parity is replaced by a discrete Z_3 baryon parity, and the dimension-five operators that are responsible for the proton decay are forbidden [37]. (Though the phenomenology of such a model is very different, it does not directly affect the discussion in this paper.) Similarly, superstring-derived models, which are not true GUT's, may not have any problems with proton decay. Finally, more (split) (super)multiplets at the high-scale boundary, within SU(5) or in a model with a larger GUT gauge group, can change the above situation as well. We discuss such a possibility in the following section.

A similar discussion applies to $\delta_3+\delta_4+\delta_5$, but here the sparticle contribution can easily pick any sign; e.g., $M_2\geq M_1$ will give a negative contribution, and thus a lower M_G. $M_1\gg M_2$ is thus favored by proton decay (which implies $M_{\rm SUSY}\ll 1$ TeV). One should also note that t will be corrected for $M_{\rm SUSY}=M_Z$.

For a more quantitative discussion, one has to choose reasonable ranges for the different parameters. We suggest the following.

(i) $m_t\leq M_i\leq 1$ TeV, and further constrain the splitting to be less than a factor of 4. $M_2\gg M_1$ and/or $M_2\gg M_3$ are excluded (and proton decay may exclude $M_3>M_1$).

(ii) $10^{-2}\times M_G\leq M_V,M_{24},M_5\leq M_G$ and constrain M_{24} and M_5 to be smaller than a few times M_V (and proton decay may further constrain M_5).

(iii) $0\leq|\eta|\leq 10$. For larger values the treatment is not perturbative. Note that $\Delta_i^{\rm NRO}$ becomes negligible for $|\eta|<1$. For example, large-radius Calabi-Yau compactification, which yield interesting neutrino masses predict $|\eta|\ll 1$ [28].

(iv) 113 GeV$\leq m_t\leq 159$ GeV from precision electroweak data.

For these ranges we present the different contributions to $s_0^2(M_Z),\alpha_s(M_Z)-(\delta_1+\delta_2)$—and to $t(\alpha,s_0^2)-(\delta_3+\delta_4+\delta_5)$—in Figs. 4–6. We also display in each figure the two-loop correction, the corresponding input error bar, and for $s_0^2(M_Z)$ the prediction uncertainty from the $\alpha_s(M_Z)$ input value error bar. [The $s_0^2(M_Z)$ error bar induces much smaller uncertainties, and those induced by the $\alpha(M_Z)$ error bar are negligible.]

The observations made above become clear if we examine once again the spectra given in Ref. [21]. The sparticle and m_t contributions to $\delta_1+\delta_2$ can be offset by $\eta\approx-4.5$ and -0.7 for the two cases. Also the M_i and m_t contributions are comparable and in the second case come with opposite signs (which explains the small $|\eta|$ required in this case). If constraints from proton decay are ignored (see the footnote above), we can also use $M_5\approx 0.1M_G$ and $0.75M_G$. (The second value corresponds to $M_5\approx 2\times 10^{16}$ GeV.) Thus, we see that for a combination of $M_{\rm SUSY}<M_Z$, $m_t<138$ GeV, and a M_5 just below the unification scale, we can still have $\delta_1+\delta_2\leq 1$.

Finally, we estimate the theoretical uncertainties for the $s_0^2(M_Z)$, $\alpha_s(M_Z)$, $t(\alpha,s_0^2)$, and $(1/\alpha_G)(\alpha,s_0^2)$ predictions. We present these in Table XI. [The prediction for $s_0^2(M_Z)$ is to be compared with the value in (4), for which

m_t does not contribute to the error bar.] For our choices of values for the different correction parameters, we obtain theoretical uncertainties to $s_0^2(M_Z)$ comparable with the one induced by the $\alpha_s(M_Z)$ error bar, and in the $\alpha_s(M_Z)$ case comparable with its error bar. They may add to or may offset each other. In order to have a more decisive observation, a better determination of $\alpha_s(M_Z)$ and elimination of some of these uncertainties are required. We also obtain $M_G \geq 1.3 \times 10^{16}$ GeV, where different corrections were added in quadrature. This is well above the limit ($\sim 10^{15}$ GeV) from proton decay via vector boson exchange [43]. Let us emphasize again that though we arbitrarily chose the different correction parameter values, our choices serve as reasonable order of magnitude estimations.

