Decomposition of weak hypercharge number and electroweak interaction model

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The lack of simplicity of the weak hypercharge Y values in the standard electroweak model suggests the possibility that Y may not be a truly fundamental number and may be expressed as the sum of the baryon number, lepton number, I_3^L, and I_3^R. Such an idea will lead to an $SU(2)_L \times U(1)_{\text{leptons}} \times U(1)_{\text{baryon}} \times U(1)_R$ electroweak model, where the lepton and baryon numbers have short-range dynamical effects, and new weak neutral vector particles of unusual coupling properties arise.

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Differently from the electric charge number, the lepton number and the baryon number are usually regarded as nondynamical quantum numbers (they do not generate gauge interactions, i.e., they do not correspond to local gauge symmetries), though all of them have been proven experimentally to be strictly conserved. Lee and Yang [1] interpreted that there is no long-range U(1) gauge interaction associated with the baryon number. However, this does not exclude the possibility of short-range interaction. If the corresponding local gauge symmetries are broken ones, the baryon number, and also the lepton number, may still be concerned with some gauge interactions. Thus whether the lepton and baryon numbers have short-range dynamical effects is a problem of much interest.

Rajpoot [2] presented a possibility for this problem, which is an extension of the standard electroweak model, namely, an

$$SU(2)_L \times U(1)_{\text{lepton}} \times U(1)_{\text{baryon}} \times U(1)_R$$

model, where the gauge symmetries corresponding to lepton and baryon numbers are broken by some Higgs fields. In the present paper we will point out such a possibility from a different viewpoint, which leads to a different extension of the standard model.

The starting point of our discussion is the weak electric charge number formula of the standard electroweak model:

$$Q = I_3^L + Y/2 \ . \tag{1}$$

It seems that the weak hypercharge number Y is not a truly fundamental quantum number, since the Y values of elementary fermions do not have enough simplicity. We believe that the really fundamental quantum numbers should be simple and clear. In formula (1), both the electric charge number Q and the component of left-handed weak isospin I_3^L possess such a kind of fundamental simplicity, which forms a sharp contrast to Y. Thus we tend to think that Y may be a composite number and may be the combination of some more general quantum numbers, e.g., the lepton and baryon numbers, which possess the fundamental status that can compare with Q and I_3^L. We hence decompose Y as

$$Y = l + b + s \ , \tag{2}$$

where l is the "weak lepton number" defined as the negative value of the sum of the usual three kinds of lepton numbers:

$$l = -(L_e + L_\mu + L_\tau) \ ; \tag{3}$$

b is the usual baryon number; and s is introduced here to ensure Eq. (2). Separating the l and b parts from Y, we find that the s indeed has very good simplicity. Actually it is easy to see that $s = 2 \times I_3^R$, where I_3^R is the component of the right-handed weak isospin,

Therefore, instead of (1) we have the electric charge formula

$$Q = I_3^L + \tfrac{1}{2}(l + b) + I_3^R \ . \tag{4}$$

We now think that the truly fundamental quantum numbers are I_3^L, l, b, and s (i.e., $2I_3^R$), and then the gauge group $U(1)_Y$ in the standard model will be replaced by $U(1)_l$, $U(1)_b$, and $U(1)_R$. This leads to an electroweak model with the gauge group

$$G = SU(2)_L \times U(1)_l \times U(1)_b \times U(1)_R \ .$$

Of course, since there are no long-range interactions associated with lepton and baryon numbers, the U(1) symmetries in G must be broken.

