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Planck scale effects in neutrino physics

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We study the phenomenology and cosmology of the Majoron (flavon) models of three active and one inert neutrino paying special attention to the possible (almost) conserved generalization of the Zeldovich-Konopinski-Mahmoud lepton charge. Using Planck scale physics effects which provide the breaking of the lepton charge, we show how in this picture one can incorporate the solutions to some of the central issues in neutrino physics such as the solar and atmospheric neutrino puzzles and the dark matter problem with the possible existence of a heavy (1–10 keV) neutrino. These gravitational effects induce tiny Majorana mass terms for neutrinos and considerable masses for flavons. The cosmological demand for the sufficiently fast decay of flavons implies a lower limit on the electron-neutrino mass in the range of 0.1–1 eV.

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I. INTRODUCTION

The central open issues in neutrino physics, according to our belief, are the following.

(a) The solar neutrino puzzle (SNP). The solar neutrino experiments under operation [1–4] indicate a deficiency of solar neutrinos pointing to neutrino properties being a source of the discrepancy between theory and experiment. The most popular and natural explanation is based on oscillations of ν_e into another neutrino in solar matter or in vacuum during the flight to Earth.

(b) The atmospheric neutrino puzzle (ANP). There is some evidence for a significant depletion of the atmospheric ν_μ flux, by almost a factor of 2 [5]. This result, if true, would point again to neutrino oscillations, this time of ν_μ into another species, with a large mixing angle and an oscillation length less than or of the order of the atmospheric height.

It is, at least in principle, possible to resolve both the SNP and ANP in the context of the usual three neutrino flavors, e.g., the SNP could be due to the $\nu_e \to \nu_\mu$ oscillations, and the ANP due to the $\nu_\mu \to \nu_\tau$ oscillations.

(c) Dark matter problem. Neutrinos with a mass in the range of 10–100 eV have been considered for many years as natural candidates for dark matter needed to explain the observed large scale structure of the Universe. This popular, so-called hot dark matter (HDM) scenario, which also enables one to explain the missing cosmological density, was disfavored in the last years due to the bounds on the primordial density fluctuations coming from the measurements of the cosmic microwave background radiation (CMBR). The recent Cosmic Background Explorer (COBE) discovery of the CMBR anisotropy [6], however, suggests at least some presence of HDM together with cold dark matter (CDM) with the latter being the dominant component [7]. This role can now be naturally played by neutrinos with a mass in the 1 eV range.

(d) A heavy neutrino. Recently, the existence of a heavy ($m_\nu \simeq 17$ keV) neutrino mixed by the angle $\theta_S \simeq 0.1$ with ν_e has been claimed [8]. However, many experiments looking for such a neutrino have not found it [9]; moreover, it was pointed out that the positive results could be due to a detector effect [10]. Thus, at present Simpson's 17 keV neutrino is more dead than alive. Still, the very existence of a neutrino with a mass in the 1 keV or 10 keV range is not ruled out by the experiment. It is therefore tempting and theoretically challenging to try to incorporate such a neutrino into our understanding of particle physics, astrophysics, and cosmology. Many theoretical models on the subject were proposed [11], inspired mainly by the experimental evidence for the existence of the 17 keV neutrino [8]. However, the difficult task of incorporating the SNP in this picture has only recently been addressed [12]. The problem is that the conventional scenario of three neutrinos ν_e, ν_μ, and ν_τ cannot reconcile laboratory constraints with the solar

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neutrino deficit. Namely, the combined restriction from the neutrinoless double β decay and $\nu_e \leftrightarrow \nu_\mu$ oscillations leads to a conserved (or at most very weakly broken) generalization of Zeldovich-Konopinski-Mahmoud (ZKM) [13] symmetry: $L_e - L_\mu + L_\tau$ [14, 15]. This in turn implies that the heavy neutrino mainly consists of ν_μ^c and ν_τ, mixed with the massless ν_e. Clearly, in this picture there is no room for the solution of the SNP due to neutrino properties.

To get a nonvanishing mass of ν_e in the framework of an extended ZKM symmetry, one needs to introduce at least one more neutrino species. The phenomenology of a four-neutrino system with the conserved lepton charge $L_e - L_\mu + L_\tau - L_\sigma$ (with σ being an active neutrino of the fourth generation) was analyzed by Lusignoli [16]. Subsequently the possibility of the existence of yet another light active neutrino was excluded by the limit [17] on Z^0 decay width reached at the CERN e^+e^- collider LEP. However, the same in general is not true for a sterile neutrino (n_R). Of course, once introduced, n (instead of ν_μ^c) can combine with ν_τ to form the heavy neutrino or just provide a missing light partner to ν_e needed for the neutrino oscillation solution to the SNP. The latter possibility has been recently advocated by the authors of Ref. [12]. In this paper we study in some detail the physics of an extra sterile neutrino in the framework of (almost) conserved generalized ZKM charges. We will show that its existence can accommodate the solution to all the above puzzles. We offer a systematic study of this scenario, paying special attention to possible effective operators that could induce neutrino masses. We consider the case of a maximal Abelian lepton flavor symmetry with n_R included, inspired by an analysis performed by Barbieri and Hall (BH) [18] for the case of three active neutrinos. The crucial characteristics of this approach is the existence of flavons, i.e., Majorons associated with spontaneous violation of extended lepton flavor symmetries. These flavons can naturally provide sufficiently fast decay of the heavy neutrino which is necessary for cosmological reasons.

