Canonical covariant quantization of the Brink-Schwarz superparticle

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A method for the covariant canonical quanitzation of a wide class of gauge theories with reducible first- and second-class constraints which includes the Brink-Schwarz superparticle is obtained. The application to the Brink-Schwarz superparticle is discussed in detail. The canonical Batalin-Fradkin-Vilkovisky construction of the correct Becchi-Rouet-Stora-Tyutin (BRST) operator and the BRST-invariant effective action is presented and shown to agree with the expressions obtained by Kallosh and Bergshoeff using other methods. The correct gauge-fixing procedure to recover the Brink-Schwarz superparticle action is discussed.

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Superstring theories are the best candidates to give a unified description for all fundamental interactions. The formulation of superstrings in the manner of Green and Schwarz [1], manifestly supersymmetric in the target manifold, presents several advantages over the perturbative Neveu-Schwarz-Ramond (NSR) formulation [2]. In particular a nonperturbative second-quantized light-cone gauge formulation was obtained in [1]. Furthermore, closure of the super-Poincaré algebra [3] and multiloop analysis of the S matrix was also performed in [4]. Nevertheless a second-quantized covariant Green-Schwarz (GS) formulation is still lacking. The first step in the construction is to perform covariantly the first quantization of the GS superstring, a problem which has not been solved. The main difficulty has been the covariant gauge fixing of the local κ supersymmetry [5]. In fact, the first-class constraints associated with the gauge symmetries appear mixed with second-class ones, and so far no local, Lorentz-covariant, and finite reducible [6,7] approach to disentangle them has been found.

The zero-mode structure of the Green-Schwarz superstring (GSS) is described by the Brink-Schwarz superparticle (BSS) [8]. Moreover the canonical structure of both theories presents a very close correspondence. In particular first- and second-class constraints in the BSS also appear mixed and similar to the GSS problems with the infinite reducibility of the covariant generators of gauge symmetries are present.

For these reasons all the approaches to the covariant quantization of superstrings have been first tested with the BSS. The physical spectrum of the BSS is easily obtained in the light-cone gauge $x^+=-p_-\tau$, $\gamma^+\theta=0$. It corresponds to an $N=1$ Yang-Mills supermultiplet.

In order to circumvent the problem presented by the mixing, in a covariant treatment, of first- and second-class constraints, other actions for the description of the superparticle, following original ideas of Siegel, have been proposed. The so-called Siegel superparticle (SS) [9] does not have the same number of degrees of freedom as the BSS. It corresponds to ignore the second-class constraints of the BSS formulation, leaving only the first-class constraints which may be covariantly projected from the original BSS set of constraints. The fact that the SS has a number of degrees of freedom different from the BSS is a consequence of nontrivial restrictions imposed by the second-class constraints. The modified Siegel superparticles (MSSI [10] and MSSII [11]) have the same physical spectrum as the BSS, allowing a formulation in terms of first-class constraints only. The formulations however are given in terms of irregular constraints, distinct from the original BSS formulation. The presence of irregular constraints [12] does not allow a straightforward application of the Batalin-Fradkin-Vilkovisky (BFV) [6] method or of the Batalin-Vilkovisky [7] approach. As a consequence the off-shell nilpotent Becchi-Rouet-Stora-Tyutin (BRST) charge for these models has not been constructed.

A direct application of the Batalin-Vilkovisky formalism to the BSS was also pursued in Refs. [13] and [14]. Although an adequate gauge-fixing procedure has been presented such that the final action is BRST invariant the cohomology of the associated BRST operator is not adequate for the BSS. The reason for this may be traced to the fact that the resultant system presents a twisted $N=2$ supersymmetry while the BSS has only $N=1$ supersymmetry.

A covariant BRST operator in terms of the same infinite set of fields introduced in Refs. [13] and [14] but with the correct cohomology for the BSS was finally obtained in Ref. [15] using a very ingeneous construction which, unfortunately, has no obvious generalization for the GSS. Later on in Ref. [16] Kallosh presented a new action from which the correct BRST operator for the BSS may be deduced using the standard BFV procedure. Nevertheless neither a demonstration of the canonical equivalence of this new action and the original Brink-Schwarz one, nor a direct construction of the correct BRST operator starting with the latter, has been presented until now.

