Meson masses from SU(3) and heavy-quark symmetry

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Pseudoscalar D and B and vector D^* and B^* meson masses are described using SU(3) and heavy-quark symmetry by considering all effects to first order in isospin and SU(3) breaking and to order m_Q^{-1}, where Q is the heavy quark. A relation between spin-dependent splittings in the $\bar{B}_s$ and $\bar{B}^0$ systems is found. It implies that the photons in $\bar{B}_s^* \to \bar{B}_s\gamma$ and $\bar{B}^{*0} \to \bar{B}^0\gamma$ are equal in energy to an accuracy of about 1 MeV.

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Mesons containing one heavy quark (c,b) display some simple features because the interactions of the heavy quark are largely independent of its mass and spin. Here we discuss relations among masses of these particles that follow from heavy-quark symmetry [1] and light-quark flavor SU(3).

We write all possible operators contributing to D, D^*, B, and B^* masses, working to first order in light-quark masses m_q, to first order in the electromagnetic fine-structure constant α, to first order in $1/m_Q$, where Q is the heavy quark, and to orders m_q/m_Q and α/m_Q. At these orders, we find 11 such operators. Moreover, five quarks masses enter the description. Despite the presence of all these parameters, we find the relation

$$[m(\bar{B}_s^*)-m(\bar{B}_s)]-[m(\bar{B}^{*0})-m(\bar{B}^0)] = (m_c/m_b)\{[m(D_s^*)-m(D_s)] - [m(D^{*+})-m(D^+)]\} .$$

Our treatment is meant to be completely systematic and general, and is complementary to those (e.g., Ref. [2]) in which certain quantities are estimated on the basis of hadronic models.

We list all mass operators contributing to the mass splitting among the pseudoscalar mesons with flavor quantum numbers $D^+\equiv c\bar{d}$, $D^0\equiv c\bar{u}$, $D_s\equiv c\bar{s}$, $\bar{B}^0\equiv b\bar{d}$, $B^-\equiv b\bar{u}$, $\bar{B}_s\equiv b\bar{s}$, as well as the corresponding vector mesons (denoted by an asterisk). We work to first order in light-quark masses (m_u,m_d,m_s), to first order in electromagnetic interactions, and to first order in inverse powers of the heavy-quark mass m_Q, including terms of order m_q/m_Q and α/m_Q. The heavy meson fields are incorporated in H_a^i, where indices $a,b,\ldots$ stand for the light-quark flavor (u,d or s); $i,j,\ldots$ denotes the heavy-quark flavor (c or b); and H is a 4×4 Dirac matrix whose Dirac indices are not exhibited explicitly. (It contains the pseudoscalar and vector fields and is the same as that used in Ref. [3].) The masses and charges of light quarks are described by diagonal 3×3 matrices $(m_q)_a^b$ and Q_a^b, while the diagonal 2×2 matrices $(m_Q^{-1})_i^j$ and Q_i^j stand for inverse masses and charges of heavy quarks. We sum over repeated indices.

The operators of interest are the following:

$$\mathcal{P}_1= \mathrm{Tr}[\bar{H}_a^i H_i^b](m_q)_b^a , \tag{1}$$

$$\mathcal{P}_2= \mathrm{Tr}[\bar{H}_a^i H_j^b](m_Q^{-1})_i^j(m_q)_b^a , \tag{2}$$

$$\mathcal{P}_3=\alpha\, \mathrm{Tr}[\bar{H}_a^i H_i^b](Q^2)_b^a , \tag{3}$$

$$\mathcal{P}_4=\alpha\, \mathrm{Tr}[\bar{H}_a^i H_j^b]Q_b^a Q_i^j , \tag{4}$$

$$\mathcal{P}_5=\alpha\, \mathrm{Tr}[\bar{H}_a^i H_j^b](m_Q^{-1})_i^j(Q^2)_b^a , \tag{5}$$

$$\mathcal{P}_6=\alpha\, \mathrm{Tr}[\bar{H}_a^i H_k^b](m_Q^{-1})_i^j Q_j^k Q_b^a , \tag{6}$$

