# $CP$ asymmetries in neutral $B$-meson decays

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(Received 6 July 1992)

The formalism of the $CP$-violating asymmetries in neutral $B$-meson decays is discussed in detail. The standard model predictions of the $CP$-violating effects are estimated. The implications of these predictions are then intensively analyzed in the phenomenology of the asymmetries in semileptonic decays, in the decay to a final $CP$ eigenstate, and in the decay of $B^0\bar{B}^0$ into two $CP$ eigenstates in the view of the next planned $B$ factories.

PACS number(s): 11.30.Er, 13.20.Jf, 13.25.+m, 14.40.Jz

## I. INTRODUCTION

Measurements of $CP$ asymmetries in $B$-meson decays will provide us with a clean test of the source of $CP$ noninvariance. The standard electroweak theory with three quark generations does not explain but merely accommodates the small amount of the $CP$-violating parameter; therefore, a deeper theoretical approach has to be considered in order to clarify its origin.

It is believed that the observed $CP$-violating phenomena are intimately related to the existence of the third generation of quarks [1]. An unavoidable complex phase $\delta$, which parametrizes the mixing matrix, is supposed to be responsible for the $CP$ impurity, provided that we base our calculation on the short-distance box diagrams. The present uncertainties in the determination of quark mixing elements and the lack of evidence for the top quark below nearly 90 GeV, leave sufficient freedom to allow several theoretical approaches which can account for the measured value of kaon oscillations, the unsolved $\Delta I=\frac{1}{2}$ puzzle, and the prediction of the $\varepsilon'_K/\varepsilon_K$ parameter. This unclear situation shades off if the theoretical amibiguities in the evaluation of the hadronic matrix elements are considered. In order to solve these problems, several nonperturbative methods have been developed to improve the accuracy, and other techniques based on fundamental theories, like $1/N_c$ expansion and chiral Lagrangians, are applied for a better control of the calculations.

On the other hand, though $CP$ violation has been investigated in the kaon system for almost thirty years, $B$ mesons represent a further sensitive laboratory for the study of such noninvariance. These mesons are more massive and have more decay modes than kaons and, in spite of our modest understanding of quark masses and mixings, the hadronic uncertainties are more reliable; moreover, next-to-leading QCD corrections [2] may now be used to predict with less ambiguity the physical observables.

In the standard model we cannot expect a larger magnitude for the $CP$-violating parameter $\varepsilon_B$ of the $B^0$ system (indeed this parameter is not really peculiar):

$$\varepsilon_B=i\frac{\mathrm{Im}(M_{12}^B)-(i/2)\mathrm{Im}(\Gamma_{12}^B)}{\mathrm{Re}(M_{12}^B)-(i/2)\mathrm{Re}(\Gamma_{12}^B)+\Delta_B}\ , \tag{1}$$

where

$$\Delta_B=\{[M_{12}^B-(i/2)\Gamma_{12}^B][M_{12}^{B*}-(i/2)\Gamma_{12}^{B*}]\}^{1/2}\ ,$$

being $[M_{12}^B-(i/2)\Gamma_{12}^B]$ the off-diagonal element of the regeneration Hamiltonian.

Within the framework of the standard model, the dispersive and absorptive parts, $M_{12}^B$ and $\Gamma_{12}^B$ respectively, can be assumed to be dominantly determined by the short-distance approach, which leads to the expressions

$$\begin{aligned}M_{12}^B&=\frac{G_F^2}{12\pi^2}M_W^2f_B^2B_Bm_B[\lambda_c^2\eta_2^BE(x_c)+\lambda_t^2\eta_2^BE(x_t)\\&\quad+2\lambda_c\lambda_t\eta_3^BE(x_c,x_t)]\\&\simeq\frac{G_F^2}{12\pi^2}M_W^2f_B^2B_Bm_B\eta_2^B\lambda_t^2E(x_t)\end{aligned} \tag{2}$$

and

$$\Gamma_{12}^B=-\frac{G_F^2}{8\pi}f_B^2B_Bm_B(\lambda_u^2X_{uu}+\lambda_c^2X_{cc}+2\lambda_u\lambda_cX_{uc})\ , \tag{3}$$

where $B_B$ is the conventional parameter which represents our lack of knowledge of the matrix element of the $\Delta B=2$ short-distance operator between the $B^0$ and $\bar{B}^0$ states, and it can be supposed to be slightly different from the value predicted in the vacuum insertion approximation [3].

The parameters $\eta_i^B$ represent the factorizable strong interaction corrections and are of order 1, almost independent of the top-quark mass. Actually, the extrapolation to $m_t\simeq M_W$ of the $\eta_i^B$ coefficients obtained in the limit of small running quarks masses [4] turns out to be in a good agreement with the results of more accurate analyses which consider next-to-leading-order corrections for the Wilson coefficients [5].

The dimensionless box-diagram loop integrals $E(x_i,x_j)$ depending on the quark masses $x_i=m_{q_i}^2/M_W^2$ can be derived from the factors of Ref. [6] by exploiting the unitarity of the quark mixing matrix $V_{ij}$ to rearrange the products $\lambda_i = V_{ib} V_{iq}^*$ $(q = d,s)$. Furthermore, the terms $X_{uu}$, $X_{cc}$, and $X_{uc}$ contain QCD corrections and depend on $z = m_c^2/m_b^2$ [7]. We derived the numerical values of $X_{uu} = 0.920$, $X_{cc} = 0.714$, and $X_{uc} = 0.820$, respectively.

The value of the pseudoscalar $B$-decay constant $f_B$ is rather controversial. It has been predicted in the range from the 120–130 MeV of the lower bound of the QCD spectral sum rules [8] to the value of more than 250 MeV for a top quark as heavy as 140 GeV in the case of the lattice computations [9], though a good value could be extracted from the experimental results of the $B \to \tau\nu$ decay.

In spite of these large hadronic uncertainties, in this paper, a global best-fit analysis let us to improve the results of a previous paper [10] by means of the feasibilities of the MINUIT minimization computer program [11] and thus to update our knowledge about the quark mixing elements necessary for understanding the $CP$-violating asymmetries.

The relative time-dependent and mainly the time-integrated asymmetries are analyzed in detail mainly in the original case of finite detecting regions to check if they could test the standard model. In fact, this paper will deal with the asymmetries which may eventual manifest the occurrence of $CP$ violation in the $B$-meson sector. The full formalism for the oscillation and decay of $B$ mesons is outlined in Sec. II, where a useful new matrix parametrization is introduced. Section III is devoted to a detailed discussion of $CP$-violating asymmetries in semileptonic decays in finite regions. Using the results of a global analysis on the present status of the quark mixing, the predictions for the charge dilepton asymmetry have been derived and then compared with the expectations of other alternative approaches to the electroweak interactions. In a following section, the $CP$ asymmetries in hadronic $B$ decays are derived and a powerful generalization of previous results is proposed. Finally, the most likely values for the hadronic asymmetries in the decays of $B_d^0$ are provided. In conclusion, the possibilities of experimental evidence of the various $CP$-violating asymmetries outside the neutral kaon system are examined. A summary and comparison of our analysis and results completes the picture.

