

## Distinguishability of superstring $E_6$ -inspired low-energy models and $SU(5)_c$ color model

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We calculate the asymmetry parameters  $A$ ,  $B$ ,  $C_{L,R}$  in the deep-inelastic  $e-d$  and  $e-p$  scattering processes in the four effective low-energy models (referred to as  $\chi$ ,  $\psi$ ,  $\eta$ , and alternative left-right-symmetric models) resulting from the breaking of the superstring-inspired  $E_6$  model and the  $SU(5)_c$  model, and compared with those in the standard model. It is seen that  $\chi$  and  $\eta$  models can be discriminated through the measurement of  $A$  and  $B$  parameters while  $C_L$  can differentiate the  $\chi$  model only from the standard model. However, the alternative left-right-symmetric model and the  $SU(5)_c$  model cannot be distinguished from the standard model through measurement of  $A$ ,  $B$ , and  $C_L$  parameters. Interestingly, all the models can be markedly distinguished from the standard model through the measurement of  $C_R$  for values of  $y$  in the range  $0.38 \leq y \leq 0.42$ .

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### I. INTRODUCTION

In spite of the spectacular success of the standard model (SM) [1] in explaining the available experimental data, theorists tend to believe that the standard rank-4 gauge group  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ , is to emerge at low energy as a result of the breaking of higher-rank gauge groups based on grand unification [2] [ $SU(5)$ ,  $SO(10)$ ,  $E_6$  or  $E_8$ ], chiral or extended color symmetries, or supersymmetry [3]. The effective rank-5 models (ER5M) embedded in

$$(a) \quad SU(3)_c \otimes SU(2)_L \otimes U(1) \otimes U(1)$$

and

$$(b) \quad SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1) \quad [\text{alternative left-right-symmetric model (ALRM)}]$$

originating at low energies from the superstring-inspired  $E_6$  model [6]. The two  $U(1)$ 's form the basis for any pair of orthogonal Abelian generators independent of the electric charge  $Q$  ( $= T_{3L} + Y/2$ ). The ER5M contains three massive neutral gauge bosons  $Z$ ,  $Z'$ ,  $Z''$  with mass hierarchy  $M_{Z''} \gg M_{Z'} > M_Z$  and, hence,  $Z''$  effectively decouples from both  $Z$  and  $Z'$ . It is assumed that  $Z'$  is relatively light to mix with  $Z$ , the weak neutral SM gauge boson. There are many possible embeddings of the two  $U(1)$ 's in  $E_6$ , parametrized by a mixing angle  $\alpha$ . The three special values of the mixing angles result in three ER5M's: (i) the  $\psi$  model ( $\alpha=0^\circ$ ); (ii) the  $\chi$  model ( $\alpha=-90^\circ$ ); (iii) the  $\eta$  model ( $\alpha=37.8^\circ$ ). The alternative left-right-symmetric model (ALRM) contains extra weak charged as well as neutral gauge bosons. The other two- $Z$  model, which was proposed recently by Foot and Hernandez [7], is based on the gauge group  $SU(3) \otimes SU(2) \otimes SU(2) \otimes U(1)$  emerging at low energy

the higher-rank gauge group can be confronted with experiment to discriminate them from one another, as well as from the SM, through detection of the effects due to extra  $Z$  bosons. Following Cahn and Gilman [4], several authors [5] have discussed the measurement of the asymmetry parameters  $A$ ,  $B$ ,  $C_L$ ,  $C_R$  in the polarized  $e-p$  and  $e-d$  deep-elastic scattering processes in order to distinguish some extensions of the SM from one another. This has motivated us to calculate these asymmetry parameters in the following extended gauge models. We first consider the ER5M based on

from the breaking of the higher-rank gauge group  $SU(5) \otimes SU(2) \otimes U(1)$ . Weak hypercharge  $U(1)_Y$  is a linear combination of the  $U(1)$  and the  $U(1)_c$  of  $SU(5)_c$ .  $SU(5)_c$  breaks down to  $SU(3)_c$  of color and  $SU(2)_c$  corresponding to a new confining force. The motivation for the study of the asymmetry parameters stems from the fact that this model unlike most extra  $Z'$  models, enlarges the color rather than the electroweak gauge sector.

In the present work we have calculated the asymmetry parameters in the  $\psi$ ,  $\chi$ ,  $\eta$ , ALRM, and  $SU(5)$  color models and compare with those in the SM. Our analysis shows that  $\chi$  and  $\eta$  models can be discriminated from the SM through the measurement of  $A$ ,  $B$ , at the level of  $5\sigma$  or greater assuming 2% accuracy in the measured values of the parameters. For a certain mixing angle  $\phi$  of the two neutral  $Z$  bosons, the measurement of  $C_L$  can only differentiate the  $\chi$  model from the SM. Such discrimination is not possible for the ALRM and  $SU(5)_c$  color mod-

els through the measurement of the  $A$ ,  $B$ , and  $C_L$  parameters. Interestingly the parameter  $C_R$  in both  $e$ - $d$  and  $e$ - $p$  processes will unambiguously distinguish all the models from the SM.

The plan of the paper is as follows. Section II discusses the models. Our calculations of the various asymmetry parameters are given in Sec. III. Results are summarized in Sec. IV. Section V contains our conclusions.

