PHYSICAL REVIEW D VOLUME 47, NUMBER 7 1 APRIL 1993

Meson decay constants from isospin mass splittings in the quark model

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(Received 21 September 1992)

Decay constants of D and B mesons are estimated within the framework of a heavy-quark approach using measured isospin mass splittings in the D, D^*, B, and B^* states to isolate the electromagnetic hyperfine interaction between quarks. A relation following from the use of splittings in kaons is also considered.

PACS number(s): 12.70.+q, 13.20.Fc, 13.40.Dk, 14.40.Jz

The decay constants f_D and f_B of mesons containing a single heavy quark are of fundamental importance for the understanding of the strong interactions, since they describe the behavior of a single light quark bound to a nearly static source of color. The constant f_B is crucial for interpreting data on particle-antiparticle mixing in the neutral B meson system, and both constants are essential if one is to anticipate and interpret new signatures for CP violation. In this Brief Report we describe a method for determination of these constants which relies on the isospin splittings of the D, D^*, B, and B^* mesons. We also consider a relation following from the use of splittings in kaons.

The isospin splittings allow one to extract the contributions of the spin-dependent electromagnetic interaction between light and heavy quarks. Additional assumptions about quark masses are required in order to interpret these contributions in terms of decay constants.

The decay constant f_M for a meson M is specified by the matrix element of the axial-vector current between the one-particle state and the vacuum: $\langle 0| A_\mu |M(q)\rangle = iq_\mu f_M$. In a nonrelativistic quark model it is related to $|\Psi(0)|$, the wave function at the origin, by [1]

$$f_M = (12/M_M)^{1/2} |\Psi(0)| \ . \tag{1}$$

While we recognize the limitations of the nonrelativistic model and relation (1), especially for D mesons [2–4], we seek independent information on $|\Psi(0)|$. We find it by comparing isospin splittings in pseudoscalar and vector meson multiplets [5,6]. These have recently been measured very precisely, with the results [7]

$$\delta m(D) \equiv M(D^+) - M(D^0) = (4.80 \pm 0.10 \pm 0.06)\ \text{MeV}\ , \tag{2}$$

$$\delta m(D^*) \equiv M(D^{*+}) - M(D^{*0}) = (3.32 \pm 0.08 \pm 0.05)\ \text{MeV}, \tag{3}$$

$$\delta m(D) - \delta m(D^*) = (1.48 \pm 0.09 \pm 0.05)\ \text{MeV}\ . \tag{4}$$

There are previously existing measurements of the mass difference

$$\delta m(B) \equiv M(\bar{B}^0) - M(B^-) = \begin{cases} (2.0 \pm 1.1 \pm 0.3)\ \text{MeV} & \text{CLEO 85 [8]}\ , \\ (-0.4 \pm 0.6 \pm 0.5)\ \text{MeV} & \text{CLEO 87 [9]}\ , \\ (-0.9 \pm 1.2 \pm 0.5)\ \text{MeV} & \text{ARGUS [10]}\ , \\ (0.12 \pm 0.58)\ \text{MeV} & \text{(average)}\ . \end{cases} \tag{5}$$

At present we do not know the value of $\delta m(B^*) \equiv M(\bar{B}^{*0}) - M(B^{*-})$. With sufficiently precise information on $\delta m(B) - \delta m(B^*)$ we would be able to determine $|\Psi(0)|$ for the D and B meson systems, and hence to calculate f_D and f_B.

A systematic expansion of D and B meson masses in terms of light-quark masses, electromagnetic effects, and $O(1/m_Q)$ terms has recently been performed [11]. Here Q is the heavy quark (c or b). The difference in isospin splittings between D and D^* or B and B^* states can be expressed purely in terms of $1/m_Q$ contributions:

$$\Delta(D) \equiv \delta m(D) - \delta m(D^*) = a + 4zQ_c\ , \tag{6}$$

$$\Delta(B) \equiv \delta m(B) - \delta m(B^*) = (m_c/m_b)(a + 4zQ_b)\ . \tag{7}$$

The term a is a sum of two contributions, one arising from the $d-u$ quark mass difference and the other proportional to the squares of light-quark charges. Notice that, aside from a ratio of heavy-quark masses, it is common to $\Delta(D)$ and $\Delta(B)$.