$$\mathcal{P}_7= \mathrm{Tr}[\bar{H}_a^i \sigma^{\mu\nu} H_j^a \sigma_{\mu\nu}](m_Q^{-1})_i^j , \tag{7}$$

$$\mathcal{P}_8= \mathrm{Tr}[\bar{H}_a^i \sigma^{\mu\nu} H_j^b \sigma_{\mu\nu}](m_Q^{-1})_i^j(m_q)_b^a , \tag{8}$$

$$\mathcal{P}_9=\alpha\, \mathrm{Tr}[\bar{H}_a^i \sigma^{\mu\nu} H_j^b \sigma_{\mu\nu}](m_Q^{-1})_i^j(Q^2)_b^a , \tag{9}$$

$$\mathcal{P}_{10}=\alpha\, \mathrm{Tr}[\bar{H}_a^i \sigma^{\mu\nu} H_k^a \sigma_{\mu\nu}](m_Q^{-1})_i^j(Q^2)_j^k , \tag{10}$$

$$\mathcal{P}_{11}=\alpha\, \mathrm{Tr}[\bar{H}_a^i \sigma^{\mu\nu} H_k^b \sigma_{\mu\nu}](m_Q^{-1})_i^j Q_j^k Q_b^a . \tag{11}$$

These terms have straightforward interpretations in a constituent quark model.

The term $\mathcal{P}_1$ is SU(3) violating, spin independent, and independent of the heavy-quark flavor. It would arise in the lowest order of a quark counting procedure. Corrections to quark counting dependent on the heavy-quark flavor are expressed by $\mathcal{P}_2$. Such corrections would arise, for example, from kinetic terms. There are four spin-independent $O(\alpha)$ terms. $\mathcal{P}_3$ and $\mathcal{P}_5$ denote electromagnetic light-quark self-energies, while $\mathcal{P}_4$ and $\mathcal{P}_6$ denote Coulomb interactions between a light quark and a heavy one. In $\mathcal{P}_3$ and $\mathcal{P}_5$ one effect will be a mass shift independent of the environment in which the light quark sits, but there will also be effects associated with exchanges of gluons with the heavy quark. We have neglected SU(3)-violating terms of order α.

All five spin-dependent terms $\mathcal{P}_7$–$\mathcal{P}_{11}$ are of order $1/m_Q$. Their relative expectation values are $-3{:}+1$ for pseudoscalars and vectors. Two terms ($\mathcal{P}_7$ and $\mathcal{P}_8$) have no factor of α, while the other three do. The usual strong hyperfine interaction term is $\mathcal{P}_7$. It is the main effect responsible for splitting the heavy pseudoscalar and vector mesons from one another. An SU(3)-violating contribution to this hyperfine splitting is expressed by $\mathcal{P}_8$, while explicit electromagnetic corrections occur in $\mathcal{P}_9$–$\mathcal{P}_{11}$. The term $\mathcal{P}_{11}$ involves an electromagnetic hyperfine interaction of a light quark with a heavy one. This term is of interest for a constituent-quark model of meson decay constants [4,5]. We have not written the contribution of heavy-quark masses explicitly, through they must be included in the masses of mesons. We have omitted the SU(3)-symmetric terms $\mathrm{Tr}[\bar{H}_a^i H_i^a]$, $\mathrm{Tr}[\bar{H}_a^i H_i^a](m_q)_b^b$, $\mathrm{Tr}[\bar{H}_a^i H_j^a](m_Q^{-1})_i^j$, and $\mathrm{Tr}[\bar{H}_a^i H_j^a](Q^2)_i^j$, which can be absorbed into overall shifts in the heavy-quark masses.

We may parametrize the effects in Eqs. (1)–(11) by writing mass formulas for D, D^*, B, and B^* mesons in terms of effective quark masses, hyperfine splitting terms, and corrections which depend on SU(3) breaking or electromagnetism. We define effective light-quark masses in terms of their values in D systems; the corresponding values in B systems will be multiplied by a constant λ as a result of the term $\mathcal{P}_2$. The terms arising from $\mathcal{P}_3$ and $\mathcal{P}_5$ are combined to give contributions proportional to a constant ξ for D mesons and to $\lambda'\xi$ for B's, while $\mathcal{P}_4$ and $\mathcal{P}_6$ combine to give a term proportional to a constant y for D's and μy for B's. For each spin-dependent term in D mesons, the corresponding term in the B's scales by the ratio $\rho \equiv m_c/m_b$. For example, the strong hyperfine splitting parameter due to $\mathcal{P}_7$ is denoted by σ for the D's; it is then $\rho\sigma$ for the B's. The effects of the terms $\mathcal{P}_8$–$\mathcal{P}_{11}$ are proportional to constants β, γ, δ, and z, respectively. Using symbols to represent the corresponding masses, we then find