In order to generate neutrino oscillations in the light sector needed for the solution of the SNP and ANP, it will turn out to be necessary to break the lepton number symmetry. We propose an interesting possibility of higher dimensional operators being responsible for this breaking [19]. These operators could naturally result from the quantum gravitational effects and should be cut off by the Planck scale. We find it encouraging that such tiny effects may be sufficient for the simultaneous solution of the above-mentioned problems. It will be shown in Sec. III that these effects induce mass splittings between the components of Dirac or ZKM neutrinos of the order of 10^{-6} eV. Since the solution to ANP seems to require $\Delta m^2 \simeq 10^{-2} - 10^{-3}$ eV^2 with large mixing angles, this in turn suggests that the mass of the heavy neutrino is of the order of a few keV. This encouraged us to seriously pursue the possibility of such a neutrino. In fact, all we need is the existence of a heavy neutrino with a mixing angle which could be much smaller than θ_S. We would like to emphasize that otherwise our analysis is quite general, and it will hold true even if the 17 keV neutrino with $\theta_S \simeq 0.1$ finally disappears. We therefore denote ν_h as a heavy neutrino with a mass in the 1 keV or 10 keV range, mixed with ν_e by an angle θ_h which can be completely different from the Simpson angle.

Furthermore, the same gravitational effects create the potential problem by inducing appreciable masses for flavons, of the order of 1 keV. Just like ν_h, they also must decay fast enough in order not to postpone the matter dominated era of the expansion of the Universe needed for the development of the cosmological large scale structure. This requirement is put on firmer ground through the COBE findings indicating rather small initial density fluctuations. Since the couplings of Majorons to neutrinos are necessarily proportional to the masses of the latter, this leads to both a phenomenologically and cosmologically important lower limit on the electron-neutrino mass $m_{\nu_e} > (0.1 - 1)$ eV. As we will show in Sec. III, the electron neutrino thus becomes a natural candidate to provide the needed 10–30 % of the hot dark matter of the Universe.

In addition to the already mentioned ANP, our motivation to consider a heavy neutrino in the 1–10 keV mass range was as follows. The lower limit of ~ 1 keV results from the requirement of having ν_h decay into flavons of mass of the order 1 keV. The motivation for the upper limit can be twofold. If n_R is a part of ν_h, for too large a mass of heavy neutrino m_h, one is potentially in conflict with the supernova 1987A bound $m_h \lesssim (1-30)$ keV due to n being sterile and taking away the energy of the supernova after a helicity flipping scattering $\nu_\tau(\nu_\mu) \to n$ [20]. The same limit does not apply, of course, when ν_h consists of only active neutrinos. However, in this case ν_μ is a part of ν_h and so the upper limit on the ν_μ mass $m_{\nu_\mu} < 270$ keV [21] applies.

Our paper is organized as follows. In the next section we offer a general study of a system of three active and one sterile neutrino with a conserved generalized ZKM lepton number. In Sec. III we study the implications of the necessary breaking of this symmetry induced through the quantum gravitational effects. In Sec. IV a specific model is offered for the sake of demonstration, and finally the last section is reserved for the discussion and outlook.

II. THE EFFECTIVE OPERATOR STUDY OF THE NEUTRINO MASSES

The introduction of a new state n_R opens up a number of new possibilities for a conserved (or approximately conserved) generalized lepton number L. We distinguish two such different classes.

(i) A case of one Dirac and one ZKM state, for which L takes the form

$$L_\pm = L_e - L_\mu \pm (L_\tau - L_{n^c})$$

or (1)

$$L' = L_e + L_\mu - L_\tau - L_{n^c},$$

where hereafter we use the convenient notation of a left-handed n^c field $(n^c)_L \equiv C\bar{n}_R^T$. In each of the L_+, L_-, and L' cases, we are still left with the freedom of having $\nu_h \simeq \nu_\tau + n_R$ or $\nu_h \simeq \nu_\tau + \nu_\mu^c$ for L_+, $\nu_h \simeq n^c + \nu_\mu^c$ or $\nu_h \simeq n^c + \nu_\tau^c$ for L_-, and $\nu_h \simeq \nu_\mu + \nu_\tau^c$ or $\nu_h \simeq \nu_\mu + n_R$ for L'. Notice that for $L = L'$ the mixing angle θ_h must be < 0.03 in order to comply with the constraints from the $\nu_e \leftrightarrow \nu_\mu$ oscillations.

(ii) A case of either a Dirac or ZKM ν_h and two massless states, corresponding to lepton charges with only one minus sign:

$$\begin{aligned} L_1 &= L_e - L_\mu + L_\tau + L_{n^c}, \\ L_2 &= L_e + L_\mu - L_\tau + L_{n^c}, \\ L_3 &= L_e + L_\mu + L_\tau - L_{n^c}. \end{aligned} \tag{2}$$

Obviously, $-L_e$ is not allowed.

In what follows, we shall analyze systematically the above possibilities, some of which were studied in the context of specific models [22]. Our aim is to extract as much model independent information as possible, but we will also present a simple model which will serve as an illustration of general considerations.

One may wonder at this point whether $\nu_i \to n$ oscillations could bring n into equilibrium before the nucleosynthesis [23–25], thereby affecting the primordial ^{4}He abundance in the Universe [26]. The situation crucially depends on the mixing angle θ_n between sterile and active neutrinos and so varies with the structure of ν_h.

The relative presence of n in the number of neutrino species at the time of nucleosynthesis is of course a function of its decoupling temperature T_n. We can thus speak of two distinct cases: $T_n > T_{\rm QCD}$ and $T_n \leq T_{\rm QCD}$, where $T_{\rm QCD}$ is the QCD phase transition temperature. In the former case, it can be shown that n counts at most 0.3 of the usual neutrino contribution due to the reheating of active neutrinos when T drops below $T_{\rm QCD}$, whereas in the latter case we expect $N_\nu \simeq 4$, since the only change below $T_{\rm QCD}$ is the annihilation of $\mu^+\mu^-$ pairs, which barely changes the temperature of the neutrino sea. Clearly, T_n depends on the mixing angle θ_n, the smaller θ_n is, the larger T_n.

