Scaling limit of a nonrelativistic model of confined "quarks"

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I calculate the structure function for scattering from the two-body bound state in its lowest level in a nonrelativistic model of confined scalar "quarks" of masses m_A and m_B. The scaling limit in $x = \mathbf{q}^2/2(m_A + m_B)q^0$ exists and is nonvanishing only for the values $x = m_A/(m_A + m_B)$ and $x = m_B/(m_A + m_B)$ which correspond to the fractions of the momentum of the two-body system carried by each of the "quarks." In the scaling limit, the interference from scattering off of the two "quarks" vanishes. Thus the scaling limit of this model agrees with the parton picture.

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I. INTRODUCTION

Since the scaling limit of the structure functions for deep inelastic scattering concerns a regime which is at an opposite extreme from the confinement regime, it is interesting to see how asymptotic freedom, which is responsible for the high-momentum-transfer scaling limit of deep inelastic scattering given by the parton model [1], coexists with confinement, which is of crucial importance for low-energy hadronic physics. In the present paper, I study the scaling regime in a nonrelativistic model of scalar "quarks" bound by a harmonic potential to get some clues as to how these disparate regimes can coexist in such models. Insight gained might be useful in studying the corresponding problem in QCD. Specifically, I calculate the structure function $W_{00}^{(AB)}$ for inelastic lepton scattering from a bound state of "quarks" of masses m_A and m_B. I find that the scaling limit in $x = \mathbf{q}^2/2(m_A+m_B)q^0$ exists and is nonvanishing only for the values $x = m_A/(m_A+m_B)$ and $x = m_B/(m_A+m_B)$ which correspond to the fractions of the momentum of the two-body system carried by each of the "quarks." For these values of x the scattering is that expected from free "quarks." Thus the scaling limit of this model agrees with the parton picture. Because particles interacting via a harmonic potential are much freer at short distances than those which interact with a potential which is linear at large distances and Coulombic with a coupling constant which decreases logarithmically, as in the case of asymptotic freedom, at short distances, one can expect a rapid approach to the scaling limit.

Section II describes the model. Section III contains the calculation of the structure function. Section IV summarizes the work and gives the outlook for future developments.

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With a view to possible later work, I carried out the calculation using the N quantum approximation (NQA), which is exact in this model. A brief review of the NQA in the form relevant to theories with a confining potential is given in the Appendix. I emphasize that the NQA is not necessary for this calculation; the usual Schrödinger theory gives the same result.

II. DESCRIPTION OF THE MODEL

The model has two species of nonrelativistic scalar "quarks" of masses m_A and m_B. (I could have called one of these a "quark" and the other an "antiquark.") The second-quantized Hamiltonian of the model is

$$H = \sum_{i=A,B} (2m_i)^{-1} \int d^3x\, \nabla\psi_i^\dagger(\mathbf{x},t) \cdot \nabla\psi_i(\mathbf{x},t) + (k/2) \int d^3x\, d^3y\, \psi_A^\dagger(\mathbf{x},t)\psi_B^\dagger(\mathbf{y},t) \times(\mathbf{x}-\mathbf{y})^2\psi_B(\mathbf{y},t)\psi_A(\mathbf{x},t). \qquad (1)$$

The Heisenberg equation of motion for ψ_A is

$$i\partial\psi_A(\mathbf{x},t)/\partial t = -(2m_A)^{-1}\nabla^2\psi_A(\mathbf{x},t) + k\int d^3y\, \psi_B^\dagger(\mathbf{y},t)(\mathbf{x}-\mathbf{y})^2 \times\psi_B(\mathbf{y},t)\psi_A(\mathbf{x},t), \qquad (2)$$

with a similar equation for ψ_B. The solution of Eq. (2) using the Haag expansion is given in the Appendix. The equation for the two-body Schrödinger amplitudes, which are identical to the Haag amplitudes described in the Appendix, in terms of the relative coordinate $\mathbf{r} = \mathbf{x} - \mathbf{y}$, is

$$\left(-\frac{\hbar^2\nabla_r^2}{2\mu_{AB}}+k\mathbf{r}^2\right)F_{AB,\mathbf{n}}(\mathbf{r})=\epsilon_{\mathbf{n}}F_{AB,\mathbf{n}}(\mathbf{r}), \tag{3}$$

$$\frac{1}{\mu_{AB}}=\frac{1}{m_A}+\frac{1}{m_B}. \tag{4}$$

To fix notation, the solution of Eq. (3) is $F_{AB,\mathbf{n}}(\mathbf{x})=\prod_1^3 F_{AB,n_i}(x_i)$,

$$F_{AB,n}(x)=\frac{1}{\sqrt{2^n n!}}\left(\frac{\mu_{AB}\omega}{\pi\hbar}\right)^{1/4}\times\exp\left(-\frac{\mu_{AB}\omega x^2}{2\hbar}\right)H_n\left(\sqrt{\frac{\mu_{AB}\omega}{\hbar}}x\right), \tag{5}$$

where H_n is the Hermite polynomial.

