

## Response of nucleons to external probes in hedgehog models. II. General formalism

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Linear response theory for SU(2) hedgehog soliton models is developed in analogy with a standard method in many-body physics. In this framework, we discuss the response of baryons to external probes, and develop expressions for polarizabilities. We discuss isospin effects (neutron-proton splitting) in polarizabilities. Methods for cases with zero modes are presented, including numerical techniques. Our approach is based on the  $1/N_c$ -expansion scheme. We work in a model with quark and meson degrees of freedom, but the basic method is valid in any hedgehog model, such as the Skyrme or the Nambu–Jona-Lasinio model in the solitonic treatment. The equations of motion for coupled random phase approximation quark-meson fluctuations are classified according to the hedgehog symmetries, and are written down explicitly in the grand-spin basis.

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### I. INTRODUCTION

In recent years various hedgehog models (chiral quark meson (CQM) models [1–6], Skyrme models [7–10], hybrid bag models [11], chiral models with confinement [5, 12–14], or the Nambu–Jona-Lasinio (NJL) model [15] in the solitonic treatment [16–22]) were extensively applied to describe the physics of low-energy baryons. Semiclassical methods for treatment of these models, such as various projection methods [23–26], or random phase approximation (RPA) method [27] were developed. Masses, various charges,  $\pi - N$  phase shifts [28], were calculated, with quite reasonable agreement with experiment, depending on the specific model, number of included fields, etc. In this article we develop the linear response formalism for hedgehog models. We work in the framework of a CQM model, since it has both quark and meson degrees of freedom, and in this respect has the essential features of both the purely mesonic Skyrme model and the NJL model, which involves quark degrees of freedom only. Our methods and final expressions can be modified straightforwardly to be applicable in these models.

Hedgehog models can be used to describe the response of nucleons to external probes, and to calculate the corresponding polarizabilities. A natural approach is the linear response method of many-body physics [29]. The underlying picture is as follows: A current interacts with the nucleon, creates an intermediate state which is an RPA phonon excitation on top of the soliton. This state interacts with another current and deexcites back into a nucleon state. The RPA phonon states are constructed from one-particle–one-hole excitations of the quarks, as well as from quantum meson excitations. Quark and me-

son fluctuations are coupled, and the resulting equations of motion for the fluctuations are solved. An example of physically important two-current observables which can be calculated in this way are the electromagnetic polarizabilities of the nucleon [30]. This topic is extensively studied in the preceding paper [31], henceforth referred to as (I). The present article is devoted to development of the necessary formalism, and contains many technical but necessary details of linear response in hedgehog models, while (I) concentrates on physical aspects.

This article is organized as follows. In Sec. II we very briefly review a CQM model [32], its soliton solutions (Sec. II A), as well as hedgehog symmetries (Sec. II B). One of the discrete symmetries, the grand-reversal symmetry [33, 23], will be particularly useful in classifying various perturbations. Section III is the core of the article, and describes the equations-of-motion approach [34] to linear response in hedgehog models. We start from deriving small-fluctuation equations of motion (Sec. III A) for coupled quark-meson systems driven by an external perturbation. These equations are classified according to hedgehog symmetries. In the static limit the grand-reversal symmetry  $\mathcal{R}$  decouples the equations into odd- $\mathcal{R}$  equations, involving quarks only, and even- $\mathcal{R}$  equations, involving both quarks and mesons. We discuss in detail the problem of zero modes (Sec. III B). These zero modes arise from breaking of the symmetries of the Lagrangian by the soliton solution. In our applications, we will have to deal with rotational zero modes (cranking) and translational zero modes [isovector electric perturbation in (I)]. We describe a numerical method to deal with the excitation of zero modes whose amplitude diverges as the frequency of the perturbation goes to zero (Sec. III C). We discuss the stability of solitons (Sec. III D). In Sec. III E we present cranking in the linear response formalism. Quantization via cranking is reviewed in Sec. III F. In Sec. III G we describe the calculation of polarizabilities in states of good spin and isospin, and obtain our basic formulas. Section IV illustrates the method by presenting the standard calculation of the  $N$ - $\Delta$  mass

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splitting, as well as the evaluation of the neutron-proton hadronic mass difference. The issues of  $N_c$  counting are discussed in Sec. V. We show how to apply the linear response in a way consistent with  $1/N_c$ -expansion scheme. Finally, Sec. VI contains remarks relevant to other models (Skyrme model, NJL).

The appendixes contain some details of the grand-spin algebra, derivation of the explicit forms of the equations of motion for fluctuations in the grand-spin basis (Appendix A), and a glossary of useful formulas with collective matrix elements (Appendix B). We also give a simple proof of equality of the soliton mass and the inertial mass parameter (Appendix C), and discuss the issue of Pauli blocking of the Dirac sea in chiral quark models (Appendix D).

## II. HEDGEHOG MODELS

In this article most of the derivations will be done in the framework of the chiral quark-meson (CQM) model of Ref. [1]. For the details, description of solutions, and the resulting phenomenology obtained with the cranking projection method, the reader is referred to Ref. [23]. The reason of choosing this particular model over other models, e.g., the Skyrme or the NJL model, is that it contains both quark and meson degrees of freedom, and formally has all essential features of a generic hedgehog model with two flavors. At the same time, it is free of the nonlinear complications of the Skyrme model, or the Dirac-sea complications of the NJL model.

### A. Soliton solutions

The Lagrangian of the model is the Gell-Mann–Lévy Lagrangian [35], with  $\psi$  denoting the quark operator, and  $\sigma$  and  $\boldsymbol{\pi}$  denoting the meson fields:

$$\mathcal{L} = \bar{\psi} [i\hat{p} + g(\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})] \psi + \frac{1}{2}(\partial^\mu \sigma)^2 + \frac{1}{2}(\partial^\mu \boldsymbol{\pi})^2 - U(\sigma, \boldsymbol{\pi}). \quad (2.1)$$

The Mexican hat potential

$$U(\sigma, \boldsymbol{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \boldsymbol{\pi}^2 - \nu^2)^2 + m_\pi^2 F_\pi \sigma, \quad (2.2)$$

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2F_\pi^2}, \quad \nu^2 = \frac{m_\sigma^2 - 3m_\pi^2}{m_\sigma^2 - m_\pi^2}$$

leads to the spontaneous breaking of the chiral symmetry in the usual way [35, 1]. Our convention for the pion decay constant is  $F_\pi = 93$  MeV. At the (time-dependent) mean-field level, only valence quarks, denoted by  $q$ , are retained in the expansion of the quark fields, and the meson fields are treated as classical,  $c$ -number fields [23] (see also Appendix D). The time-dependent equations of motion have the form

$$(h[\phi] - i\partial_t)q = 0, \quad (2.3)$$

$$-\square\phi = \frac{\delta U[\phi]}{\delta\phi} - gN_c \bar{q} M q, \quad (2.4)$$

where  $\phi = (\sigma, \boldsymbol{\pi})$  denotes the meson fields,  $M = (\beta, i\gamma_5 \boldsymbol{\tau})$  describes the quark-meson coupling, and the Dirac Hamiltonian is  $h[\phi] = -i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} - gM\phi$ . Equations (2.4) have a stationary solution of the form

$$\sigma(\mathbf{r}, t) = \sigma_h(r), \quad \boldsymbol{\pi}(\mathbf{r}, t) = \hat{\mathbf{r}}\pi_h(r), \quad q(t) = q_h(\mathbf{r})e^{-i\varepsilon t}, \quad (2.5)$$

$$q_h = \begin{pmatrix} G_h(r) \\ i\boldsymbol{\sigma} \cdot \hat{\mathbf{r}} F_h(r) \end{pmatrix} (|u \downarrow\rangle - |d \uparrow\rangle)/\sqrt{2},$$

where  $\varepsilon$  is the quark eigenvalue. For a discussion of this solution, plots of the radial functions  $\sigma_h$ ,  $\pi_h$ ,  $G_h$ , and  $F_h$ , and other details, the reader is referred to Refs. [1, 23].

### B. Hedgehog symmetries

The solution (2.5) has the hedgehog form, which breaks the spin  $\mathbf{J}$  and isospin  $\mathbf{I}$  symmetries of the Lagrangian (2.1), leaving as a good symmetry the grand spin  $\mathbf{K} = \mathbf{I} + \mathbf{J}$ . There are also two discrete symmetries which are very useful in classifying solutions and perturbations. One is parity  $\mathcal{P}$ ; the other is the ‘‘grand-reversal’’ symmetry  $\mathcal{R}$  discussed in Refs. [33, 27]. Formally,  $\mathcal{R}$  is defined as the time reversal, followed by an isorotation by angle  $\pi$  about the two-axis in isospin. Explicitly, it transforms the quark spinors and mean meson fields as follows [23]:

$$q(\mathbf{r}, t) \rightarrow \sigma_z \tau_2 q^*(\mathbf{r}, -t), \quad (2.6)$$

$$\sigma(\mathbf{r}, t) \rightarrow \sigma^*(\mathbf{r}, -t), \quad \boldsymbol{\pi}(\mathbf{r}, t) \rightarrow \boldsymbol{\pi}^*(\mathbf{r}, -t).$$

We denote the action of  $\mathcal{R}$  on an object by the superscript  $\mathcal{R}$ . The soliton solution has  $K^{\mathcal{P}\mathcal{R}} = 0^{++}$ .