From an analysis of Ref. [23] one can get (for a mass difference $\Delta m \simeq 10$ keV) the relation between θ_n and T_n:

$$T_n^3 \simeq \frac{(3\ {\rm MeV})^3}{\frac{1}{2}\sin^2 2\theta_n} \tag{3}$$

and $T_n \geq T_{\rm QCD} \simeq 200$ MeV requires $\theta_n \leq 10^{-3}$.

Now it is readily seen that for L_- the mixing angle θ_n coincides with θ_h and therefore in this case one predicts $N_\nu \simeq 4$ (since $T_n \ll T_{\rm QCD}$) for $\theta_h > 10^{-3}$. In other cases the situation depends on the details of the model, i.e., on the structure of ν_h; we will return to them later when we discuss the neutrino mass matrix.

Before proceeding, we wish to recall the fact that ν_h must decay fast enough in order to comply with cosmological constraints, and it appears that the simplest mechanism is provided by the Majoron, the Goldstone boson of a spontaneously broken lepton number (or lepton flavor) symmetry. We therefore assume large global symmetry G spontaneously broken down to L. In particular, this implies the existence of some new scalar fields, generically denoted by S, which transform nontrivially under G. Since the Majoron (one or more) is a phase of S, due to already mentioned LEP constraints on the Z^0 decay width, S fields should be singlets under ${\rm SU(2)}_L\times{\rm U(1)}$. Furthermore, any effective mass term invariant under G will necessarily involve S fields (assuming that the lepton flavor numbers, including n, are distinct). When an illustration is needed, we discuss the straightforward extension of the lepton number based on $G = {\rm U(1)}_e\times{\rm U(1)}_\mu\times{\rm U(1)}_\tau\times{\rm U(1)}_n$.

In order to generate small masses naturally, we allow no tree-level $d = 4$ operators that could lead to neutrino masses. In particular, this implies that (a) the only scalar fields are ${\rm SU(2)}_L$ doublets and singlets, and (b) no singlet carries such quantum numbers under G which allow direct ($d = 4$) Yukawa couplings.

In the context of the above example we allow only S_{ab}, $a \neq b$ $(a, b = e, \mu, \tau, n)$ singlet fields. These fields give naturally rise to "flavons," i.e., Majorons which change lepton flavor and provide fast decay of ν_h [18].

Consistent with the above rules, the leading effective Yukawa neutrino operators invariant under ${\rm SU(2)}_L\times{\rm U(1)}\times G$ are

$$\alpha_{ij}(l_i^T C \tau_2 \boldsymbol{\tau} l_j)\frac{H^T\tau_2\boldsymbol{\tau}H}{M^2}S_{ij}^*, \quad \alpha_{in}(l_i^T C n^c)\frac{\tau_2 H S_{ij}^* S_{jn}}{M^2}, \tag{4}$$

where $l_i^T = (\nu_{iL}^T,\ e_{iL}^T)$ are the leptonic weak doublets, H is the usual ${\rm SU(2)}_L\times{\rm U(1)}$ Higgs doublet, M is a regulator (cutoff) scale which is an input parameter and should be above $\langle H\rangle \sim M_W$ and $\langle S\rangle$ (S generically denotes S_{ij} and S_{in}), and α_{ab} are dimensionless factors expected to arise from the loop expansion, $\alpha_{ab} \lesssim 10^{-3} - 10^{-4}$. The quantum numbers of S_{ab} fields under $G = {\rm U(1)}_e\times{\rm U(1)}_\mu\times{\rm U(1)}_\tau\times{\rm U(1)}_n$ are

$$\begin{aligned} &S_{e\mu}\ (1,1,0,0), \quad S_{en}\ (1,0,0,1), \\ &S_{\mu\tau}\ (0,1,1,0), \quad S_{\mu n}\ (0,1,0,1), \\ &S_{e\tau}\ (1,0,1,0), \quad S_{\tau n}\ (0,0,1,1). \end{aligned} \tag{5}$$

Leptons carry their usual flavor charges, n^c carries -1 unit of ${\rm U(1)}_n$, and the Higgs doublet H of course carries no lepton flavor. From the constraints on ν_h flavon decay, one can deduce the limit 30 GeV$\lesssim \langle S\rangle \lesssim$ 300 GeV [18], where the nonvanishing $\langle S\rangle$ conserve lepton number L (for any L defined above there corresponds a certain set of $\langle S\rangle$).

Before one specifies the form of ν_h in the sense discussed above, one cannot decide the value of M and $\langle S\rangle$. For example, if $\nu_h \simeq \nu_\tau + n$, both $\langle S\rangle$ and M could be as large as desired, whereas in the case $\nu_h \simeq \nu_\tau + \nu_\mu^c$ both M and $\langle S\rangle$ should be close to M_W [see Eq. (21) below]. We come back to this question in the specific examples, suffice it to say that the operators (4) give the leading contributions to neutrino masses. We start for definiteness with $L_+ = L_e - L_\mu + L_\tau - L_{n^c}$, in which case the nonvanishing vacuum expectation values (VEV's) are $\langle S_{e\mu}\rangle$, $\langle S_{\mu\tau}\rangle$, and $\langle S_{\mu n}\rangle$. The neutrino mass matrix in the Dirac basis takes then the form

$$M_\nu \;=\; \begin{array}{c} \\ \nu_e \\ \nu_\tau \end{array}\!\!\begin{array}{c} n^c\;\nu_\mu \\ \begin{pmatrix} m_{en} & m_{e\mu} \\ m_{\tau n} & m_{\tau\mu} \end{pmatrix} \end{array}. \tag{6}$$