III. CALCULATION OF THE STRUCTURE FUNCTION

A general formula for the structure function is

$$\begin{aligned}W_{\mu\nu}&=\sum_\alpha\int d^3k\langle\mathbf{P},\mathbf{0}|j_\mu(\mathbf{0})|\mathbf{k},\alpha\rangle\langle\mathbf{k},\alpha|j_\nu(\mathbf{0})|\mathbf{P},\mathbf{0}\rangle(2\pi)^4\delta^4(q+P-P_{\mathbf{k},\alpha})\\&=\sum_\alpha\langle\mathbf{P},\mathbf{0}|j_\mu(\mathbf{0})|\mathbf{P}+\mathbf{q},\alpha\rangle\langle\mathbf{P}+\mathbf{q},\alpha|j_\nu(\mathbf{0})|\mathbf{P},\mathbf{0}\rangle(2\pi)^4\delta(q^0+E_0(\mathbf{P})-E_\alpha(\mathbf{P}+\mathbf{q}));\end{aligned} \tag{6}$$

α labels the quantum numbers of the intermediate states aside from the momentum. For the nonrelativistic model, I calculate only W_{00}. I work in the rest frame of the target, $\mathbf{P}=\mathbf{0}$; however, I will put $\mathbf{P}$ in the label of the states, and set $\mathbf{P}=\mathbf{0}$ in the δ functions and in the final results, because putting $\mathbf{0}$ in the labels of the states might confuse the states with the vacuum. The requirement that the charges $Q_i=\int d^3x\rho_i(x)$ obey $\langle\mathbf{p},\mathbf{n}|Q_i|\mathbf{p}',\mathbf{n}'\rangle=\delta_{\mathbf{n},\mathbf{n}'}\delta(\mathbf{p}-\mathbf{p}')$ provides the normalization of the bound-state amplitudes,

$$\int d^3r\,F^*_{AB,\mathbf{n}}(\mathbf{r})F_{AB,\mathbf{n}'}(\mathbf{r})=\delta_{\mathbf{n},\mathbf{n}'}. \tag{7}$$

The structure function for scattering from a single particle A or B "quark" is

$$\begin{aligned}W^i_{00}&=\int d^3k\langle\mathbf{P},i|\rho(\mathbf{0})|\mathbf{k},i\rangle\langle\mathbf{k},i|\rho(\mathbf{0})|\mathbf{P},i\rangle(2\pi)^4\delta^4(P+q-k)\\&=\langle\mathbf{P},i|\rho_i(\mathbf{0})|\mathbf{P}+\mathbf{q},i\rangle\langle\mathbf{P}+\mathbf{q},i|\rho_i(\mathbf{0})|\mathbf{P},i\rangle(2\pi)^4\delta\left(q^0-\frac{\mathbf{q}^2}{2m_i}\right)\\&=(2\pi)^{-2}\delta\left(q^0-\frac{\mathbf{q}^2}{2m_i}\right),\quad i=A,B.\end{aligned} \tag{8}$$

The structure function for scattering from the ground state of the two-body bound AB system is

$$\begin{aligned}W^{(AB)}_{00}&=\sum_\mathbf{n}\int d^3k\langle\mathbf{P},\mathbf{0}|\rho(\mathbf{0})|\mathbf{k},\mathbf{n}\rangle\langle\mathbf{k},\mathbf{n}|\rho(\mathbf{0})|\mathbf{P},\mathbf{0}\rangle(2\pi)^4\delta^4(q+P-P_{\mathbf{k},\mathbf{n}})\\&=\sum_\mathbf{n}\langle\mathbf{P},\mathbf{0}|\rho(\mathbf{0})|\mathbf{P}+\mathbf{q},\mathbf{n}\rangle\langle\mathbf{P}+\mathbf{q},\mathbf{n}|\rho(\mathbf{0})|\mathbf{P},\mathbf{0}\rangle(2\pi)^4\delta(q^0+E_\mathbf{0}(\mathbf{P})-E_\mathbf{n}(\mathbf{P}+\mathbf{q})),\end{aligned} \tag{9}$$