## III. LINEAR RESPONSE IN HEDGEHOG MODELS

In this section the basic formalism of linear response in hedgehog models is developed. We use the equation-of-motion approach [34], which is based on solving equations of motion for small oscillation on top of the ground-state solutions. This method is equivalent to the particle-hole formalism [29], in which one introduces a quantum RPA state, quasiboson RPA phonon operators, etc. Methods such as cranking, projection, or quantization of zero modes, can be described in this framework, and have a definite quantum-mechanical interpretation. For simplicity of notation, we present our formalism in the equations-of-motion method.

### A. Equations of motion for small fluctuations

Let us consider a small oscillation problem in our system. We introduce shifts in the valence quark spinor and in the meson fields:

$$\delta q(\mathbf{r}, t) = [X(\mathbf{r})e^{-i\omega t} + Y^{\mathcal{R}}(\mathbf{r})e^{i\omega t}] e^{-i\varepsilon t}, \quad (3.1)$$

$$\delta\phi_a(\mathbf{r}, t) = Z_a(\mathbf{r})e^{-i\omega t} + Z_a^{\mathcal{R}}(\mathbf{r})e^{i\omega t},$$

where  $X$  and  $Y$  describe the shift in the valence quark spinor, and  $\delta\phi_0$  and  $\delta\phi$  are the shifts in the  $\sigma$  and  $\pi$  fields, respectively. Note that in Eqs. (3.1) the  $\mathcal{R}$  transformation has taken the place of the usual [29] complex conjugation. This is because in hedgehog systems the grand reversal replaces the usual time-reversal symmetry. According to definition (2.6), the meson shifts  $\delta\phi$  are even under grand reversal, but the quark shifts have in general both even and odd components. We linearize Eqs. (2.4) about the solitonic solution (2.5), and obtain the quark-meson RPA equations. When external perturbations are present, these equations are in general driven by a quark source  $j_q$  and a meson source  $j_\phi$ ;

$$\begin{aligned} j_q &= j_X e^{-i\omega t} + j_Y^{\mathcal{R}} e^{i\omega t}, \\ j_\phi &= j_Z e^{-i\omega t} + j_Z^{\mathcal{R}} e^{i\omega t}, \end{aligned} \quad (3.2)$$

Again, the meson source is even under  $\mathcal{R}$ , whereas the quark source has in general even- and odd- $\mathcal{R}$  components. Using the fact that  $h[\phi_h]$ ,  $M$  and  $q_h$  are even under  $\mathcal{R}$  (in fact they are  $K^{\mathcal{P}\mathcal{R}} = 0^{++}$  objects), we obtain a general form of the linear response equations for our hedgehog system:

$$\begin{aligned} (h[\phi_h] - \varepsilon) X - g \sum_a M_a q_h Z_a - \omega X &= j_X, \\ (h[\phi_h] - \varepsilon) Y - g \sum_a M_a q_h Z_a + \omega Y &= j_Y, \\ -\nabla^2 Z_a + \sum_b \left. \frac{\delta^2 U}{\delta\phi_a \delta\phi_b} \right|_{\phi=\phi_h} Z_b & \\ -g N_c (q_h^\dagger M_a X + Y^\dagger M_a q_h) - \omega^2 Z_a &= j_Z. \end{aligned} \quad (3.3)$$

Introducing auxiliary meson momentum variables  $P_a = -i\omega Z_a$ , we observe that Eqs. (3.3) can be written in the symplectic form [29]

$$\mathcal{H}\xi - \omega\Lambda\xi = j, \quad (3.4)$$

where  $\mathcal{H}$  is the RPA Hamiltonian, and  $\Lambda$  is the symplectic RPA metric, satisfying  $\Lambda^2 = 1$ . In the grand-spin basis (Appendix A),  $\mathcal{H}$  is real. Our problem (3.3) can then be written as

$$\mathcal{H} = \begin{pmatrix} N_c(h-\varepsilon) & 0 & -gN_c M q_h & 0 \\ 0 & N_c(h-\varepsilon) & -gN_c M q_h & 0 \\ -gN_c q_h^\dagger M & -gN_c q_h^\dagger M & -\nabla^2 + U'' & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (3.5)$$

$$\Lambda = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}, \quad \xi = \begin{pmatrix} X \\ Y \\ Z \\ P \end{pmatrix}, \quad j = \begin{pmatrix} N_c j_X \\ N_c j_Y \\ j_Z \\ j_P \end{pmatrix}. \quad (3.6)$$

Note the appearance of an odd- $\mathcal{R}$  momentum component

in the source  $j_P$ , which arises in some cases (cranking).

We are interested in the limit of vanishing frequency of the external perturbation,  $\omega \rightarrow 0$ . If zero modes are excited by an even- $\mathcal{R}$  perturbation (Sec. III B), then the full equations (3.3) have to be solved. Otherwise, one can set  $\omega = 0$  and deal with simplified cases. At this point the grand-reversal classification becomes very useful. Acting with  $\mathcal{R}$  on Eq. (3.4) effectively replaces  $X \leftrightarrow Y$ ,  $j_X \leftrightarrow j_Y$ ,  $Z \leftrightarrow Z$ ,  $j_Z \leftrightarrow j_Z$ ,  $P \leftrightarrow P$ , and  $j_P \leftrightarrow j_P$ . Let us introduce odd and even grand-reversal combinations:  $\delta q^\pm = X \pm Y$ ,  $j_q^\pm = j_X \pm j_Y$ , and rewrite Eq. (3.4) by adding and subtracting the first two equations. We get, for the case of an odd- $\mathcal{R}$  perturbation,

$$(h - \varepsilon)\delta q^- = j_q^-, \quad P = j_P, \quad (3.7)$$

and, for the case of an even- $\mathcal{R}$  perturbation,

$$(h - \varepsilon)\delta q^+ - 2gMq_h Z = j_q^+, \quad (3.8)$$

$$(-\nabla^2 + U'')Z - N_c g q_h^\dagger M \delta q^+ = j^Z.$$

The odd- $\mathcal{R}$  equations (3.7) involve a quark field equation, and a trivial equation for  $P$ . The even- $\mathcal{R}$  equations (3.8) involve coupled quark and meson fluctuations. Equations (3.4) or (3.7), and (3.8) are further decomposed by grand spin  $K$  and parity  $\mathcal{P}$  (Appendix A).

In models with vector mesons, such as [32], the odd- $\mathcal{R}$  equations may also involve mesonic shifts. For example, the space components of the  $\omega$  meson and the time component of the  $\rho$  meson enter into the cranking equations of motion [36].

## B. Zero modes

First consider the undriven problem (3.4), with  $j = 0$ , which determines the RPA spectrum and eigenmodes. A complication arises whenever a continuous symmetry of the Lagrangian is broken by the solitonic solution, e.g., translation, or rotational symmetry. For each broken symmetry the small fluctuation equations have a pair of zero modes [29]:  $\xi_0$ , the symmetry mode, obtained by acting with a symmetry generator on the solitonic solution, and a conjugate zero mode,  $\xi_1$ . They satisfy the equations

$$\mathcal{H}\xi_0 = 0, \quad (3.9)$$

$$\mathcal{H}\xi_1 = -i\Lambda\xi_0. \quad (3.10)$$

The remaining ‘‘physical’’ modes  $\xi_i$  satisfy the equation

$$\mathcal{H}\xi_i = \omega_i \Lambda \xi_i. \quad (3.11)$$

One can easily show that the symplectic norms satisfy the conditions

$$\begin{aligned} \xi_0^\dagger \Lambda \xi_0 &= \xi_1^\dagger \Lambda \xi_1 = 0, \quad \xi_0^\dagger \Lambda \xi_1 = -\frac{i}{4} \mathcal{M}, \\ \xi_i^\dagger \Lambda \xi_j &= \frac{1}{4} \delta_{ij} \mathcal{N}_i, \\ \xi_0^\dagger \Lambda \xi_i &= \xi_1^\dagger \Lambda \xi_i = 0, \quad i = 2, 3, \dots \end{aligned} \quad (3.12)$$

where  $\mathcal{M}$  is the appropriate inertia parameter (mass, moment of inertia) parameter, and  $\mathcal{N}_i$  are the symplectic norms of the physical modes. The factors of  $\frac{1}{4}$  are con-

ventional, and factors of  $i$  are inserted for convenience. Expanding the solution of Eq. (3.4) in RPA eigenmodes,

$$\xi = \sum_{\mu=0,1,2,\dots} c_{\mu} \xi_{\mu}, \quad (3.13)$$

introducing ‘‘charges’’:  $Q_0 = 4i\xi_0^{\dagger}j$ ,  $Q_{\mu} = 4\xi_{\mu}^{\dagger}j$ ,  $\mu = 1, 2, \dots$ , and using Eqs. (3.12), we find that

$$\omega c_1 \mathcal{M} + Q_0 = 0, \quad i\omega c_0 \mathcal{M} + Q_1 - c_1 \mathcal{M} = 0, \quad (3.14)$$

$$c_i(\omega_i - \omega) \mathcal{N}_i = Q_i.$$

We consider two cases which arise in practical applications: (1)  $Q_1 = 0$  and (2)  $Q_1 \neq 0$ ,  $Q_0 = 0$ .