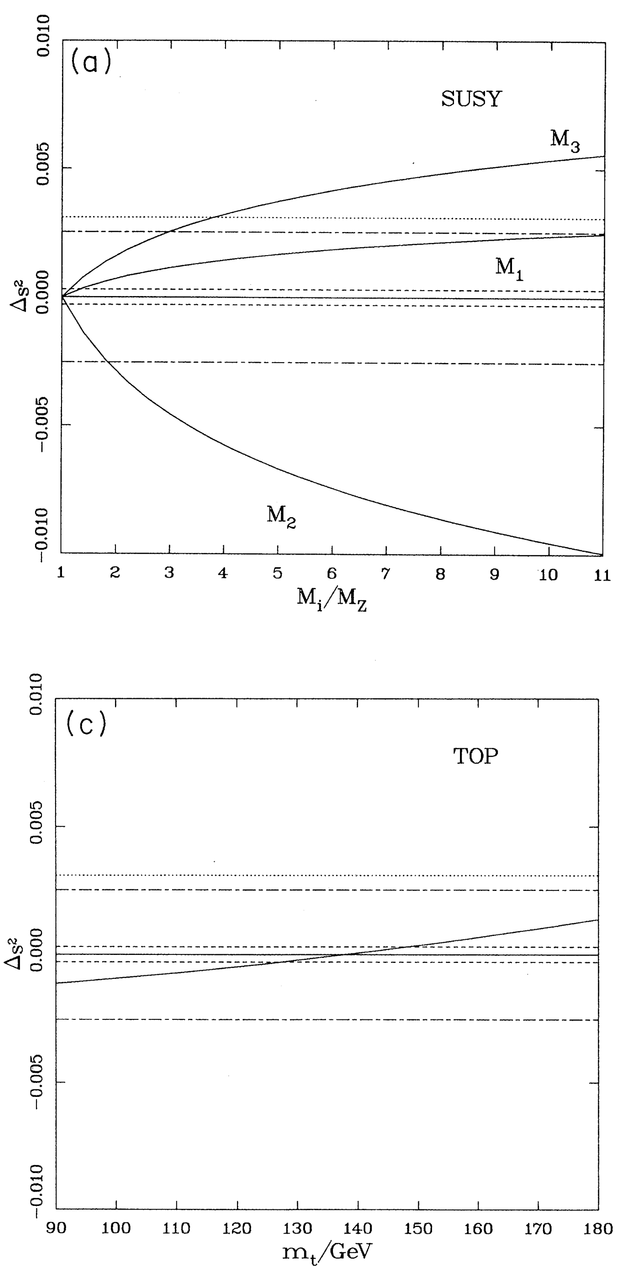

FIG. 4. Contributions of individual correction terms—the SUSY effective mass parameters M_i (a), the heavy thresholds at the high scale (b), the top (c), and NRO's at the high scale (d)—to the $s_0^2(M_Z)$ prediction [via the last term of (27)]. The NRO term changes sign for $\eta < 0$. The error bar on $s_0^2(M_Z)$ (dashed line), the uncertainty induced by the error bar on $\alpha_s(M_Z)$ (dash-dotted line), and the two-loop contribution to the $s_0^2(M_Z)$ prediction (dotted line) are given for comparison.

V. A GENERAL TREATMENT OF THRESHOLD CORRECTION AT THE HIGH-SCALE

Above, we assigned explicit mass parameters to the different (super)multiplet classes at the high scale, M_V, M_{24}, M_5, while at the low-scale boundary we parametrized the threshold corrections using three effective mass parameters M_1, M_2, M_3, which can be computed in any model. Similar effective parameters can also be defined at the high scale, i.e.,

$$\sum_{\zeta} \frac{b_i^{\zeta}}{2\pi} \ln\left[\frac{M_{\zeta}}{M_G}\right] \equiv \frac{b_i^{\text{matter}}}{2\pi} \ln\left[\frac{M_i'}{M_G}\right] \quad \text{for } i=1,2,3 \ . \tag{41}$$