From the smallness of θ_h and the absence of the $\nu_\mu \to X$ oscillations it follows that only one entry of M_ν, either $m_{\tau n}$ or $m_{\tau\mu}$, can be $\sim$10 keV, whereas the other entries must be at least an order of magnitude smaller. As we mentioned before, there is still freedom for ν_h to consist of either (a) $\nu_\tau+n$ or (b) $\nu_\tau+\nu_\mu^c$, depending on whether $m_{\tau n}$ or $m_{\tau\mu}$ is large, respectively. In the former case, $\theta_h \simeq m_{en}/m_{\tau n}$ while in the latter, $\theta_h \simeq m_{e\mu}/m_{\tau\mu}$. The angle $\theta_n \simeq m_{\tau\mu}/m_{\tau n}$ (a) or $\theta_n \simeq m_{\tau n}/m_{\tau\mu}$ (b) determines the abundance of n during the nucleosynthesis. If it is less than 10^{-3}, we expect $N_\nu \le 3.3$ and, if not, N_ν is close to 4. The analysis for other choices of L can easily be performed along the same lines and we do not present it here.

III. THE ONLY GOOD GLOBAL SYMMETRY IS A BROKEN GLOBAL SYMMETRY

As we have seen up to now, in the limit of exact L the neutrino spectrum prevents oscillations in the light sector and so leaves the SNP unresolved. On the other hand, the belief in exact global symmetries is becoming increasingly less popular. This is certainly encouraged by the fact that the virtual black holes and wormholes, while preserving local gauge invariance, can destroy the meaning of global quantum numbers. It is not unlikely then that there could be higher dimensional operators cut off by the Planck scale which violate our lepton number symmetry. Barring the possibility of accidental cancellations and assuming that the symmetry G is not a part of a larger local gauge symmetry, we expect this breaking to occur at the $d = 5$ effective operator level.

Neutrino mass. Without further ado then, we list the leading operators that could induce corrections to the neutrino mass matrix [19]:

$$(\nu_i^T C\tau_2\boldsymbol{\tau}\nu_i)\,\frac{H^T\tau_2\boldsymbol{\tau}H}{M_{\rm Pl}}, \quad (n^T C n)\left[\frac{H^\dagger H}{M_{\rm Pl}}+\frac{S^2}{M_{\rm Pl}}\right], \tag{7}$$

where S^2 stands for any bilinear combinations of the S_{ab} fields, and we list only the flavor-diagonal terms since their effect on M_ν is most dramatic. Namely, they induce the mass splittings between the components of Dirac and ZKM neutrinos and open up new channels for oscillations.

The split Δm coming through the above operators when the relevant fields get nonvanishing VEVs can be estimated as

$$\Delta m \lesssim \frac{M_W^2}{M_{\rm Pl}} \simeq 10^{-6}\ \text{eV}, \tag{8}$$

where the number 10^{-6} eV is probably an upper limit, since there could be further dimensionless suppressions in (8) (certainly an order of magnitude suppression should not come out as a surprise). We remind the reader that our mass scales are expected to lie close to the electroweak scale. An important point is that the gravitational effects are expected to be flavor blind. This implies that the mass splits in the light and heavy sectors should be of the same order of magnitude. These splits Δm being small, much less than any Dirac mass term, lead to the prediction of two pseudo Dirac states with the mixing between the partners in each state being maximal ($\simeq 45°$).

The squared mass differences in the light and heavy sectors will be

$$\Delta m^2_{\rm light} \simeq m_{\nu_e}\Delta m, \quad \Delta m^2_{\rm heavy} \simeq m_h \Delta m, \tag{9}$$

where Δm is given by Eq. (8). The experimental upper limit is $m_{\nu_e} < 9.4$ eV [27] and, as we shall see below from the discussion of the Majoron decays, there is a lower limit $m_{\nu_e} > 0.1$ eV implying $10^{-8}\ \text{eV}^2 < \Delta m^2_{\rm light} < 10^{-5}$ eV^2. This range allows for the neutrino oscillations being naturally the solution of the SNP.

The oscillations in the heavy sector $\nu_\mu \to \nu_\tau(n^c)$ can be relevant for the recently reported deficiency of the atmospheric ν_μ [5]. From Eq. (9) it follows that for $m_h \sim 1\text{--}10$ keV $\Delta m^2_{\rm heavy}$ can naturally be $\sim 10^{-2}\text{--}10^{-3}$ eV^2 which with the mixing angle being 45° perfectly fits the required parameter range [5].

The induced mass splittings of the pseudo Dirac neutrinos open up new channels of oscillations that can bring the sterile neutrino n into the equilibrium at the time of nucleosynthesis. Although the number of allowed light species at that epoch is still debated [28], the frequently quoted limit $N_\nu < 3.4$ [26], if taken seriously, would imply $\Delta m^2_{\rm light} < 5\times 10^{-9}$ eV^2 if n is a part of $\nu_{\rm light}$ and $\Delta m^2_{\rm heavy} < 8\times 10^{-6}$ eV^2 when n is a component of ν_h [25]. From the limit $\Delta m^2_{\rm light} \gtrsim 10^{-8}$ eV^2 it is clear that we are dangerously close to the prediction of four light neutrino species in equilibrium, i.e., N_ν=4. However, because of the uncertainties in the estimation of the gravitational effects, any conclusion would be premature; all we can say is that N_ν could be lying anywhere between 3 and 4. The physical, astrophysical, and cosmological implications of different generalized ZKM lepton charges are summarized in Table I.