In this paper we present a canonical approach which leads to the covariant quantization of the BSS and resolves the above-mentioned problems. The formulation generalizes further a canonical approach for dynamical systems restricted by reducible first- and second-class constraints presented in Ref. [17]. In this approach, which is closely related to the work in [18], the phase space is extended to a larger manifold where all the relevant extended constraints are first class. By an appropriate gauge fixing one may reduce the functional integral to a functional integral on the original constrained manifold, with the correct functional measure. It is an off-shell approach allowing the systematic construction of the off-shell nilpotent BRST charge and of the BRST-invariant effective action. Additionally in the enlarged phase space where all constraints are first class the operatorial quantization construction of Batalin and Fradkin [6] may be directly implemented. The generalization of this approach presented in this paper may be applied to the BSS and in principle also to the GSS. For the BSS as we show below it introduces naturally the correct infinite set of fields and leads directly to the BRST charge previously found in Ref. [15]. It also gives a systematic method for the construction of the superparticle action, proposed in Ref. [16]. A detailed and more general discussion of the approach we use in this paper will be presented in [19].

Another approach which also uses ideas close to the ones in Ref. [18] to handle the second-class constraints in the BSS may be found in Ref. [20]. There, representations of the BSS system are constructed using twistorial variables and harmonic superfields. This approach is an interesting alternative for the quantization of the BSS. Unfortunately this treatment has not been yet applied to the GSS in a completely satisfactory way. For other treatments of the BSS using twistorial variables see Ref. [21].

The first-order action for the 10-dimensional BS superparticle is

$$S=\langle P_\mu \partial_\tau \chi^\mu + \bar{\xi} \not{P} \partial_\tau \xi + eP^2 \rangle \ , \tag{1}$$

where e is a Lagrange multiplier associated with the constraint

$$P^2=0 \ . \tag{2}$$

Let η be the momenta canonically conjugate to ξ. Since the action (1) is first order in $\partial_\tau \xi$ its dynamics is restricted by

$$\phi=\eta-\not{P}\xi=0 \ . \tag{3}$$

The canonical Hamiltonian action of the system is

$$S=\langle P_\mu \partial_\tau \chi^\mu + \bar{\eta}\partial_\tau \xi + eP^2 + \bar{\psi}(\eta - \not{P}\xi) \rangle \ , \tag{4}$$

where $\bar{\psi}$ are Lagrange multipliers associated with the constraints (3).

Constraints (3) are a combination of first- and second-class ones. This is best observed computing the Poisson algebra of the constraints which yields

$$\{\phi,\phi\}=2\not{P} \ . \tag{5}$$

Over the manifold defined by (2), $\not{P}$ is noninvertible and in fact conservation of (3) fixes only half of the multipliers $\bar{\psi}$. The constraints associated with the other half are first class.

One can covariantly project from (3) the first-class constraints by application of $\not{P}$:

$$\varphi \equiv \not{P}\eta = 0 \ , \tag{6}$$

$$\{\varphi,\varphi\}=0=\{\varphi,\phi\} \ . \tag{7}$$

The price one has to pay is that (6) is a set of infinite reducible constraints since

$$\not{P}\varphi \equiv 0 \ . \tag{8}$$

Over the manifold defined in (2) and (6) constraints (3) become reducible since

$$\not{P}\phi=0 \tag{9}$$

holds identically. The manifold defined by (2) and (3) may thus be equivalently described by the first-class infinite reducible constraint (2), (6), and the reducible second-class constraints (3). In order to write the effective action of the system one has to include the ghost fields adequate for the first-class reducible constraints φ [6] and devise a method to handle the second-class reducible constraints (3).

This decomposition of the constraints into first class and reducible second class ones, over the manifold of first-class constraints, is best understood by introducing a transverse + longitudinal $(T+L)$ decomposition of the geometrical objects. Although $\not{P}$ is not an invertible matrix it serves to define such decomposition of spinors. Because of the identity

$$\not{P}\gamma^+ + \gamma^+ \not{P} = P_- 1 \ , \tag{10a}$$

for any spinors ξ one has the $(T+L)$ decomposition

$$\xi=\xi_T+\gamma^+\xi_L \tag{10b}$$

with

$$\not{P}\xi_T=0 \ . \tag{10c}$$

ξ_L is not uniquely defined but $\gamma^+\xi_L$ and ξ_T are uniquely determined.