For definiteness we identify $M_V \equiv M_G$, where M_V here is the mass of the vector (super)fields, which we assume are degenerate. [(41) can be easily generalized to include nondegenerate vector masses.] The summation is then over all massive matter, scalar and fermion (Majorana, chiral, and Dirac), (super)fields at the high-scale bound-

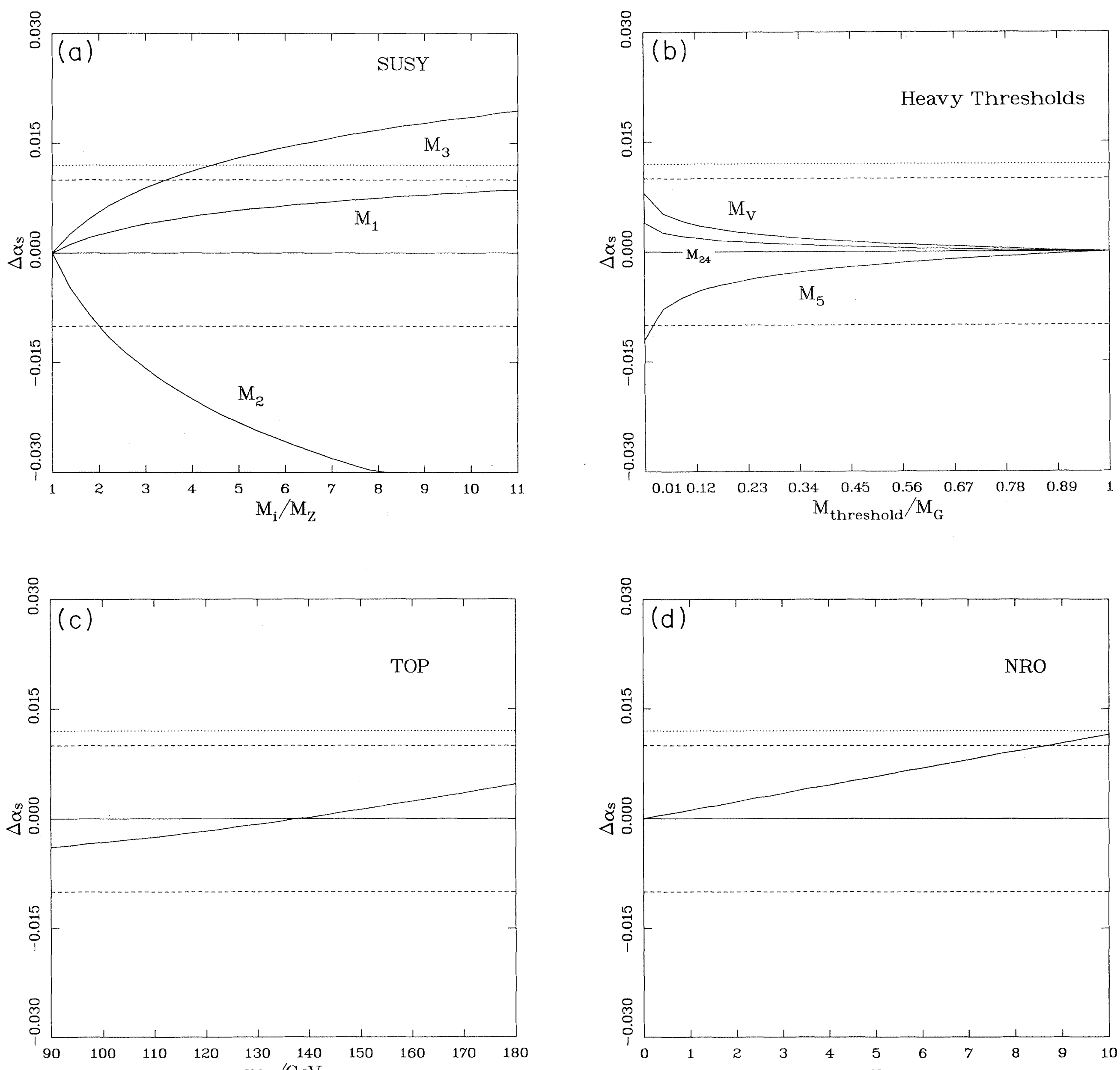

FIG. 5. Contributions of individual correction terms—the SUSY effective mass parameters M_i (a), the heavy thresholds at the high scale (b), the top (c), and NRO's at the high scale (d)—to the $\alpha_s(M_Z)$ prediction [via the last term of (28)]. The error bar on $\alpha_s(M_Z)$ (dashed line) and the two-loop contribution to the $\alpha_s(M_Z)$ prediction (dotted line) are given for comparison.