where all states are AB bound states. If the difference $\epsilon_0-\epsilon_\mathbf{n}$ could be neglected so that the δ function could be taken out of the sum, completeness of the intermediate states would lead to

$$W^{(AB)}_{00}=\sum_n\langle\mathbf{P},\mathbf{0}|[\rho_A(\mathbf{0})+\rho_B(\mathbf{0})]|\mathbf{P}+\mathbf{q},\mathbf{n}\rangle\langle\mathbf{P}+\mathbf{q},\mathbf{n}|[\rho_A(\mathbf{0})+\rho_B(\mathbf{0})]|\mathbf{P},\mathbf{0}\rangle(2\pi)^4\delta\left(q^0-\frac{\mathbf{q}^2}{2(m_A+m_B)}\right) \tag{10}$$

and each direct term would have the form

$$\sum_n\langle\mathbf{P},\mathbf{0}|\rho_i(0)|\mathbf{P}+\mathbf{q},\mathbf{n}\rangle\langle\mathbf{P}+\mathbf{q},\mathbf{n}|\rho_i(0)|\mathbf{P},\mathbf{0}\rangle(2\pi)^4\delta\left(q^0-\frac{\mathbf{q}^2}{2(m_A+m_B)}\right)=(2\pi)^{-2}\delta\left(q^0-\frac{\mathbf{q}^2}{2(m_A+m_B)}\right), \tag{11}$$

which is the structure function for scattering from a particle of mass m_A+m_B. This serves as a useful check on the calculation. Taking into account momentum conservation, the relevant matrix element is

$$\begin{aligned}\langle \mathbf{P},\mathbf{0}|\rho_A(0)|\mathbf{P}+\mathbf{q},\mathbf{n}\rangle &= (2\pi)^{-3}\int F^*_{AB,\mathbf{0}}(\mathbf{y})\exp\left(-\frac{im_B\mathbf{q}\cdot\mathbf{y}}{m_A+m_B}\right)F_{AB,\mathbf{n}}(\mathbf{y})d^3y\\ &= (2\pi)^{-3}\int F_{AB,0}(y)\exp\left(-\frac{im_Bqy}{m_A+m_B}\right)F_{AB,n}(y)dy\\ &= \frac{(-i)^n}{(2\pi)^3\sqrt{n!}}\phi_A^{(n)},\end{aligned} \tag{12}$$

$$\phi_A^{(n)} = \left(\frac{m_Bq^2}{2m_A(m_A+m_B)\hbar\omega}\right)^{n/2}\exp\left(-\frac{m_Bq^2}{4m_A(m_A+m_B)\hbar\omega}\right); \tag{13}$$

$\mathbf{q}$ was chosen in the z direction, $q=|\mathbf{q}|$, and y is the z coordinate of $\mathbf{y}$. With this choice, only the z-direction modes of the higher oscillator states are excited. A similar formula holds for $\phi_B^{(n)}$. Inserting Eq. (12) in Eq. (9),

$$W_{00}^{(AB)} = (2\pi)^{-2}\sum_n \frac{1}{n!}(\phi_A^{(n)}+\phi_B^{(n)})^2\delta\left(q^0-n\hbar\omega-\frac{\mathbf{q}^2}{2(m_A+m_B)}\right). \tag{14}$$

Since, if the n in the δ function could be neglected, the sum on n would cancel the exponential in each of the direct terms, this verifies the check of Eq. (11) above. For this nonrelativistic problem, I define the scaling variable to be

$$x = \mathbf{q}^2/2(m_A+m_B)q^0. \tag{15}$$

In terms of x, the structure function is

$$\begin{aligned}W_{00}^{(AB)} &= (2\pi)^{-2}\sum_n\frac{1}{n!}(\phi_A^{(n)}+\phi_B^{(n)})^2\delta\left(\frac{q^2}{2(m_A+m_B)}\frac{(1-x)}{x}-n\hbar\omega\right)\\ &\approx (2\pi)^{-2}\{\exp[-\mathcal{Q}^2f_A(x)]+\exp[-\mathcal{Q}^2f_B(x)]\}^2\sum_n\delta\left[\left(\mathcal{Q}^2\frac{(1-x)}{x}-n\right)\hbar\omega\right],\end{aligned} \tag{16}$$