Majoron mass. As much as in the case of the neutrinos, we expect $d = 5$ effective operators explicitly violating lepton number in the scalar sector:

$$\frac{S}{M_{\rm Pl}}\left[S^4 + S^2H^\dagger H + (H^\dagger H)^2\right], \tag{10}$$

where we only give a typical example. Therefore the Majorons (in our case flavons) acquire nonvanishing masses, i.e., become pseudo Goldstone bosons. Since we take $< S > \sim M_W$, we get an order of magnitude estimate:

$$m_F \simeq \left(\frac{M_W^3}{M_{\rm Pl}}\right)^{1/2} \simeq 1\ \text{keV}. \tag{11}$$

The above is the typical value of the elements of the mass matrix of flavons F_{ab} which are expected to have large mixings with each other.

Since $m_F \ll m_h$, the decay rate of ν_h is almost unaffected by the generated flavon masses. However, the issue now becomes whether flavons themselves manage

TABLE I. Summary of heavy neutrino composition and solutions for the SNP and ANP for generalized lepton charges L_+, L_-, L', and $L_{1,2,3}$. SW and JS stand for the solutions of the SNP through short-wavelength (averaged) vacuum oscillations ($\Delta m^2 \simeq 10^{-8} - 10^{-5}$ eV^2) and "just so" oscillations ($\Delta m^2 \simeq 10^{-10} - 10^{-11}$eV^2), respectively. Also shown are the effective number of neutrino species at the time of nucleosynthesis N_ν and the composition of HDM. Question marks indicate the problem with the decay of massive flavons explained in Sec. V.

Generalized ZKM lepton number	Content of heavy neutrino ν_h	ANP	SNP	N_ν	HDM
L_+ ($\theta_h \simeq \theta_{e\tau}$)	$\nu_\tau + \nu_\mu^c$	$\nu_\mu \leftrightarrow \nu_\tau$	$\nu_e \leftrightarrow n^c$ (SW)	$3-4$	ν_e
	$\nu_\tau + n$		$\nu_e \leftrightarrow \nu_\mu$ (SW)	4	ν_e
L_- ($\theta_h \simeq \theta_{en^c}$)	$n^c + \nu_\mu^c$	$\nu_\mu \leftrightarrow n^c$	$\nu_e \leftrightarrow \nu_\tau$ (SW)	4	ν_e
	$n^c + \nu_\tau^c$		$\nu_e \leftrightarrow \nu_\mu$ (SW)	4	ν_e
L' ($\theta_h \simeq \theta_{e\mu}$)	$\nu_\mu + \nu_\tau^c$	$\nu_\mu \leftrightarrow \nu_\tau$	$\nu_e \leftrightarrow n^c$ (SW)	$3-4$	ν_e
	$\nu_\mu + n$	$\nu_\mu \leftrightarrow n^c$	$\nu_e \leftrightarrow \nu_\tau$ (SW)	4	ν_e
L_1 ($\theta_h \simeq \theta_{e\tau}$)	$\nu_\tau + \nu_\mu^c$	$\nu_\mu \leftrightarrow \nu_\tau$	$\nu_e \leftrightarrow n^c$ (JS)	3	?
L_1 ($\theta_h \simeq \theta_{en^c}$)	$n^c + \nu_\mu^c$	$\nu_\mu \leftrightarrow n^c$	$\nu_e \leftrightarrow \nu_\tau$ (JS)	4	
L_2 ($\theta_h \simeq \theta_{en^c}$)	$n^c + \nu_\tau^c$		$\nu_e \leftrightarrow \nu_\mu$ (JS)	4	?
L_2 ($\theta_h \simeq \theta_{e\mu}$)	$\nu_\mu + \nu_\tau^c$	$\nu_\mu \leftrightarrow \nu_\tau$	$\nu_e \leftrightarrow n^c$ (JS)	3	
L_3 ($\theta_h \simeq \theta_{e\tau}$)	$\nu_\tau + n$		$\nu_e \leftrightarrow \nu_\mu$ (JS)	4	?
L_3 ($\theta_h \simeq \theta_{e\mu}$)	$\nu_\mu + n$	$\nu_\mu \leftrightarrow n^c$	$\nu_e \leftrightarrow \nu_\tau$ (JS)	4	

to decay fast enough to be in accord with cosmological limits. Let us recall here the estimate of the ν_h lifetime due to the decay $\nu_h \to \nu_e + F$:

$$\tau_h \simeq 16\pi \left(\theta_h \frac{m_h}{\langle S\rangle}\right)^{-2} m_h^{-1} \simeq (10^{-3} - 10^{-1})\theta_h^{-2} \text{ sec} \tag{12}$$

for $\langle S\rangle \simeq M_W$, which is obviously cosmologically acceptable for $\theta_h \gtrsim 10^{-4} - 10^{-3}$. We should stress that ν_h is relativistic at the cosmological time $t \simeq (0.1-1)$ sec and so the time dilation effect makes the actual decay time in the comoving reference frame bigger than 1 sec. Therefore, flavons appear only after the nucleosynthesis [29]. However, the cosmological problems related to ν_h now get replaced by the presence of massive flavons which are produced in the ν_h decay with the concentration being equal to that of the active neutrinos. The only possible mechanism to solve the problem of an overabundance of massive flavons is their decay into light neutrinos ν_e. Recall that the coupling of Majorons to neutrinos is proportional to the neutrino mass and this decay cannot take place for massless ν_e. This poses a serious problem for any Majoron-type models of the heavy neutrino in which ν_e stays massless or very light [e.g., for $L = L_{1,2,3}$, Eq. (2) or in the absence of sterile neutrinos for $L = L_e - L_\mu + L_\tau$]. This question was recently raised by Grasso *et al.* [30] in the context of the BH picture. However, in our case all we know is that $m_{\nu_e} < 10$ eV [27] and so flavons are free to decay into light neutrinos. As we show now, this provides a lower limit on the ν_e mass [31]. It is easy to estimate the flavon lifetime due to the decay into two light neutrinos:

$$\tau_F \simeq 8\pi \left(\frac{m_{\nu_e}}{\langle S\rangle}\right)^{-2} m_F^{-1} \tag{13}$$

which using Eq. (11) becomes

$$\tau_F \simeq 8\pi \frac{(\langle S\rangle M_{\rm Pl})^{1/2}}{m_{\nu_e}^2} \simeq 10^6 \left(\frac{\text{eV}}{m_{\nu_e}}\right)^2 \text{ sec.} \tag{14}$$