### 1. Case $Q_1 = 0$

Using Eq. (3.14) we find

$$c_0 = \frac{iQ_0}{\mathcal{M}\omega^2}, \quad c_1 = -\frac{Q_0}{\mathcal{M}\omega}, \quad c_i = \frac{Q_i}{\mathcal{N}_i(\omega_i - \omega)}. \quad (3.15)$$

The second-order energy shift  $\kappa$  corresponding to a given perturbation (a ‘‘polarizability’’ is equal to  $2\kappa$ ) is given by the usual perturbation theory result

$$\kappa = 2\xi^{\dagger}j = \sum_{\mu} c_{\mu}^* Q_{\mu} = \kappa^{\text{zero}} + \kappa^{\text{phys}}, \quad (3.16)$$

$$\kappa^{\text{zero}} = -\frac{1}{2} \frac{Q_0^2}{\mathcal{M}\omega^2}, \quad \kappa^{\text{phys}} = \frac{1}{2} \sum_i \frac{Q_i^2}{\mathcal{N}_i(\omega_i - \omega)}.$$

In the limit  $\omega \rightarrow 0$ , the coefficients  $c_0$ ,  $c_1$  and the zero-mode part of  $\kappa$  diverge, as long as  $Q_0 \neq 0$ . This has a physical interpretation: for instance, in the case of translation the center of mass of the system moves, and the amplitude of this motion,  $c_0$ , as well as ‘‘velocity’’  $c_1$ , diverge. In (I) we show how this feature of the linear response formalism leads to the Thompson limit of the Compton scattering amplitude.

### 2. Case $Q_1 \neq 0$ , $Q_0 = 0$

In this case we can take the limit  $\omega \rightarrow 0$  on the outset, and from Eq. (3.14) we get

$$c_1 = \frac{Q_1}{\mathcal{M}}, \quad c_i = \frac{Q_i}{\mathcal{N}_i \omega_i}. \quad (3.17)$$

The amplitude of the symmetry mode  $c_0$  remains undetermined. The second-order energy shift is

$$\kappa = \frac{1}{2} \frac{Q_1^2}{\mathcal{M}} + \frac{1}{2} \sum_i \frac{Q_i^2}{\mathcal{N}_i \omega_i}. \quad (3.18)$$

### C. Numerical methods in presence of diverging zero modes

Numerically, the excitation of amplitude-growing zero modes (Sec. III B 1) creates special difficulties in extract-

ing the ‘‘physical’’ parts of observables, e.g., electromagnetic polarizabilities. The problem can be remedied as follows. We solve Eqs. (3.4) for a small value of  $\omega$ . Next, we project out the zero-mode part from  $\xi$ , obtaining  $\xi^{\text{phys}} = \xi - c_0 \xi_0$ , and calculate physical parts of observables. The procedure is repeated with decreasing  $\omega$ , until the results no longer change. In practice, a very high accuracy of the soliton solution, as well as the fluctuation solutions, is required for this procedure to be feasible. A better method is to project the part of the source  $j$ , which couples to the zero mode, and solve equations

$$\mathcal{H}\xi^{\text{phys}} - \omega\Lambda\xi^{\text{phys}} = j^{\text{phys}}, \quad (3.19)$$

where  $j^{\text{phys}} = j - (Q_0/\mathcal{M})\Lambda\xi_1$ , and  $\xi_1$  is obtained by solving Eq. (3.10) first. Equations (3.19) do not excite the zero mode, and directly lead to the physical part of the solution. The advantage of the method with the projected source over the direct method described previously follows from the fact that in numerical solutions to Eqs. (3.19) the admixtures of the zero mode arise only from numerical noise. Their amplitude is small, such that we can easily control numerical precision in the physical mode. Because of these admixtures, a small nonzero value of  $\omega$  should be kept as a regulator in Eqs. (3.19), and the zero-mode contamination has to be projected out after the numerical solution is found.

### D. Stability of solitons

Since in our problem  $\mathcal{H}$  and  $\Lambda$  are Hermitian, one finds that  $\mathcal{H}^2\xi_i = \omega_i^2\xi_i$  is a Hermitian eigenvalue problem. Therefore in our case  $\omega_i^2$  are real, and  $\omega_i$  can either be purely real, or purely imaginary. The modes appear in conjugated pairs  $(\xi_i, \xi_j)$ , with  $\omega_i = -\omega_j$ . If the spectrum contains an imaginary eigenvalue, we have an instability (in the Lyapunov sense [37]) of the ground-state (soliton) solution [38, 39], and of course linear response on top of an unstable system makes no sense. In Ref. [27] we have shown that the soliton of Ref. [1] is stable with respect to breathing modes, i.e., the  $K^P = 0^+$  excitations. With the explicit forms of the equations in Appendix A, stability could be checked numerically for any  $K^P$  vibrational mode. It is generally believed that the hedgehog solitons are indeed stable, although it has not been proved analytically or numerically.

### E. Cranking as linear response

Cranking [23] may be viewed as a linear response. In a frame isorotating with a small angular velocity  $\hat{\lambda}$ , we discover equations of the form (3.10), with  $\omega = 0$  and  $j = -i\lambda\Lambda\xi_0(\hat{\lambda})$ . In this case  $\xi_0(\hat{\lambda})$  is the symmetry mode obtained by acting on the soliton fields with the generator of isorotation about the axis  $\hat{\lambda}$ :

$$\xi_0(\hat{\lambda}) = \frac{1}{2} \begin{pmatrix} (i/2) \tau \cdot \hat{\lambda} q_h \\ (i/2) \tau \cdot \hat{\lambda} q_h \\ -\hat{\lambda} \times \pi_h \\ 0 \end{pmatrix}. \quad (3.20)$$

Next, we have to find the conjugated mode, by solving the second of Eqs. (3.10). We notice, that this is an odd- $R$  case (3.7). We immediately get  $\mathbf{P} = \frac{1}{2}\boldsymbol{\lambda} \times \boldsymbol{\pi}_h$ . For the quark shift,  $\delta q_{cr}$ , a differential equation of the form (3.7) is solved [23]. The problem is of the type discussed in Sec. III B 2, where  $\mathcal{M}$  is the moment of inertia,  $\Theta$ , and the ‘‘charges’’ are  $\mathbf{Q}_1 = \boldsymbol{\lambda}\Theta$ ,  $Q_\mu = 0$  for  $\mu \neq 1$ . The second-order energy shift is  $\kappa = \frac{1}{2}\boldsymbol{\lambda}^2\Theta$ . Explicitly, one finds

$$\begin{aligned}\Theta &= \Theta_m + \Theta_q, \\ \Theta_m &= \int d^3x (\widehat{\boldsymbol{\lambda}} \times \boldsymbol{\pi}_h)^2 = (8\pi/3) \int dr r^2 \pi_h^2, \\ \Theta_q &= 2 \int d^3x \delta q_{cr}^\dagger \widehat{\boldsymbol{\lambda}} \cdot \boldsymbol{\tau} q_h\end{aligned}\quad (3.21)$$

### F. Quantization

The simplest approach to quantization via cranking is to recognize that in the frame isorotating with velocity  $\boldsymbol{\lambda}$ , in which we solve the cranking equations of motion (Sec. III E), we still have the freedom of (iso)rotating the soliton by an arbitrary (time-independent) angle. This is an example of the freedom of choice in the  $c_0$  coefficient in Sec. III B 2, which in this case corresponds to three Euler angles, or, in the commonly used Cayley-Klein notation [8], to the matrix  $B = b_0 + \mathbf{b} \cdot \boldsymbol{\tau}$  [23]. In our mean-field approach, the corresponding fields carry these (time-independent)  $B$  matrices, and in the rotating frame they assume the form

$$\sigma \rightarrow \sigma, \quad \boldsymbol{\pi} \rightarrow B\boldsymbol{\pi}B^\dagger, \quad q \rightarrow Bq. \quad (3.22)$$

Matrix  $B$  plays the role of coordinate variables conjugated to  $\boldsymbol{\lambda}$ , which upon quantization becomes a differential operator [8, 23]. The quantization is straightforwardly implemented in two steps. (1) One identifies the collective spin and isospin operators, as done in Ref. [23]. Then

$$\boldsymbol{\lambda}\Theta = \mathbf{J}, \quad I_a = c_{ab}J_b, \quad (3.23)$$

where  $J$  and  $I$  are the spin and isospin operators, satisfying appropriate commutation relations, and  $c_{ab}$ , defined in Appendix B, is the transformation matrix from the body-fixed to the laboratory frame [29]. (2) Corresponding collective wave functions are introduced. Expectation values of operators are calculated by first identifying in the semiclassical expression for an operator its collective part (dependent on  $\boldsymbol{\lambda}$ ,  $c_{ab}$ , etc.), and an intrinsic part (dependent on the meson and quark fields  $\sigma$ ,  $\boldsymbol{\pi}$ ,  $q$ ). Then, the matrix element factorizes into a collective matrix element in the wave functions of Appendix B [this is an integral over the collective coordinates, viz. Euler angles, or  $(b_0, \mathbf{b})$ ], and an intrinsic matrix element, which is a space integral over the quark and meson fields. For details, see Ref. [23].

