Dynamical mass generation in (2+1)-dimensional QED with a Chern-Simons term

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We analyze the effect of a Chern-Simons term in the dynamical generation of masses for fermions in (2+1)-dimensional QED in the large-flavor (N) limit. When a Chern-Simons term is present, both the magnitude and the critical flavor number for the parity-even (four-component) mass are reduced.

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One of the outstanding problems in gauge theories is to understand nonperturbative phenomena such as dynamical symmetry breaking or confinement. (2+1)-dimensional QED has served as a field-theoretical model to study such nonperturbative phenomena [1,2]. Recently, three-dimensional QED (QED_3) has been extensively studied in another respect. Namely, it might help to understand some interesting phenomena in planar condensed-matter physics such as high-temperature superconductivity, where a Chern-Simons term plays an important role [3]. In field-theoretic contexts, the Chern-Simons term is also responsible for a topological mass for gauge fields in 2+1 dimensions [4,5] in addition to changing the statistics of elementary excitations [6].

Being one of a few systematic field-theoretical methods, the $1/N$ expansion has been a useful tool to study nonperturbative phenomena that cannot be seen in the usual weak-coupling expansion [7]. This $1/N$ method has been employed to study dynamical symmetry breaking in QED_3 without the Chern-Simons term [1,2]. The spontaneous breakdown of the chiral symmetry of the four-component fermions has been shown to occur when N is less than a critical value N_c [1,8], which was confirmed by a numerical analysis [9]. The higher-order corrections were shown to preserve the nature of this symmetry breaking [10]. Also, the fermion mass has been shown to be generated dynamically such that parity is preserved [2].

In this paper, we analyze the effect of the Chern-Simons term in dynamical mass generation for fermions and spontaneous parity breaking in QED_3 by solving the Dyson-Schwinger equation in the large-flavor (N) limit. When the Chern-Simons term is present, the parity is always spontaneously broken. The fermion condensate $\langle \bar{\psi}\psi(x) \rangle$ is, therefore, nonzero. The dynamical fermion condensate, which may be generated even though the Chern-Simons term is not present, can also be affected by the Chern-Simons term in a nonperturbative way. We attempt to analyze this effect in our paper.

In general, when massive fermions are present, the Chern-Simons term is generated by a one-loop vacuum polarization [11,4]. The effect of the Chern-Simons term on fermion mass was recently studied in the weak-coupling expansion [12]. When the Chern-Simons term is present in QED_3, the photon will get a (topological) mass [4] and it will induce a (parity-violating) mass term to the fermions coupled to the photon, since the Chern-Simons term violates the parity invariance that guarantees the masslessness of the two-component fermions. In addition to these perturbative effects, as we shall see later, the Chern-Simons term affects a parity-preserving part of the dynamical fermion mass in a nonperturbative way, and it also modifies the critical flavor number for the generation of the parity-preserving mass, found by Appelquist *et al.*

The Lagrangian density that describes QED_3 with a Chern-Simons term is given as

$$\mathcal{L}=\sum_{i=1}^{N}\bar{\psi}_i(i\partial\!\!\!/-eA\!\!\!/)\psi_i-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\kappa\epsilon_{\mu\nu\lambda}A^{\mu}\partial^{\nu}A^{\lambda}\ , \tag{1}$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and ψ is a two-component spinor. The flavor number N is taken to be even. The three 2×2 Dirac matrices were chosen as $\gamma^0=\sigma_3$, $\gamma^1=i\sigma_1$, and $\gamma^2=i\sigma_2$, where σ_i are the Pauli matrices. Requiring the kinetic terms to be parity invariant, we see that mass terms $m\bar{\psi}\psi$ and $\kappa\epsilon_{\mu\nu\lambda}A^{\mu}\partial^{\nu}A^{\lambda}$ are parity odd. But, when the number of fermions is even, we may extend the definition of parity by combining it with the Z_2 symmetry which exchanges a pair of fermions: $(\psi_1,\psi_2)\to(\psi_2,\psi_1)$. This is just the parity symmetry for the four-component fermions in 2+1 dimensions. With this definition of parity, fermions can have a parity-even mass term $m\bar{\psi}_1\psi_1-m\bar{\psi}_2\psi_2$. To have a well-defined field theory in the large-N limit, we keep $\alpha=e^2N$ and κ to be finite when N goes to infinity.