It is clear from the above result that no useful limit on τ_F (i.e., on m_{ν_e}) emerges from the requirement that the Universe is not overclosed. A much more serious constraint follows, however, from the galaxy formation. The recent COBE measurements of CMBR anisotropy [6] imply the small initial density fluctuations $\delta\rho/\rho \sim 10^{-5}$. This, in turn, requires a sufficiently long matter dominated epoch for the linear growth of $\delta\rho/\rho$ to form the observed large scale structure of the Universe. Therefore the decay products of the flavons have to be redshifted sufficiently in order not to dominate the nonrelativistic matter (CDM) density at the time $t_{\rm eq}$ of a radiation dominated universe turning into a matter dominant one in the standard picture. We, therefore, demand

$$m_F n_\nu(t_{\rm eq}) \left(\frac{\tau_F}{t_{\rm eq}}\right)^{1/2} < \rho_M(t_{\rm eq}), \tag{15}$$

where $n_\nu(t_{\rm eq})$ is the neutrino number density at that time and $\rho_M(t_{\rm eq})$ is the matter density at the same time. Using Eq. (11) one obtains the limit $\tau_F < 10^7$ sec. This along with Eq. (14) leads to the promised lower limit on the electron neutrino mass

$$m_{\nu_e} > 0.3 \text{ eV}, \tag{16}$$

where, due to uncertainties in the flavon masses and mixings, this limit should be read as some number between 0.1 and 1 eV.

We should stress here that increasing the scale $\langle S\rangle$ of the lepton symmetry breaking only increases the lower limit on m_{ν_e} since both m_F and τ_F become larger. Moreover, at the scale $\langle S\rangle \gg 1$ TeV flavons become heavier than ν_h and therefore ν_h itself cannot decay.

It is rather encouraging that the limit in (16) is not too far from the laboratory upper limit on m_{ν_e}. This provides even more impetus for the direct experimental search of electron neutrino mass in β decays. It should be remembered that the almost Dirac nature of ν_e in our work implies prediction of a negative result in the experiments on neutrinoless double β decay.

The cosmological implication of our result is also rather important. Let us notice that the concentration of light neutrinos today is eight times that of a normal two-component neutrino. Recall that before ν_h decay there are four light neutrino species, and this number will not change with just the decay of ν_h. However, the subsequent decay of flavons adds yet another four species to the light neutrino concentration of the present day Universe [32]. So we can estimate the present day light neutrino concentration to be

$$n_\nu = 8 \times \frac{3}{11} n_\gamma \simeq 870/\mathrm{cm}^3, \tag{17}$$

where $n_\gamma \simeq 400/\mathrm{cm}^3$ is the relic photon density of the Universe. Then from (16) we find that at least a fraction of the present day critical density of the Universe is in the form of neutrinos, i.e., hot dark matter. This observation may be rather important, since the standard model of CDM with bias $b \simeq 2-3$ seems to be disfavored in view of recent COBE measurements, if one takes the Harrison-Zeldovich "flat" spectrum for initial density fluctuations, which is motivated (up to some small corrections) by inflation. In this case the value of the CMBR quadrupole anisotropy is more than 2σ below the one that can be derived from the COBE data [6]. The latter is consistent with $b \simeq 1$, which seems not to be in agreement with observed large scale structure, showing an excess of a small scale power.

As was shown in [7], the partial (10–30%) replacement of CDM by HDM increases the large scale power and decreases the small scale one compared to pure CDM case. So, such a CDM+HDM model with $b \simeq 1$ and inflationary (flat) spectrum can be made compatible with COBE data. Our situation, however, differs somewhat from the one studied in Ref. [7], since there it was assumed only one light neutrino species with a mass $\sim 5-10$ eV, whereas we end up with 8 times larger concentration and a mass ~ 1 eV. It would follow that in our case one predicts the power spectrum to be more flat than that in [7] for the same percentage of HDM. In any case this issue deserves further consideration.

IV. THE MODEL

Although most of our analysis was to a large extent model independent, for the sake of illustration we present a simple model based on $G=U(1)_e \times U(1)_\mu \times U(1)_\tau \times U(1)_n$. The model is a straightforward extension of that of BH [18], which is based on the lepton flavor symmetry in the Zee model [33]. On top of S_{ab} fields, one introduces a set of $SU(2)_L$ singlet charged scalars h^-_{ab} $(a \neq b)$ transforming as S_{ab} under G, which have the following couplings to leptons:

$$\Delta L_Y = f_{ij} l_i^T C i\tau_2 l_j h^*_{ij} + f_{in}(n^c)^T C e^c_i h_{in} + \mathrm{H.c.}, \tag{18}$$

where e^c_i are the charge conjugates of the right-handed leptons, singlets under $SU(2)_L$. Note that here all the fermion fields are left handed.