Under G the elementary fermions' transformation properties (I^L, l, b, s) are

$$\begin{bmatrix} \nu & \nu' & \nu'' \\ e & \mu & \tau \end{bmatrix}_L : \quad \left[\frac{1}{2}, -1, 0, 0\right] ,$$

$$e_R \mu_R \tau_R : \quad (0, -1, 0, -1) ,$$

$$\begin{bmatrix} u & c & t \\ d' & s' & b' \end{bmatrix}_L : \quad \left[\frac{1}{2}, 0, \frac{1}{3}, 0\right] ,$$

$$u_R c_R t_R : \quad (0, 0, \tfrac{1}{3}, 1) ,$$

$$d_R s_R b_R : \quad (0, 0, \tfrac{1}{3}, -1) ,$$

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where d', s', b' are the Cabibbo-Kobayashi-Maskawa (CKM) mixing of d, s, b. The covariant derivative required by G is

$$D_\mu = \partial_\mu - ig_L I_j^L W_\mu^j(x) - i\frac{l}{2}g_1 L_\mu(x) - i\frac{b}{2}g_2 B_\mu(x) - i\frac{s}{2}g_R S_\mu(x) , \tag{5}$$

where I_j^L, $l/2$, $b/2$, $s/2$ are the generators of G, corresponding to the gauge fields W_μ^j, L_μ, B_μ, S_μ and couplings g_L, g_1, g_2, g_R. Differently from the usual $SU(2)_L \times SU(2)_R \times U(1)$ models with manifest left-right symmetry [3], here we do not take $g_L \equiv g_R$.

In order to produce $G \to U(1)_Q$ symmetry breaking, we introduce one left-handed doublet Higgs field ϕ and two singlet fields φ_1, φ_2:

$$\phi\text{: } (\tfrac{1}{2},0,0,1), \quad \varphi_1\text{: } (0,-1,0,1), \quad \varphi_2\text{: } (0,0,1,-1) .$$

The properties of these particles are very different from those of Ref. [2]. There both Higgs singlets have simultaneously nonzero baryon and lepton numbers; i.e., they are both baryons and leptons, and hence are not the fundamental particles at the same level as $\nu, e, u, d, \ldots$. (As in the case of atoms, they are some kind of "mixed" particles.) This choice of Higgs fields will lead to different physical results from that of Ref. [2].

Assuming that the neutral components of Higgs fields have nonzero vacuum averages,

$$\langle \phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v \end{bmatrix}_L , \quad \langle \varphi_k \rangle_0 = \frac{1}{\sqrt{2}} v_k , \quad k = 1,2 , \tag{6}$$

after the symmetry breaking, we have

$$D_\mu = \partial_\mu - iQeA_\mu - i\frac{e}{\sin\theta} I_\pm^L W_\mu^\pm - i\frac{e}{\sin\theta\cos\theta}(I_3^L - Q\sin^2\theta)Z_\mu - i\frac{l}{2}\frac{e}{\sin\theta_1'\cos\theta}Z'_{1\mu} - i\frac{b}{2}\frac{e}{\sin\theta_2'\cos\theta}Z'_{2\mu} , \tag{7}$$

where the free parameters of the theory are

$$e = g_L g_R / \sqrt{g_L^2 + g_R^2} , \tag{8}$$

$$\tan\theta = g_R / g_L , \quad \tan\theta_k' = g_k / g_L , \tag{9}$$

and three massive neutral vector particles arise:

$$Z = \cos\theta W^3 - \sin\theta S , \quad M_Z = \tfrac{1}{2} v \sqrt{g_L^2 + g_R^2} , \tag{10}$$

$$Z_k = \cos\theta_k' L_k - \sin\theta_k' S , \quad M_{Z_k'} = \tfrac{1}{2} v_k \sqrt{g_k^2 + g_R^2} , \tag{11}$$

where $L_{k\mu} = L_\mu, B_\mu$. The field A_μ orthogonal to Z_μ is the usual electromagnetic field, and those orthogonal to Z_k' are nonindependent and can be expressed with Z, A, and Z_k'. In addition, the mass of $W_L^\pm$ is $M_W = \frac{1}{2} g_L v$.

The fermions obtain masses through their Yukawa couplings with ϕ. Thus ϕ corresponds to the standard model Higgs field.