First, we will examine the pattern of the spontaneous breaking of parity, and then elaborate on the dynamical mass generation for fermions. An order parameter for the spontaneous breaking of parity is the vacuum condensate for the fermion bilinear, $\langle \bar{\psi}\psi(x) \rangle$, which can be determined once one finds the asymptotic behavior of the fermion propagator [13]. At the leading order in the $1/N$ expansion, the Dyson-Schwinger gap equation is, in Eu-

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clidean notation,

$$-[Z(p)-1]\not{p}+\Sigma(p)=\frac{\alpha}{N}\int\frac{d^3k}{(2\pi)^3}D_{\mu\nu}(p-k)\gamma_\nu\frac{Z(k)\not{k}-\Sigma(k)}{Z^2(k)k^2+\Sigma^2(k)}\gamma_\mu\ , \tag{2}$$

where $D_{\mu\nu}^{-1}=\Delta_{\mu\nu}^{-1}-\Pi_{\mu\nu}$ and $\Delta_{\mu\nu}$ is the free Landau gauge propagator. Σ is the dynamically generated mass matrix for fermions. The vacuum polarization tensor is given as

$$\Pi_{\mu\nu}=\alpha\int\frac{d^3k}{(2\pi)^3}\mathrm{tr}\left[\gamma_\nu\frac{\not{k}-\Sigma(k)}{k^2+\Sigma^2(k)}\times\gamma_\mu\frac{\not{p}+\not{k}-\Sigma(p+k)}{(p+k)^2+\Sigma^2(p+k)}\right] \tag{3}$$

$$=\Pi_{\rm even}(p)\left[\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right]+\Pi_{\rm odd}(p)\epsilon_{\mu\nu\lambda}p_\lambda\ . \tag{4}$$

The detailed form of $\Pi_{\rm even}(p)$ and $\Pi_{\rm odd}(p)$ will be determined once we solve the above coupled Dyson-Schwinger equation, Eqs. (2) and (3). Since the wave-function renormalization constant is $Z(p)=1+O(1/N)$, we may take $Z(p)=1$ consistently in Eq. (2). From Eq. (2), taking trace over the γ matrix, we get

$$\Sigma(p)=\frac{\alpha}{N}\int\frac{d^3k}{(2\pi)^3}\frac{2\Pi_1(p-k)}{(p-k)^2}\frac{\Sigma(k)}{k^2+\Sigma^2(k)}+\frac{\alpha}{N}\int\frac{d^3k}{(2\pi)^3}\frac{(p-k)\cdot k}{|p-k|^3}\frac{\Pi_2(p-k)}{k^2+\Sigma^2(k)}\ , \tag{5}$$

where Π_1 and Π_2 contain the quantum corrections to the parity-even part and the parity-odd part of the photon propagator:

$$D_{\mu\nu}(p)=\frac{\delta_{\mu\nu}-p_\mu p_\nu/p^2}{p^2}\Pi_1(p)+\frac{\epsilon_{\mu\nu\lambda}p^\lambda}{|p|^3}\Pi_2(p)\ , \tag{6}$$

where

$$\Pi_1(p)=\frac{1-\Pi_{\rm even}(p)/p^2}{[\Pi_{\rm even}(p)/p^2]^2+[\kappa-\Pi_{\rm odd}(p)]^2/p^2}\ , \tag{7}$$

$$\Pi_2(p)=-\frac{[\kappa-\Pi_{\rm odd}(p)]/|p|}{[\Pi_{\rm even}(p)/p^2]^2+[\kappa-\Pi_{\rm odd}(p)]^2/p^2}\ . \tag{8}$$

In the large-N approximation the dynamically generated mass will be at most of the order of $1/N$, compared to the scale of the theory, Λ. (Λ is of same order as α or κ.) In fact, when there is no Chern-Simons term, the magnitude of the dynamically generated mass is exponentially small, compared to the scale Λ of the theory: $m\simeq\Lambda e^{-4N/N_c}$, where $N_c=64/\pi^2$ [2]. So, we may assume $\Sigma(p)\ll p$. The vacuum polarization tensor then takes a simple form:

$$\Pi_{\rm even}(p)=-\frac{\alpha}{16}|p|\ , \tag{9}$$

$$\Pi_{\rm odd}(p)=\frac{1}{N}\sum_{i=1}^{N}M_i\frac{\alpha}{4|p|}\ , \tag{10}$$

where $M_i\simeq\Sigma_i(0)$, the mass of the ith fermion. [In general $\Sigma(p)$ will depend on the flavor but we will suppress the index i for simplicity.] Now, the gap equation becomes

$$\Sigma(p)=\frac{\alpha}{N}\int_0^\infty\frac{k^2}{4\pi^2}dk\frac{\Sigma(k)}{k^2+\Sigma^2(k)}I_1(p,k)+\frac{\alpha}{N}\int_0^\infty\frac{k^2}{4\pi^2}dk\frac{1}{k^2+\Sigma^2(k)}I_2(p,k) \tag{11}$$