## II. THE MODELS

In this section we briefly summarize the essential features of the models. They contain one or more additional neutral gauge bosons whose couplings with fermions influence the asymmetry parameters.

### A. $E_6$ -inspired models

We first consider the rank-6 model originating from the flux breaking [6] as discussed earlier. The mass eigenstates (not true mass eigenstates as  $Z_\psi, Z_\chi$  can mix) of the gauge fields ( $Z_\psi$  and  $Z_\chi$ ) corresponding to  $U(1)_\psi$  and  $U(1)_\chi$  are given by

$$Z'(\alpha) = Z_\psi \cos \alpha - Z_\chi \sin \alpha, \quad (1a)$$

$$Z''(\alpha) = Z_\chi \cos \alpha + Z_\psi \sin \alpha, \quad (1b)$$

where  $\alpha$  depends on the vacuum expectation values (VEV's) of the Higgs fields and the gauge couplings  $g_{\psi, \chi}$ , corresponding to  $U(1)_{\psi, \chi}$  symmetry. In the rank-5 limit  $Z''$  becomes significantly more massive than  $Z'$ . This implies that either  $Z''$  is very heavy with a mass characteristic of a large intermediate mass scale ( $\sim 10^{10}$  GeV) or perhaps relatively light ( $\sim 10$  TeV) but still sufficiently heavy so that it effectively decouples from both  $Z$  and  $Z'$ , and hence we have two neutral gauge bosons  $Z$  and  $Z'$ . Furthermore, it is to be noted that  $Z' = Z_\psi$  ( $\psi$  model) and  $Z' = Z_\chi$  ( $\chi$  model) states can be recovered for  $\alpha = 0$  and  $\alpha = -90$ , respectively. However, the couplings of the true rank-5 model ( $\eta$  model) are recovered for  $\alpha \sim 37.8$ . To discuss further the properties of the  $Z'$  in these models, we need to examine the mixing between the  $Z'$  and  $Z = Z_{SM}$ . Including the usual  $Z$  boson the coupling of the extra  $Z'$  boson can be expressed as

$$\left[ \frac{g}{\cos \theta_w} \right] \left[ J_Z Z + \sqrt{5/3} \sin \theta_w Q' Z' \right], \quad (2)$$

with  $J_Z = (T_3 - \sin^2 \theta_w Q_f^{em})$  and  $Q' = Q_\psi \cos \alpha - Q_\chi \sin \alpha$  in the ER5M resulting from Eq. (1a). In general, two doublets and two singlets of Higgs fields are introduced in the ER5M [8] which are as follows:

$$\phi_1 = \begin{bmatrix} \phi_1^0 \\ \phi_1^0 \end{bmatrix}, \quad \phi_2 = \begin{bmatrix} \phi_2^0 \\ \phi_2^0 \end{bmatrix}, \quad \phi_3 = \phi_3^0, \quad \phi_4 = \phi_4^0, \quad (3)$$

with  $\langle \phi_i^0 \rangle = v_i / \sqrt{2}$  and  $v^2 = v_1^2 + v_2^2$ . The neutral gauge boson mass matrix is given by

$$M^2 = \begin{bmatrix} M_Z^2 & \delta M^2 \\ \delta M^2 & M_{Z'}^2 \end{bmatrix}, \quad (4)$$

where  $M_Z$  is the usual SM  $Z$ -boson mass in the absence of mixing between  $Z$  and  $Z'$  and

$$\frac{\delta M^2}{M_Z^2} = 2 \frac{Q'_1 v_1^2 - Q'_2 v_2^2}{v^2} \left[ \frac{5}{3} \sin^2 \theta_w \right]^{1/2}, \quad (5a)$$

$$\frac{M_{Z'}^2}{M_Z^2} = \frac{20}{3} \sin^2 \theta_w \frac{1}{v^2} \sum_{i=1}^4 (Q'_i v_i)^2, \quad (5b)$$

with  $Q' = Q'(\phi_i^0)$ . The mass matrix in Eq. (4) is diagonalized by an orthogonal transformation

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} Z \\ Z' \end{bmatrix}, \quad (6)$$

producing the mass eigenstates  $Z_1, Z_2$  with masses  $M_{Z_1}$  and  $M_{Z_2}$ , where  $\phi$  is the mixing angle between  $Z - Z'$  and is given by

$$\tan^2 \phi = \frac{M_Z^2 - M_{Z_1}^2}{M_{Z_2}^2 - M_Z^2}. \quad (7)$$

For large VEV's  $v_{3,4}$ ,  $M_{Z'}^2 \rightarrow \infty$  and  $\phi \rightarrow 0$  so that  $Z_1 = Z$ .

In the ALRM the couplings of the extra neutral gauge bosons  $Z_R$  [arising from  $SU(2)_R$ ] differ from that of other  $Z$ 's which arise in rank-5 or ER5 models. In the weak interaction basis,  $Z$  and  $Z_R$  are found to couple via ( $g_L = g_R = g$ )

$$L = \frac{g}{\cos \theta_w} (J_Z Z + J_R Z_R), \quad (8)$$

with

$$J_R = (1 - 2 \sin^2 \theta_w)^{-1/2} [\sin^2 \theta_w T_{3L} + (1 - \sin^2 \theta_w) T_{3R} - \sin^2 \theta_w Q_f^{em}], \quad (9)$$

where

$$Q_f^{em} = T_{3L} + T_{3R} + \frac{V}{2} \quad (10)$$

and  $V$  is the quantum number of the extra  $U(1)$  group. Neutral current data can be used to bound  $M_{Z_2}$  as function of  $\phi$  (mixing angle between weak states  $Z$  and  $Z_R$ ) as in the other ER5M.