The transformation properties of h_{ab} and S_{ab} fields allow for an important additional term in the scalar potential

$$\Delta V = \lambda_{ab}(\phi_1^T i\tau_2 \phi_2) h_{ab} S^*_{ab} + \lambda_{abcd} h^*_{ab} h_{cd} S_{ab} S^*_{cd} + \mathrm{H.c.}, \tag{19}$$

where the two scalar doublets ϕ_1 and ϕ_2 are necessary because of the antisymmetry of the $\phi\phi h$ coupling [33].

We illustrate here the case $L_+ = L_e - L_\mu + L_\tau - L_{n^c}$, which implies the only nonvanishing VEV's of the S fields to be

$$\langle S_{e\mu}\rangle \neq 0 \neq \langle S_{\mu\tau}\rangle, \quad \langle S_{\mu n}\rangle \neq 0. \tag{20}$$

The leading radiatively induced neutrino masses are then (see Fig. 1)

$$\begin{aligned} m_{\mu\tau} &\simeq \frac{1}{16\pi^2}(f_{\mu\tau} g_\tau \lambda_{\mu\tau}) m_\tau \frac{\langle S_{\mu\tau}\rangle\langle H\rangle}{M^2}, \\ m_{e\mu} &\simeq \frac{1}{16\pi^2}(f_{e\mu} g_\mu \lambda_{e\mu}) m_\mu \frac{\langle S_{e\mu}\rangle\langle H\rangle}{M^2}, \\ m_{en} &\simeq \frac{1}{16\pi^2}(f_{e\tau} f_{\tau n} \lambda_{e\tau\tau n}) m_\tau \frac{\langle S_{e\mu}\rangle\langle S_{\mu n}\rangle}{M^2}, \\ m_{\tau n} &\simeq \frac{1}{16\pi^2}(f_{\tau\mu} f_{\mu n} \lambda_{\tau\mu\mu n}) m_\mu \frac{\langle S_{\tau\mu}\rangle\langle S_{\mu n}\rangle}{M^2}, \end{aligned} \tag{21}$$

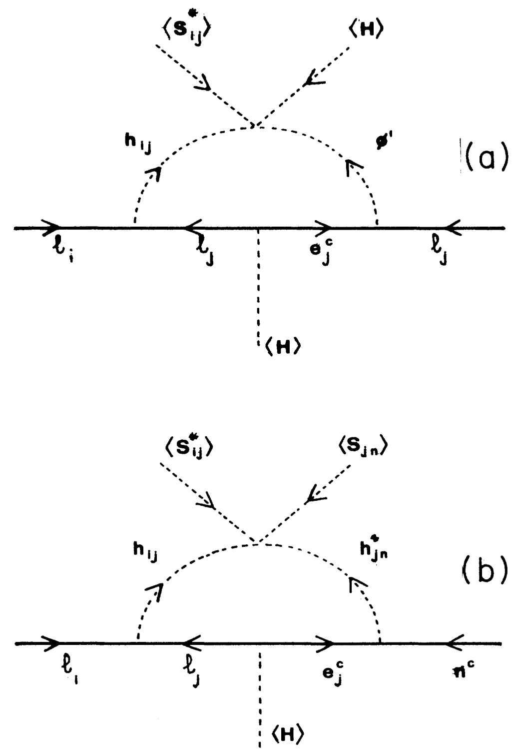

FIG. 1. One-loop diagrams which induce the neutrino mass terms m_{ij} (a) and m_{in} (b); i,j take the values allowed by the L symmetry. H and ϕ' are the linear combinations of ϕ_1 and ϕ_2 with nonvanishing and vanishing VEV's, respectively.

where H is a linear combination of ϕ_1 and ϕ_2, chosen so as to have $\langle H\rangle \neq 0$ and g_i $(i = e, \mu, \tau)$ are the Yukawa couplings of the orthogonal combination ϕ' with a vanishing VEV. For simplicity we assume all the scalar masses to be the same (M) [34]. Recall that the above pattern of VEV's can always be achieved in the absence of additional symmetries.

Notice that for $M \simeq M_W, \langle S_{\mu\tau}\rangle \simeq M_W$, with the phenomenological limit $f_{\mu\tau} \lesssim 10^{-1}$, $m_{\mu\tau}$ is naturally of the order 10 keV. Another way of phrasing this is that M and $\langle S\rangle$ must be close to M_W in order to reproduce ν_h. It is easy to deduce an upper limit $\langle S\rangle \lesssim M \lesssim 1$ TeV, which implies that all the new particles in the model can be detectable in the near future. This is the most appealing feature of the these types of models.

If $f_{e\mu} \sim f_{\mu\tau}$, $\lambda_{e\mu} \sim \lambda_{\mu\tau}$ one would predict $\theta_h \sim m_{e\mu}/m_{\mu\tau} \sim m_\mu/m_\tau \simeq 0.1$. However, the predictions are obscured by the complete arbitrariness of the couplings g_i of the second doublet. Furthermore, the model as it stands would not lead to the natural flavor conservation in the quark sector [35]. The simplest way out is to have the doublets ϕ_α to couple to up and down quarks separately. This requires the existence of a discrete symmetry D, such as

$$\phi_u \to -\phi_u, \quad u_R \to -u_R, \quad S \to -S \tag{22}$$

and the rest of the fields invariant. Now obviously both ϕ_u and ϕ_d should have nonvanishing VEV's (due to an above symmetry, one cannot rotate one of the VEV's away). The spontaneous symmetry breaking of D through $<\phi_u>\neq 0$ leads at first glance to the catastrophic existence of the domain walls. Fortunately, because of an anomaly, these walls can be shown to decay away before dominating the energy density of the Universe [36].