### G. External perturbations

The quark and meson field profiles in Eq. (3.22) are in general not equal to the hedgehog profiles. We have

demonstrated in Sec. III E that the quarks develop shifts upon cranking. If some other (external) interaction is present, then the profiles are additionally shifted. These shifts are obtained by solving the linear response equations, as described in Sec. III. We introduce a resolvent for the  $\mathcal{H} - \omega\Lambda$  operator in Eq. (3.4) (RPA propagator) and solve formally Eq. (3.4), obtaining

$$\xi = \mathcal{G}j, \quad \mathcal{G} = (\mathcal{H} - \omega\Lambda)^{-1}. \quad (3.24)$$

In the presence of cranking and some other external perturbation, we have

$$\xi = \xi_{cr} + \xi_{ext} = \mathcal{G}(j_{cr} + j_{ext}), \quad (3.25)$$

where subscripts cr and ext refer to cranking and an external perturbation, respectively. The second-order energy shift corresponding to a perturbation can be written as

$$\kappa = 2\xi^\dagger j = 2j^\dagger \mathcal{G}j. \quad (3.26)$$

The difference between this expression and the generic expression (3.16) is that in the present case the source carries collective degrees of freedom,  $j = j^{\text{coll}}j^{\text{intr}}$ . Thus, the matrix element of  $\kappa$  in a baryon state  $|b\rangle$  is (see the example in Sec. IV A)

$$\begin{aligned}\kappa^b &= 2\langle \text{coll} | j^{\text{coll}} j^{\text{coll}} | \text{coll} \rangle \\ &\quad \times \int d^3x d^3x' j^{\text{intr}\dagger}(x) \mathcal{G}(x, x') j^{\text{intr}}(x'), \\ &= 2\langle \text{coll} | j^{\text{coll}} j^{\text{coll}} | \text{coll} \rangle \int d^3x \xi^{\text{intr}\dagger}(x) j^{\text{intr}}(x),\end{aligned}\quad (3.27)$$

where  $|\text{coll}\rangle$  represents the collective wave function (Appendix B) associated with the baryon state  $|b\rangle$ .

It is possible to have isospin-dependent effects in linear response of the nucleon. For example, if the external interaction has  $K^P = 1^+$  (the same quantum numbers as in cranking) we pick up cross terms between cranking and the external perturbation (see the example in Sec. IV B):

$$\begin{aligned}\kappa^b &= 2\langle \text{coll} | j_{cr}^{\text{coll}} j_{ext}^{\text{coll}} | \text{coll} \rangle \\ &\quad \times \int d^3x d^3x' j_{cr}^{\text{intr}\dagger}(x) \mathcal{G}(x, x') j_{ext}^{\text{intr}}(x') + \text{H.c.} \\ &= 2\langle \text{coll} | j_{cr}^{\text{coll}} j_{ext}^{\text{coll}} | \text{coll} \rangle \int d^3x \xi_{cr}^{\text{intr}\dagger}(x) j_{ext}^{\text{intr}}(x) + \text{H.c.}\end{aligned}\quad (3.28)$$

Expressions (3.27), and (3.28) are just second-order perturbation results. We may formally continue to higher order in perturbation theory, which leads to chains of the form

$$\begin{aligned}\kappa_{i_1, \dots, i_n} &= 2\langle \text{coll} | j_{i_1}^{\text{coll}} \mathcal{V}_{i_2}^{\text{coll}} \dots j_{i_n}^{\text{coll}} | \text{coll} \rangle \\ &\quad \times \int d^3x_1 \dots d^3x_n j_{i_1}^{\text{intr}\dagger}(x_1) \mathcal{G}(x_1, x_2) \mathcal{V}_{i_2}(x_2) \\ &\quad \times \mathcal{G}(x_2, x_3) \dots \mathcal{V}_{i_{n-1}}(x_{n-1}) \mathcal{G}(x_{n-1}, x_n) \\ &\quad \times j_{i_n}^{\text{intr}}(x_n),\end{aligned}\quad (3.29)$$

where  $\mathcal{V}_{i_k}$  is interaction of  $k$ th type. The total energy shift is the sum over all possible orderings of  $(i_1, \dots, i_n)$

in (3.29). Because the ground state has  $K^{\mathcal{P}} = 0^+$ , the matrix element in Eq. (3.29) is nonzero only if one can compose the  $K^{\mathcal{P}}$  quantum numbers of  $j_{i_1}, \nu_{i_2}, \dots, j_{i_n}$  to  $K^{\mathcal{P}} = 0^+$ . In (I) we show an application of Eq. (3.29) with two RPA propagators in the analysis of the neutron-proton splitting of electromagnetic polarizabilities. In Sec. V we discuss whether perturbation theory is consistent with  $N_c$  counting, which is our basic principle in organizing the perturbation expansion in hedgehog models.

#### IV. SIMPLE EXAMPLES

In this section we give some simple application of the described formalism. A more advanced and physically important case of electromagnetic polarizabilities is given in (I).

##### A. $N$ - $\Delta$ mass splitting

As an illustration of the application of Eq. (3.27), consider the  $N$ - $\Delta$  mass splitting. In this case  $\kappa^b$  is the energy shift of the baryon  $|b\rangle$  due to the cranking perturbation. From Eqs. (3.23), (3.21), and (3.27) we obtain immediately the usual expression for the  $N$ - $\Delta$  mass splitting:

$$M_{\Delta} - M_N = \frac{1}{2}(\langle \Delta | \lambda^2 | \Delta \rangle - \langle N | \lambda^2 | N \rangle) \Theta = \frac{3}{2\Theta}. \quad (4.1)$$

##### B. Hadronic $p$ - $n$ mass splitting

As an example of an isospin-dependent effect, consider the neutron-proton mass difference due to the difference of the up- and down-quark masses. The perturbation in the Lagrangian has the form  $\mathcal{L}_m = \frac{1}{2}(m_d - m_u)\bar{\psi}\tau_3\psi$ . It has  $K^{\mathcal{P}\mathcal{R}} = 1^{+-}$ , exactly as cranking; hence, a mixed perturbation of the form (3.28) appears. Passing to an isorotating frame, we find the source corresponding to the quark mass splitting, which arises in Eqs. (3.7):  $j_m = \frac{1}{2}(m_d - m_u)N_c\gamma_0\mathbf{c} \cdot \boldsymbol{\tau}q_h$ , where  $\mathbf{c}$  is defined in Appendix B. Since we have already solved the cranking equation, we do not have to solve the new equation with source  $j_m$ . We simply calculate the overlap of  $j_m$  with the shift in the fields due to cranking,  $\delta q_{cr}$ , according to Eq. (3.28). Using the fact that  $\langle N | \boldsymbol{\lambda} \cdot \mathbf{c} | N \rangle = -\langle N | I_3 | N \rangle$ , we obtain the following expression for the hadronic splitting of the neutron and proton masses:

$$\begin{aligned} (M_n - M_p)^{\text{hadr}} &= (m_d - m_u) \int d^3x d^3x' [\langle n | j_m^\dagger \delta q_{cr} | n \rangle \\ &\quad - (n \leftrightarrow p)] \\ &= \frac{m_d - m_u}{\Theta} \int d^3x j_m^{\text{intr}\dagger} \delta q_{cr}^{\text{intr}}. \end{aligned} \quad (4.2)$$

The numerical value, obtained for the solution of Ref. [23], gives  $(M_n - M_p)^{\text{hadr}} = 0.4 \times (m_d - m_u)$ , which for typical values of  $(m_d - m_u)$  gives a number around 2 MeV. The electromagnetic mass difference can also be studied in hedgehog models [40].

#### V. $N_c$ COUNTING

The basic organizational principle behind hedgehog models is the  $1/N_c$  expansion of QCD [41–43]. In the  $N_c \rightarrow \infty$  limit, masses of baryons diverge as  $N_c$ , and can be calculated using mean-field theory [42]. It should be noted that the assumption of the spin-isospin correlated wave function, which is essential in hedgehog models, does not follow from the large- $N_c$  limit alone—it is an additional assumption of the hedgehog approach. By analogy with nuclear physics, in systems with many nucleons we may have nuclei with intrinsic deformations, but we may also have spherically symmetric nuclei, and it is the dynamics which determines whether or not the wave function is deformed. In hedgehog models the hedgehog wave function is assumed to be deformed in the spin-isospin space, and the nucleon and the  $\Delta$  masses, which are of the order  $N_c$ , are degenerate in the leading- $N_c$  order.

When cranking is used, these masses split as  $\sim N_c^{-1}$ . In fact, cranking becomes an exact projection method in the large- $N_c$  limit, since it may be viewed as a Peierls-Yoccoz projection with  $\delta$ -function overlaps between rotated wave functions [29]. Thus we obtain the hedgehog result for the mass splitting, Eq. (4.1). It would not be consistent, however, to conclude that the nucleon or  $\Delta$  masses individually are given by the hedgehog soliton mass plus the cranking piece. There are other effects (center-of-mass correction, centrifugal stretching, etc.) which enter at the same level as the cranking term. Also, the effective Lagrangian may be supplemented by subleading terms in  $N_c$ , which we did not have to include to obtain the leading piece in the hedgehog mass. Therefore, it is useless to write down  $M_J = M_h + J(J+1)/(2\Theta) + O(N_c^{-1})$ , since the last term, which we do not calculate, enters at the same level as the cranking term. We can only trust the leading piece,  $M_J = M_h + O(N_c^{-1})$ , and, in order to maintain consistency with the  $N_c$  counting, the mass formula should not be “improved” by adding the cranking term. The mentioned effects of center-of-mass corrections, centrifugal stretching, etc., are at the leading level the same for the nucleon and for the  $\Delta$ , therefore for the  $N - \Delta$  mass splitting we get the formula  $M_{\Delta} - M_N = 3/(2\Theta) + O(N_c^{-2})$ .