with

$$I_1(p,k)=\int_0^{2\pi}\sin\theta\,d\theta\frac{2}{(p-k)^2}\frac{(p-k)^2+(\alpha/16)|p-k|}{(|p-k|+\alpha/16)^2+\kappa^2}\ , \tag{12}$$

$$I_2(p,k)=\int_0^{2\pi}\sin\theta\,d\theta\frac{(p-k)\cdot k}{|p-k|^3}\frac{-\kappa|p-k|}{(|p-k|+\alpha/16)^2+\kappa^2}\ . \tag{13}$$

In the deep ultraviolet region, $p\gg\alpha$ (or κ), the second term in Eq. (11) is negligible. This is consistent with the fact that at high energy the Maxwell term is dominant, compared to the Chern-Simons term. One thing we should note here is that the integration on the right-hand side in Eq. (11) is convergent and we can expand it in powers of p. We find, neglecting the second term, in the deep ultraviolet region, $p\gg\alpha,\kappa$:

$$\Sigma(p)\simeq\frac{\alpha}{4\pi^2Np}\int_0^\infty dk\frac{k\Sigma(k)}{k^2+\Sigma^2(k)}\times\ln\left[\frac{\kappa^2+(\alpha/16+p+k)^2}{\kappa^2+(\alpha/16+|p-k|)^2}\right]\ . \tag{14}$$

To study Eq. (14) analytically we break the momentum integration into two regions and expand the logarithm appropriately for each region as was done by Appelquist *et al.* [1]:

$$\Sigma(p)=\frac{\alpha}{4\pi^2Np}\int_0^p dk\frac{k\Sigma(k)}{k^2+\Sigma^2(k)}\left[\frac{4k(\alpha/16+p)}{\kappa^2+(\alpha/16+p)^2}+O\left[\left[\frac{4k(\alpha/16+p)}{\kappa^2+(\alpha/16+p^2)}\right]^3\right]+\cdots\right]+\frac{\alpha}{4\pi^2Np}\int_p^\infty dk\frac{k\Sigma(k)}{k^2+\Sigma^2(k)}\left[\frac{4p(\alpha/16+k)}{\kappa^2+(\alpha/16+k)^2}+O\left[\left[\frac{4p(\alpha/16+k)}{\kappa^2+(\alpha/16+k)^2}\right]^3\right]+\cdots\right]\ . \tag{15}$$

For high momentum, retaining only the first term in the perturbative expansion of the logarithm, we may convert the integral equation into a second-order differential equation:

$$\frac{d}{dp}\left[\frac{d\Sigma(p)}{dp}\cdot\frac{p^2(\kappa^2+(\alpha/16+p)^2)^2}{2p^3+(5/16)\alpha p^2+(1/64)\alpha^2p+(\alpha/16)(\kappa^2+(\alpha/16)^2)}\right]=-\frac{\alpha}{\pi^2N}\frac{p^2\Sigma(p)}{p^2+\Sigma^2(p)}\ . \tag{16}$$

When $p \gg \alpha,\kappa$, Eq. (16) becomes a linear differential equation:

$$\Sigma''(p)+3p\Sigma'(p)=0 , \tag{17}$$

where a prime denotes differentiation with respect to p. The asymptotic solutions are either

$$\Sigma_s(p)=\frac{C_1}{p^2}+O\left[\frac{1}{p^3}\right]$$

or (18)

$$\Sigma_e(p)=C_2+O\left[\frac{1}{p}\right] .$$

To find the meaning of the solutions, we compare them with the operator product expansion for the fermion propagator in momentum space:

$$\langle \bar{\psi}\psi(p)\rangle=\frac{A(p/\mu)}{\not p}+\frac{C(p/\Lambda)\overline{m}(\mu)\langle 1\rangle}{p^2}+\frac{D(p/\mu)\langle \bar{\psi}\psi(x)\rangle}{p^3}+\cdots , \tag{19}$$

where A, C, and D are the coefficient functions.[1] The direct substitution of the self-energy in the propagator gives

$$\langle \bar{\psi}\psi(p)\rangle=\frac{1}{\not p-\Sigma(p)}=\frac{1}{\not p}+\frac{\Sigma(p)}{p^2}+\cdots . \tag{20}$$