### B. $SU(5)_c$ color model

This model [7] based on gauge group  $G = SU(5)_c \otimes SU(2)_L \otimes U(1)'$  enlarges the color group rather than the electroweak sector, with the hypercharge  $U(1)_Y$  being a linear combination of  $U(1)'$  and  $U(1)'_c$  generated by the  $SU(5)_c$  generator  $\sim \text{diag}(2, 2, 2, -3, -3)$ . The breaking of  $SU(5)_c \otimes SU(2)_L \otimes U(1)' \rightarrow SU(3)_c \otimes SU(2)_c \otimes SU(2)_L \otimes U(1)_Y$  is assumed to be due to a new Higgs field  $\chi$  of VEV  $\omega / \sqrt{2}$  transforming as an antisymmetric  $\mathbf{10}$  of  $SU(5)_c$ . The breaking of  $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$  takes place through a standard Higgs

doublet with a VEV  $u/\sqrt{2}$ . This theory contains charge  $\pm\frac{1}{2}$  exotic fermions. Following the notation of Glashow and Sarid [9] we have the gauge fields  $W_1, W_2, W_3$  corresponding to  $U(1)_{e'}$ ,  $U(1)'$  and the third component of  $SU(2)_L$  which couple with fermions as

$$g_1 T_1 W_1 + g_2 T_2 W_2 + g_3 T_3 W_3, \quad (11)$$

where  $g_3$  is the ordinary  $SU(2)_L$  coupling constant,  $g_2$  is approximately the hypercharge coupling constant, and  $g_1$  denotes the strong coupling constant up to normalization. The electric charge is defined to be  $Q = T_1 + T_2 + T_3$ , where the quantum numbers are assigned as shown in Table I. The  $Z$ - $Z_1$  mass matrix is obtained in two steps. First we rotate  $W_1$  and  $W_2$  through an angle  $\beta$  to form the linear combinations of  $W_4$  and  $Z'$  in such a way that  $W_4$  again couples to  $W_3$  through the conventional electroweak angle  $\theta_W$  to form the standard  $Z$  boson and the photon field  $A$ . The mass matrix of  $Z$ - $Z'$  gauge bosons is given by

$$M^2 = M_Z^2 \begin{pmatrix} 1 & -\sin\theta_W \cot\beta \\ -\sin\theta_W \cot\beta & \sin^2\theta_W \cot^2\beta + \frac{4\sin^2\theta_W}{25\cos^2\beta\sin^2\beta} \end{pmatrix} \frac{\omega^2}{u^2}, \quad (12)$$

where,  $M_Z^2 = \frac{1}{4}(g^2 + g'^2)u^2$  and  $g_1 \cos\beta = g_2 \sin\beta = g'$ . In the weak eigenstates basis the gauge coupling is defined as

$$\frac{e}{\sin\theta_W \cos\theta_W} [J_Z Z + \sin\theta_W J_C Z'], \quad (13)$$

with  $J_C = (-T_1 \tan\beta + T_2 \cot\beta)$  and,

$$\cos^2\beta \sim 0.0063. \quad (14)$$

The mass matrix in Eq. (12) is diagonalized by the orthogonal transformation

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix}, \quad (15)$$

producing the mass eigenstates  $Z_1$  and  $Z_2$ . In the limit  $\omega \rightarrow \infty$ , i.e.,  $M_{Z'} \rightarrow \infty$ ,  $Z_1$  will be identified with the SM  $Z$  boson and  $\phi \rightarrow 0$ .

### III. CALCULATION OF ASYMMETRY PARAMETERS

The asymmetry parameters in the  $e$ - $d$  and  $e$ - $p$  scattering are defined [4,5] as

$$A = \frac{d\sigma(e_R^-) - d\sigma(e_L^-)}{d\sigma(e_R^-) + d\sigma(e_L^-)}, \quad (16)$$

$$B = \frac{d\sigma(e_R^-) - d\sigma(e_L^+)}{d\sigma(e_R^-) + d\sigma(e_L^+)}, \quad (17)$$

$$C_{L,R} = \frac{d\sigma(e_{L,R}^+) - d\sigma(e_{L,R}^-)}{d\sigma(e_{L,R}^+) + d\sigma(e_{L,R}^-)}, \quad (18)$$

TABLE I. Quantum numbers for the fields under the groups  $U(1)_{e'}$ ,  $U(1)'$ , and  $SU(2)_L$ , respectively.