$$D^0 = c + u + \tfrac{4}{9}\xi - \tfrac{4}{9}y - 3\sigma - 3\beta u - \tfrac{4}{3}\gamma - \tfrac{4}{3}\delta - \tfrac{4}{3}z \ , \quad (12)$$

$$D^+ = c + d + \tfrac{1}{9}\xi + \tfrac{2}{9}y - 3\sigma - 3\beta d - \tfrac{1}{3}\gamma - \tfrac{4}{3}\delta + \tfrac{2}{3}z \ , \quad (13)$$

$$D_s = c + s + \tfrac{1}{9}\xi + \tfrac{2}{9}y - 3\sigma - 3\beta s - \tfrac{1}{3}\gamma - \tfrac{4}{3}\delta + \tfrac{2}{3}z \ , \quad (14)$$

$$D^{*0} = c + u + \tfrac{4}{9}\xi - \tfrac{4}{9}y + \sigma + \beta u + \tfrac{4}{9}\gamma + \tfrac{4}{9}\delta + \tfrac{4}{9}z \ , \quad (15)$$

$$D^{*+} = c + d + \tfrac{1}{9}\xi + \tfrac{2}{9}y + \sigma + \beta d + \tfrac{1}{9}\gamma + \tfrac{4}{9}\delta - \tfrac{2}{9}z \ , \quad (16)$$

$$D_s^* = c + s + \tfrac{1}{9}\xi + \tfrac{2}{9}y + \sigma + \beta s + \tfrac{1}{9}\gamma + \tfrac{4}{9}\delta - \tfrac{2}{9}z \ , \quad (17)$$

$$B^- = b + \lambda u + \tfrac{4}{9}\lambda'\xi + \tfrac{2}{9}\mu y + \rho(-3\sigma - 3\beta u - \tfrac{4}{3}\gamma - \tfrac{1}{3}\delta + \tfrac{2}{3}z) \ , \quad (18)$$

$$\bar{B}^0 = b + \lambda d + \tfrac{1}{9}\lambda'\xi - \tfrac{1}{9}\mu y + \rho(-3\sigma - 3\beta d - \tfrac{1}{3}\gamma - \tfrac{1}{3}\delta - \tfrac{1}{3}z) \ , \quad (19)$$

$$\bar{B}_s = b + \lambda s + \tfrac{1}{9}\lambda'\xi - \tfrac{1}{9}\mu y + \rho(-3\sigma - 3\beta s - \tfrac{1}{3}\gamma - \tfrac{1}{3}\delta - \tfrac{1}{3}z) \ , \quad (20)$$

$$B^{*-} = b + \lambda u + \tfrac{4}{9}\lambda'\xi + \tfrac{2}{9}\mu y + \rho(\sigma + \beta u + \tfrac{4}{9}\gamma + \tfrac{1}{9}\delta - \tfrac{2}{9}z) \ , \quad (21)$$

$$\bar{B}^{*0} = b + \lambda d + \tfrac{1}{9}\lambda'\xi - \tfrac{1}{9}\mu y + \rho(\sigma + \beta d + \tfrac{1}{9}\gamma + \tfrac{1}{9}\delta + \tfrac{1}{9}z) \ , \quad (22)$$

$$\bar{B}_s^* = b + \lambda s + \tfrac{1}{9}\lambda'\xi - \tfrac{1}{9}\mu y + \rho(\sigma + \beta s + \tfrac{1}{9}\gamma + \tfrac{1}{9}\delta + \tfrac{1}{9}z) \ . \quad (23)$$