## II. THE FORMALISM FOR THE PARTICLE MIXING

Although this subject has been often proposed [12], an attempt at generalization is now introduced also for a request of internal completeness. As in the $K^0$-$\bar{K}^0$ system, $B^0$ and $\bar{B}^0$ may be mixed and lead to the two physical eigenstates of the regeneration Hamiltonian denoted by

$$|B_{H,L}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle , \tag{4}$$

being

$$\frac{p}{q} = \frac{1+\varepsilon_B}{1-\varepsilon_B} = \left[ \frac{M_{12}^B - (i/2)\Gamma_{12}^B}{M_{12}^{B*} - (i/2)\Gamma_{12}^{B*}} \right]^{1/2} . \tag{5}$$

In the following considerations we shall adopt the conventions $B_d^0 \equiv (\bar{b}d)$ and $\mathcal{CP}|B^0\rangle = -|B^0\rangle$. The time development of states which are initially pure $B^0$ and $\bar{B}^0$ is given by

$$\begin{aligned} |B^0(t)\rangle &= g_1(t)|B^0\rangle + \frac{q}{p} g_2(t)|\bar{B}^0\rangle , \\ |\bar{B}^0(t)\rangle &= \frac{p}{q} g_2(t)|B^0\rangle + g_1(t)|\bar{B}^0\rangle , \end{aligned} \tag{6}$$

where

$$g_{1,2}(t) \equiv \frac{1}{2} \left\{ \exp\left[ -it \left[ m_H - \frac{i}{2}\Gamma_H \right] \right] \pm \exp\left[ -it \left[ m_L - \frac{i}{2}\Gamma_L \right] \right] \right\} . \tag{7}$$

The time-dependent wave function for a $B\bar{B}$ pair at rest can be written as

$$|B\bar{B}\rangle = \frac{1}{\sqrt{2}} [ |B^0(\mathbf{k},t)\rangle \otimes |\bar{B}^0(-\mathbf{k},t)\rangle + (-1)^L |B^0(-\mathbf{k},t)\rangle \otimes |\bar{B}^0(\mathbf{k},t)\rangle ] , \tag{8}$$

where $\mathbf{k}$ is the three-momentum vector of the $B$ mesons and $L$ is the relative angular momentum of the $B\bar{B}$ pair.

Supposing one meson decaying into a general final state $|X_a(\mathbf{k})\rangle$ at time $t$ and the other into $|X_b(-\mathbf{k})\rangle$ at time $\tilde{t}$, the time-dependent decay amplitude is given by

$$\begin{aligned} \langle X_a X_b|\mathcal{H}|B\bar{B}\rangle &= \frac{1}{\sqrt{2}} [ \langle X_a|\mathcal{H}|B^0(\mathbf{k},t)\rangle \langle X_b|\mathcal{H}|\bar{B}^0(-\mathbf{k},\tilde{t})\rangle + (-1)^L \langle X_a|\mathcal{H}|\bar{B}^0(\mathbf{k},t)\rangle \langle X_b|\mathcal{H}|B^0(-\mathbf{k},\tilde{t})\rangle ] \\ &= \frac{1}{\sqrt{2}} \left[ \left[ g_1(t)\langle X_a|\mathcal{H}|B^0\rangle + \frac{q}{p} g_2(t)\langle X_a|\mathcal{H}|\bar{B}^0\rangle \right] \left[ \frac{p}{q} g_2(\tilde{t})\langle X_b|\mathcal{H}|B^0\rangle + g_1(\tilde{t})\langle X_b|\mathcal{H}|\bar{B}^0\rangle \right] \right. \\ &\quad \left. + (-1)^L \left[ \frac{p}{q} g_2(t)\langle X_a|\mathcal{H}|B^0\rangle + g_1(t)\langle X_a|\mathcal{H}|\bar{B}^0\rangle \right] \left[ g_1(\tilde{t})\langle X_b|\mathcal{H}|B^0\rangle + \frac{q}{p} g_2(\tilde{t})\langle X_b|\mathcal{H}|\bar{B}^0\rangle \right] \right] . \end{aligned} \tag{9}$$

Thus, in general, the time-dependent decay rate is

$$\Gamma(B^0(t)\bar{B}^0(\bar{t})\rightarrow X_aX_b)=|\langle X_aX_b|\mathcal{H}|B\bar{B}\rangle|^2 , \tag{10}$$

and the observable total width becomes

$$\hat{\Gamma}(B^0\bar{B}^0\rightarrow X_aX_b)=\int dt\, d\bar{t}|\langle X_aX_b|\mathcal{H}|B\bar{B}\rangle|^2 . \tag{11}$$

After having defined

$$\Delta m_B\equiv m_H-m_L , \quad \Delta\Gamma_B\equiv\Gamma_L-\Gamma_H ,$$
$$\Gamma_B\equiv\tau_B^{-1}\equiv\frac{\Gamma_H+\Gamma_L}{2} , \tag{12}$$

with

$$x\equiv\left[\frac{\Delta m_B}{\Gamma_B}\right] , \quad y\equiv\left[\frac{\Delta\Gamma_B}{2\Gamma_B}\right] , \tag{13}$$

it is worth introducing the following original notations mainly in the view of further applications. The time-dependent decay rates of a physical $B$ meson can always be described in terms of the matrix

$$(\mathcal{M}_{ij})\equiv(g_ig_j^*)=\begin{bmatrix} Z_1+Z_2 & W_1+iW_2 \\ W_1-iW_2 & Z_1-Z_2\end{bmatrix} , \tag{14}$$

where the time-evolution functions turn out to be

$$Z_1(\tau)=\tfrac{1}{2}e^{-\tau}\cosh(y\tau) , \quad Z_2(\tau)=\tfrac{1}{2}e^{-\tau}\cos(x\tau) ,$$
$$W_1(\tau)=\tfrac{1}{2}e^{-\tau}\sinh(y\tau) , \quad W_2(\tau)=\tfrac{1}{2}e^{-\tau}\sin(x\tau) , \tag{15}$$

being $\tau\equiv t/\tau_B$, an adimensional parameter. In order to shorten the expressions for the time evolution of two distinct $B$ mesons, in the following we will indicate the $\mathcal{M}$-matrix elements as

$$\mathcal{M}_{ij}\equiv\mathcal{M}_{ij}(\tau) , \quad \tilde{\mathcal{M}}_{ij}\equiv\mathcal{M}_{ij}(\bar{\tau}) . \tag{16}$$

## III. CP ASYMMETRIES IN SEMILEPTONIC DECAYS

By the use of the previous formalism, $CP$-violating asymmetries can now be reexamined on a general ground for each interesting decay channel. The $CP$ nonconservation in the neutral meson systems is crucially entangled with the relative mixing. At present, the study of $B\bar{B}$ mixing is one of the main challenges for $e^+e^-$ colliders ($B$ factories) with the $\Upsilon(4S)$ production and its subsequent decay in half of the time into a $B^0\bar{B}^0$ pair. The investigation of the decay modes can select whether the original meson was a $B$ or a $\bar{B}$. Some of the most direct evidences could depend on the reconstruction of semileptonic decays of the $B^0\bar{B}^0$ pair:

$$e^+e^-\rightarrow\Upsilon(B^0\bar{B}^0)_L\rightarrow(\ell^-\bar{\nu}X^+)(\ell^+\nu Y^-) .$$

In calculating the "dilepton numbers,"

$$\mathcal{N}^{ij}=|\langle\ell^i\ell^j|\mathcal{H}|B\bar{B}\rangle|^2 , \tag{17}$$

we can now deal with three different situations.