where $\mathcal{Q}^2 \equiv q^2/2(m_A+m_B)\hbar\omega$ is dimensionless, and

$$2f_A(x) = \frac{m_A+m_B}{m_A}-\frac{1}{x}+\frac{1-x}{x}\ln\left(\frac{m_A}{m_B}\frac{1-x}{x}\right), \tag{17}$$

and f_B is the same formula, except A and B are interchanged. The $\approx$ sign is because Stirling's approximation for $n!$ holds only for large n. Each term of the sum comes from a different excitation of the AB bound state. These excitations play the role of resonances in hadronic physics. The different "resonances" contribute on disjoint lines of slope one in the q^0-$q^2/2(m_A+m_B)$ plane. In order to discuss the scaling limit, I replace the sum on n by an integral over n. Then

$$W_{AB}^{(00)} \approx \frac{1}{(2\pi)^2\hbar\omega}\{\exp[-\mathcal{Q}^2f_A(x)] + \exp[-\mathcal{Q}^2f_B(x)]\}^2. \tag{18}$$

The details of the averaging procedure do not matter. For example, averaging over q^2 at fixed x gives the same result. The functions f_A and f_B are positive between $0\le x\le 1$, except for quadratic zeros at

$$x = m_A/(m_A+m_B)\ \text{ and }\ x = m_B/(m_A+m_B), \tag{19}$$

respectively; thus the structure function vanishes for large q, except at these values of x which are the fractions of the momentum of the target bound state carried by the respective "quarks," as expected by the parton picture of deep inelastic scattering. Note also that, unless $m_A=m_B$, f_A+f_B has no zeroes; thus the interference term vanishes in this limit, which verifies the incoherence assumption of the parton picture [2]. The rapid Gaussian decrease of $W_{00}^{(AB)}$ with q^2 away from the values of x at which scaling occurs reflects the rapid vanishing of the "quark"-"quark" potential at short distance mentioned in Sec. I.

To evaluate x moments of $W_{AB}^{(00)}$ in the large-q^2 limit, approximate f_A by

$$f_A(x) \approx \frac{(m_A+m_B)^4}{4m_A^3m_B}\left(x-\frac{m_A}{m_A+m_B}\right)^2. \tag{20}$$

Then

$$\int_0^1 W_{00}^{(AB)}dx \approx \frac{2\sqrt{\pi m_Am_B}}{(2\pi)^2\hbar\omega(m_A+m_B)\mathcal{Q}}; \tag{21}$$

thus moments of $q^0W_{00}^{(AB)}$ remain finite and nonvanishing in the scaling limit. This result holds for either order of doing $\int dx$ and replacing $\sum_n$ by $\int dn$. However, if the scaling limit, $q^2\to\infty$, is (improperly) taken before calculating the x moments, then, since $W_{00}^{(AB)}$ is bounded and vanishes in this limit, except at the two isolated points given by Eq. (19), the x moments would appear to vanish [3].

Scaling in deep inelastic scattering from the deuteron considered as a bound state of a proton and a neutron, with emphasis on the effect of Fermi motion, was discussed in [4].

The calculation of $W_{00}^{(AB)}$ in Eq. (14) corresponds to

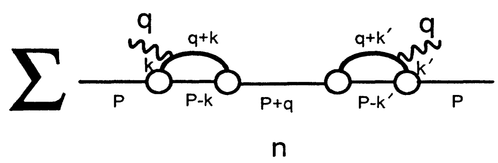

FIG. 1. Graph for W_{00} in which light lines are on shell, heavy lines are off shell, and the sum stands for the integrals over $\mathbf{k}$ and $\mathbf{k}'$ and a sum over harmonic levels n. The line labeled by the momenta P and $P+q$ are two-body bound states; the other solid lines are single-particle states. The wiggly lines are currents.

Fig. 1; the parton model limit corresponds to Fig. 2. Note that Fig. 1 is a two-loop graph while Fig. 2 is a one-loop graph; thus in the scaling limit the graph with two independent momentum integrations reduces to a single such integration. Figure 3 shows the fixed-x and fixed-resonance mass lines in the q^0-$q^2/2(m_A+m_B)$ plane.