ary. b_i^{matter} is the (decoupled) contribution of these (super)fields to b_i. By using the M_i', we lose some sensitivity to the fine details of the heavy spectrum but are able to examine models in which there are more and larger supermultiplets. (We will limit ourselves, however, to consideration of simple extensions in which additional supermultiplets are decoupled at the high-scale boundary.)

Assuming the heavy supermultiplets of the minimal (SUSY) SU(5) model,

$$\Delta_1=\frac{5}{4\pi}\ln\left[\frac{M_1}{M_Z}\right]+\frac{1}{5\pi}\ln\left[\frac{M_1'}{M_G}\right]+\cdots\,, \tag{42}$$

$$\Delta_2=\frac{25}{12\pi}\ln\left[\frac{M_2}{M_Z}\right]+\frac{1}{\pi}\ln\left[\frac{M_2'}{M_G}\right]+\cdots\,, \tag{43}$$

$$\Delta_3=\frac{2}{\pi}\ln\left[\frac{M_3}{M_Z}\right]+\frac{2}{\pi}\ln\left[\frac{M_3'}{M_G}\right]+\cdots\,, \tag{44}$$

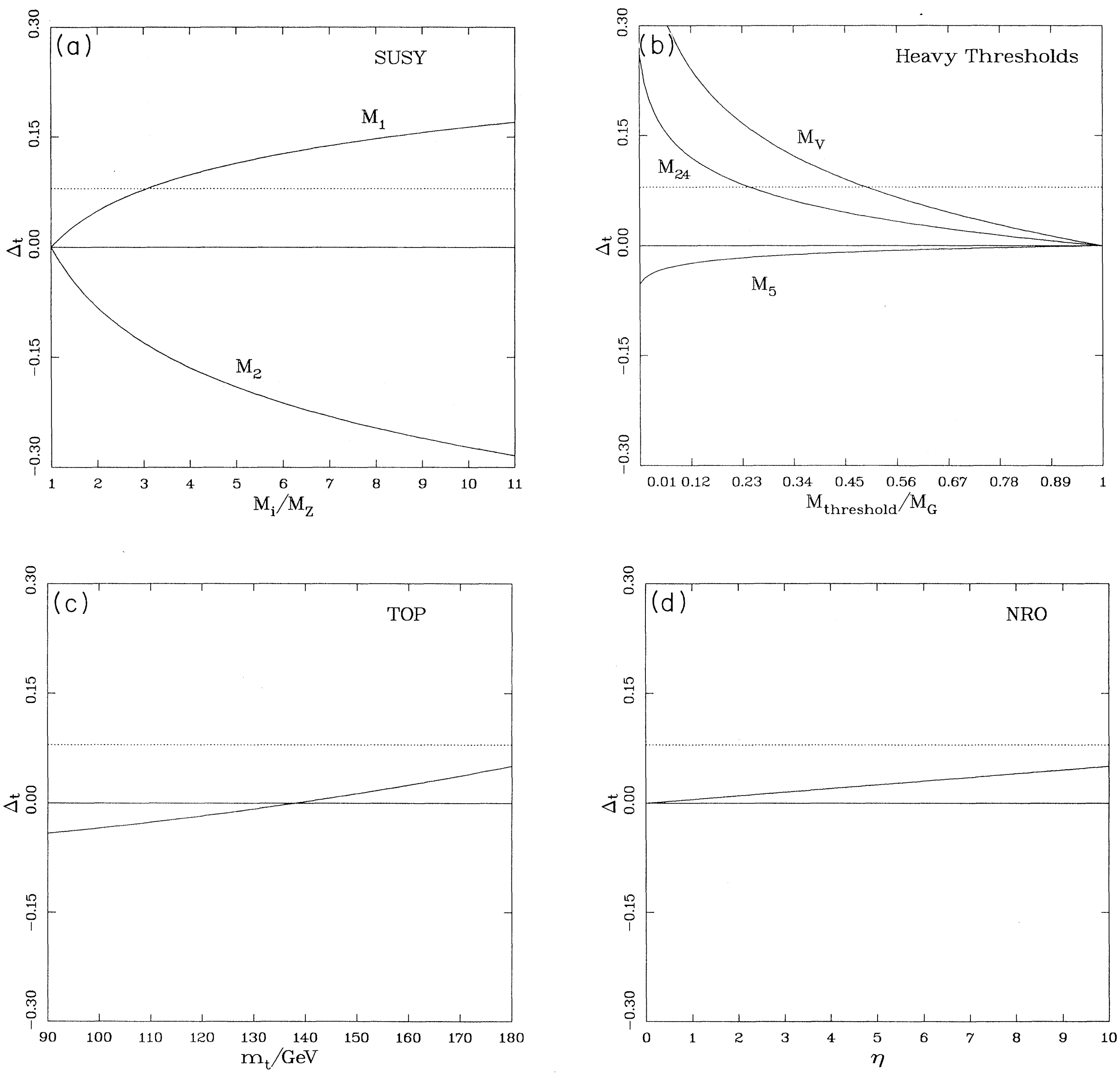

FIG. 6. Contributions of individual correction terms—the SUSY effective mass parameters M_i (a), the heavy thresholds at the high scale (b), the top (c), and NRO's at the high scale (d)—to the prediction of the scale parameter t [via the last term of (29)]. The two-loop contribution to t (dotted line) is given for comparison.

where we wrote explicitly only the M_i and M_i' contributions. δ_1 can then be rewritten as

$$\delta_1 = 1 + 4\ln\left[\frac{M_1'}{M_G}\right] - 48\ln\left[\frac{M_2'}{M_G}\right] + 56\ln\left[\frac{M_3'}{M_G}\right] + 25\ln\left[\frac{M_1}{M_Z}\right] - 100\ln\left[\frac{M_2}{M_Z}\right] + 56\ln\left[\frac{M_3}{M_Z}\right] . \quad (45)$$

We can further define a new effective parameter $M_{\rm heavy}$ (in analogy with $M_{\rm SUSY}$):