	$T_1$	$T_2$	$T_3$
$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\frac{1}{15}$	$\frac{1}{10}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$
$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	0	$-\frac{1}{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$
$u_L^c$	$-\frac{1}{15}$	$-\frac{3}{5}$	0
$d_L^c$	$-\frac{1}{15}$	$\frac{2}{5}$	0
$e_L^c$	0	1	0
$\phi^+$	0	$\frac{1}{2}$	$\frac{1}{2}$
$\phi^0$	0	$\frac{1}{2}$	$-\frac{1}{2}$
$\chi$	$-\frac{1}{5}$	$\frac{1}{5}$	0

where,  $d\sigma_R = \sum (d\sigma_{RL}^i + d\sigma_{RR}^i) q_i(x)$  and  $d\sigma_L = \sum (d\sigma_{LL}^i + d\sigma_{LR}^i) q_i(x)$ ; the summation is over all quarks and antiquarks. Assuming the Dirac structure of photons and the  $Z$ -fermion vertex to be

$$\left[ \frac{1}{2} \gamma_\mu (1 - \gamma_5) Q_{Lj}^{\gamma, Z(n)} + \frac{1}{2} \gamma_\mu (1 + \gamma_5) Q_{Rj}^{\gamma, Z(n)} \right] [A^\mu, Z(n)^\mu], \quad (19)$$

where  $Q_{Lj}^{\gamma, Z}$  and  $Q_{Rj}^{\gamma, Z}$  are the strength by which photons or  $Z(n)$  couple with the left- and right-handed fermions, respectively, one obtains the cross sections

$$d\sigma_{RR}^i \propto \left| \frac{Q_{ke}^\gamma Q_{ki}^\gamma}{Q^2} + \sum \frac{Q_{Re}^{Z(n)} Q_{Ri}^{Z(n)}}{Q^2 + M_{Z(n)}^2} \right|^2, \quad (20)$$

$$d\sigma_{RL}^i \propto \left| \frac{Q_{ke}^\gamma Q_{Li}^\gamma}{Q^2} + \sum \frac{Q_{Re}^{Z(n)} Q_{Li}^{Z(n)}}{Q^2 + M_{Z(n)}^2} \right|^2 (1-y)^2, \quad (21)$$

$$d\sigma_{LL}^i \propto \left| \frac{Q_{Le}^\gamma Q_{Li}^\gamma}{Q^2} + \sum \frac{Q_{Le}^{Z(n)} Q_{Li}^{Z(n)}}{Q^2 + M_{Z(n)}^2} \right|^2, \quad (22)$$

and

$$d\sigma_{LR}^i \propto \left| \frac{Q_{Le}^\gamma Q_{ki}^\gamma}{Q^2} + \sum \frac{Q_{Le}^{Z(n)} Q_{Ri}^{Z(n)}}{Q^2 + M_{Z(n)}^2} \right|^2 (1-y)^2, \quad (23)$$

where  $d\sigma_{L(R)L(R)}^i(e^-)$  is the cross section for the scattering of left- (right-) handed electrons from the left- (right-) handed quarks of type  $i$ ;  $y$  is the fraction of the incident lepton's energy which is transferred to the nucleons.

To get the total contribution we multiply each  $d\sigma^i$  by

corresponding weight factors given by the quark-antiquark distribution function obtainable from deep inelastic scattering. Since the electromagnetic current in every gauge model remains unchanged we have  $Q_{L_e}^Y = Q_{R_e}^Y = -e$ ,  $Q_{L_u}^Y = Q_{R_u}^Y = \frac{2}{3}e$ ,  $Q_{L_d}^Y = Q_{R_d}^Y = -\frac{1}{3}e$ . The electromagnetic coupling  $e$  and the weak Fermi coupling  $G_F$  are related as

$$e = g \sin \theta_w, \quad (24a)$$

$$\frac{G_F}{2\sqrt{2}\pi\alpha} = \frac{1}{4 \sin^2 \theta_w \cos^2 \theta_w M Z_Z^2}. \quad (24b)$$

The weak charges of the fermions are given by

$$Q_L^Z = \frac{e}{\sin \theta_w \cos \theta_w} (T_{3L} - Q_{L_f}^Y \sin^2 \theta_w), \quad (25a)$$

$$Q_R^Z = \frac{e}{\sin \theta_w \cos \theta_w} (T_{3R} - Q_{R_f}^Y \sin^2 \theta_w). \quad (25b)$$

Here  $T_{3L}$  and  $T_{3R}$  are the third components of weak isospin for left- and right-handed fermions, respectively. Similar expressions for the weak charges  $Q_R^{Z(n)}$ , etc., for the other models can be easily computed by considering the relations  $Q_L^{Z(n)} = V^{Z(n)} - A^{Z(n)}$  and  $Q_R^{Z(n)} = V^{Z(n)} + A^{Z(n)}$  where  $V^{Z(n)}$  and  $A^{Z(n)}$  are the appropriate weak vector and axial-vector charges for the respective  $Z(n)$ 's.

### A. $E_6$ model

The couplings of the two neutral gauge bosons of mass eigenstates  $Z_1$  and  $Z_2$  to the fermions in the  $\psi$ ,  $\chi$ , and  $\eta$  models are

$$Q^{Z_1} = \frac{e}{\sin \theta_w \cos \theta_w} [J_Z \cos \phi + \sqrt{5/3} Q' \sin \theta_w \sin \phi], \quad (26a)$$

$$Q^{Z_2} = \frac{e}{\sin \theta_w \cos \theta_w} [-J_Z \sin \phi + \sqrt{5/3} Q' \sin \theta_w \cos \phi], \quad (26b)$$

and for the ALRM the couplings are

$$Q^{Z_1} = \frac{e}{\sin \theta_w \cos \theta_w} [J_Z \cos \phi + 2(1 - 2 \sin^2 \theta_w)^{-1/2} J_R \sin \phi], \quad (27a)$$

$$Q^{Z_2} = \frac{e}{\sin \theta_w \cos \theta_w} [-J_Z \sin \phi + (1 - 2 \sin^2 \theta_w)^{-1/2} J_R \cos \phi] \quad (27b)$$

The quantum numbers  $Q_\psi$ ,  $Q_\chi$ ,  $V/2$ ,  $T_{3L}$ ,  $T_{3R}$  are given in Table II.