Equation (9) imposes that the longitudinal part of ϕ, more precisely $\gamma^+\phi_L$, be identically zero over the manifold of first class constraints. The true content of the reducible constraints (3) is then only

$$\phi_T=0 \ . \tag{11}$$

Let us translate the above situation to a general notation. We have a constrained system with Hamiltonian H_0 subject to a set of reducible constraints ϕ_{a_1} $(a_1=1,\ldots,n)$ and a set of first-class constraints φ_i $(i=1,\ldots,k)$ which we omit in the explicit construction that follows. We limit ourselves to remark on the modifications to be done when included. So we have

$$\phi_{a_1}=0 \ , \tag{12}$$

$$a_{a_2}^{a_1}\phi_{a_1}=0 \ , \quad a_1=1,\ldots,n \ , \ a_2=1,\ldots,m \ . \tag{12a}$$

We will not suppose $a_{a_2}^{a_1}$ to be of maximal rank. Instead we will impose that a $(T+L)$ decomposition similar to (10) is allowed.

We have then, for any object V_{a_1},

$$V_{a_1}=V^T_{a_1}+A^{a_2}_{a_1}V^L_{a_2}\ ,$$
$$a^{a_1}_{a_2}V^T_{a_1}=0\ ,\quad V^L_{a_2}=a^{a_1}_{a_2}V_{a_1}\ , \tag{13a}$$

and, for any object W^{a_1},

$$W^{a_1}=W^{a_1}_T+a^{a_1}_{a_2}W^{a_2}_L\ ,$$
$$A^{a_2}_{a_1}W^{a_1}_T=0\ ,\quad W^{a_2}_L=A^{a_2}_{a_1}W_{a_1}\ . \tag{13b}$$

It follows that

$$V^L_{a_2}=a^{a_1}_{a_2}A^{b_2}_{a_1}V^L_{b_2}\ ,\quad W^{a_2}_L=A^{a_2}_{a_1}a^{a_1}_{b_2}W^{b_2}_L\ , \tag{13c}$$

and

$$W^{a_1}V_{a_1}=W^{a_1}_TV^T_{a_1}+W^{a_2}_LV^L_{a_2}\ . \tag{13d}$$

In the irreducible case ($a^{a_1}_{a_2}$ of a maximum rank) $A^{a_2}_{a_1}$ is the inverse of $a^{a_1}_{a_2}$. In the finite reducible case this decomposition may always be done in a unique way for a given pair A,a. For an infinite reducible system, we will assume that there exists such a decomposition. The constraints (12) are second class in the sense that they have an invertible Poisson brackets matrix in the transverse subspace.

Following Refs. [17] and [18] let us enlarge the phase space using a set of auxiliary variables ξ^{a_1} and η_{b_1} conjugate to each other. We also introduce the combinations

$$\Phi_{a_1}=\eta_{a_1}-\tfrac{1}{2}\omega_{a_1b_1}(p,q)\xi^{b_1}\ ,$$
$$\overline{\Phi}_{a_1}=\eta_{a_1}+\tfrac{1}{2}\omega_{a_1b_1}(p,q)\xi^{b_1}\ . \tag{14}$$

Here ω_{ab} is an antisymmetric matrix with vanishing Poisson brackets with itself to be fixed by the procedure. $\overline{\Phi}$ and Φ satisfy

$$\{\Phi_{a_1},\Phi_{b_1}\}=-\omega_{a_1b_1}\ ,$$
$$\{\overline{\Phi}_{a_1},\overline{\Phi}_{b_1}\}=\omega_{a_1b_1}\ , \tag{15}$$
$$\{\Phi_{a_1},\overline{\Phi}_{b_1}\}=0\ .$$

In order to introduce only the complications necessary to deal with the case of the BS superparticle we will suppose in the following that $\omega_{a_1b_1}$ is transverse,

$$a^{a_1}_{a_2}\omega_{a_1b_1}=0\ ,\quad a_1=1,\ldots,n\ ,\ a_2=1,\ldots,m\ , \tag{16}$$

and invertible in transverse space. More general cases will be discussed in Ref. [19].