As we can see easily, the constant C_2 in the second solution in Eq. (18) corresponds to a renormalized mass $\overline{m}(\mu)$ of the fermions. But this second solution $\Sigma_e(p)$, corresponding to explicit parity breaking, is not compatible with the homogeneous integral equation (14). It would be a solution to the integral equation if we had included a mass term in the Lagrangian. By comparing two equations, (19) and (20), we see that the first solution in Eq. (18) corresponds to spontaneous parity breaking and C_1 measures the magnitude of the vacuum condensate $\langle \bar{\psi}\psi\rangle$. This solution is compatible with the gap equation, where no mass term is included. Thus, we have shown that in the limit of $\overline{m}(\mu)=0$ the parity is also spontaneously broken in QED_3 with a Chern-Simons term. The value C_1 is a parameter of the theory in a sense that it is given as the boundary condition of the differential equation Eq. (18). But it should be chosen to give the minimum of the effective potential, which is, in the large-N limit,

$$V(\Sigma)=\frac{N}{2\pi^2}\int_0^\infty p^2dp\left[\frac{\Sigma^2}{p^2+\Sigma^2}-\ln\left[1+\frac{\Sigma^2}{p^2}\right]\right] . \tag{21}$$

The Chern-Simons term does not contribute to the vacuum energy, as it should, since it is a topological term. It contributes to the vacuum energy only indirectly by modifying the solution to the gap equation. It can be easily seen from Eq. (21) that any nonzero C_1 solution has a lower energy than the perturbative vacuum solution, $\Sigma(p)=0$. Therefore, the parity is always spontaneously broken, whether the Chern-Simons term is present or not, once such a parity-breaking solution is found.

To find the physical mass of the fermions, we have to solve the equation

$$p^2+\Sigma^2(p)=0 \quad \text{at } p^2=-m_{\rm phy}^2 . \tag{22}$$

But since $\Sigma(p)$ is quite small, compared to the scale of the theory, α or κ, we may approximate $m_{\rm phy}\simeq\Sigma(0)$. From the gap equation (5), the fermion self-energy at zero momentum is given by

$$\Sigma_i(0)=\frac{\alpha}{N}\int\frac{d^3k}{(2\pi)^3}\left[\frac{2\Pi_1(k)M_i}{k^2(k^2+M_i^2)}-\frac{\Pi_2(k)}{|k|(k^2+M_i^2)}\right] , \tag{23}$$

where the flavor index i is restored and the fermion self-energy in the integrand is taken to be $\Sigma_i(k)\simeq M_i$. The first term on the right-hand side of Eq. (23) makes a negative or positive contribution to $\Sigma_i(0)$, depending on the sign of M_i, while the sign of the second term depends only on that of the coefficient of the Chern-Simons term κ. We see that the parity is maximally broken when the second term is dominant, which happens precisely when the Chern-Simons term is dominant, i.e., $\kappa\gg\alpha$. The mass is found to be, in this limit,

$$M_i\simeq\frac{\alpha}{2\pi^2N}\frac{\Lambda}{\kappa} , \tag{24}$$

where Λ is the UV cutoff, which will be of order of κ. On the other hand, when the Chern-Simons term is not present, the mass will be generated in a parity-invariant way. Namely, half the fermions get positive mass m and the other half negative mass $-m$. Therefore, when both of the Chern-Simons term and the Maxwell term are present, we assume

$$M_i=M+m_i \tag{25}$$

with

$$m_i=\begin{cases} m, & i=1,\ldots,N-L,\\ -m, & i=N-L+1,\ldots,N.\end{cases}$$

We then find from Eq. (23), taking $\Sigma_i(0)\simeq M_i$, that

$$M_i=\frac{\alpha}{2\pi^2N}\int_m^\Lambda dk\frac{2M_i}{k^2+M^2}\frac{k^2[1+(\alpha/16)(1/|k|)]}{(\alpha/16)^2+\kappa^2}+\frac{\alpha}{2\pi^2N}\int_m^\Lambda dk\frac{[\kappa-(M+\theta m)(\alpha/4|k|)]k^2}{(k^2+M^2)[(\alpha/16)^2+\kappa^2]} , \tag{26}$$

where, in the denominator, we approximate $M_i\simeq M$ and $\kappa-(\alpha/4|k|)(M+\theta m)\simeq\kappa$, since the main contribution to the integration comes from the high momentum. This approximation is consistent in the leading order in the $1/N$ expansion. $\theta(=1-2L/N)$ measures the strength of the parity violation when $\kappa=0$. In (26), the flavor-independent part and the flavor-dependent part of the dynamical mass should satisfy separate equations, since,

[1] In general, $\bar{\psi}\psi(x)$ can get mixed with an operator $\epsilon_{\mu\nu\lambda}A^\mu\partial^\nu A^\lambda$ in renormalization. But we ignore this possibility.