TABLE II. Quantum numbers for the fields in the 27 representation in  $E_6$  under the groups  $U(1)_\eta$ ,  $U(1)_\psi$ ,  $U(1)_\chi$ ,  $SU(2)_L$ ,  $SU(2)_R$ ,  $U(1)_V$ , and  $Q_f^{\text{em}}$ , respectively.

	$2\sqrt{15}Q_\eta$	$2\sqrt{6}Q_\psi$	$2\sqrt{10}Q_\chi$	$T_{3L}$	$T_{3R}$	$\frac{V}{2}$	$Q_{\text{em}}$
$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	2	1	-1	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	0	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$
$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	-1	1	3	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	0	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$u_L^c$	2	1	-1	0	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{2}{3}$
$d_L^c$	-1	1	3	0	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{3}$
$e_L^c$	2	1	-1	0	$\frac{1}{2}$	$\frac{1}{2}$	1
$\nu_L^c$	5	1	-5	0	$-\frac{1}{2}$	$\frac{1}{2}$	0
$H = \begin{pmatrix} N_L \\ E_L \end{pmatrix}$	-1	-2	-2	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	0	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$H^c = \begin{pmatrix} E_L^c \\ N_L^c \end{pmatrix}$	-4	-2	2	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$	0	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$h_L$	-4	-2	2	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$h_L^c$	-1	-2	-2	0	0	$\frac{1}{3}$	$\frac{1}{3}$
$S_L^c$	5	4	0	0	0	0	0

### B. $SU(5)_c$ color model

In the  $SU(5)_c$  color model the corresponding weak charges for two neutral gauge bosons with mass eigenstates  $Z_1$  and  $Z_2$  are

$$Q^{Z_1} = \frac{e}{\sin\theta_W \cos\theta_W} [J_Z \cos\phi + \sin\theta_W J_C \sin\phi], \quad (28a)$$

$$Q^{Z_2} = \frac{e}{\sin\theta_W \cos\theta_W} [-J_Z \sin\phi + \sin\theta_W J_C \cos\phi]. \quad (28b)$$

## IV. RESULTS

In the present work we have calculated the asymmetry parameters  $A/Q^2$ ,  $B/Q^2$ ,  $C_L/Q^2$ , and  $C_R/Q^2$  as a function of  $y$  for polarized  $e$ - $d$  and  $e$ - $p$  deep inelastic scattering processes in the ER5M ( $\eta, \chi, \psi$ , and ALRM) emerging at low energies from the  $E_6$  model. The same asymmetry parameters have also been calculated in the  $SU(3)_C \otimes SU(2)_C \otimes SU(2)_L \otimes U(1)_Y$  resulting from the breaking of the extended color model based on  $SU(5)_C \otimes SU(2)_L \otimes U(1)'$ . The weight factors are  $q_p^u(x) = 2q_p^d(x)$  for proton target and  $q_d^u(x) = q_d^d(x)$  for deuteron target. We have ignored the QCD corrections to the quark distribution functions and the contributions of the quark-antiquark sea.

### A. $E_6$ models

It is to be noted that, for all models with two effective neutral gauge bosons  $Z_1$  and  $Z_2$ , the constraint on the mixing angle  $\phi$  arises from the  $Z$  mass hierarchy  $M_{Z_2} > M_{Z_1}$ ,  $M_{Z_1} \approx M_Z$ . In the ER5M we have chosen  $M_{Z_2} \approx 91.174$  GeV,  $M_{Z_1} \approx 91.17$  GeV, and  $M_{Z_2} > 500$  GeV [10] which implies that the lighter  $Z$  mass agrees with the SM  $Z$  mass [11] and the heavier one lies within the phenomenological bound [12]. The lower bound on  $M_{Z_2}$  implies an upper bound on the mixing angle  $\phi \leq 0.09$ . Since the effect of the extra  $Z$  boson on the asymmetry parameters increases with  $\phi$ , we shall present our study for  $\phi = 0.09$  (which corresponds to  $M_{Z_2} \sim 500$  GeV) for  $y$  within the range  $0.1 \leq y \leq 1.0$ .

For the ALRM, stronger bound on  $M_{Z_2} > 800$  GeV exists from an analysis of the data from the CERN  $e^+e^-$  collider LEP [13]. This will imply  $\phi \leq 0.05$  and we shall present our analysis for  $\phi = 0.05$  and  $y$  within the range  $0.1 \leq y \leq 1.0$ .