We continue the application of our method [18] and extend the constraints in the enlarged space to

$$\tilde{\phi}_{a_1}=\phi_{a_1}+V^{c_1}_{a_1}\Phi_{c_1}=0\ , \tag{17}$$

where $V^{c_1}_{a_1}(q,p)$ is also to be fixed. In general the first-class constraints φ may also have to be extended in order that the complete set of extended constraints be first class. In the case of the superparticle, however, the extension is not necessary. We assume $V^{b_1}_{a_1}$ to be invertible. In this case we impose the constraints (17) to be irreducible, first class and with structure functions at most linear in Φ_{a_1}. We then have

$$\{\tilde{\phi}_{a_1},\tilde{\phi}_{b_1}\}=U^{c_1}_{a_1b_1}\tilde{\phi}_{c_1}=-2(u^{c_1}_{a_1b_1}+v^{c_1d_1}_{a_1b_1}\Phi_{d_1})\tilde{\phi}_{c_1}\ . \tag{18}$$

The structure functions $U^{c_1}_{a_1b_1}$ may depend on the phase-space variables p and q. Substitution of (17) in (18) yields

$$\{\phi_{a_1},\phi_{b_1}\}-V^{c_1}_{a_1}V^{d_1}_{b_1}\omega_{c_1d_1}+2u^{c_1}_{a_1b_1}\phi_{c_1}=0\ ,$$
$$\{\phi_{a_1},V^{c_1}_{b_1}\}+\{V^{c_1}_{a_1},\phi_{b_1}\}+2v^{d_1c_1}_{a_1b_1}\phi_{d_1}+2u^{d_1}_{a_1b_1}V^{c_1}_{d_1}=0\ , \tag{19}$$
$$\{V^{c_1}_{a_1},V^{d_1}_{b_1}\}+\{V^{d_1}_{a_1},V^{c_1}_{b_1}\}+2V^{c_1}_{e_1}v^{e_1d_1}_{a_1b_1}+2V^{d_1}_{e_1}v^{e_1c_1}_{a_1b_1}=0\ .$$

We suppose here that

$$\{\phi_{a_1},\Phi_{a_1}\}=0\ ,\quad \{V^{b_1}_{1a_1},\Phi_{c_1}\}=0\ . \tag{20}$$

Let us suppose that we are able to find a solution to (20) with all the required conditions. As first discussed for irreducible systems in Ref. [17], in order to demonstrate the equivalence of our system in the enlarged phase space to the original system we have to impose an additional restriction besides (17). A counting of the degrees of freedom suggests which ones should be chosen in this generalized situation. The original model has $2N$ phase space variables p,q restricted by $(n-m_L)$ transverse constraints with m_L the rank of $a^{a_2}_{a_1}$. The enlarged model has $2N$ variables p,q and $2n$ variables ξ, η restricted by n constraints $\tilde{\phi}_{a_1}$ and n gauge-fixing conditions $\tilde{\chi}_{a_1}$. To match we need $(n-m_L)$ additional constraints. We take them to be

$$\overline{\Phi}^T_{a_1}=0\ . \tag{21}$$

Since

$$[\overline{\Phi}^T_{a_1},\overline{\Phi}^T_{a_2}]=\omega^T_{a_1a_2} \tag{22}$$

the constraints (21) are in our hypothesis second class. The advantage of this formulation is that the field dependence in $\omega^T_{a_1a_2}$ (whose determinant will appear in functional measure) may be simpler than in $\{\phi_{a_1},\phi_{b_1}\}$ since $V^{a_2}_{a_1}$ may be also a field-dependent object. This justifies the enlarging of the phase space and the modification of the constraints. Moreover, iterating the process one can hope to obtain a field-independent functional measure and pure gauge model. For the BS superparticle, as we will show below, infinitely many iterations are needed to this end, but in other cases only finite steps may be necessary.

A gauge-invariant extension of the Hamiltonian H_0 may be written in the form [17]

$$\tilde{H}=H_0+h^{a_1}\Phi_{a_1}\ . \tag{23}$$

h^{a_1} is fixed imposing

$$\{\tilde{H},\tilde{\phi}_{a_1}\}=W^{b_1}_{a_1}\tilde{\phi}_{b_1} \; . \tag{24}$$

Introducing the ghost variables C^{a_1} and μ_{a_1} the BRST operator is obtained by solving [6]

$$\{\Omega,\Omega\}=0 \; , \tag{25a}$$

$$\left.\frac{\partial\Omega}{\partial C^{a_1}}\right|_{\mu=0}=\tilde{\phi}_{a_1} \; . \tag{25b}$$

It takes the form

$$\Omega=C^{a_1}\tilde{\phi}_a+C^{a_1}U^{c_1}_{a_1b_1}C^{b_1}\mu_{c_1}+\cdots \tag{25c}$$

with a nontrivial tail, when the algebra of first-class constraints has structure functions of a higher order. In the general case when first-class constraints φ are also present, one has to include, of course, associated ghost fields and condition (25b) must also be satisfied for the extended first-class constraints $\tilde{\varphi}$.