*Case (a)*: $B^0\bar{B}^0\rightarrow(\ell^+\nu X^-)(\ell^+\nu Y^-)$, ($X$, $Y$ hadrons with a given total charge). On account of the $\Delta B=\Delta Q$ rule, both events derive from a $\bar{B}^0$ decay, so that

$$\langle\ell^+\nu X^-|\mathcal{H}|B^0\rangle=\langle\ell^+\nu Y^-|\mathcal{H}|B^0\rangle=0 . \tag{18}$$

*Case (b)*: $B^0\bar{B}^0\rightarrow(\ell^-\bar{\nu}X^+)(\ell^-\bar{\nu}Y^+)$. In this configuration, the decays come from a $B^0$, and

$$\langle\ell^-\bar{\nu}X^+|\mathcal{H}|\bar{B}^0\rangle=\langle\ell^-\bar{\nu}Y^+|\mathcal{H}|\bar{B}^0\rangle=0 . \tag{19}$$

Since $CPT$ invariance implies

$$|\langle\ell^-\bar{\nu}X^+|\mathcal{H}|B^0\rangle|^2=|\langle\ell^+\nu Y^-|\mathcal{H}|\bar{B}^0\rangle|^2 , \tag{20}$$

we can neglect, onward, the squared amplitudes which never appear in the $CP$ asymmetries. Thus, the time-dependent dilepton numbers relative to like-sign leptons may be written as

$$\mathcal{N}^{\pm\pm}(\tau,\bar{\tau})=\frac{1}{2}\begin{Bmatrix}|q/p|^2\\|p/q|^2\end{Bmatrix}\{\mathcal{M}_{11}\tilde{\mathcal{M}}_{22}+\mathcal{M}_{22}\tilde{\mathcal{M}}_{11}+(-1)^L[\mathcal{M}_{12}\tilde{\mathcal{M}}_{21}+\mathcal{M}_{21}\tilde{\mathcal{M}}_{12}]\}$$
$$=\begin{Bmatrix}|q/p|^2\\|p/q|^2\end{Bmatrix}\{[Z_1\tilde{Z}_1+(-1)^LW_1\tilde{W}_1]-[Z_2\tilde{Z}_2-(-1)^LW_2\tilde{W}_2]\} . \tag{21}$$

*Case (c)*: $B^0\bar{B}^0\rightarrow(\ell^-\bar{\nu}X^+)(\ell^+\nu Y^-)$. In this case the $\Delta B=\Delta Q$ rule implies

$$\langle\ell^-\bar{\nu}X^+|\mathcal{H}|\bar{B}^0\rangle=\langle\ell^+\nu Y^-|\mathcal{H}|B^0\rangle=0 , \tag{22}$$

and

$$\mathcal{N}^{+-}(\tau,\bar{\tau})=\tfrac{1}{2}\{\mathcal{M}_{11}\tilde{\mathcal{M}}_{11}+\mathcal{M}_{22}\tilde{\mathcal{M}}_{22}+(-1)^L[\mathcal{M}_{12}\tilde{\mathcal{M}}_{12}+\mathcal{M}_{21}\tilde{\mathcal{M}}_{21}]\}$$
$$=[Z_1\tilde{Z}_1+(-1)^LW_1\tilde{W}_1]+[Z_2\tilde{X}_2-(-1)^LW_2\tilde{W}_2] , \tag{23}$$

whereas $\mathcal{N}^{-+}(\tau,\bar{\tau})=\mathcal{N}^{+-}(\tau,\bar{\tau})$. Since

$$Z_1\tilde{Z}_1\pm W_1\tilde{W}_1=\tfrac{1}{4}e^{-(\tau+\bar{\tau})}\cosh[y(\tau\pm\bar{\tau})] ,$$
$$Z_2\tilde{Z}_2\mp W_2\tilde{W}_2=\tfrac{1}{4}e^{-(\tau+\bar{\tau})}\cos[x(\tau+\bar{\tau})] , \tag{24}$$

one finally obtains

$$\mathcal{N}^{\pm\pm}_{L=\pm}(\tau,\bar{\tau})=\frac{1}{4}\begin{Bmatrix}|q/p|^2\\|p/q|^2\end{Bmatrix}e^{-(\tau+\bar{\tau})}\{\cosh[y(\tau\pm\bar{\tau})]-\cos[x(\tau\pm\bar{\tau})]\}\ ,$$
$$\mathcal{N}^{+-}_{L=\pm}(\tau,\bar{\tau})=\tfrac{1}{4}e^{-(\tau+\bar{\tau})}\{\cosh[y(\tau\pm\bar{\tau})]+\cos[x(\tau\pm\bar{\tau})]\}\ . \tag{25}$$

We stress that these formulas, we propose are quite general. Actually, they correctly describe the semileptonic decays of any pseudoscalar meson. Thus, the mixing properties relative to each particular meson system are determined once we fix the parameters $|p/q|$, $x$, and $y$. In the case of $B_d^0$ mesons, one has $|p/q|\simeq 1$, $x_d\simeq 0.66$, and $y\simeq 0$.

Furthermore, time-integrated decay rates are governed by

$$(\hat{\mathcal{M}}_{ij})\equiv\int d\tau(\mathcal{M}_{ij})=\begin{bmatrix}\hat{Z}_1+\hat{Z}_2 & \hat{W}_1+i\hat{W}_2\\ \hat{W}_1-i\hat{W}_2 & \hat{Z}_1-\hat{Z}_2\end{bmatrix}\ , \tag{26}$$

where

$$\hat{Z}_1=\frac{1}{2(1-y^2)}\ ,\quad \hat{Z}_2=\frac{1}{2(1+x^2)} \tag{27}$$

and $\hat{W}_1+i\hat{W}_2=y\hat{Z}_1+ix\hat{Z}_2$.

With this general notation, the total number of observed events with assigned leptonic charges (dilepton numbers) are given by

$$\hat{\mathcal{N}}^{\pm\pm}=\begin{Bmatrix}|q/p|^2\\|p/q|^2\end{Bmatrix}\{[\hat{Z}_1^2+(-1)^L\hat{W}_1^2]-[\hat{Z}_2^2-(-1)^L\hat{W}_2^2]\}$$
$$=\frac{1}{4}\begin{Bmatrix}|q/p|^2\\|p/q|^2\end{Bmatrix}\left[\frac{1+(-1)^Ly^2}{(1-y^2)^2}-\frac{1-(-1)^Lx^2}{(1+x^2)^2}\right]\ , \tag{28}$$
$$\hat{\mathcal{N}}^{+-}=[\hat{Z}_1^2+(-1)^L\hat{W}_1^2]+[\hat{Z}_2^2-(-1)^L\hat{W}_2^2]$$
$$=\frac{1}{4}\left[\frac{1+(-1)^Ly^2}{(1-y^2)^2}+\frac{1-(-1)^Lx^2}{(1+x^2)^2}\right]\ .$$

Notice that expressions with carets always refer to time-integrated quantities.