The  $SU(5)_c$ , in addition to the weak mixing angle  $\theta_W$ , has two extra angles  $\phi$  and  $\beta$  (we take  $\beta = 85.45^\circ$ ) which mix the neutral gauge bosons. The bound on  $M_{Z_2}$  for the  $SU(5)_c$  model again comes from the direct search [12] and  $M_{Z_2} > 500$  GeV implies  $\phi \leq 0.24$ . So we work with  $\phi = 0.24$  for  $0.1 \leq y \leq 1.0$ .

#### 1. $\eta$ , $\psi$ , and $\chi$ models

Figures 1(a)–3(a) show the ratio of  $A/A_0$  in the deep-inelastic  $e$ - $d$  ( $e$ - $p$ ) scattering. The ratio varies from  $\sim 1.085$  (1.114) at  $y = 0.1$  to 1.747 (1.966) at  $y = 1.0$  in the

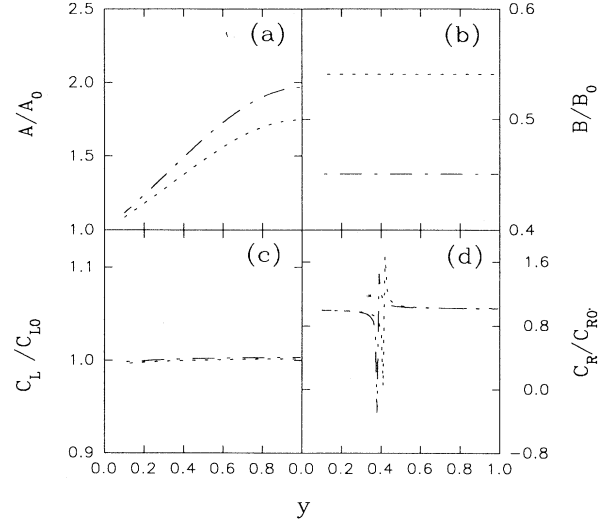


FIG. 1. The ratios (a)  $A/A_0$ , (b)  $B/B_0$ , (c)  $C_L/C_{L0}$ , and (d)  $C_R/C_{R0}$  are plotted against  $y$  for  $e$ - $d$  scattering (dotted lines) and  $e$ - $p$  scattering (dash-dot lines) for the  $\eta$  model.

$\eta$  model ( $\alpha = 37.8^\circ$ ) [Fig. 1(a)]. This ratio changes from  $\sim 0.615$  (0.822) at  $y = 0.1$  to 1.963 (2.247) at  $y = 1.0$  for the  $\chi$  model ( $\chi = -90^\circ$ ) [Fig. 2(a)]. However, the ratio remains almost independent of  $y$  with constant value  $\sim 0.99$  for the  $\psi$  model ( $\alpha = 0^\circ$ ) [Fig. 3(a)]. The experimental value of the asymmetry parameter  $A \approx (-11.8 \pm 4.5) \times 10^{-5}$  of Prescott *et al.* [14] for  $e$ - $d$  scattering at  $y = 0.22$  agrees within the error with that predicted from the  $\chi$  and the  $\psi$  models. Figures 1(b)–3(b) exhibit the variation of the ratio of the  $B$  parameter with  $y$  for  $\eta$ ,  $\chi$  and  $\psi$  models. The ratios are independent of  $y$  and have the values 0.542 (0.45), 0.542 (0.545), 0.91

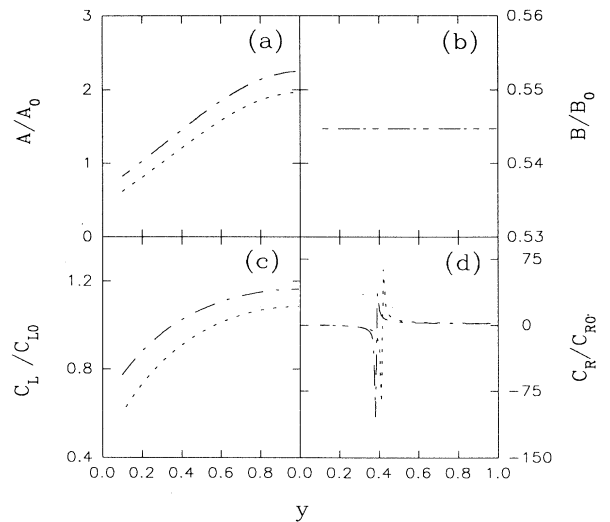


FIG. 2. The ratios (a)  $A/A_0$ , (b)  $B/B_0$ , (c)  $C_L/C_{L0}$ , and (d)  $C_R/C_{R0}$  are plotted against  $y$  for  $e$ - $d$  scattering (dotted lines) and  $e$ - $p$  scattering (dash-dot lines) for the  $\chi$  model.

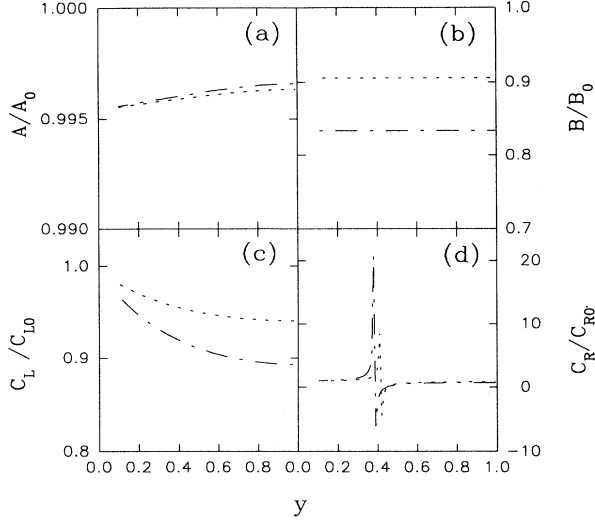


FIG. 3. The ratios (a)  $A/A_0$ , (b)  $B/B_0$ , (c)  $C_L/C_{L0}$ , and (d)  $C_R/C_{R0}$  are plotted against  $y$  for  $e$ - $d$  scattering (dotted lines) and  $e$ - $p$  scattering (dash-dot lines) for the  $\psi$  model.