The term z arises from the electromagnetic hyperfine interaction between the light (nonstrange) quark q and the heavy quark Q. Using Eqs. (6) and (7), with an assumption about the ratio of m_c/m_b, one can separate out the effects of this term. In a constituent-quark model [12] it is related to the same wave function which appears in (1):

$$z=\frac{2\pi\alpha|\Psi(0)|^2}{3m_q m_c} . \tag{8}$$

In order to extract $|\Psi(0)|^2$ from (8), we need an estimate of m_q. Fits to meson masses based on constituent quark masses, with account taken of spin-spin interactions, are remarkably successful [13,14]. They lead to an estimate [14] of $m_q\equiv(m_u+m_d)/2=310$ MeV, $m_c=1662$ MeV, $m_b=5$ GeV. We use these masses in what follows.

We can express the expected values of f_D and f_B, via Eqs. (1), (4), and (6)–(8), as functions of the unknown quantity $\Delta(B)$. The results are shown in Fig. 1. For the meson masses in (1) we have taken the spin-averaged values $m_D=1974$ MeV, $m_B=5315$ MeV. The results are compatible with the experimental upper limit [15] $f_D\leq 290$ MeV (90% C.L.) and with the positivity of $|\Psi(0)|^2$ only for a rather narrow range of $\Delta(B)$:

$$-0.05 \text{ MeV}\leq\Delta(B)\leq 0.49 \text{ MeV} . \tag{9}$$

This set of inequalities represents one consistency check of our approach. One sees from Fig. 1 that it will require a measurement of $\Delta(B)$ with accuracy comparable to that of $\Delta(D)$, i.e., to 0.1 MeV, in order to make a useful prediction of f_D and f_B. Fortunately, if we are willing to enlist the support of isospin splittings in the kaon system, we can estimate $\Delta(B)$ with the required accuracy.

It has been suggested by Cohen and Lipkin [16,17] that the ratio of isospin and strong hyperfine splittings in hadrons should depend only on the charges and colors of the constituents. Let us apply this observation to the kaon and B systems, for which the charges of the constituents are identical. We define

$$\Delta(K)\equiv[m(K^{*-})-m(\bar{K}^{*0})]-[m(K^-)-m(\bar{K}^0)] ,$$

$$\Sigma(K)\equiv[m(K^{*-})+m(\bar{K}^{*0})]-[m(K^-)+m(\bar{K}^0)] ,$$

$$\Sigma(B)\equiv[m(B^{*-})+m(\bar{B}^{*0})]-[m(B^-)+m(\bar{B}^0)] . \tag{10}$$

Then one expects

$$\Delta(B)=\Sigma(B)\Delta(K)/\Sigma(K)=-0.056\pm0.043 \text{ MeV} , \tag{11}$$

where we have used $\Sigma(B)=91.7\pm1.3$ MeV from the averages of CUSB and CLEO results [18], and $\Delta(K)=-0.49\pm0.37$ MeV and $\Sigma(K)=796.37\pm0.37$ MeV from the compilation of Ref. [19]. The result (11) lies just within in the bounds (9), and implies that

$$z=(0.41\pm0.04) \text{ MeV} . \tag{12}$$

Using (1) and (8) and assuming the values of quark masses mentioned above, we then find that f_D is near its experimental upper limit:

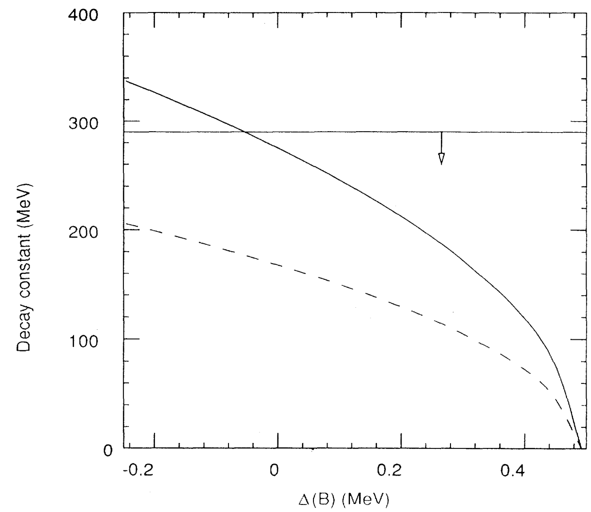

FIG. 1. Predicted decay constants f_D (solid curve) and f_B (dashed curve) as functions of difference $\Delta(B)$ in isospin splittings between B^* and B mesons. The horizontal line with the arrow pointing downward denotes the upper limit of Ref. [15] on f_D.