IV. SUMMARY AND OUTLOOK

In a simple model, I verified that the deep inelastic limit of the structure function approaches the limit of incoherent elastic scattering off its constituents as though the constituents were free. This shows the way in which the scaling limit can coexist with confinement. In the present two-body model, each constituent "quark" carries a fixed part of the total momentum. It would be interesting to study a three-body bound state in which the "quarks" can carry variable fractions of the total momentum, as well as a model in which there are a variable number of constituents. The three "quark" masses, the Lagrangian mass which occurs in the kinetic term in the Hamiltonian (or the corresponding Lagrangian), Eq. (1), the constituent mass which occurs in the (analog of the) Schrödinger equation, Eq. (3), and the current mass which occurs in the parton model limit, Eq. (15), are all the same in this simple model. These masses will differ in more realistic models.

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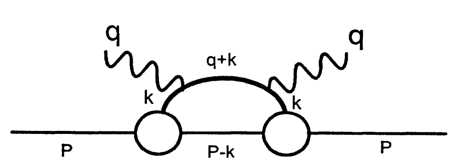

FIG. 2. Graph for the parton model limit of W_{00}. Here there is just one integration, over $\mathbf{k}$. The line labeled by P is the two-body ground state; the other solid lines are single-particle states. The wiggly lines are again currents.

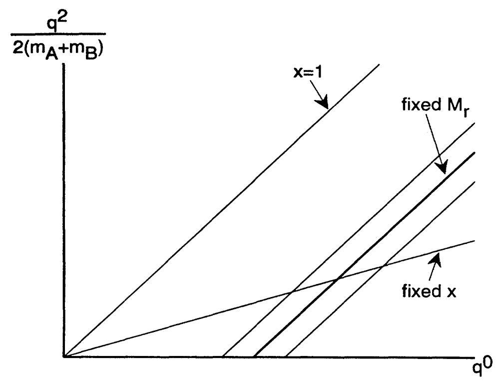

FIG. 3. The allowed kinematic domain for the structure function is between the 45°, line $x=1$, and the q^0 axis, $x=0$. Generic fixed x and generic fixed resonance mass M_r are indicated.

APPENDIX: REVIEW OF THE CONFINED IN FIELD VERSION OF THE N QUANTUM APPROXIMATION

The basic idea of the N quantum approximation is that the complete and irreducible set of Heisenberg fields which appear in the Lagrangian of a theory can be expanded, following the seminal paper of Haag [5], in (normal-ordered, if one chooses) products of the complete and irreducible set of asymptotic fields for the stable particles of the theory, both the particles which correspond to the Heisenberg fields and the particles corresponding to any bound states which are present. The c-number coefficients of the Haag expansion (I call these the Haag amplitudes) are retarded (if in fields are chosen) or advanced (if out fields are chosen) amplitudes with the legs corresponding to the asymptotic fields on shell and the single leg corresponding to the Heisenberg field off shell. Thus only one leg in any Haag amplitude is off shell. This formalism is as close to being on shell as can be achieved in a field theory. The Haag amplitudes are closely related to scattering and bound-state amplitudes. For theories with confined particles, I assume that a state with only one confined particle is allowed, but that scattering states of more than one confined particle are prohibited. In order to study the modifications necessary in the Haag expansion of the Heisenberg fields in normal-ordered in (or out) fields for the case of theories with confinement, I studied a model [6] in which nonrelativistic particles, called "quarks," interact with a harmonic potential. Solution of the model required that the in (or out) fields be replaced by "confined in (or out) fields" which differ from the usual asymptotic fields by having vacuum projectors Λ_0 to the left of the annihilation parts of the asymptotic fields and to the right of the creation parts of the asymptotic fields. The insertion of vacuum [or in quantum chromodynamics (QCD), color-singlet] projectors enforces the prohibition of multiparticle scattering states. For the model of [6] with confinement by a harmonic potential there are no analogs of the unconfined mesons and baryons of QCD, so the in and out fields are identical. For a realistic model with color and antiquarks, the vacuum projectors should be replaced by projectors onto the color-singlet states.

Although I use the confined in field formalism, as emphasized in Sec. I, the results do not depend on the use of this formalism.