The prescription, which we tried to illustrate above, is that with semiclassical methods we can only get the leading- $N_c$  term for a given observable. The power of  $N_c$  varies, depending on the quantity we are investigating. The same is true for the calculation of polarizabilities, described in this paper. We can easily obtain the  $N_c$  behavior of various terms in Eqs. (3.27), (3.28), and (3.29), but only the leading- $N_c$  piece corresponding to a particular polarizability should be retained. As an illustration, consider the electric polarizability of the nucleon, discussed extensively in [30] and in (I). The electric field polarizes the hedgehog. The electric charge of the quark has an isoscalar component, of order  $N_c^{-1}$ , and isovector component, of order 1. We immediately see from Eq. (3.27) that the leading part of the electric polarizability of the nucleon is obtained from interactions with two

isovector sources, and the term with two isoscalar sources is two powers of  $N_c$  suppressed. The nondispersive seagull effects also enter at the level of  $N_c$  (I); hence, the nucleon polarizability goes as  $N_c$ . Quite analogously to the problem of the  $N - \Delta$  mass splitting, the neutron-proton splitting of the electric polarizability is a  $N_c^{-1}$  effect (I), and we can calculate it consistently only to this order.

In principle, one might try to perform a calculation which consistently takes into account the subleading pieces. The appropriate scheme would be the Kerman-Klein method [44], but its application would involve a complicated fully quantum-mechanical calculation.

## VI. OTHER MODELS

Techniques described in this paper are applicable to other model after straightforward modifications. In the Skyrme model, the described RPA method involves fluctuations of the meson fields which do not satisfy the nonlinear constraint for the  $\sigma$  and  $\pi$  field operators. This linearization may be viewed as an approximation to the fully nonlinear dynamics. The RPA dynamics, obviously, involves mesons only, and the higher-derivative terms are manifest in the equations of motion for the fluctuations.

In the case of the (partly bosonized) NJL model [45], the mesonic potential has the simple form  $\frac{1}{2}\mu^2(\sigma^2 + \pi^2)$ . The sea quarks are present explicitly, and the number of quark equations is infinite. Standard methods of solving these equations numerically may encounter problems for the case when the translational zero mode is excited, since extremely good accuracy is necessary in this case.

## VII. CONCLUSION

We have presented the linear response method in hedgehog soliton models. We have shown that the method is consistent with the basic philosophy of these models, namely, the  $1/N_c$  expansion, if its application is restricted to obtaining the leading- $N_c$  order of a given quantity. We have discussed many technical points which are encountered in practical calculations, especially the treatment of zero modes, which create special problems. Appropriate equations of motion have been classified according to hedgehog symmetries, and derived explicitly for the model of Ref. [1]. Our method, after straightforward modifications, is directly applicable to other hedgehog models. A physical application of the approach is described in the preceding paper (I), where we study the electromagnetic polarizabilities of the nucleon.

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## APPENDIX A: EQUATIONS OF MOTION FOR SMALL FLUCTUATIONS IN THE GRAND-SPIN BASIS

We compose the basis of Dirac spinors with good  $K$  quantum numbers using the coupling scheme in which the isospin,  $I = \frac{1}{2}$ , and spin  $S = \frac{1}{2}$ , are first coupled to a quantum number  $\Lambda$ , and then orbital angular momentum  $L$  and  $\Lambda$  are coupled to  $K$ . Since there is no confusion concerning the value of  $K$  or  $K_3$ , we use the notation

$$|L, \Lambda\rangle = |K, (L, \Lambda(I = \frac{1}{2}, S = \frac{1}{2})), K_3\rangle. \quad (\text{A1})$$

States with parity  $\mathcal{P} = (-)^K$  [ $\mathcal{P} = -(-)^K$ ] are called normal [abnormal] parity states. The basis of Dirac spinors is

$$q^{L,\Lambda} = \begin{pmatrix} G^{L,\Lambda}(r) \\ i\boldsymbol{\sigma} \cdot \hat{\mathbf{r}} F^{L,\Lambda}(r) \end{pmatrix} |L, \Lambda\rangle. \quad (\text{A2})$$

Spinors  $X$  and  $Y$  are expressed in states (A2). The quark sources are decomposed into  $(L, \Lambda)$  components:

$$j^{L,\Lambda} = \begin{pmatrix} j_G^{L,\Lambda}(r) \\ i\boldsymbol{\sigma} \cdot \hat{\mathbf{r}} j_F^{L,\Lambda}(r) \end{pmatrix} |L, \Lambda\rangle. \quad (\text{A3})$$

Tables I – III list the matrix elements which arise in deriving the quark parts of perturbation equations. It is clear from Table III that unless  $K = 0$ , the kinetic term mixes the  $\Lambda = 0$  and  $\Lambda = 1$  components of the  $L = K$  states (normal parity case). Diagonalization is made through the substitution

$$G^a = \sqrt{\frac{K+1}{2K+1}} G^{K,0} - \sqrt{\frac{K}{2K+1}} G^{K,1}, \quad (\text{A4})$$

$$G^b = \sqrt{\frac{K}{2K+1}} G^{K,0} + \sqrt{\frac{K+1}{2K+1}} G^{K,1},$$

and similarly for the  $F$  components, and the sources.

The basis for the meson fluctuations is composed by coupling isospin to  $L$ . For a given value of  $K$ , the  $\sigma$  and

TABLE I. Matrix elements of  $\tau \cdot \hat{\mathbf{r}}$ .

	$ K, 0\rangle$	$ K, 1\rangle$	$ K-1, 1\rangle$	$ K+1, 1\rangle$ <sup>a</sup>
$\langle K, 0 $	0	0	$\sqrt{\frac{K}{2K+1}}$	$-\sqrt{\frac{K+1}{2K+1}}$
$\langle K, 1 $	0	0	$-\sqrt{\frac{K+1}{2K+1}}$	$-\sqrt{\frac{K}{2K+1}}$
$\langle K-1, 1 $	$\sqrt{\frac{K}{2K+1}}$	$-\sqrt{\frac{K+1}{2K+1}}$	0	0
$\langle K+1, 1 $	$-\sqrt{\frac{K+1}{2K+1}}$	$-\sqrt{\frac{K}{2K+1}}$	0	0

TABLE II. Matrix elements of  $\sigma \cdot \hat{r}$ 

	$ K, 0\rangle$	$ K, 1\rangle$	$ K-1, 1\rangle$	$ K+1, 1\rangle$
$\langle K, 0 $	0	0	$-\sqrt{\frac{K}{2K+1}}$	$\sqrt{\frac{K+1}{2K+1}}$
$\langle K, 1 $	0	0	$-\sqrt{\frac{K+1}{2K+1}}$	$-\sqrt{\frac{K}{2K+1}}$
$\langle K-1, 1 $	$-\sqrt{\frac{K}{2K+1}}$	$-\sqrt{\frac{K+1}{2K+1}}$	0	0
$\langle K+1, 1 $	$\sqrt{\frac{K+1}{2K+1}}$	$-\sqrt{\frac{K}{2K+1}}$	0	0

$\pi$  fluctuations can be expressed through functions

$$\sigma^L(r)|K, (L, 0), K_3\rangle, \quad \pi^L(r)|K, (L, 1), K_3\rangle, \quad (\text{A5})$$

Obviously,  $L = K$  for  $\sigma$ , and  $L = K-1, K, K+1$  for  $\pi$ , such that, for a given  $K$  and  $K_3$ ,

$$\delta\sigma = \sigma^K(r)|K, (L, 0), K_3\rangle, \quad (\text{A6})$$

$$\delta\pi = \sum_{L=K-1, K, K+1} \pi^L(r)|K, (L, 1), K_3\rangle.$$

Using standard Racah algebra, it is straightforward to derive the general equations (3.4) for a given  $K$  perturbation. In the notation of this appendix,  $G_{\{X,Y\}}^a$ , etc., correspond to the  $X$  and  $Y$  spinors from Eq. (3.4), and  $G_{\{X+Y\}}^a = G_X^a + G_Y^a$ , etc. The functions describing meson fluctuation,  $\sigma^L, \pi^L$ , have the meaning of the  $Z$  functions of Eq. (3.4).