The extended Hamiltonian is then obtained by solving [6]

$$\{\hat{H},\Omega\}=0 \; , \quad \hat{H}|_{\mu=0}=\tilde{H} \; . \tag{26}$$

The BRST invariant effective action in a phase-space representation is given by [6,22]

$$S_{\text{eff}}=\langle p\dot{q}+\mu_{a_1}\dot{C}^{a_1}+\eta_{a_1}\dot{\xi}^{a_1}-\hat{H}+\hat{\delta}(\lambda^{a_1}\mu_{a_1})+\hat{\delta}(\bar{C}_{a_1}\chi^{a_1})\rangle, \tag{27}$$

where χ^{a_1} are the gauge-fixing conditions and $\hat{\delta}$ is defined by

$$\hat{\delta}F=[\Omega,F] \tag{28}$$

for any function F of the canonical variables of the enlarged super-phase-space. For the noncanonical sector we have

$$\hat{\delta}\lambda^{a_1}=\theta^{a_1} \; , \quad \hat{\delta}\theta^{a_1}=0 \; , \tag{29a}$$

$$\hat{\delta}\bar{C}_{a_1}=B_{a_1} \; , \quad \hat{\delta}B_{a_1}=0 \; , \tag{29b}$$

$$\delta\mu_{a_1}=\tilde{\phi}_{a_1} \; . \tag{29c}$$

We claim that the gauge invariant system defined by (27) and constrained by (21) is canonically equivalent to the original system. To prove this, we will show that with an adequate gauge-fixing condition one can reduce the path integral corresponding to the enlarged system to the Senjanovic-Fradkin expression for the original system.

The classical gauge transformation law for ξ is

$$\delta\xi^{a_1}=\{\xi^{a_1},\epsilon^{b_1}\tilde{\phi}_{b_1}\}=V^{a_1}_{b_1}\epsilon^{b_1} \; , \tag{30}$$

where ϵ^{b_1} are the infinitesimal parameters of the transformation.

We may then choose the gauge conditions

$$\chi^{a_1}=\xi^{a_1} \; . \tag{31}$$

Using (25), (28), and (29) we have

$$\begin{aligned}\hat{\delta}(\bar{C}_{a_1}\chi^{a_1})&=B_{a_1}\chi^{a_1}-\bar{C}_{a_1}\frac{\delta\chi^{a_1}}{\delta\epsilon^{b_1}}C^{b_1}+O(\mu)\\&=B_{a_1}\xi^{a_1}-\bar{C}_{a_1}V^{a_1}_{b_1}C^{b_1}+O(\mu) \; ,\end{aligned} \tag{32}$$

where $O(\mu)$ may appear if the structure functions depend explicitly on the phase-space coordinates.

We also have

$$\begin{aligned}\hat{\delta}(\lambda^{a_1}\mu_{a_1})&=\lambda^{a_1}\tilde{\phi}_{a_1}+\theta^{a_1}\mu_{a_1}\\&=\lambda^{a_1}_T\tilde{\phi}_{a_1}+\lambda^{a_2}_L a^{a_1}_{a_2}\tilde{\phi}_{a_1}+\theta^{a_1}\mu_{a_1}\\&=\lambda^{a_1}_T\tilde{\phi}^T_{a_1}+\lambda^{a_2}_L a^{a_1}_{a_2}V^{b_1}_{a_1}\Phi_{b_1}+\theta^{a_1}\mu_{a_1} \; .\end{aligned} \tag{33}$$

The functional integral is

$$I(\chi)=\int\mathcal{D}z\,\delta(\bar{\Phi}^T)(\det\omega^T)^{1/2}e^{-S_{\text{eff}}} \; , \tag{34}$$

where $\mathcal{D}z$ is the Liouville measure

$$\mathcal{D}z=\mathcal{D}p\,\mathcal{D}q\,\mathcal{D}C\,\mathcal{D}\bar{C}\,\mathcal{D}\mu\,\mathcal{D}B\,\mathcal{D}\theta\,\mathcal{D}\eta\,\mathcal{D}\xi \; . \tag{35}$$