On the other hand, the eventual presence of $CP$ violation in the mixing is measured by a nonzero value of the dilepton charge asymmetry [13]

$$\hat{\mathcal{A}}_{SL}\equiv\frac{\hat{\mathcal{N}}^{++}-\hat{\mathcal{N}}^{--}}{\hat{\mathcal{N}}^{++}+\hat{\mathcal{N}}^{--}}=\frac{|q/p|^4-1}{|q/p|^4+1}$$
$$=-\frac{4\mathcal{R}e(\varepsilon_B)(1+|\varepsilon_B|^2)}{(1+|\varepsilon_B|^2)^2+4[\mathcal{R}e(\varepsilon_B)]^2}$$
$$=\frac{\mathrm{Im}(\Gamma_{12}^B/M_{12}^B)}{1+\frac{1}{4}|\Gamma_{12}^B/M_{12}^B|^2}\simeq\mathrm{Im}\left[\frac{\Gamma_{12}^B}{M_{12}^B}\right]\ , \tag{29}$$

which is independent of the relative angular momentum $L$. The last approximated expression derives from the smallness of the ratio $|\Gamma_{12}^B/M_{12}^B|$ as an effect of the hierarchy in quark masses $m_b^2\ll m_t^2$. This tiny effect is a consequence of a rather surprising cancellation of the principal contributions in the standard model [14].

Taking into account Eqs. (2) and (3), the allowed values for this charge asymmetry consistent with the experimental result of the $B_d^0$-$\bar{B}_d^0$ mixing are illustrated in Fig. 1, which severely restricts previous bounds. Thus, it can be inferred that in the standard model, the most promising prediction for the charge asymmetry is, respectively,

$$\hat{\mathcal{A}}_{SL}\sim\begin{cases}10^{-3}-10^{-2} & \text{for } B_d^0\ ,\\ 10^{-4} & \text{for } B_s^0\ ,\end{cases} \tag{30}$$

whose observation in the near future, indeed, is a hard experimental task. On the other side, $\hat{\mathcal{A}}_{SL}$ represents a good tool to put in evidence the effect of new physics. In fact, in the extended electroweak models the value of $\hat{\mathcal{A}}_{SL}^{d,s}$ could be constrained to a bound of the order of several percent [15], which is greater than the most favorable estimate in the standard model and would become a good test for an indirect manifestation of a new mechanism of $CP$ impurity.

Another sensible observable of the meson mixings is represented by the same (opposite)-sign dilepton ($ee$, $\mu\mu$, and $e\mu$) ratio coming from the decay of $B^0\bar{B}^0$ pairs and parametrized by

$$\hat{\mathcal{R}}\equiv\frac{\hat{\mathcal{N}}^{++}+\hat{\mathcal{N}}^{--}}{\hat{\mathcal{N}}^{+-}+\hat{\mathcal{N}}^{-+}}\ .$$

For completeness, it can be written as

$$\hat{\mathcal{R}}_{\text{odd}}=\frac{1}{2}\left[\left|\frac{q}{p}\right|^2+\left|\frac{p}{q}\right|^2\right]\frac{x^2+y^2}{2+x^2-y^2}\simeq\frac{x^2}{2+x^2} \tag{31}$$

$$\hat{\mathcal{R}}_{\text{even}}=\frac{1}{2}\left[\left|\frac{q}{p}\right|^2+\left|\frac{p}{q}\right|^2\right]\frac{3x^2+x^4+y^2(3+x^4)+y^4(x^2-1)}{(1+y^2)(1+x^2)^2+(1-x^2)(1-y^2)^2}\simeq\frac{3x^2+x^4}{2+x^2+x^4} \tag{32}$$

respectively, whether the $B^0$-$\bar{B}^0$ system is produced coherently in a $L=1$ (or $C=-1$) antisymmetric state [like at the threshold region of $\Upsilon(4S)$ decays] or if the $B\bar{B}$ system is obtained with a $B^{0*}\bar{B}^0$ ($L$ even) production and the consequent $B^0\bar{B}^0\gamma$ decay (in this case a mixture of $L=0$ and $L=1$ states is unavoidable). The last simplified expressions overlap

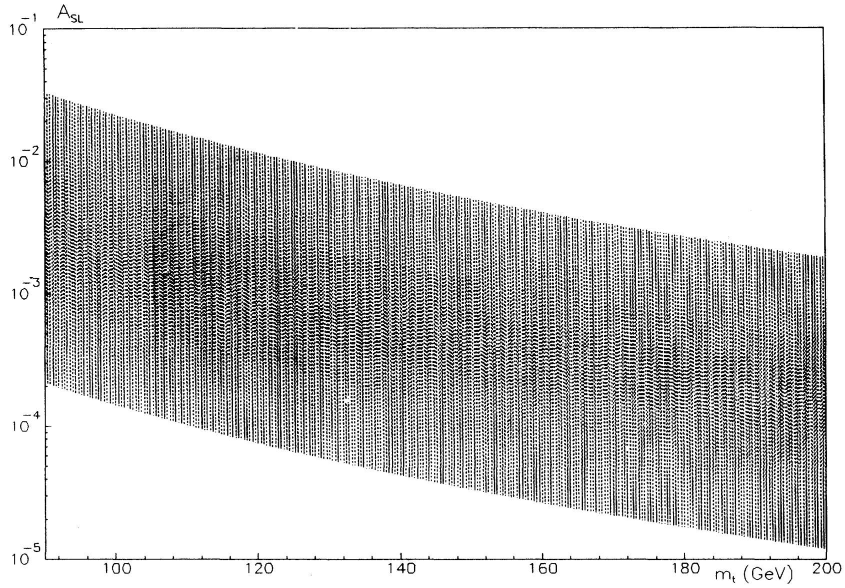


FIG. 1. The allowed ranges for the dilepton charge asymmetry $\hat{\mathcal{A}}^{d}_{SL}$ relative to the $B_d$-meson as a function of $m_t$. The quark mixing parameters are assumed in their experimental intervals of Refs. [3,21].

the standard formulation for the $B$-meson system which is obtained in the limiting case of $y \ll x$ and $|p/q| \simeq 1$.

In the face of the overwhelming success of the study of the $B^0_d\bar{B}^0_d$ mixing phenomenon at the $\Upsilon(4S)$ threshold [16,17], we want, here, to analyze the clear semileptonic signature of any $B^0$ decay mode in a finite detecting region and to show the influence on the observable $\hat{\mathcal{R}}_{\rm odd}$.

Actually, it is useful to perform the integration of the dilepton numbers in such away to distinguish the time order of the two $B$ decays. Specifically, we integrate over finite regions $\Omega$ of the time-flights, with

$$\Omega \equiv \{[\tau_1,\tau_2]\times[\bar\tau_2,\bar\tau_3]\} \cup \{[\tau_2,\tau_3]\times[\bar\tau_1,\bar\tau_2]\} \ . \tag{33}$$

The numbers of events in the region $\Omega$ presenting two leptons with an assigned charge in the final state are given by

$$\hat{\mathcal{N}}^{\pm\pm}(\Omega)=\int_\Omega d\tau\, d\bar\tau |\langle \ell^i \ell^j|\mathcal{H}|B\bar{B}\rangle|^2 \ . \tag{34}$$