in the leading order, they are independent to each other. We find, in the leading order,

$$M=\frac{\alpha}{2\pi^2 N}\int_m^\Lambda dk \frac{k^2}{k^2+M^2}\frac{\kappa}{(\alpha/16)^2+\kappa^2}\ , \tag{27}$$

$$m_i=\frac{1}{\pi^2 N}\frac{(\alpha/16)^2}{(\alpha/16)^2+\kappa^2}\int_m^\Lambda dk\left[\frac{16m_i}{k}-\frac{64}{k}\theta m\right]\ . \tag{28}$$

For Eq. (27) to have a consistent solution, $\theta=0+O(1/N)$. In this case, we see, reshuffling the flavor indices i, the fermion masses are

$$M_i\simeq\frac{\Lambda}{2\pi^2 N}\frac{\alpha\kappa}{(\alpha/16)^2+\kappa^2}+(-1)^i\Lambda\exp(-4N/\tilde{N}_c)\ , \tag{29}$$

where $\tilde{N}_c=N_c/[1+(16\kappa/\alpha)^2]$. The value for the parity-violating mass M is a perturbative one in the $1/N$ expansion, while the parity-preserving mass is nonperturbative. The effect of the Chern-Simons term is now clear. It induces a parity-violating mass perturbatively and it decreases in a nonperturbative way the magnitude of the parity-preserving mass m:

$$\frac{m(\kappa\neq 0)}{m(\kappa=0)}=\exp\left[-\frac{4N}{N_c}\frac{\kappa^2}{(\alpha/16)^2}\right]\ . \tag{30}$$

Since $\theta=0$, half the fermions get (positive) mass $M+m$ and the other half $M-m$. The pattern of the flavor-symmetry breaking is same whether or not the Chern-Simons term is present.

Finally, we mention briefly what happens when $N<\tilde{N}_c$. Since the parity-breaking mass M is perturbative in $1/N$, it will occur for any large value of N as long as the Chern-Simons term is present. But, since the parity-preserving (four-component) mass m is nonperturbative, it is not clear at all whether it will be generated for any large value of N. When there is no Chern-Simons term in QED_3, the mass is generated for $N<N_c$ [1]. To find what will happen to the parity-preserving mass in QED_3 with a Chern-Simons term, let us go back to the gap equation (14). Neglecting $\Sigma(k)$ in the denominator in the integrand of the right-hand side, we get an approximate linear integral equation. Then, the parity-even piece of $\Sigma(p)$ will satisfy a separate equation, which is just Eq. (14), since the second term in the gap equation (11) does not contribute to the parity-even piece $\Sigma_e(p)$. For $\Sigma(p)\ll p\ll\alpha,\kappa$, the parity-even piece satisfies

$$\frac{d}{dp}\left[p^2\frac{d\Sigma_e(p)}{dp}\right]=-\frac{16}{\pi^2 N}\frac{(\alpha/16)^2}{(\alpha/16)^2+\kappa^2}\Sigma_e(p)\ . \tag{31}$$

Equation (31) has solutions of the form

$$\Sigma_e(p)=Ap^a\ ,$$

where

$$a=-\frac{1}{2}\pm\frac{1}{2}\left[1-\frac{\tilde{N}_c}{N}\right]^{1/2}\ , \tag{32}$$

where $\tilde{N}_c=N_c[1+(16\kappa/\alpha)^2]$. For $N<\tilde{N}_c$, $\Sigma_e(p)$ oscillates like $\sin[\gamma\ln(p/\Lambda)]$, where $\gamma=\sqrt{\tilde{N}_c/N-1}$. This is the same pathological behavior found for the strong QED_4, where $\Sigma(p)$ oscillates when the coupling is larger than the critical coupling $\alpha_c(\simeq\pi/3)$ for chiral-symmetry breaking [14,15]. As was argued in Ref. [8], for $N<\tilde{N}_c$, chiral symmetry (of a four-component fermion) will be broken spontaneously and the parity-even mass will be generated dynamically. When there is no Chern-Simons term, the critical number was $N_c\simeq 64/\pi^2\simeq 6.4$. But with the Chern-Simons term the critical number decreases; $N_c\to\tilde{N}_c\simeq N_c/[1+(16\kappa/\alpha)^2]$.

In conclusion, we find that, when the Chern-Simons term is present, the parity tends to be maximally broken; the magnitude of the parity-even (four-component) mass for the fermions gets smaller, and the critical number for the generation of the parity-even mass decreases. But, in the large-N limit, the pattern of the flavor-symmetry breaking does not depend on the Chern-Simons term.

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