Integrating in θ one gets $\delta(\mu)$ so that in particular $O(\mu)$ in (32) does not contribute. Integrating in B_{a_1}, $\bar{C}_{a_1}$, and $\lambda^{a_2}_L$ and using Eq. (19) the factor in the measure of (34) becomes

$$\begin{aligned}&(\det\omega^T)^{1/2}\delta(\bar{\Phi}^T)\delta(\xi)\delta(V^{Ta_1}_{Tb_1}C^{b_1}_T)\delta(a^{b_1}_{b_2}V^{a_1}_{b_1}A^{a_2}_{a_1}C^{b_2}_L)\delta(a^{a_1}_{a_2}V^{b_1}_{a_1}\Phi_{b_1})\\&\qquad=(\det\omega^T)^{1/2}\delta(\eta^T)\delta(\xi)\delta(V^{Ta_1}_{Tb_1}C^{b_1}_T)\delta(a^{b_1}_{b_2}V^{a_1}_{b_1}A^{a_2}_{a_1}C^{b_2}_L)\delta(a^{a_1}_{a_2}V^{b_1}_{a_1}A^{a_2}_{b_1}\eta^L_{a_2}) \; .\end{aligned} \tag{36}$$

In (36) the arguments of the last two factors have opposite statistics. Hence

$$\delta(a^{b_1}_{b_2}V^{a_1}_{b_1}A^{a_2}_{a_1}C^{b_2}_L)\delta(a^{a_1}_{a_2}V^{b_1}_{a_1}A^{a_2}_{b_1}\eta^L_{b_2})=\delta(C^{b_2}_L)\delta(\eta^L_{a_2}) \; . \tag{37}$$

This can be taken as valid even in the case of $a^{a_1}_{a_2}$ and $A^{b_1}_{b_2}$ being noninvertible. The factor in the measure reduces to

$$(\det\omega^T)^{1/2}\delta(\eta)\delta(\xi)\delta(C)\det V^T_T \; . \tag{38}$$

Now we note from (19) that

$$(\det\omega^T)^{1/2}\det V^T_T=(\det\{\phi_T,\phi_T\})^{1/2} \; . \tag{39}$$

Doing the trivial integrations in η, ξ, and C, we finally obtain

$$\begin{aligned}I=\int&\mathcal{D}q\,\mathcal{D}p\,\mathcal{D}\lambda^T(\det\{\phi_T,\phi_T\})^{1/2}\\&\times\exp(-\langle p\dot{q}-H+\lambda_T\phi^T\rangle)\end{aligned} \tag{40}$$

which is the correct Senjanovic-Fradkin expression of the functional integral of this system.

The discussion above only supposes the uniqueness of decomposition (13). For the superparticle we identify $a_{a_2}^{a_1}$ with $\boldsymbol{P}$ and $A_{a_1}^{a_2}$ with γ^+/P_-. The $T+L$ decomposition is given by (11). Solving (19) by taking V as a Dirac δ we have

$$\omega=-2\boldsymbol{P} \ . \tag{41}$$

In terms of the auxiliary Majorana spinors η_1 and ξ_1 we have

$$\Phi_1=\eta_1+\boldsymbol{P}\xi_1 \ , \tag{42a}$$

$$\overline{\Phi}_1=\eta_1-\boldsymbol{P}\xi_1 \ . \tag{42b}$$

The enlarged constraints (17) are, in this case,

$$\widetilde{\phi}_0=\eta-\boldsymbol{P}\xi+\Phi_1 \ . \tag{43}$$

The additional restrictions corresponding to (21) are

$$\overline{\Phi}_1^T=0 \ . \tag{44}$$

Since we choose $V_{a_1}^{b_1}$ to be field independent the factor $\det\{\overline{\Phi}_1^T,\overline{\Phi}_1^T\}^{1/2}$ in the measure of functional integral appears in principle in this case as problematic as the factor $\det\{\phi^T,\phi^T\}^{1/2}$ in the direct approach. Nevertheless we observe that constraint (44) is equivalent to the reducible constraint

$$\boldsymbol{P}\overline{\Phi}_1|_{\text{first class}}=\boldsymbol{P}\eta_1\equiv 0 \ , \tag{45a}$$