After an ugly computation, this observable can be written in a compact and completely general way (i.e., without approximations) as

$$\begin{aligned}\hat{\mathcal{N}}^{\pm\pm}_L(\Omega)&=\begin{Bmatrix}|q/p|^2\\ |p/q|^2\end{Bmatrix}\left\{\sum_{i,j=1,3}^{i\neq j}(-1)^{i+j+1}\mathcal{D}^{(1)}_L(\tau_i,\bar\tau_j)-2\mathcal{D}^{(1)}_L(\tau_2,\bar\tau_2)\right\} ,\\ \hat{\mathcal{N}}^{+-}_L(\Omega)&=\left\{\sum_{i,j=1,3}^{i\neq j}(-1)^{i+j+1}D^{(2)}_L(\tau_i\bar\tau_j)-2\mathcal{D}^{(2)}_L(\tau_2,\bar\tau_2)\right\} ,\end{aligned} \tag{35}$$

where

$$\begin{aligned}\mathcal{D}^{(k)}_{L=-}(\tau,\bar\tau)&=\frac{1}{4}e^{-(\tau+\bar\tau)}\left\{\frac{\cosh[y(\tau-\bar\tau)]}{1-y^2}+(-1)^k\frac{\cos[x(\tau-\bar\tau)]}{1+x^2}\right\} ,\\ \mathcal{D}^{(k)}_{L=+}(\tau,\bar\tau)&=-\frac{1}{4}e^{-(\tau+\bar\tau)}\left\{\frac{(1+y^2)\cosh[y(\tau+\bar\tau)]+2y\sinh[y(\tau+\bar\tau)]}{(1-y^2)^2}\right.\\ &\qquad\left.+(-1)^k\frac{(1-x^2)\cos[x(\tau+\bar\tau)]-2x\sin[x(\tau+\bar\tau)]}{(1+x^2)^2}\right\} .\end{aligned} \tag{36}$$

Under the symmetric assumption of detectable regions ($\tau_i=\bar\tau_i$), these dilepton numbers are depicted in Fig. 2 for an indicative fiducial volume with ($\tau_1=0$, $\tau_3=5$). The previous quantities determine the total number of events within the fiducial volume for which one $B$ meson decays after $\tau_2$, while the other decayed before that time. If we restrict to the $B^0_d\bar{B}^0_d$ pair produced at $\Upsilon(4S)$ in the $P$ wave, the dilepton ratio calculated in the region $\Omega$, $\hat{\mathcal{R}}^{\Omega}_{\rm odd}$, shows a set of peaks independent of the fiducial volume (the same conclusion, anyway, can be derived for $\hat{\mathcal{R}}^{\Omega}_{\rm even}$). This phenomenon, which is illustrated in Fig. 3, can be ascribed to the coherent decays of the produced $B\bar{B}$ pairs. The effect is intimately connected

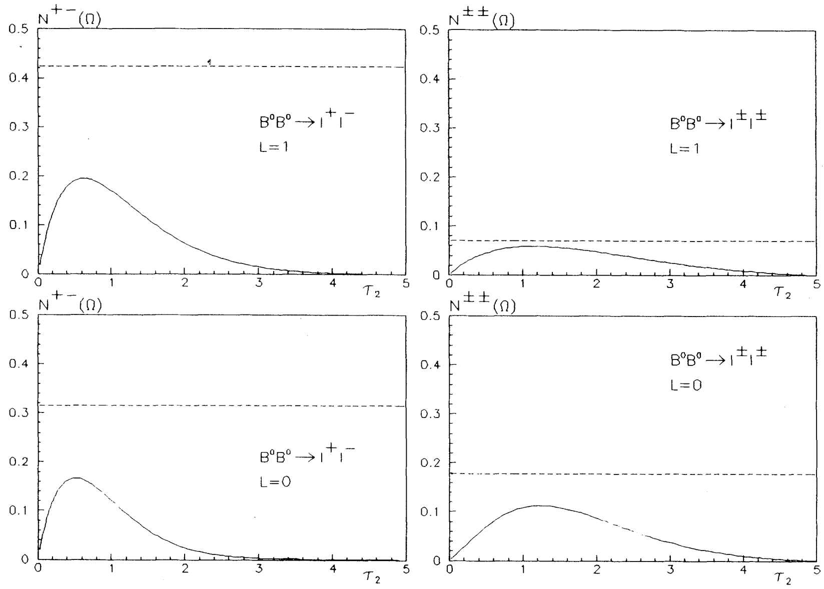


FIG. 2. The dilepton numbers $\hat{\mathcal{N}}^{\pm\pm}$ and $\hat{\mathcal{N}}^{+-}$ for every decay of $L=1$ and $L=0$ $B\bar{B}$ correlated pairs within an indicative fiducial volume $(\tau_1,\tau_3)=(0,5)$ (dashed lines) and inside the corresponding region $\Omega$ (solid lines). Therefore, the total number of decays after $\tau_2$ of one $B$, with its correlated one already decayed, are depicted.

with the peculiar physical conditions which characterize the $B$ system, i.e., the negligible difference between the lifetimes of $B_H$ and $B_L$ ($y \ll x$), and the absence of $CP$ violation in the mixing matrix ($|p/q|\simeq 1$). In fact, an analogous study of a similar resonant effect in the dilepton charge ratio for $K$ system has given negative results. It is worth stressing that the measurements of this sizable increasing of a factor 30 for $\hat{\mathcal{R}}^{\Omega}_{\rm odd}$ is strictly connected with the discrimination of the decay regions with the difficulties to detect the production points of the $B\bar{B}$ pairs. Therefore, the shape of the first peaks of the ratio in the region $\Omega$, $\hat{\mathcal{R}}^{\Omega}_{\rm odd}$, appears to be, in principle, an alternative way to determine the physical parameters of the $B$ system.

## IV. *CP* ASYMMETRIES IN HADRONIC DECAYS

*CP*-violating effects in nonleptonic decays of $B^0$ mesons are expected to be quite large, so they are a good place to test the standard model. In particular, *CP* violation is easily displayed if the final state has a definite *CP* parity. Thus, the decays of neutral $B$ mesons into a hadronic *CP* eigenstate $|X_{\mathcal{CP}}\rangle$ suggests us to define the proper-time evolving amplitudes

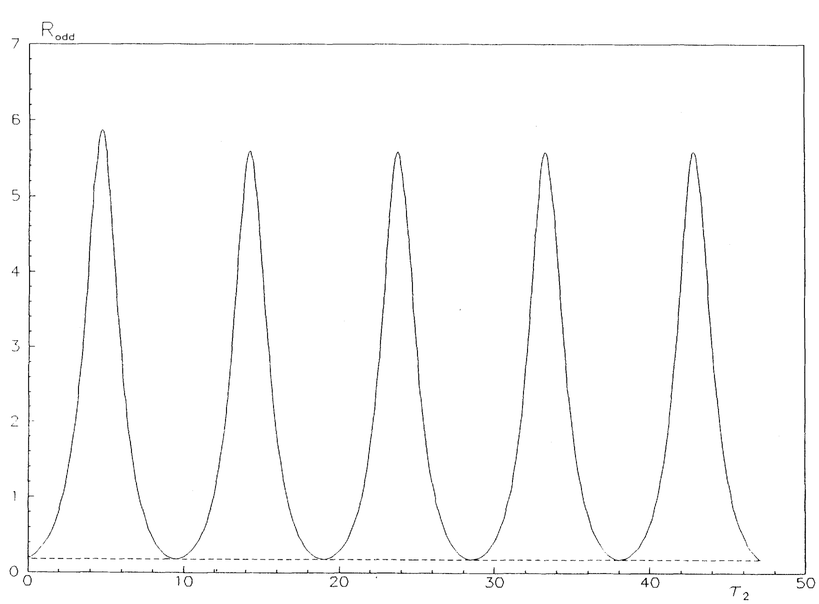


FIG. 3. The dilepton ratio $\hat{\mathcal{R}}^{\Omega}_{\rm odd}$ in the region $\Omega$ (solid line). Independent of the fiducial volume, several peaks are displayed. The asymmetry can be experimentally detected through the observation of the first one. The dashed line represents the average value of $\hat{\mathcal{R}}_{\rm odd}$ obtained in Refs. [18,19].