(0.834) for  $e$ - $d$  ( $e$ - $p$ ) scattering processes. Figures 1(c)–3(c) depict the plots of  $C_L/C_{L0}$  versus  $y$ . Although the ratio is almost independent of  $y$  for the  $\eta$  model and has a value  $\sim 1.0$ , the ratio for the  $\chi$  model increases with increasing  $y$  from a value 0.603 (0.774) at  $y=0.1$  to a value 1.086 (1.165) at  $y=1.0$  for  $e$ - $d$  ( $e$ - $p$ ) scattering, while for the  $\psi$  model the ratio decreases from 0.993 (0.989) at  $y=0.1$  to 0.94 (0.893) at  $y=1.0$  for the  $e$ - $d$  ( $e$ - $p$ ) processes. Figures 1(d)–3(d) show the marked variation of the ratio  $C_R/C_{R0}$  from unity for the values of  $y$  in the range  $0.41$  ( $0.38$ )  $\leq y \leq 0.42$  ( $0.39$ ). It is seen that while the ratio has the values  $\sim 0.046$  ( $-0.287$ ),  $-88.3$  ( $-103.67$ ),  $8.473$  ( $20.752$ ) at  $y=0.41$  ( $0.38$ ), it has values  $\approx 1.67$  ( $1.45$ ),  $63.74$  ( $37.86$ ),  $-4.339$  ( $-6.058$ ) at  $y=0.42$  ( $0.39$ ) for polarized  $e$ - $d$  ( $e$ - $p$ ) deep-inelastic scattering in the  $\eta$ ,  $\chi$ , and  $\psi$  models, respectively. However, the ratio becomes almost unity for the values of  $y$  in the range  $0.1 \leq y \leq 0.25$  and  $0.55 \leq y \leq 1.0$  for  $\psi$  and  $\eta$  models.

## 2. Alternative left-right-symmetric model (ALRM)

We have calculated the  $y$  dependence of the ratio of the asymmetry parameters  $A/A_0$ ,  $B/B_0$ ,  $C_L/C_{L0}$ , and  $C_R/C_{R0}$  in the ALRM for polarized  $e$ - $d$  and  $e$ - $p$  deep-inelastic scattering processes for  $\phi=0.05$  and  $0.1 \leq y \leq 1.0$  (Fig. 4). The range of the mixing angle and the masses of the  $Z$  bosons are the same as that mentioned in the previous models. Figure 4(a) depicts the variation of the ratio, which remains nearly constant for both processes, and for values of  $y$  with  $A/A_0 \sim 0.994$ . Figure 4(b) shows that the ratio  $B/B_0$  is also nearly constant with its value  $\sim 1.004$ . The plots of the ratio of  $C_L/C_{L0}$  versus  $y$  are shown in Fig. 4(c). The ratio varies from  $\sim 0.996$  at  $y=0.1$  to a value  $\sim 1.0$  at  $y=1.0$  for both the processes. Figure 4(d) reveals the marked departure of the  $C_R$  parameter from that of SM for  $e$ - $d$  as well

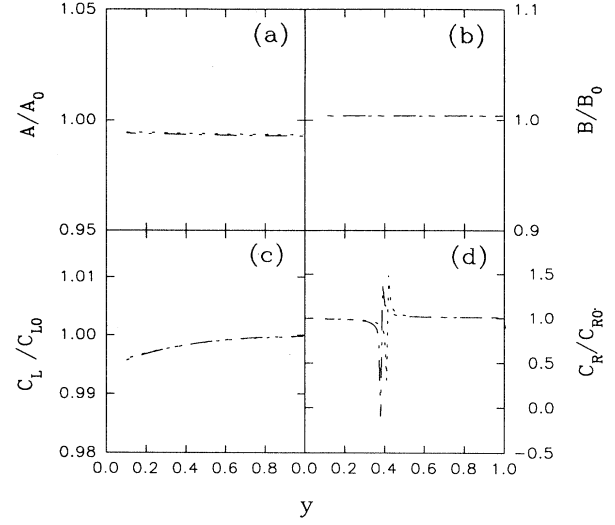


FIG. 4. The ratios (a)  $A/A_0$ , (b)  $B/B_0$ , (c)  $C_L/C_{L0}$ , and (d)  $C_R/C_{R0}$  are plotted against  $y$  for  $e$ - $d$  scattering (dotted lines) and  $e$ - $p$  scattering (dash-dot lines) for the ALRM.

as  $e$ - $p$  scattering processes. The variation of the ratio  $C_R/C_{R0}$  is more sensitive in the region  $0.38 \leq y \leq 0.42$ . In  $e$ - $d$  and  $e$ - $p$  processes the ratio decreases from 0.96 at  $y=0.33$  to 0.31 ( $-0.09$ ) at  $y=0.41$  ( $0.38$ ) and then reaches the maximum  $\sim 1.48$  ( $1.38$ ) at  $y=0.42$  ( $0.39$ ).