For a normal parity equation we get

$$\begin{aligned} \partial_r G_{\{X,Y\}}^a &= \frac{K}{r} G_{\{X,Y\}}^a + (g\sigma_h - \varepsilon \mp \omega) F_{\{X,Y\}}^a + g\pi_h \left( -\frac{1}{2K+1} G_{\{X,Y\}}^a - \frac{2\sqrt{K(K+1)}}{2K+1} G_{\{X,Y\}}^b \right) \\ &\quad + g \left( \sqrt{\frac{K+1}{2K+1}} F_h \sigma^K + G_h \pi^{K+1} \right) - j_{F,\{X,Y\}}^a, \\ \partial_r G_{\{X,Y\}}^b &= -\frac{K+1}{r} G_{\{X,Y\}}^b + (g\sigma_h - \varepsilon \mp \omega) F_{\{X,Y\}}^b + g\pi_h \left( -\frac{2\sqrt{K(K+1)}}{2K+1} G_{\{X,Y\}}^a + \frac{1}{2K+1} G_{\{X,Y\}}^b \right) \\ &\quad + g \left( \sqrt{\frac{K}{2K+1}} F_h \sigma^K - G_h \pi^{K-1} \right) - j_{F,\{X,Y\}}^b, \\ \partial_r F_{\{X,Y\}}^a &= -\frac{K+2}{r} F_{\{X,Y\}}^a + (g\sigma_h + \varepsilon \pm \omega) G_{\{X,Y\}}^a + g\pi_h \left( \frac{1}{2K+1} F_{\{X,Y\}}^a + \frac{2\sqrt{K(K+1)}}{2K+1} F_{\{X,Y\}}^b \right) \\ &\quad + g \left( \sqrt{\frac{K+1}{2K+1}} G_h \sigma^K - F_h \pi^{K+1} \right) + j_{G,\{X,Y\}}^a, \\ \partial_r F_{\{X,Y\}}^b &= \frac{K-1}{r} F_{\{X,Y\}}^b + (g\sigma_h + \varepsilon \pm \omega) G_{\{X,Y\}}^b + g\pi_h \left( \frac{2\sqrt{K(K+1)}}{2K+1} F_{\{X,Y\}}^a - \frac{1}{2K+1} F_{\{X,Y\}}^b \right) \\ &\quad + g \left( \sqrt{\frac{K}{2K+1}} G_h \sigma^K + F_h \pi^{K-1} \right) + j_{G,\{X,Y\}}^b, \\ \left( \partial_r^2 + \frac{2}{r} \partial_r - \frac{(K-1)K}{r^2} \right) \pi^{K-1} &= \lambda^2 (\sigma_h^2 + \pi_h^2 - \nu^2 - \omega^2) \pi^{K-1} \\ &\quad + 2\lambda^2 \left( \frac{K^2}{2K+1} \pi_h^2 \pi^{K-1} + \sqrt{\frac{K}{2K+1}} \sigma_h \pi_h \sigma^K - \frac{\sqrt{K(K+1)}}{2K+1} \pi_h^2 \pi^{K+1} \right) \\ &\quad - gN_c (F_h G_{\{X+Y\}}^b + G_h F_{\{X+Y\}}^b) + j_\pi^{K-1}, \\ \left( \partial_r^2 + \frac{2}{r} \partial_r - \frac{K(K+1)}{r^2} \right) \sigma^K &= \lambda^2 (\sigma_h^2 + \pi_h^2 - \nu^2 - \omega^2) \sigma^K \\ &\quad + 2\lambda^2 \left( \sqrt{\frac{K}{2K+1}} \sigma_h \pi_h \pi^{K-1} + \sigma_h^2 \sigma^K - \sqrt{\frac{K+1}{2K+1}} \sigma_h \pi_h \pi^{K+1} \right) \end{aligned} \quad (\text{A7})$$



$$\begin{aligned}
& -gN_c \left[ G_h \left( \sqrt{\frac{K+1}{2K+1}} G_{\{X+Y\}}^a + \sqrt{\frac{K}{2K+1}} G_{\{X+Y\}}^b \right) \right. \\
& \quad \left. - F_h \left( \sqrt{\frac{K+1}{2K+1}} F_{\{X+Y\}}^a + \sqrt{\frac{K}{2K+1}} F_{\{X+Y\}}^b \right) \right] + j_\sigma^K, \tag{A8}
\end{aligned}$$

$$\begin{aligned}
\left( \partial_r^2 + \frac{2}{r} \partial_r - \frac{(K+1)(K+2)}{r^2} \right) \pi^{K+1} &= \lambda^2 (\sigma_h^2 + \pi_h^2 - \nu^2 - \omega^2) \pi^{K+1} \\
&+ 2\lambda^2 \left( -\frac{\sqrt{K(K+1)}}{2K+1} \pi_h^2 \pi^{K-1} - \sqrt{\frac{K+1}{2K+1}} \sigma_h \pi_h \sigma^K - \frac{K+1}{2K+1} \pi_h^2 \pi^{K+1} \right) \\
&- gN_c (-F_h G_{\{X+Y\}}^a - G_h F_{\{X+Y\}}^a) + j_\pi^{K+1}.
\end{aligned}$$

The abnormal parity equations have the form

$$\begin{aligned}
\partial_r G_{\{X,Y\}}^{K-1,1} &= \frac{K-1}{r} G_{\{X,Y\}}^{K-1,1} + (g\sigma_h - \varepsilon \mp \omega) F_{\{X,Y\}}^{K-1,1} + g\pi_h \left( \frac{1}{2K+1} G_{\{X,Y\}}^{K-1,1} + \frac{2\sqrt{K(K+1)}}{2K+1} G_{\{X,Y\}}^{K+1,1} \right) \\
&\quad - g\sqrt{\frac{K+1}{2K+1}} G_h \pi^K - j_{F,\{X,Y\}}^{K-1,1}, \\
\partial_r G_{\{X,Y\}}^{K+1,1} &= -\frac{K+2}{r} G_{\{X,Y\}}^{K+1,1} + (g\sigma_h - \varepsilon \mp \omega) F_{\{X,Y\}}^{K+1,1} + g\pi_h \left( \frac{2\sqrt{K(K+1)}}{2K+1} G_{\{X,Y\}}^{K-1,1} - \frac{1}{2K+1} G_{\{X,Y\}}^{K+1,1} \right) \\
&\quad - g\sqrt{\frac{K}{2K+1}} G_h \pi^K - j_{F,\{X,Y\}}^{K+1,1}, \tag{A9}
\end{aligned}$$

$$\begin{aligned}
\partial_r F_{\{X,Y\}}^{K-1,1} &= -\frac{K+1}{r} F_{\{X,Y\}}^{K-1,1} + (g\sigma_h + \varepsilon \pm \omega) G_{\{X,Y\}}^{K-1,1} + g\pi_h \left( -\frac{1}{2K+1} F_{\{X,Y\}}^{K-1,1} - \frac{2\sqrt{K(K+1)}}{2K+1} F_{\{X,Y\}}^{K+1,1} \right) \\
&\quad + g\sqrt{\frac{K+1}{2K+1}} F_h \pi^K + j_{G,\{X,Y\}}^{K-1,1}, \\
\partial_r F_{\{X,Y\}}^{K+1,1} &= \frac{K}{r} F_{\{X,Y\}}^{K+1,1} + (g\sigma_h + \varepsilon \pm \omega) G_{\{X,Y\}}^{K+1,1} + g\pi_h \left( -\frac{2\sqrt{K(K+1)}}{2K+1} F_{\{X,Y\}}^{K-1,1} + \frac{1}{2K+1} F_{\{X,Y\}}^{K+1,1} \right) \\
&\quad + g\sqrt{\frac{K}{2K+1}} F_h \pi^K + j_{G,\{X,Y\}}^{K+1,1},
\end{aligned}$$

$$\begin{aligned}
\left( \partial_r^2 + \frac{2}{r} \partial_r - \frac{K(K+1)}{r^2} \right) \pi^K &= \lambda^2 (\sigma_h^2 + \pi_h^2 - \nu^2 - \omega^2) \pi^K \\
&- gN_c \left[ G_h \left( \sqrt{\frac{K+1}{2K+1}} F_{\{X+Y\}}^{K-1,1} + \sqrt{\frac{K}{2K+1}} F_{\{X+Y\}}^{K+1,1} \right) + F_h \left( \sqrt{\frac{K+1}{2K+1}} G_{\{X+Y\}}^{K-1,1} + \sqrt{\frac{K}{2K+1}} G_{\{X+Y\}}^{K+1,1} \right) \right] + j_\pi^K. \tag{A10}
\end{aligned}$$

For the  $\omega = 0$ , odd- $\mathcal{R}$  case, meson fluctuations vanish, and appropriate equations have the form of Eqs. (A7) and (A9), with the meson fluctuations set to zero. The  $X$  and  $Y$  equations can be combined to a single equation of the form (3.7).

In the case of an even- $\mathcal{R}$  source which does not excite a zero mode (case  $Q_0 = 0$  in Sec. III B 1), we can set  $\omega = 0$ , in the above equations. We can combine the  $X$  and  $Y$  equations, and obtain the form (3.8). If the zero mode is excited ( $Q_0 \neq 0$ ), we have to solve full equations (A7) and (A8), or (A9) and (A10), depending on parity.

For the special case of  $K = 0$ ,  $G^a = G^{0,0}$ ,  $G^b = 0$ , etc., and only equations for the  $a$  components in Eqs. (A7) remain. Fields with negative (i.e.,  $K - 1$ ) superscripts, and equations for these fields are eliminated.

The boundary conditions in Eqs. (A7)–(A10) are such that the solutions are everywhere finite. At the origin, radial derivatives of  $S$ -wave fields vanish, and the values of higher- $L$  fields vanish. At  $r \rightarrow \infty$ , the appropriate boundary conditions follow from solutions of the equations in the asymptotic region.

TABLE III. Matrix elements of  $\sigma \cdot L$ .