$$\frac{\sum_{i=1}^{3} w_i \ln(M_i'/M_G)}{\sum_{i=1}^{3} w_i} \equiv \ln\left[\frac{M_{\rm heavy}}{M_G}\right] , \quad (46)$$

where $w_i = \frac{5}{2}\sum_{j,k=1}^{3}\frac{1}{2}\epsilon_{jki}(b_j - b_k)b_i^{\rm matter}$, and where ϵ_{ijk} is the Levi-Civita symbol, and the factor of $\frac{5}{2}$ is introduced for consistency with (31).

In the minimal (SUSY) SU(5) model this gives

$$4\ln\left[\frac{M_1'}{M_G}\right] - 48\ln\left[\frac{M_2'}{M_G}\right] + 56\ln\left[\frac{M_3'}{M_G}\right] \equiv 12\ln\left[\frac{M_{\rm heavy}}{M_G}\right] . \quad (47)$$

While $\ln(M_i/M_Z)$ are always positive, $\ln(M_i'/M_G)$ can have either sign. If indeed $M_5 \approx 3M_V$, then $\ln(M_1'/M_G)$ is positive, $\ln(M_2'/M_G)$ is more probably negative, and $\ln(M_3'/M_G) > \frac{3}{4}\ln(M_2'/M_G)$, which is a restatement of the high-scale threshold positive contribution to δ_1 discussed above. If we introduce more matter supermultiplets in (41), this situation may change. Let us assume n_{10} (n_5) additional **10** (**5**) of SU(5) chiral supermultiplets.[17] Each **10** (**5**) consists of $(3,1,\frac{2}{3})\oplus(3,2,\frac{1}{6})\oplus(1,1,1)$ $[(3,1,-\frac{1}{3})\oplus(1,2,\frac{1}{2})]$ S superfields, and we further allow an arbitrary split among the different S thresholds introduced here. For illustration, we will also assume that the new superfluids are not constrained by proton decay limits. In practice, the extent to which they are constrained is determined by their couplings to the MSSM superfields, and by discrete symmetries (e.g., R parity) and their quantum number assignments. Then, $\delta b_i^{\rm matter} = \frac{3}{2}n_{10} + \frac{1}{2}n_5$ for $i = 1,2,3$, and