$$\overline{\Phi}_1=0 \ . \tag{45b}$$

We iterate now the process and introduce ξ_2, η_2, and ω_2. We obtain again

$$\begin{aligned} &\omega_2=-2\boldsymbol{P} \ , \\ &\Phi_2=\eta_2+\boldsymbol{P}\xi_2 \ , \\ &\overline{\Phi}_2=\eta_2-\boldsymbol{P}\xi_2 \ , \end{aligned} \tag{46}$$

and we have the new constraints

$$\widetilde{\phi}_1=\overline{\Phi}_1+\Phi_2 \ , \tag{47a}$$

$$\overline{\Phi}_2^T=0 \ . \tag{47b}$$

For the same reason as above we take instead of (47b) the reducible constraint

$$\boldsymbol{P}\overline{\Phi}_2|_{\text{first class}}=\boldsymbol{P}\eta_2\equiv 0 \ , \tag{48a}$$

$$\overline{\Phi}_2=0 \ , \tag{48b}$$

and continue the process. After l steps we have

$$\begin{aligned} &\widetilde{\phi}_{i-1}=\overline{\Phi}_{i-1}+\overline{\Phi}_i \ , \quad i=1,\dots,l \ , \\ &\overline{\Phi}_l^T=0 \end{aligned} \tag{49}$$

with $\overline{\Phi}_0\equiv\phi$.

At this level the classical action may be written in terms of the canonical variables in the form

$$S_l=\left\langle P_\mu\dot{x}^\mu+\sum_{i=0}^{l}\eta_i\dot{\xi}^i+\lambda P^2+\sum_{i=0}^{l-1}\overline{\psi}^i\boldsymbol{P}\eta_i+\sum_{i=0}^{l}\lambda^i\widetilde{\phi}_{i-1}\right\rangle \ , \tag{50a}$$

subject to the second-class constraints

$$\overline{\Phi}_l^T=0 \ . \tag{50b}$$

In (50a) $\overline{\psi}^i$ is the Lagrange multiplier associated with the ith analogue to (45a) and (48a). The constraints $\widetilde{\phi}_l$ in (49) are irreducible, the other constraints being infinite reducible. At each level l the formulation (50) is not Lorentz covariant due to the transverse projection in (50b).

In order to avoid this problem we may introduce infinite auxiliary fields. We then obtain

$$S_\infty=\left\langle P_\mu\dot{x}^\mu+\sum_{i=0}^{\infty}\eta_i\dot{\xi}^i+\lambda P^2+\sum_{i=0}^{\infty}\overline{\psi}^i\boldsymbol{P}\eta_i+\sum_{i=0}^{\infty}\lambda^i\widetilde{\phi}_{i-1}\right\rangle \ .$$

This is the action proposed by Kallosh in Ref. [16], which is associated with the BRST charge with the correct cohomology for the BSS. The effective action associated with (51) may be truncated at any level l by imposing the gauge-fixing conditions (31) for $i=l+1,\dots,\infty$, and the effective action associated with (50) is regained. In the limit case $\boldsymbol{P}\eta_i=0$ and $\widetilde{\phi}_i=0$, $i=0,\dots,\infty$ are regular infinite reducible first-class constraints.

The final action (51) contains an infinite number of auxiliary fields. An adequate treatment of them requires the introduction of appropriate generating functions. This is reminiscent of the analogous situation in the construction of an unconstrained off-shell covariant formulation of some super Yang-Mills and supergravity theories. There, in order to circumvent the no-go theorems [23], it was necessary to introduce in the off-shell construction an infinite set of auxiliary fields. An adequate handling of them was obtained by replacing ordinary superspace by harmonic superspace [24]. A superfield in harmonic superspace is equivalent to an infinite set of ordinary superfields. In particular the quantization of $N=3$, $D=4$ Yang-Mills theory was explicitly performed in [24].

The reformulation of the full effective action associated to (51) in terms of generating functions has not been performed. Progress in this direction has still to be made. However, as we already mentioned the infinite fields of (51) coincides with the minimal sector of fields introduced in Ref. [13]. This sector has been analyzed using representations of the orthosymplectic supergroup Osp(10/4) [25]. Moreover in Ref. [25] it has been shown how to perform Osp($2n/2n$)-covariant computations in this infinite sector using supergroup techniques. This allows the systematic BFV [6] construction of the off-shell nilpotent BRST charge for this system which, by construction, has the correct cohomology for the BSS [16].

To conclude let us say that the construction presented in this work suggests a concrete and systematic way to handle the problem of covariant quantization of the GSS.

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