	$ K, 0\rangle$	$ K, 1\rangle$	$ K-1, 1\rangle$	$ K+1, 1\rangle$
$\langle K, 0 $	0	$-\sqrt{K(K+1)}$	0	0
$\langle K, 1 $	$-\sqrt{K(K+1)}$	-1	0	0
$\langle K-1, 1 $	0	0	$K-1$	0
$\langle K+1, 1 $	0	0	0	$-K-2$

### APPENDIX B: COLLECTIVE MATRIX ELEMENTS

Suppose a space rotation  $R$  is described by Euler angles  $\alpha$ ,  $\beta$ , and  $\gamma$ :

$$R = e^{-i\alpha J_z} e^{-i\beta J_y} e^{-i\gamma J_z}. \quad (\text{B1})$$

Then, matrix  $B$  from Sec. III F is given by

$$B = e^{i\gamma\tau_3/2} e^{i\beta\tau_2/2} e^{i\alpha\tau_3/2}. \quad (\text{B2})$$

The matrix transforming from the body-fixed to laboratory frame,  $c_{ab}$ , is defined as

$$c_{ab} = \frac{1}{2} \text{Tr}[\tau_a B \tau_b B^\dagger] = D_{ba}^1(\alpha, \beta, \gamma), \quad (\text{B3})$$

where the first (second) subscript in the Wigner  $D$ -matrix is connected to the spin (isospin) space. It follows that  $\sum_b c_{ab} J_b = -I_a$ . The spin operator and the matrix  $c$  commute:  $[c_{ab}, J_k] = 0$ . We also introduce a vector  $c$  defined as

$$c = \frac{1}{2} \text{Tr}[\tau_3 B \tau B^\dagger], \quad c_\mu = D_{\mu 0}^1. \quad (\text{B4})$$

The collective baryon states with spin  $J$ , isospin  $I = J$ , and projections  $m$  and  $I_3$  are

$$|J = I; m, I_3\rangle = \sqrt{\frac{2J+1}{8\pi^2}} D_{m, -I_3}^J. \quad (\text{B5})$$

In formulas below we do not display  $m$  or  $I_3$  in labels of the states, and use the notation  $|N\rangle = |\frac{1}{2}; m, I_3\rangle$ , and  $|\Delta\rangle = |\frac{3}{2}; m, I_3\rangle$ . The following useful formulas can be easily derived (no implicit summation over repeated indices):

$$(c_\mu)^* c_\mu = \frac{1}{3} + \left(\frac{2}{3} - \mu^2\right) D_{00}^2, \quad \sum_\mu (c_\mu)^* c_\mu = 1, \quad (\text{B6})$$

from which follows that

$$\begin{aligned} \langle N | (c_\mu)^* c_\mu | N \rangle &= \frac{1}{3}, \quad \text{any } \mu, \\ \langle \Delta | (c_\mu)^* c_\mu | \Delta \rangle &= \frac{1}{3} + \frac{2 - \mu^2}{5} \begin{cases} +1, & |m| = |I_3|, \\ -1; & |m| \neq |I_3|, \end{cases} \quad (\text{B7}) \\ \langle N | (c_\mu)^* c_\mu | \Delta \rangle &= \frac{\sqrt{2}(\frac{2}{3} - \mu^2)}{5} \begin{cases} +1, & m = I_3, \\ -1, & m = -I_3. \end{cases} \end{aligned}$$

One also finds

$$\langle N | [(J_\mu)^* c_\mu + (c_\mu)^* J_\mu] | N \rangle = -\frac{2}{3} I_3, \quad \text{any } \mu. \quad (\text{B8})$$

One also derives

$$\langle N | c_0 | \Delta \rangle = \frac{\sqrt{2}}{3}. \quad (\text{B9})$$

For our analysis of the  $\Delta$  states in hadronic loops in (I) the following formulas are important:

$$\sum_{\mu, m', I'_3} \langle N | (c_\mu)^* | N; m', I'_3 \rangle \langle N; m', I'_3 | c_\mu | N \rangle = \frac{1}{3}, \quad (\text{B10})$$

$$\sum_{\mu, m', I'_3} \langle N | (c_\mu)^* | \Delta; m', I'_3 \rangle \langle \Delta; m', I'_3 | c_\mu | N \rangle = \frac{2}{3}.$$

The sum of the above formulas gives unity, in accordance to the sum rule (B6).

### APPENDIX C: EQUALITY OF INERTIAL AND SOLITON MASSES

For the case of translations, the inertia parameter  $\mathcal{M}$  is equal to the soliton mass  $M_{\text{sol}}$ . This result, required by Lorentz invariance, can be verified explicitly as follows: Consider a boost in the  $z$  direction, with small velocity  $v$ . The fields transform as

$$\phi \rightarrow \phi_h(\mathbf{r} - v\mathbf{t}), \quad (\text{C1})$$

$$e^{-i\epsilon t} \rightarrow e^{-i\epsilon(t-vz) + \frac{1}{2}v\alpha_z} q_h(\mathbf{r} - v\mathbf{t}),$$

which lead to the following shifts linear in the velocity:

$$\delta\phi = -vt\partial_z\phi_h, \quad (\text{C2})$$

$$\delta q_h = v e^{-i\epsilon t} (i\epsilon z + \frac{1}{2}\alpha_z - t\partial_z) q_h.$$

Using identities  $[h, z] = -i\alpha_z$  and  $\{h, \alpha_z\} = -2i\partial_z$ , we easily derive the equation

$$[h - \epsilon](-\epsilon z + \frac{1}{2}i\alpha_z) q_h = \partial_z q_h. \quad (\text{C3})$$

After integrating by parts we get the expression for the energy shift of a moving soliton:

$$\delta\mathcal{E} = \frac{1}{2}v^2 \left( \frac{1}{3}T_q + \frac{2}{3}T_\phi + N_c\epsilon \right), \quad (\text{C4})$$

where  $T_q$  and  $T_\phi$  are kinetic energies in the soliton, carried by the quarks and mesons, respectively.

Next, we use a virial relation. Consider scale change of the radial coordinate,  $r \rightarrow sr$ . The soliton energy scales as  $\mathcal{E}(s) = T_q/s + V_{q\phi} + sT_\phi + s^3V_\phi$ , where  $V_{q\phi}$  and  $V_\phi$  are the quark-meson and meson-meson interaction energies. Stationarity of the solution imposes  $\partial_s \mathcal{E}|_{s=1} = 0$ , which, together with the relation  $N_c\epsilon = T_q + V_{q\phi}$ , leads to the

virial relation

$$M_{\text{sol}} = \frac{1}{3}T_q + \frac{2}{3}T_\phi + N_c \varepsilon. \quad (\text{C5})$$

Comparing Eq. (C4) and Eq. (C5) completes the proof that  $\delta\mathcal{E} = \frac{1}{2}v^2 M_{\text{sol}}$ . Using similar methods, one can show the equality of inertial and soliton masses in other models [46], including nonlocal theories, such as the NJL model [47].

#### APPENDIX D: BOSONIZATION AND PAULI BLOCKING OF THE DIRAC SEA

In this section we return to the question whether the Dirac sea should be ‘‘Pauli-blocked’’ in our model. Effective chiral models are believed to result from bosonizing QCD, which, of course, can only be done approximately. For definiteness, we discuss the issue of Pauli blocking in the framework of the partly bosonized [45] NJL model, but the result is more general. In the presence of an external source  $J$ , the action of the model is

$$S_{\text{NJL}} = -i\text{Tr} \ln[i\cancel{D} - gU - J] - \text{vac}, \quad (\text{D1})$$

$$gU = g(\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}),$$

A cutoff is understood,  $\text{Tr}$  denotes a functional trace, and  $\text{vac}$  means the vacuum subtraction. For simplicity, we assume the nonlinear constraint  $\sigma^2 + \boldsymbol{\pi}^2 = F_\pi^2$ . The source  $J$  may represent interactions with external probes (e.g., electromagnetic) or result from cranking (Sec. III E). For definiteness, let us evaluate the moment of inertia. In this case  $J = \frac{1}{2}\boldsymbol{\lambda} \cdot \boldsymbol{\tau}$ , and expanding the action to second order in  $\lambda$  we obtain

$$\Delta S_{\text{NJL}} = \frac{1}{2}\lambda^2 \Theta \int dt, \quad (\text{D2})$$

where the moment of inertia  $\Theta$  is given by

$$\Theta = \frac{i}{4}N_c \int \frac{d\omega}{2\pi} \text{Sp} \frac{1}{\omega - h} \tau_3 \frac{1}{\omega - h} \tau_3, \quad (\text{D3})$$

where  $\text{Sp}$  denote the trace over space, spin, and isospin, and  $h$  is the Dirac Hamiltonian. The pole structure and the contour of the  $\omega$  integration in Eq. (D3) is given in Fig. 1. Note that the contour goes above the occupied valence state, as well as above all the negative-energy sea states. Performing the integration over  $\omega$  in Eq. (D3),

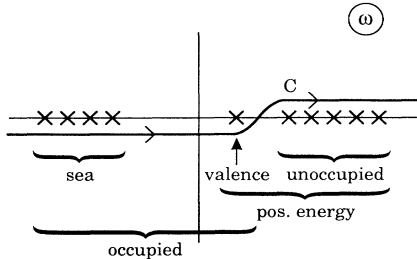


FIG. 1. Contour of integration,  $C$ , for the total (sea- and valence-quark) contribution:  $C$  cannot be Wick rotated without picking up the valence-quark contribution. Notation for various labels used in the text is visualized.

we obtain the usual spectral expression for

$$\Theta = \frac{1}{2}N_c \sum_{\substack{i \in \text{occ} \\ j \in \text{unocc}}} \frac{|\langle i | \tau_3 | j \rangle|^2}{\varepsilon_i - \varepsilon_j}, \quad (\text{D4})$$

where  $\text{occ}$  denotes all occupied states, i.e., the valence as well as the sea states, and  $\text{unocc}$  denotes the unoccupied positive energy states (see Fig. 1 for the meaning of labels). The expression under the sum is antisymmetric with respect to exchanging  $i$  and  $j$ ; therefore, the sum as in Eq. (D4) over  $i$  and  $j$  belonging to the same set of indices vanishes. Using this trick we can replace the ranges of summation indices as follows:

$$\sum_{\substack{i \in \text{occ} \\ j \in \text{unocc}}} = \sum'_{\substack{i \in \text{occ} \\ j \in \text{all}}} = \sum'_{\substack{i \in \text{val} \\ j \in \text{all}}} + \sum'_{\substack{i \in \text{sea} \\ j \in \text{all}}} = \sum'_{\substack{i \in \text{val} \\ j \in \text{all}}} + \sum_{\substack{i \in \text{sea} \\ j \in \text{pos en}}}, \quad (\text{D5})$$

where the prime means the exclusion of  $i = j$  term,  $\text{all}$  denotes all states, and  $\text{pos en}$  denotes the positive energy states. According to Eq. (D5), the moment of inertia can be decomposed into the valence and sea parts:

$$\Theta = \Theta_{\text{val}} + \Theta_{\text{sea}},$$

$$\Theta_{\text{val}} = \frac{1}{2}N_c \sum'_{\substack{i \in \text{val} \\ j \in \text{all}}} \frac{|\langle i | \tau_3 | j \rangle|^2}{\varepsilon_i - \varepsilon_j}, \quad \Theta_{\text{sea}} = \frac{1}{2}N_c \sum_{\substack{i \in \text{sea} \\ j \in \text{pos en}}} \frac{|\langle i | \tau_3 | j \rangle|^2}{\varepsilon_i - \varepsilon_j}. \quad (\text{D6})$$