Similar to the previous cases [2,4], there are triangle anomalies in the model since the U(1)'s quantum numbers do not satisfy the conditions of triangle anomaly cancellation. A simple way is to introduce mirror fermions [2]. They carry opposite chiralities of the conventional leptons and quarks and thus counteract the latter's contribution to the anomalies. At present these mirror companions of conventional fermions are assumed to have unknown masses and remain to be searched.

From (7) we see that the weak neutral current (WNC) interaction is

$$\mathcal{L}_{\rm WNC} = -i\frac{e}{\cos\theta} \left[\frac{1}{\sin\theta} J_\mu Z_\mu + \sum_k \frac{1}{\sin\theta_k'} J_{k\mu} Z'_{k\mu} \right] , \tag{12}$$

where

$$J_\mu = \sum_\psi (I_3^L - Q\sin^2\theta)\bar\psi\gamma_\mu\psi , \tag{13}$$

$$J_{k\mu} = \sum_\psi \frac{k}{2}\bar\psi\gamma_\mu\psi, \quad k = l,b , \tag{14}$$

and $\psi = (f_L, f_R)$ indicates summing up the right- and left-handed states of all elementary fermions. The Z particle may be expected to be the same as the Z^0 in the standard model, while Z_k' are new neutral vector bosons of unusual properties. From (14) we see that Z_1' couples only to corresponding leptons and Z_2' only to baryons. In addition, neither of the new Z's will directly interact with Z (the standard model Z^0). Thus differently from the new Z' particles of other extension models, such as the superstring-inspired models [5] or preon-inspired models [6], the limits on the mass of the new Z's in our model cannot be obtained through the usual ways, such as the precision measurements of the Z^0 properties and $p\bar{p} \to Z \to l\bar{l}$ production [7,8] or $e^+e^- \to Z \to \mu^+\mu^-, \tau^+\tau^-$ production [9].

The new Z's mass limits may be obtained by indirect means, e.g., through the deviation values of the parameters in the low-energy effective weak neutral current (WNC) interaction $\mathcal{L}_{\rm WNC}^{\rm eff}$ the lower bounds on mass may be estimated. The $\mathcal{L}_{\rm WNC}^{\rm eff}$ is written directly from (12):

$$\mathcal{L}_{\rm WNC}^{\rm eff} = -\frac{4}{\sqrt{2}} \left[G_F J_\mu J_\mu + \sum_k G_k' J_{k\mu} J_{k\mu} \right] , \tag{15}$$

where

$$G_k' = e^2 / 4\sqrt{2} M(Z_k') \sin^2\theta_k' \cos^2\theta \quad (k = 1,2) .$$

Using the experimental value [10,11] of the parameters $g_v(e)$, within one standard deviation, we have

$$M(Z_1') > \frac{1}{|\sin\theta_1'|} \times 250 \text{ GeV} .$$

The limit of $M(Z_2')$ will depend on the experimental values of the quark-quark WNC interaction parameters.

There are also three physical Higgs particles leftover from the symmetry breaking in the model proposed here.

One of them, left from the ϕ field, corresponds to the standard model Higgs particle, and the other two will not interact directly with quarks and leptons and thus the concrete properties of these particles are still considerably uncertain.

At last we will point out another possibility. There are three kinds of leptons and three kinds of independently conserved lepton numbers. Thus it seems that the above l quantum number should be replaced by three "weak lepton numbers" l_1, l_2, l_3 $(=-L_e, -L_\mu, -L_\tau)$. The electric charge number formula then becomes

$$Q = I_3^L + \tfrac{1}{2}(l_1 + l_2 + l_3 + b) + I_3^R \ , \tag{16}$$

which indicates a large rank electroweak model $SU(2)_L \times U(1)_{l_1} \times U(1)_{l_2} \times U(1)_{l_3} \times U(1)_b \times SU(2)_R$. By an appropriate choice of Higgs mechanism the unusual coupling properties of new Z's will be retained; i.e., each new Z' particle couples only to corresponding e, μ, τ leptons or baryons, while the situation is more complicated.

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