$$\langle X_{CP}|\mathcal{H}|B^0(\tau)\rangle=\langle X_{CP}|\mathcal{H}|B^0\rangle\{g_1(\tau)+\xi g_2(\tau)\}\ ,$$
$$\langle X_{CP}|\mathcal{H}|\bar{B}^0(\tau)\rangle=\frac{p}{q}\langle X_{CP}|\mathcal{H}|B^0\rangle\{g_2(\tau)+\xi g_1(\tau)\}\ , \tag{37}$$

where we further choose

$$\xi=\frac{q}{p}\frac{\langle X_{CP}|\mathcal{H}|\bar{B}^0\rangle}{\langle X_{CP}|\mathcal{H}|B^0\rangle}\ . \tag{38}$$

Using the notation introduced above, the time-dependent rates of the decays into final $CP$ eigenstates can be given by

$$\Gamma(B^0(\tau)\to X_{CP})=|\langle X_{CP}|\mathcal{H}|B^0\rangle|^2\{(1+|\xi|^2)Z_1+(1-|\xi|^2)Z_2+2[\mathrm{Re}(\xi)W_1+\mathrm{Im}(\xi)W_2]\} \tag{39}$$

and

$$\Gamma(\bar{B}^0(\tau)\to X_{CP})=\left|\frac{p}{q}\right|^2|\langle X_{CP}|\mathcal{H}|B^0\rangle|^2\{(1+|\xi|^2)Z_1-(1-|\xi|^2)Z_2+2[\mathrm{Re}(\xi)W_1-\mathrm{Im}(\xi)W_2]\}\ . \tag{40}$$

Again, the above expressions are quite general; i.e., they are valid for any pseudo-scalar meson system. In the physical case of interest of the $B$-meson system, the difference of the lifetimes between the two physical states is negligible, $\Gamma_H\simeq\Gamma_L\simeq\Gamma_B$ ($y\simeq 0$), and under this assumption we recover the formulas of Ref. [18]. Moreover, if we assume that the decay channel is dominated by a single amplitude, then the interference term $\xi$ is a pure phase, that is $|\xi|\simeq 1$. Under these approximations we yield

$$\Gamma(B^0(\tau)\to X_{CP})\propto e^{-\tau}[1+\mathrm{Im}(\xi)\sin(x\tau)]\ ,$$
$$\Gamma(\bar{B}^0(\tau)\to X_{CP})\propto e^{-\tau}[1-\mathrm{Im}(\xi)\sin(x\tau)]\ , \tag{41}$$

so that the deviation from the exponential law manifests the $CP$ violation effects both in the time-dependent asymmetry correlation,

$$\mathcal{A}_{NL}(\tau)\equiv\frac{\Gamma(B^0(\tau)\to X_{CP})-\Gamma(\bar{B}^0(\tau)\to X_{CP})}{\Gamma(B^0(\tau)\to X_{CP})+\Gamma(\bar{B}^0(\tau)\to X_{CP})}=\frac{(1-|\xi|^2)Z_2+2\,\mathrm{Im}(\xi)W_2}{(1+|\xi|^2)Z_1+2\,\mathrm{Re}(\xi)W_1}$$
$$=\frac{(1-|\xi|^2)\cos(x\tau)+2\,\mathrm{Im}(\xi)\sin(x\tau)}{(1+|\xi|^2)\cosh(y\tau)+2\,\mathrm{Re}(\xi)\sinh(y\tau)}\simeq\mathrm{Im}(\xi)\sin(x\tau)\ , \tag{42}$$

and mainly in the full time-integrated asymmetry:

$$\hat{\mathcal{A}}_{NL}\equiv\frac{\hat{\Gamma}(B^0\to X_{CP})-\hat{\Gamma}(\bar{B}^0\to X_{CP})}{\hat{\Gamma}(B^0\to X_{CP})+\hat{\Gamma}(\bar{B}^0\to X_{CP})}=\left[\frac{1-y^2}{1+x^2}\right]\frac{1-|\xi|^2+2\,\mathrm{Im}(\xi)x}{1+|\xi|^2+2\,\mathrm{Re}(\xi)y}\simeq\mathrm{Im}(\xi)\frac{x}{1+x^2}\ . \tag{43}$$

$CP$ asymmetries in nonleptonic decays provide a clean way to measure the angles of the unitarity triangle [19], defined by

$$\alpha\equiv\arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right]=\arctan\left[\frac{\eta}{\rho(\rho-1)+\eta^2}\right]\ ,$$
$$\beta\equiv\arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]=\arctan\left[\frac{\eta}{1-\rho}\right]\ , \tag{44}$$
$$\gamma\equiv\arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]=\arctan\left[\frac{\eta}{\rho}\right]\ ,$$

which represent the argument of the interference term $\xi$, and have been expressed using the Wolfenstein parametrization for the quark mixing [20]. The experimental constraints on the value of the semileptonic $B$ width, on $\varepsilon_K$ (the $K^0$-$\bar{K}^0$ mixing parameter), on $x_d$ (the observable of $B^0$-$\bar{B}^0$ mixing), as well as on the present knowledge of other well established quark mixing elements, yield a distinctive feature of two disconnected best-fit solutions in the space of the Wolfenstein parameters $\rho$ and $\eta$, dependent on the sign of the $CP$-violating phase and rather sensitive to the value of the decay constant $f_B$. It is interesting to note that from an updated analysis, we recover as the most likely solutions for the $B$-meson decay constant the values $f_B=(224\pm 28)$ MeV for a positive value of $\rho$ and $f_B=(125\pm 14)$ MeV for a negative one, respectively.

The forecasts for the asymmetries occurring in $\bar{b}\to\bar{u}u\bar{d}$ processes like the decay $B^0\to\pi^+\pi^-$ and in the $\bar{b}\to\bar{c}c\bar{s}$ case as in the channel $B^0\to\psi K_S$ are

$$\hat{\mathcal{A}}_{NL}(B_d^0\to\pi^+\pi^-)=-\sin(2\alpha)\frac{x_d}{1+x_d^2}\simeq\begin{cases}0.400 & \text{for } \alpha\simeq 120^\circ ,\\ -0.253 & \text{for } \alpha\simeq 17^\circ ,\end{cases}$$

$$\hat{\mathcal{A}}_{NL}(B_d^0\to\psi K_S)=-\sin(2\beta)\frac{x_d}{1+x_d^2}\simeq\begin{cases}-0.278 & \text{for } \beta\simeq 19^\circ ,\\ -0.107 & \text{for } \beta\simeq 7^\circ ,\end{cases} \tag{45}$$

where the upper expectation is linked with a positive value of $\rho$ (larger $f_B$) and the lower one with the negative $\rho$ solution (smaller $f_B$). Similar results are derived in Refs. [3,21].

In $e^+e^-$ or $p\bar{p}$ collisions, $B\bar{B}$ mixing could be put in evidence only if the produced pairs of $B$ hadrons are reconstructed completely or at least their flavor contents are tagged. We would stress that this tagging problem can be overcome by selecting two final states with an assigned $CP$ parity. In this way, $CP$-violating effects are transferred from an asymmetry to the selection of a reconstructed final state of known $CP$ parity.