$$(15n_{10} + 5n_5 + 4)\ln\left[\frac{M_1'}{M_G}\right] - (36n_{10} + 12n_5 + 48)\ln\left[\frac{M_2'}{M_G}\right] + (21n_{10} + 7n_5 + 56)\ln\left[\frac{M_3'}{M_G}\right] \equiv 12\ln\left[\frac{M_{\rm heavy}}{M_G}\right] . \quad (48)$$

$M_{\rm heavy} < M_G$ is now possible if the split is such that $M_1'(M_3') \ll M_2', M_G$ or $M_1', M_3' < M_2', M_G$. Note that now $\delta_1 - 1$ can easily pick either a negative or a positive sign. For example, $M_{\rm heavy} \approx 0.1 M_G$ and $M_{\rm SUSY} \approx 0.25 M_Z$ would imply $\delta_1 \approx 0$.

VI. CONCLUSIONS

In this paper we considered various correction terms. We introduced the effective parameters M_i (which sum the low-scale threshold corrections), realized the naive parameter $M_{\rm SUSY}$ in terms of the M_i, and pointed out that $M_{\rm SUSY}$ can differ significantly from the actual sparticle masses. We then introduced similar parameters M_i' at the high scale, and a different and more explicit set of high-scale parameters when we considered the minimal SU(5) model, in which the colored-triplet Higgs superfield threshold is strongly constrained. The parameters M_i and M_i' can be used to conveniently compare threshold correction terms in different models.

The central predictions of the MSSM are slightly high, but lie well within the experimental error bars. Z-pole determinations of $\alpha_s(M_Z)$ favor no correction or a positive correction to the $\alpha_s(M_Z)$ prediction, while low-energy determinations favor a negative correction. However, we showed that the magnitude and sign of the corrections to the two-loop predictions are determined by an interplay among various comparable terms. Of these terms, only one has a fixed sign: the contribution from the high-scale thresholds in the minimal SUSY-SU(5) model is positive when proton decay constraints are imposed. We pointed out that once simple extensions are considered, i.e., more heavy supermultiplets or replacing R parity with baryon parity, the above sign is no longer fixed. The sparticle contribution can be either positive (as for the two spectra of Ref. [21]) or negative, and so are the contributions from m_t and NRO's. Therefore we concluded that elaboration on the sign of any of these correction terms cannot be well justified at the present.

The MSSM then agrees well with experiment, and a theoretical uncertainty of $\sim +0.0026 - 0.0023$ ($+0.010 - 0.008$) has to be assigned to the $s_0^2(M_Z)$ [$\alpha_s(M_Z)$] prediction of the model. This is not the case when the SM is considered. Neither perturbative correction terms nor additional Higgs doublets can reverse the failure of coupling constant unification in this model. For example, the equivalent theoretical uncertainties in the SM are roughly $\sim \pm 0.0007$ and $\sim \pm 0.001$, respectively. The correction terms discussed, though negligible

[17]Such a situation can arise in models in which all matter is embedded in **27** supermultiplets of E_6 at some scale μ, $\mu \geq M_G$. Our assumptions imply that additional massive vector superfields are irrelevant, and that there will be no additional Majorana massive superfields. If the E_6 model is derived from the string, then usually there are no adjoint representations, and therefore no Majorana supermultiplets.

in the SM, may play an important role in the MSSM once more precise data is available.

Finally, we would like to mention once again that we have used an alternative definition of the $\overline{\text{MS}}$ weak angle [32], which differs slightly from the canonical one in the way the t quark is treated.

ACKNOWLEDGMENTS

This work was supported by the Department of Energy Grant No. DE-AC02-76-ERO-3071. It is a pleasure to thank Mirjam Cvetič and Alberto Sirlin for useful discussions.