## B. $SU(5)_c$ color model

We have computed the various asymmetry parameters as a function of  $y$  in this model and compared with those in the SM (Fig. 5). Figure 5(a) shows the plot of the ratio

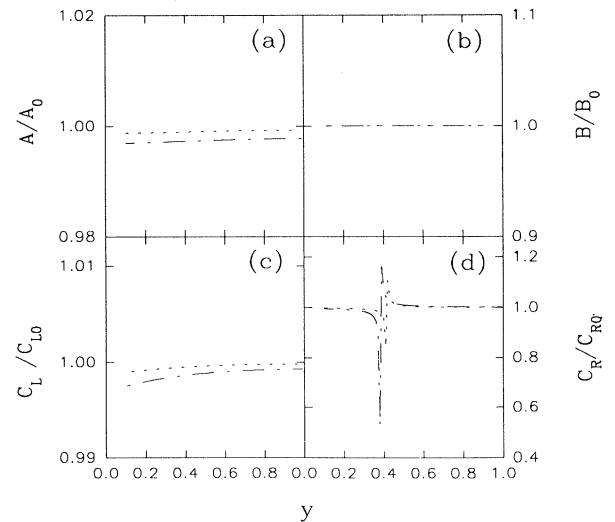


FIG. 5. The ratios (a)  $A/A_0$ , (b)  $B/B_0$ , (c)  $C_L/C_{L0}$ , and (d)  $C_R/C_{R0}$  are plotted against  $y$  for  $e$ - $d$  scattering (dotted lines) and  $e$ - $p$  scattering (dash-dot lines) for the extended  $SU(5)_c$  color model.

$A/A_0$  which is almost independent of  $y$  and has the value  $\sim 0.997$  (0.995) for  $e-d$  ( $e-p$ ) scattering processes for the values of  $y$  in the range  $0.1 \leq y \leq 1.0$ . Figure 5(b) unambiguously demonstrates that the ratio of  $B/B_0$  is independent of  $y$  and almost close to unity in both the processes. The ratio of  $C_L/C_{L0}$  does not depend upon  $y$  as shown in Fig. 5(c) and has a constant value  $\sim 0.998$  in both the processes. The variation of the ratio  $C_R/C_{R0}$  as a function of  $y$  [Fig. 5(d)] is quite spectacular in the range  $0.38 \leq y \leq 0.42$ . The value decreases from  $\sim 0.991$  (0.994) at  $y=0.3$  (0.2) to  $\sim 0.748$  (0.238) at  $y=0.41$  (0.38), then sharply increases to  $\sim 1.174$  (1.264) at  $y=0.42$  (0.39) for  $e-d$  ( $e-p$ ) scattering processes.

## V. CONCLUSION

We have calculated asymmetry parameters  $A$ ,  $B$  and  $C_{L,R}$  in the deep inelastic  $e-d$  and  $e-p$  scattering processes in the context of five effective low-energy models ( $\chi$ ,  $\psi$ ,  $\eta$  and ALRM) emerging from the breaking of the superstring inspired  $E_6$  model and the extended  $SU(5)_c$  color model and compared those with the SM. Our analysis shows that although the  $\chi$  and  $\eta$  models can be discriminated at the level of  $5\sigma$  or greater from the SM through the measurement of the  $A$  parameter for the values of  $y$  in the range  $0.6 \leq y \leq 1.0$  ( $0.4 \leq y \leq 1.0$ ), assuming 2% accuracy in measured values of the parameters, the  $\psi$  model cannot be distinguished from the SM for all values of  $y$  ( $0.1 \leq y \leq 1.0$ ) in both the processes. It is possible to discriminate the  $\chi$  and  $\eta$  models from the SM for all values of  $y$  in the range mentioned above from the mea-

surement of  $B$  parameter in both  $e-d$  and  $e-p$  scattering processes. Furthermore, the  $\psi$  and  $\eta$  models cannot be discriminated from the SM by measuring  $C_L$  although the  $\chi$  model can be discriminated from the SM through the measurement of  $C_L$  for the values of  $y$  in the range  $0.1 \leq y \leq 0.15$  in both the scattering processes. It is seen that the ALRM is not distinguishable from the SM through the measurement of any one of the parameters  $A$ ,  $B$ ,  $C_L$  in the  $e-d$  and  $e-p$  scattering processes. Interestingly, all the models ( $\chi$ ,  $\psi$ ,  $\eta$  and ALRM) can be unambiguously distinguished from SM through the measurement of the parameter  $C_R$  at low energy in both  $e-d$  and  $e-p$  scattering processes. The extended  $SU(5)_c$  color model cannot be discriminated from the SM from measurements of the parameters  $A$ ,  $B$ , and  $C_L$  for all values of  $y$ , and for both the processes  $e-d$  and  $e-p$ . However, the measurement of  $C_R$  can discriminate this model from the SM for both the processes and for values of  $y$  in the range  $0.38 \leq y \leq 0.42$ .

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