With this D symmetry one has

$$g_i = \frac{m_i}{\langle\phi_d\rangle} \tag{23}$$

and so for $\frac{f_{e\mu}}{f_{\mu\tau}}\frac{\lambda_{e\mu}}{\lambda_{\mu\tau}} \sim 1$ we get $\theta_h \simeq (m_\mu/m_\tau)^2$. Thus, in our model the natural value of θ_h is $\sim 10^{-2}$ and not $\theta_S \sim 10^{-1}$. To explain the hierarchy m_{en}, $m_{\tau n} \ll m_{\mu\tau}$ which may be needed to comply with the cosmological limits on the number of neutrino species, a slight adjustment of the coupling constants can be necessary; there is enough freedom to accommodate this requirement through the unknown λ_{ijkl} couplings.

V. DISCUSSION

In short, our model is a natural and straightforward extension of the BH picture of flavons, i.e., Majorons associated with lepton flavors. In the limit of conserved L, the model is basically phenomenologically indistinguishable from that of BH, except for the possible cosmological role of n. We do not repeat their analysis here, but suffice it to mention their central results: (a) The "flavon"-type models incorporate naturally a 10 keV mass range neutrino without requiring any new mass scales; (b) the most interesting prediction of BH which also holds here seems to be the potentially observable $\tau \to eF$ (F is a flavon) decay: $B(\tau \to eF) \simeq 10^{-4}$.

The principal motivation of our work was to attempt to shed some more light on other central issues of neutrino physics, such as the problems of solar and atmospheric neutrinos and the dark matter problem by adding a light sterile neutrino. Of course, as long as the generalized ZKM lepton number stays unbroken, one ends up with two four-component neutrinos, one ν_h and another ν_e with mass $\lesssim 10$ eV, and so no oscillations relevant to the SNP and ANP are possible.

Once again we would like to stress the crucial nature of our gravitationally induced breaking of L. In addition to providing necessary mass splittings of the order of 10^{-6} eV in both heavy (ν_h) and light (ν_e) sectors, it also induces a substantial mass of flavons, of the order of 1 keV. The requirement of sufficiently fast decay of flavons yields a lower bound on the ν_e mass $m_{\nu_e} > (0.1-1)$ eV which, in turn, implies at least a few percent of dark matter being hot.

Another important feature of our work is that squared mass difference in the heavy neutrino sector is $\Delta m^2_{\text{heavy}} \sim 10^{-2} - 10^{-3}$ eV^2, which together with maximal mixing is in the right range for the solution of the ANP. This, however, can only work if $\nu_h \simeq \nu_\tau + \nu_\mu^c$ since then $\nu_\mu \to \nu_\tau$ oscillations can do the job [5]. If ν_h really exists, the ANP can provide the necessary insight into its structure. We would like to emphasize, though, that its existence is by no means crucial for our work. It is true that without ν_h none of the other issues under consideration require the existence of a light sterile state. It is only when gravitationally induced effects are the source of the splittings of neutrino masses that n is necessary for a simultaneous solution of the SNP and ANP. We can even reverse the logic of our analysis and say that the solution of the ANF in the context of Planck scale physics tends to suggest the existence of a neutrino in the 10 keV mass range. Of course, its mixing angle with ν_e could easily be 2 orders of magnitude smaller than θ_S.

As was shown in Sec. III, $\Delta m^2_{\text{light}}$ lies in the range $10^{-8} - 10^{-5}$ eV^2. This overlaps with the Δm^2 domain of the Mikheyev-Smirnov-Wolfenstein (MSW) solution [37] of the SNP. We should stress, however, that the MSW effect is anyway irrelevant for the SNP in our scenario since the mixing angle is practically equal to 45°. This means that we have the short-wavelength vacuum oscillation $\nu_e \to n^c$ solution of the SNP since $\Delta m^2_{\text{light}} \gg 10^{-10}$ eV^2. Therefore one gets a universal suppression factor $\simeq 1/2$ for all the solar neutrino experiments. This is in a good agreement with the results of the Kamiokande [2], SAGE [3], and GALLEX [4] but is at variance with the Homestake data [1]. Further experiments are needed to clarify the situation. The oscillation into a sterile state predicts suppressed neutrino signals in the neutral-current channels in the forthcoming Sudbury Neutrino Observatory [38] experiment.

Our discussion up to now was almost exclusively devoted to the choice L_+ of the conserved generalized ZKM symmetry. It is clear that the situation in the case of L_- or L' is almost identical; some distinct features are listed in Table I (we should mention that all cases with n in the heavy state are in the potential conflict with the SN 1987A constraints [20], but we appeal to new supernovae to resolve this issue). As far as the other choices L_1, L_2, and L_3 are concerned, they lead to one heavy and two massless neutrinos (up to tiny gravitational effects inducing $\sim 10^{-6} - 10^{-5}$ eV masses for the latter) and so do not allow for the Majoron decays. Their properties are still listed in Table I, since they naturally allow for the so-called "just so" oscillation solution of the SNP, with $\Delta m^2_{\rm light} \simeq 10^{-10} - 10^{-11}$ eV^2 [39]. The natural way out of the Majoron stability for these cases remains a challenge, since we do not wish to pursue the unappealing possibility of fine tuning the flavon masses to be sufficiently small.

Last but not least we wish to emphasize the relevance of the predicted electron neutrino mass in the range 0.1–1 eV. To obtain what appears to be a favored amount of about 20 % hot dark matter in the present day Universe, the ν_e mass should be approximately 1 eV which is in the reach of a future direct observation.

Note added. While this paper was in print we became aware of the paper by D. O. Caldwell, Phys. Lett. B **289**, 389 (1992) in which the same phenomenological picture was discussed, without, however, any particular model being proposed. We thank the author for bringing his work to our attention.

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