In general, if a $B^0\bar{B}^0$ pair is produced coherently with an assigned relative angular momentum $L$, then the time-dependent rate into $CP$ eigenstates $X^a_{\mathcal{CP}}$ and $X^b_{\mathcal{CP}}$ is given by

$$\Gamma(B^0\bar{B}^0|_L\to X^a_{\mathcal{CP}}(\tau)X^b_{\mathcal{CP}}(\bar{\tau}))=\frac{1}{2}\left|\frac{p}{q}\right|^2 |X^a_{\mathcal{CP}}|\mathcal{H}|B^0\rangle|^2|\langle X^b_{\mathcal{CP}}|\mathcal{H}|B^0\rangle|^2 .$$

$$\times\{[\mathcal{M}_{11}+|\xi_a|^2\mathcal{M}_{22}+\xi_a\mathcal{M}_{21}+\xi_a^*\mathcal{M}_{12}][\mathcal{M}_{22}+|\xi_b|^2\tilde{\mathcal{M}}_{11}+\xi_b\tilde{\mathcal{M}}_{12}+\xi_b^*\tilde{\mathcal{M}}_{21}]$$
$$+(-1)^L[\mathcal{M}_{12}+|\xi_a|^2\mathcal{M}_{21}+\xi_a\mathcal{M}_{22}+\xi_a^*\mathcal{M}_{11}][\tilde{\mathcal{M}}_{21}+|\xi_b|^2\tilde{\mathcal{M}}_{12}+\xi_b\tilde{\mathcal{M}}_{11}+\xi_b^*\tilde{\mathcal{M}}_{22}]$$
$$+((11)\leftrightarrow(22)\ ,\ (12)\leftrightarrow(21))\}\ , \tag{46}$$

where

$$\xi_i=\frac{q}{p}\frac{\langle X^i_{\mathcal{CP}}|\mathcal{H}|\bar{B}^0\rangle}{\langle X^i_{\mathcal{CP}}|\mathcal{H}|B^0\rangle} \tag{47}$$

are the interference terms relative to each final state. Then, using the properties of the $\mathcal{M}$ matrix, an explicitly new and completely general formula can be derived as

$$\Gamma(B^0\bar{B}^0\Big|_{L=\pm}\to X^a_{\mathcal{CP}}(\tau)X^b_{\mathcal{CP}}(\bar{\tau}))=\left|\frac{p}{q}\right|^2|\langle X^a_{\mathcal{CP}}|\mathcal{H}|B^0\rangle|^2|\langle X^b_{\mathcal{CP}}|\mathcal{H}|B^0\rangle|^2$$
$$\times e^{-(\tau+\bar{\tau})}\left\{\left[\left[\frac{1+|\xi_a|^2}{2}\right]\left[\frac{1+|\xi_b|^2}{2}\right]\pm\mathrm{Re}(\xi_a)\mathrm{Re}(\xi_b)\right]\cosh[y(\tau\pm\bar{\tau})]\right.$$
$$+\left[\left[\frac{1+|\xi_b|^2}{2}\right]\mathrm{Re}(\xi_a)\pm\left[\frac{1+|\xi_a|^2}{2}\right]\mathrm{Re}(\xi_b)\right]\sinh[y(\tau\pm\bar{\tau})]$$
$$-\left[\left[\frac{1-|\xi_a|^2}{2}\right]\left[\frac{1-|\xi_b|^2}{2}\right]\mp\mathrm{Im}(\xi_a)\mathrm{Im}(\xi_b)\right]\cos[x(\tau\pm\bar{\tau})]$$
$$\left.-\left[\left[\frac{1-|\xi_b|^2}{2}\right]\mathrm{Im}(\xi_a)\pm\left[\frac{1-|\xi_a|^2}{2}\right]\mathrm{Im}(\xi_b)\right]\sin[x(\tau\pm\bar{\tau})]\right\}. \tag{48}$$

Again, the interference terms are entirely connected with the angles of the unitarity relationships for the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Finally, the observed branching ratios can be obtained through the integration of the decay rate over positive values of $\tau$, $\bar{\tau}$. For an odd relative angular momentum $L$ of the $B\bar{B}$ pair, the result is

$$B(B^0\bar{B}^0|_{L=-}\to X^a_{\mathcal{CP}}X^b_{\mathcal{CP}})=\left|\frac{p}{q}\right|^2 B(B^0\to X^a_{\mathcal{CP}})B(B^0\to X^b_{\mathcal{CP}})\left\{\frac{\left[\frac{1+|\xi_a|^2}{2}\right]\left[\frac{1+|\xi_b|^2}{2}\right]-\mathrm{Re}(\xi_a)\mathrm{Re}(\xi_b)}{1-y^2}\right.$$
$$\left.-\frac{\left[\frac{1-|\xi_a|^2}{2}\right]\left[\frac{1-|\xi_b|^2}{2}\right]+\mathrm{Im}(\xi_a)\mathrm{Im}(\xi_b)}{1+x^2}\right\}. \tag{49}$$

On the other hand, if a pure even $L$ state becomes clearly resolved, then

$$B(B^0\bar{B}^0|_{L=+}\to X^a_{\mathcal{CP}}X^b_{\mathcal{CP}})=\left|\frac{p}{q}\right|^2 B(B^0\to X^a_{\mathcal{CP}})B(B^0\to X^b_{\mathcal{CP}})$$
$$\times\left\{\left[\left[\frac{1+|\xi_a|^2}{2}\right]\left[\frac{1+|\xi_b|^2}{2}\right]+\mathrm{Re}(\xi_a)\mathrm{Re}(\xi_b)\right]\frac{1+y^2}{(1-y^2)^2}\right.$$
$$+\left[\left[\frac{1+|\xi_b|^2}{2}\right]\mathrm{Re}(\xi_a)+\left[\frac{1+|\xi_a|^2}{2}\right]\mathrm{Re}(\xi_b)\right]\frac{2y}{(1-y^2)^2}$$
$$-\left[\left[\frac{1-|\xi_a|^2}{2}\right]\left[\frac{1-|\xi_b|^2}{2}\right]-\mathrm{Im}(\xi_a)\mathrm{Im}(\xi_b)\right]\frac{1-x^2}{(1+x^2)^2}$$
$$\left.-\left[\left[\frac{1-|\xi_b|^2}{2}\right]\mathrm{Im}(\xi_a)+\left[\frac{1-|\xi_a|^2}{2}\right]\mathrm{Im}(\xi_b)\right]\frac{2x}{(1+x^2)^2}\right\}. \quad (50)$$

As it has previously pointed out in the paper, in the framework of the standard model the interference term $\xi_a$, $\xi_b$ may be expressed as pure phases in most processes. Therefore we set

$$\xi_a\equiv-\eta_a e^{-2i\varphi_a}, \quad \mathcal{CP}|X^a_{\mathcal{CP}}\rangle=\eta_a|X^a_{\mathcal{CP}}\rangle,$$
$$\xi_b\equiv-\eta_b e^{-2i\varphi_b}, \quad \mathcal{CP}|X^b_{\mathcal{CP}}\rangle=\eta_b|X^b_{\mathcal{CP}}\rangle, \quad (51)$$

so the $CP$ parity of the final state will be $\zeta\equiv(-1)^L\eta_a\eta_b$:

$$\mathcal{CP}|X^a_{\mathcal{CP}}X^b_{\mathcal{CP}}\rangle=\zeta|X^a_{\mathcal{CP}}X^b_{\mathcal{CP}}\rangle. \quad (52)$$