---

[1] P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991).
[2] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
[3] H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).
[4] For reviews, see P. Langacker, Phys. Rep. **72**, 185 (1981); in *Proceedings of the Ninth Workshop on Grand Unification,* Aix les Bains, France, 1988, edited by R. Barloutaud (World Scientific, Singapore, 1988), p. 3.
[5] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981); E. Witten, Nucl. Phys. **B188**, 513 (1981).
[6] For reviews, see H. P. Nilles, Phys. Rep. **110**, 1 (1984); *Testing the Standard Model,* edited by M. Cvetič and P. Langacker (World Scientific, Singapore, 1991), p. 633.
[7] For review, see H. E. Haber and G. L. Kane, Phys. Rep. **117**, 75 (1985).
[8] For review, see R. Barbieri, Riv. Nuovo Cimento **11**, 1 (1988).
[9] For reviews, see G. G. Ross, *Grand Unified Theories* (Benjamin, New York, 1984); R. N. Mohapatra, *Unification and Supersymmetry* (Springer, New York, 1992).
[10] L. E. Ibáñez and G. G. Ross, in *Perspectives in Higgs Physics,* edited by G.L. Kane (World Scientific, Singapore, in press).
[11] J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **249**, 441 (1990); U. Amaldi, W. de Boer, and H. Fürstenau, *ibid.* **260**, 447 (1991); F. Anselmo, L. Cifarelli, A. Peterman, and A. Zichichi, Nuovo Cimento **104A**, 1817 (1991).
[12] W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Phys. Rev. D **18**, 3998 (1978), and references therein.
[13] A. Sirlin, Phys. Lett. B **232**, 123 (1989); S. Fanchiotti and A. Sirlin, Phys. Rev. D **41**, 319 (1990); A. Sirlin, Nucl. Phys. **B332**, 20 (1990), and references therein.
[14] P. Langacker and S. Uma Sankar, Phys. Rev. D **40**, 1569 (1989), and references therein.
[15] R. N. Mohapatra and M. K. Parida, University of Maryland Report No. UMD-PP-92-170, 1992 (unpublished). Phys. Rev. D **47**, 264 (1992).
[16] I. Antoniadis, J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **272**, 31 (1991); T. G. Rizzo, Phys. Rev. D **45**, 3903 (1992); U. Amaldi, W. de Boer, P. H. Frampton, H. Fürstenau, and J. T. Liu, Phys. Lett. B **281**, 374 (1992). The definition of $M_{\rm SUSY}$ by Amaldi *et al.* differs from both the naive definition and the effective scale defined in this paper. There, $M_{\rm SUSY}$ is the smaller of a common mass of the strongly interacting sparticles and of the nonstrongly interacting ones. It is determined by means of best fit.
[17] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); W. J. Marciano and G. Senjanović, *ibid.* **25**, 3092 (1982); L. E. Ibáñez and G. G. Ross, Phys. Lett. **105B**, 439 (1982).
[18] M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B196**, 475 (1982).
[19] U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1987).
[20] See, for example, LEP Collaborations, Phys. Lett. B **276**, 247 (1992).
[21] G. G. Ross and R. G. Roberts, Nucl. Phys. **B377**, 571 (1992).
[22] R. Arnowitt and P. Nath, Phys. Rev. Lett. **69**, 725 (1992); Phys. Lett. B **287**, 89 (1992); Phys. Rev. D **46**, 3981 (1992); Texas A&M University Report No. CTP-TAMU-39/92, 1992 (unpublished). Proton decay via dimension-five operators was also studied more recently by J. Hisano, H. Murayama, and T. Yanagida, Phys. Rev. Lett. **69**, 1014 (1992); Tohoku University Report No. TU-400, 1992 (unpublished). Their conclusions are similar to those of Arnowitt and Nath, though are less constraining due to the inclusion of threshold effects.
[23] S. Kelley, J. L. Lopez, D. V. Nanopoulos, H. Pois, and K. Yuan, CERN Report No. TH.6498/92, 1992 (unpublished).
[24] R. Barbieri and L. J. Hall, Phys. Rev. Lett. **68**, 752 (1992).
[25] A. E. Faraggi, B. Grinstein, and S. Meshkov, Phys. Rev. D (to be published).
[26] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity,* Proceedings of the Workshop, Stony Brook, New York, 1979, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe,* Tsukuba, Japan, 1979, edited by A. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979).
[27] S. A. Bludman, D. C. Kennedy, and P. Langacker, Phys. Rev. D **45**, 1810 (1992); Nucl. Phys. **B373**, 498 (1992); S. A. Bludman, N. Hata, D. C. Kennedy, and P. Langacker, Phys. Rev. D **47**, 2220 (1993).
[28] M. Cvetič and P. Langacker, Phys. Rev. D **46**, R2759 (1992).
[29] J. Ellis, S. Kelley, and D. V. Nanopoulos, Nucl. Phys. **B373**, 55 (1992); Phys. Lett. B **287**, 95 (1992).
[30] F. Anselmo, L. Cifarelli, A. Peterman, and A. Zichichi, CERN Report No. PPE/92-103, 1992 (unpublished)..
[31] K. Hagiwara and Y. Yamada, Phys. Rev. Lett. **70**, 709 (1993).
[32] G. Degrassi, S. Fanchiotti, and A. Sirlin, Nucl. Phys. **B351**, 49 (1991).
[33] S. Bethke and S. Catani, CERN Report No. TH.6484/92, 1992 (unpublished).
[34] T. Hebbeker, Aachen Report No. PITHA-91-17, 1991 (unpublished).
[35] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).
[36] L. Clavelli *et al.*, University of Alabama Report No. UAHEP9210, 1992 (unpublished).
[37] L. E. Ibáñez and G. G. Ross, Nucl. Phys. **B368**, 3 (1992).
[38] S. Weinberg, Phys. Lett. **91B**, 51 (1980).
[39] L. Hall, Nucl. Phys. **B178**, 75 (1981).