The spin-dependent splittings in the D^+ and D_s systems are nearly equal [6,7]:

$$D^{*+} - D^+ = 4\sigma + 4\beta d + \tfrac{4}{9}\gamma + \tfrac{16}{9}\delta - \tfrac{8}{9}z = 140.64 \pm 0.08 \pm 0.06 \text{ MeV} \ , \quad (24)$$

$$D_s^* - D_s = 4\sigma + 4\beta s + \tfrac{4}{9}\gamma + \tfrac{16}{9}\delta - \tfrac{8}{9}z = 141.5 \pm 1.9 \text{ MeV} \ . \quad (25)$$

Taking the difference between these two expressions, we find $4\beta(s-d) = 0.9 \pm 1.9$ MeV. However, we can estimate $s-d$ by comparing D_s and D^+ masses [7]:

$$D_s - D^+ = (s-d)(1-3\beta) = 99.5 \pm 0.6 \text{ MeV} \ . \quad (26)$$

Thus, at the 2σ level, we find that β is less than about a percent. In a constituent-quark model the explicit quark mass dependence in the strong hyperfine splittings of D^+ and D_s mesons due to the chromomagnetic moments of the light quarks is apparently canceled by a change in the wave function at zero interquark separation.

The corresponding difference between strong hyperfine splittings in the $\bar{B}_s$ and $\bar{B}^0$ systems is

$$(\bar{B}_s^* - \bar{B}_s) - (\bar{B}^{*0} - \bar{B}^0) = (m_c/m_b)[(D_s^* - D_s) - (D^{*+} - D^*)] \approx (0.3 \pm 0.6) \text{ MeV} \quad (27)$$

for $m_c/m_b \approx \frac{1}{3}$. Although we have worked only to linear order in m_s/m_Q, it is easy to see that heavy-quark flavor symmetry implies that Eq. (27) is valid to all orders in m_s (with one factor of m_Q^{-1}). It will, however, be violated at order m_s/m_Q^p, where m_Q is a heavy-quark mass and $p > 1$. Such corrections multiply the right-hand side by $f(\Lambda_{\rm QCD}/m_c)/f(\Lambda_{\rm QCD}/m_b)$, where f is an unknown function. Thus, we expect the photons in $\bar{B}_s^* \to \bar{B}_s\gamma$ and $\bar{B}^{*0} \to \bar{B}^0\gamma$ to be equal in energy to an accuracy of better than one MeV.

Heavy-quark symmetry underlies the simple relation (27). All electromagnetic effects have been canceled by the comparison of hyperfine splittings in systems with the same light-quark charges. If one wishes, one can replace m_c/m_b by m_D/m_B or by the ratio of $B^* - B$ to $D^* - D$ hyperfine splittings; these ratios are equal up to corrections from electromagnetism or higher orders in m_Q^{-1}.

The hyperfine splittings of nonstrange B mesons are (45.4±1.0) MeV (Ref. [8]) or (46.2±0.3±0.8) MeV (Ref. [9]). These quantities stand for a weighted average of $B^{*-} - B^-$ and $\bar{B}^{*0} - \bar{B}^0$ mass splittings. The splitting for strange B mesons is inferred to be $\bar{B}_s^* - \bar{B}_s = (47.0 \pm 2.6)$ MeV on the basis of an indirect method [8]. In order to check (27), one has to measure the hyperfine splittings of the charged and neutral nonstrange B mesons separately.

One can isolate the z term for D and B mesons, which is of use in a constituent-quark picture for estimating meson decay constants [4,5] by comparing isospin splittings in pseudoscalar and vector multiplets. The difference in these splittings has been measured very precisely for charmed mesons [6] (we neglect β):

$$(D^{*0}-D^{*+})-(D^0-D^+) = \tfrac{4}{3}\gamma+\tfrac{8}{3}z=(1.48\pm0.09\pm0.05)\ \text{MeV} . \quad (28)$$

The corresponding relation for B mesons (again neglecting β) is

$$(B^{*-}-\bar{B}^{*0})-(B^- -\bar{B}^0)=\rho(\tfrac{4}{3}\gamma-\tfrac{4}{3}z) . \quad (29)$$

At present all we know [5,10] is that $B^- -\bar{B}^0=(-0.12\pm0.58)$ MeV.

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