The branching ratio of the coherent process will then be given by

$$B(B^0\bar{B}^0|_L\to X^a_{\mathcal{CP}}X^b_{\mathcal{CP}})=B(B^0\to X^a_{\mathcal{CP}})B(B^0\to X^b_{\mathcal{CP}})\mathcal{L}_{\zeta,L}[X^a_{\mathcal{CP}},X^b_{\mathcal{CP}}], \quad (53)$$

where

$$\mathcal{L}_{\zeta,L}[X^a_{\mathcal{CP}},X^b_{\mathcal{CP}}]=\begin{cases}2\sin^2(\varphi_a-\varphi_b)+\sin(2\varphi_a)\sin(2\varphi_b)\mathcal{S}_L(x), & \text{for } \zeta=-1,\\ 2\cos^2(\varphi_a-\varphi_b)-\sin(2\varphi_a)\sin(2\varphi_b)\mathcal{S}_L(x), & \text{for } \zeta=+1,\end{cases} \quad (54)$$

and

$$\mathcal{S}_-(x)=\frac{x^2}{1+x^2}, \quad \mathcal{S}_+(x)=\frac{3x^2+x^4}{(1+x^2)^2}. \quad (55)$$

If a decay comes from a $\Upsilon(4S)$, then the $B\bar{B}$ pair is produced in a pure $L=-$ state. In this case, $CP$ violation manifests itself through a nonzero branching ratio for those processes in which $\zeta=\eta_a\eta_b=-1$, i.e., the final states have the same intrinsic $CP$ parity. A remarkable example is given by the decay into $X^a_{\mathcal{CP}}=(\psi K_L)$, $X^b_{\mathcal{CP}}=(\pi^+\pi^-)$, for which we have $\xi(\psi K_L)=e^{-2i\beta}$ and $\xi(\pi^+\pi^-)=-e^{2i\alpha}$, that is $\varphi_a\equiv\beta$, $\varphi_b\equiv-\alpha$. The $CP$-violating effect is particularly sizable for small values of $|\rho|$, as it is illustrated by the shape of the dilution term $\mathcal{L}_{\zeta,L}[(\psi K_L),(\pi^+\pi^-)]$ in Fig. 4.

## V. CONCLUSIONS

Our detailed analysis is justified by the fact that $B$-meson asymmetries represent a powerful tool to increase the knowledge of one of the inner aspects of the standard model: the origin of $CP$ violation. In the standard Cabibbo-Kobayashi-Maskawa approach, the appearance of decay asymmetries is directly related to the inability to absorb one complex observable phase into the six quark

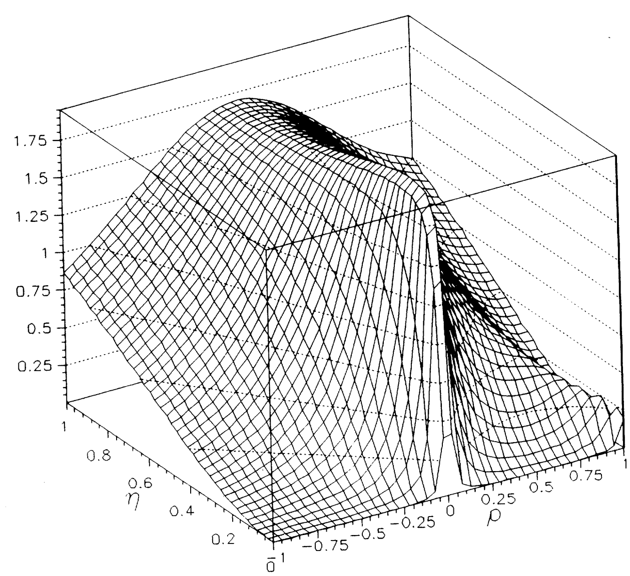


FIG. 4. The dilution factor $\mathcal{L}_{\zeta,L}$ for $B\bar{B}\to(\psi K_L)(\pi^+\pi^-)$ at the threshold as a function of the mixing parameters $\rho$, $\eta$. It represents the enhancement of the correlated branching ratio as a function of the Wolfenstein variables $\rho$, $\eta$ which are considered in a wide but sensible range.

phases. This yields an imaginary coupling which to date may explain all the observed *CP*-violating effects. No explanation of the fundamental source of the phase $\delta$ is offered in the CKM model.

The full formalism, we introduced, shows that *CP*-violating asymmetries represent a clean test to understand the quark sector and its mixing. In fact, the global fit we performed severely restricts the ranges for the CKM parameters. It emerges that the mass of the top quark of about 140 GeV is consistent with the two best likely configurations corresponding to two different values of $f_B$ and providing the values of $\delta \simeq 41°$ for $f_B \simeq 224$ MeV and $\delta \simeq 156°$ for $f_B \simeq 125$ MeV. These results suggest us the conditions under which the proposed asymmetries are in the reach of an experimental observation.

In order to improve our knowledge, further experimental information is expected from the results of the clean leptonic $B \rightarrow \tau\nu$ process, which has a branching ratio of order $10^{-4}$ and is proportional to $f_B$ with no dependence on the hadronic matrix elements. First of all, we discuss the semileptonic asymmetry arising from the productions of $B\bar{B}$ pairs. This superweak asymmetry is completely determined by the *CP* violation in $B^0$-$\bar{B}^0$ mixing due to a nonvanishing relative phase between $\Gamma_{12}^B$ and $M_{12}^B$. However, if we wish to establish a nonzero effect of this dilepton charge asymmetry to three standard deviations taking into account a pessimistic 9% experimental global efficiency, then a total number of $(2-8)\times 10^8$ $B\bar{B}$ events are required, corresponding to an integrated luminosity of nearly 700 $fb^{-1}$ in a dedicated $B$ factory. Anyway, such a numerical comparison is a redundant indicator of *CP* violation, and strongly depends on the effective lepton detection and resolution. On the other hand, the asymmetry in Fig. 1 requires a mean flight path of nearly 60 $\mu$m to be resolved, and it is not unlikely that an improved vertex measurement technique could reach this precision.

From the point of view of testing the *CP*-odd phases for hadronic $B$ decays, and consequently the unitarity triangle, it is a hard task at the moment to define completely the quark mixing parameters from known values of $|\varepsilon_K|$, the semileptonic branching ratios for $b \rightarrow u$, $c$, and $x_d$. The particularly clean *CP* asymmetry associated with the decay $B^0 \rightarrow \psi K_S$ has a size which can be measured if one observes a neutral $b$ decaying into $\psi K_S$ and determines whether it was a $B^0$ or a $\bar{B}^0$ by tagging the other $B$ in the event [22]. In a $B$ factory this physical configuration requires an integrated luminosity $L = 4/\sin^2(2\beta)$ $fb^{-1}$ to perform a three standard deviations measurement of the *CP* asymmetry $\hat{\mathcal{A}}_{NL}(B_d^0 \rightarrow \psi K_S)$ with a realistic global efficiency of 21%. Then, $(1-9)\times 10^7$ $B\bar{B}$ events are required for the best estimate of $\sin(2\beta)$. We cannot omit that a considerable uncertainty (that does not exceed one order of magnitude) affects our predictions. The required luminosity for the measurement of $\sin(2\alpha)$ is almost equal. Thus, it turns out that the *CP* asymmetries which are estimated in the framework of the standard model all appear to be detectable with the same accuracy, and therefore a promising field where the last shortcomings of the theory could be resolved.

## ACKNOWLEDGMENTS

We are obliged to G. M. Prosperi for useful advice and to I. Dunietz, G. C. Moneti, A. Pich, A. Pullia, and A. Stocchi for discussions.


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