[40] J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. **B176**, 61 (1980); C. H. Llewellyn Smith, G. G. Ross, and J. Wheater, *ibid.* **B177**, 263 (1981); P. Binetruy and T. Schucker, *ibid.* **B178**, 293 (1981); I. Antoniadis, C. K. Kounnas, and C. Roiesnel, *ibid.* **B198**, 317 (1982).
[41] D. R. T. Jones, Phys. Rev. D **25**, 581 (1982).
[42] M. E. Machacek and M. T. Vaughn, Nucl. Phys. **B222**, 83 (1983).
[43] For reviews, see Refs. [4,9] and R. Barloutaud, Saclay Report No. DPHPE-91-16, 1991 (unpublished).
[44] I. Antoniadis, C. Kounnas, and K. Tamvakis, Phys. Lett. **119B**, 377 (1982).
[45] W. Siegel, Phys. Lett. **84B**, 193 (1979); D. M. Capper, D. R. T. Jones, and P. van Nieuwenhuizen, Nucl. Phys. **B167**, 479 (1980).
[46] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. **67**, 1889 (1982).
[47] S. P. Li and M. Sher, Phys. Lett. **140B**, 339 (1984).
[48] For recent discussions see, for example, M. A. Diaz and H. E. Haber, Phys. Rev. D **45**, 4246 (1992); A. Brignole, Phys. Lett. B **281**, 284 (1992), and references therein.
[49] L. E. Ibáñez and C. López, Nucl. Phys. **B233**, 511 (1984).
[50] N. K. Falck, Z. Phys. C **30**, 247 (1986), and references therein. For reviews see also Refs. [6,8,10].
[51] M. S. Fischler and C. T. Hill, Nucl. Phys. **B193**, 53 (1981).
[52] D. R. T. Jones and L. Mezincescu, Phys. Lett. **136B**, 242 (1984); A. Parkes and P. West, *ibid.* **138B**, 99 (1984).
[53] J. E. Björkman and D. R. T. Jones, Nucl. Phys. **B259**, 533 (1985).
[54] See, for example, V. Barger, M. S. Berger, and P. Ohmann, Phys. Rev. D **47**, 1093 (1993), and references therein.
[55] B. Pendleton and G. G. Ross, Phys. Lett. **98B**, 291 (1981); C. T. Hill, Phys. Rev. D **24**, 691 (1981).
[56] M. Cvetič and C. R. Preitschopf, Nucl. Phys. **B272**, 490 (1986).
[57] C. T. Hill, Phys. Lett. **135B**, 47 (1984).
[58] Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52**, 875 (1984).