Note that the ‘‘full’’ expression (D4) obeys the Pauli exclusion principle; hence, using Eq. (D5) we have broken the original expression into two parts, each of which violates the Pauli principle. In fact, an analogous decomposition is used in the treatment of the relativistic fermion propagator in fermion matter [48]. Below we explain why this is useful. First, the expression for  $\Theta_{\text{val}}$  corresponds to our quark part of the moment of inertia calculated in Sec. III E. Second, the sea part of the moment of inertia can be simply approximated only if it is written as in Eq. (D6). Indeed, we can write down

$$\Theta_{\text{sea}} = \frac{i}{4}N_c \int \frac{d\omega}{2\pi} \text{Sp} \frac{1}{\omega - h} \tau_3 \frac{1}{\omega - h} \tau_3, \quad (\text{D7})$$

where the contour of integration is given in Fig. 2. This

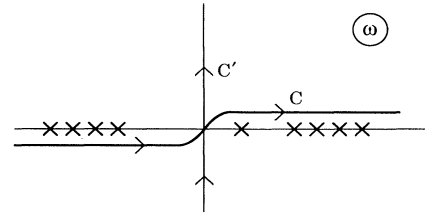


FIG. 2. Contour of integration for the sea-quark contribution:  $C$  can be Wick rotated to the contour  $C'$ . Upon bosonization, the sea-quark effects can be described by mesonic degrees of freedom.

contour can be Wick rotated without picking up any pole contributions, and we obtain an expression in Euclidean space. We can then perform standard gradient expansion methods [49–51, 17] to rewrite  $\Theta_{\text{sea}}$  as an integral over the classical pion field. The first term, with no derivatives, is just our expression for the pion part of the moment of inertia, Eq. (3.21). Furthermore, this term does not

depend on the NJL cutoff, since the normalization factor is the same as in the pion wave function normalization [15]. Further terms in the gradient expansion do depend on the cut-off. If we tried to perform the Wick rotation on the original expression (D3), we would pick up a pole contribution from the occupied valence level, and our final expression (D6) would also follow.

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- [1] M. C. Birse and M. K. Banerjee, *Phys. Lett.* **136B**, 284 (1984); *Phys. Rev. D* **31**, 118 (1985).
- [2] G. R. S. Kahana and V. Soni, *Nucl. Phys.* **A415**, 351 (1984).
- [3] S. Kahana and G. Ripka, *Nucl. Phys.* **A419**, 462 (1984).
- [4] G. Kalberman and J. M. Eisenberg, *Phys. Lett.* **139B**, 337 (1984).
- [5] M. K. Banerjee, W. Broniowski, and T. D. Cohen, in *Chiral Solitons*, edited by K.-F. Liu (World Scientific, Singapore, 1987).
- [6] M. C. Birse, *Prog. Part. Nucl. Phys.* **25**, 1 (1990).
- [7] T. H. R. Skyrme, *Proc. R. Soc. London* **A260**, 127 (1961); *Nucl. Phys.* **31**, 556 (1962).
- [8] G. S. Adkins, C. R. Nappi, and E. Witten, *Nucl. Phys.* **B228**, 552 (1983).
- [9] G. S. Adkins and C. R. Nappi, *Nucl. Phys.* **B233**, 109 (1984).
- [10] I. Zahed and G. E. Brown, *Phys. Rep.* **142**, 1 (1986), and references therein.
- [11] L. Vepstas and A. D. Jackson, *Phys. Rep.* **187**, 109 (1990), and references therein.
- [12] M. K. Banerjee, in *Nuclear Chromodynamics: Quarks and Gluons in Particles and Nuclei*, Proceedings of the ITP Workshop, Santa Barbara, California 1985, edited by S. Brodsky and E. Moniz (World Scientific, Singapore, 1985).
- [13] W. Broniowski, in *Proceedings of the Workshop on Mesonic Degrees of Freedom in Nuclei*, Bled/Ljubljana, Yugoslavia, 1987 [*Fiz.* **19** (Suppl. 2), 38 (1987)].
- [14] I. Duck, *Phys. Rev. D* **34**, 1493 (1986).
- [15] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961).
- [16] D. I. Dyakonov and V. Y. Petrov, *Pis'ma Zh. Eksp. Teor. Fiz.* **43**, 75 (1986) [*JETP Lett.* **43**, 57 (1986)].
- [17] D. I. Dyakonov, V. Yu. Petrov, and P. V. Pobylytsa, *Nucl. Phys.* **B306**, 809 (1988).
- [18] Th. Meissner, E. Ruiz Arriola, F. Grümer, K. Goeke, and H. Mavromatis, *Phys. Lett. B* **214**, 312 (1988).
- [19] H. Reinhardt and R. Wünsch, *Phys. Lett. B* **215**, 577 (1988).
- [20] Th. Meissner, F. Grümer, and K. Goeke, *Phys. Lett. B* **227**, 296 (1989).
- [21] D. I. Dyakonov, V. Yu. Petrov, and M. Praszalowicz, *Nucl. Phys.* **B323**, 53 (1989).
- [22] R. Alkofer, *Phys. Lett. B* **236**, 310 (1990).
- [23] T. D. Cohen and W. Broniowski, *Phys. Rev. D* **34**, 3472 (1986).
- [24] B. Golli and M. Rosina, *Phys. Lett.* **165B**, 347 (1985).
- [25] M. C. Birse, *Phys. Rev. D* **33**, 1934 (1986).
- [26] C.-Y. Ren and M. K. Banerjee, *Phys. Rev. C* **43**, 1981 (1991).
- [27] W. Broniowski and T. D. Cohen, *Nucl. Phys.* **A458**, 652 (1986).
- [28] For a review, see B. Schwesinger, H. Weigel, G. Holzwarth, and A. Hayashi, *Phys. Rep.* **173**, 173 (1989).
- [29] See, for example, P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, Berlin, 1980).
- [30] W. Broniowski, M. K. Banerjee, and T. D. Cohen, *Phys. Lett. B* **283**, 22 (1992).
- [31] W. Broniowski and T. D. Cohen, preceding article, *Phys. Rev. D* **47**, 299 (1993).
- [32] W. Broniowski and M. K. Banerjee, *Phys. Lett.* **158B**, 335 (1985); *Phys. Rev. D* **34**, 849 (1986).
- [33] T. D. Cohen and M. K. Banerjee, *Phys. Lett.* **167B**, 21 (1986).
- [34] See, for example, D. J. Rowe, *Nuclear Collective Motion* (Methuen, London, 1970).
- [35] M. Gell-Mann and M. Lévy, *Nuovo Cimento* **16**, 705 (1960).
- [36] W. Broniowski and T. D. Cohen, *Phys. Lett. B* **177**, 141 (1986).
- [37] See, for example, V. I. Arnold, *Mathematical Methods of Classical Mechanics* (Springer, Berlin, 1978).
- [38] R. Friedberg, T. D. Lee, and A. Sirlin, *Phys. Rev. D* **13**, 2739 (1976).
- [39] R. Jackiw, L. Jacobs, and C. Rebbi, *Phys. Rev. D* **20**, 474 (1979).
- [40] M. Fiolhais, C. Christov, T. Neuber, M. Bergmann, and K. Goeke, *Phys. Lett. B* **269**, 43 (1991).
- [41] G. 't Hooft, *Nucl. Phys.* **B75**, 461 (1974).
- [42] E. Witten, *Nucl. Phys.* **B156**, 269 (1979).
- [43] E. Witten, *Nucl. Phys.* **B160**, 57 (1979).
- [44] A. Kerman and A. Klein, *Phys. Rev.* **132**, 1326 (1963). For soliton models, this method was introduced by J. Goldstone and R. Jackiw, *Phys. Rev. D* **11**, 1486 (1975).
- [45] T. Eguchi, *Phys. Rev. D* **14**, 2755 (1975).
- [46] R. Jackiw, *Rev. Mod. Phys.* **49**, 681 (1977).
- [47] P. V. Pobylytsa *et al.*, Technical Report No. RUB-TPH/1991, Ruhr-Universität Bochum, 1991 (unpublished).
- [48] S. A. Chin, *Ann. Phys. (N.Y.)* **108**, 301 (1977).
- [49] I. J. R. Aitchison and C. M. Frazer, *Phys. Lett.* **146B**, 63 (1984); *Phys. Rev. D* **31**, 2605 (1985); **32**, 2190 (1985).
- [50] R. I. Nepomechi, *Ann. Phys. (N.Y.)* **158**, 67 (1984).
- [51] J. A. Zuk, *Z. Phys. C* **29**, 303 (1985).