The relevant terms in the Haag expansion, with confined in fields, are

$$\psi_A(\mathbf{x},t) = \Lambda_0\psi_{A,\text{in}}(\mathbf{x},t) + \sum_{\mathbf{n}}\int F_{AB,\mathbf{n}}(\mathbf{x}-\mathbf{y})\psi^\dagger_{B,\text{in}}(\mathbf{y},t)\Lambda_0 B_{\mathbf{n},\text{in}}(\mathbf{R},t)d^3y, \tag{A1}$$

where $\mathbf{R} = (m_A\mathbf{x}+m_B\mathbf{y})/(m_A+m_B)$, $\psi_{A,\text{in}}$ and $\psi_{B,\text{in}}$ are Fermi in fields and obey Fermi equal-time anticommutation relations (for this nonrelativistic model, either Fermi or Bose statistics could be chosen), and $B_{\mathbf{n},\text{in}}$ is the Bose in field for the two-body bound state in oscillator level $\mathbf{n}$ and obeys Bose equal-time commutation relations. There is a similar equation for the field ψ_B with A and B interchanged and $F_{BA,\mathbf{n}}(\mathbf{x}) = -F_{AB,\mathbf{n}}(-\mathbf{x})$. The form of the expansion for the terms with bound states is dictated by the requirements of translation and Galilean invariance. Focusing attention on the two-body bound states, the Haag amplitudes $F_{AB,\mathbf{n}}$, $\mathbf{n} = (n_1, n_2, n_3)$, satisfy the Schrödinger equation, and thus are the Schrödinger amplitudes for the oscillator bound states with energy $\epsilon_{\mathbf{n}} = \sum_i (n_i + 3/2)\hbar\omega$, $\hbar\omega = \sqrt{k/2\mu_{AB}}$, at rest.

The expansions of the fields in annihilation operators are

$$\psi_i(\mathbf{x},t) = (2\pi)^{-3/2}\int d^3p\, dE\, a_i(\mathbf{p},E)\exp(-iEt+i\mathbf{p}\cdot\mathbf{x}),\quad i = A,B, \tag{A2}$$

$$\psi_{i,\text{in}}(\mathbf{x},t) = (2\pi)^{-3/2}\int d^3p\, a_{i,\text{in}}(\mathbf{p})\exp(-i\mathbf{p}^2t/2m_i+i\mathbf{p}\cdot\mathbf{x}),\quad i = A,B, \tag{A3}$$

$$B_{\mathbf{n},\text{in}}(\mathbf{x},t) = (2\pi)^{-3/2}\int d^3p\,\Lambda_0 b_{\mathbf{n},\text{in}}(\mathbf{p})\exp[-iE_{\mathbf{n}}(\mathbf{p})t+i\mathbf{p}\cdot\mathbf{x}], \tag{A4}$$

$E_{\mathbf{n}}(\mathbf{p}) = \mathbf{p}^2/2(m_A+m_B)+\epsilon_{\mathbf{n}}$ for the two-body bound states. The equal-time anticommutation relations for the in fields, $[\psi_i(\mathbf{x},t),\psi^\dagger_j(\mathbf{y},t)]_+ = \delta_{ij}\delta(\mathbf{x}-\mathbf{y})$, lead to anticommutation relations for the annihilation and creation operators, $[a_{i,\text{in}}(\mathbf{p}), a^\dagger_{j,\text{in}}(\mathbf{q})]_+ = \delta_{ij}\delta(\mathbf{p}-\mathbf{q})$. The charge density $j_0(x)\equiv\rho(x)$ is $\rho(x) = \rho_A(x)+\rho_B(x)$,

$$\begin{aligned}\rho_A(x) &= \psi^\dagger_A(x)\psi_A(x)\\ &= \psi^\dagger_{A,\text{in}}(x)\Lambda_0\psi_{A,\text{in}}(x) + \sum_{\mathbf{n},\mathbf{n}'}\int F^*_{AB,\mathbf{n}}(\mathbf{x}-\mathbf{y})B^\dagger_{\mathbf{n},\text{in}}(\mathbf{R},t)\Lambda_0 B_{\mathbf{n}',\text{in}}(\mathbf{R},t)F_{AB,\mathbf{n}'}(\mathbf{x}-\mathbf{y})d^3y,\end{aligned} \tag{A5}$$

where I have kept only the relevant terms in ρ_A.

[1] For a general survey of the parton model and related issues, see B.L. Ioffe, V.A. Khoze, and L.N. Lipatov, *Hard Processes* (North-Holland, Amsterdam, 1984).
[2] R.L. Jaffe made a similar calculation for a particle bound to a force center in 1970 (private communication).
[3] S. Wallace (private communication) emphasized that an appropriate power of q times $W_{00}^{(AB)}$ should have finite x moments.
[4] G.B. West, Ann. Phys. (N.Y.) **74**, 464 (1972).
[5] R. Haag, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. **29**(12) (1955).
[6] O.W. Greenberg, Phys. Rev. D **17**, 2576 (1978).