$$f_D=(290\pm15) \text{ MeV} , \quad f_B=(177\pm9) \text{ MeV} , \tag{13}$$

corresponding to

$$|\Psi(0)|^2=(13.8\pm1.4)\times10^{-3} \text{ GeV}^3 . \tag{14}$$

The values (13) should of course be viewed with some caution, as they involve assumptions which go beyond the heavy-quark treatment of Ref. [11]. The most questionable of these involves the use of a specific light-quark mass in Eq. (8). The errors on light-quark masses used in extracting $|\Psi(0)|^2$ from the electromagnetic hyperfine interaction energy are probably about 20%, based on the spread in values obtained in various constituent-quark fits to mesons and baryons [14]. These errors alone would lead to uncertainties in f_D and f_B of about 10%.

The relation of Ref. [16] itself goes beyond what we have used in obtaining Eq. (11). It is proposed there that

$$\Delta(D)=\Sigma(D)\Delta(\Xi)/\Sigma(\Xi) , \tag{15}$$

where

$$\Sigma(D)\equiv[m(D^{*+})+m(D^{*0})]-[m(D^+)+m(D^0)] , \tag{16}$$

$$\Delta(\Xi)=[m(\Xi^{*0})-m(\Xi^{*-})]-[m(\Xi^0)-m(\Xi^-)] , \tag{17}$$

$$\Sigma(\Xi)=[m(\Xi^{*0})+m(\Xi^{*-})]-[m(\Xi^0)+m(\Xi^-)] . \tag{18}$$

The relation (15) compares electromagnetic and chromomagnetic forces on a charmed quark with similar forces on a nonstrange diquark. By contrast, the comparison in Eq. (11) merely involves charge $-\frac{1}{3}$ quarks of two different masses. Thus we would not expect violations of (15) necessarily to entail those of (11).

At present there is no evidence that (15) is violated. The new value (4) of $\Delta(D)=(1.48\pm0.10)$ MeV is slightly more than 1σ away from the prediction $\Delta(D)=(2.10\pm0.59)$ MeV of Ref. [17].

An independent check of the light-quark mass in (8), in a very similar context, would be its contribution to the magnetic transitions in $D^{*+}\rightarrow D^{*}\gamma$ and $D^{*0}\rightarrow D^{0}\gamma$. Both these magnetic transitions and the electromagnetic hyperfine interaction term (7) involve quark magnetic moments, so that it might even (in principle) be possible to dispense altogether with a discussion of quark masses.

At present there are no measurements of the absolute rates for radiative D^* decay, but errors on branching ratios have recently been reduced significantly [20]. One can make use of these results by assuming that isospin invariance applies to the decays $D^*\rightarrow D\pi$. Correcting for phase space with factors of $(p^{\pi}_{\mathrm{c.m.}})^3$, we find

$$\Gamma_{\mathrm{tot}}(D^{*0})/\Gamma_{\mathrm{tot}}(D^{*+})=0.69\pm0.05 \ . \qquad (19)$$

The measured ratio of branching ratios for charged and neutral $D^*\rightarrow D\gamma$ decays then can be interpreted in terms of a ratio of decay rates. In a quark model, this ratio is given by

$$\frac{\Gamma(D^{*0}\rightarrow D^0\gamma)}{\Gamma(D^{*+}\rightarrow D^+\gamma)} =(\omega_0/\omega_+)^3\left[\frac{2}{m_u}+\frac{2}{m_c}\right]^2\Bigg/\left[-\frac{1}{m_d}+\frac{2}{m_c}\right]^2 , \qquad (20)$$

where $\omega_0=137.1$ MeV and $\omega_+=135.7$ MeV are the photon energies in the respective radiative decays.

With $m_u=m_d=310$ MeV, $m_c=1662$ MeV, and with [20]

$$B_0\equiv B(D^{*0}\rightarrow D^0\gamma)=(36.4\pm2.3\pm3.3)\% \ ,$$

Eqs. (15) and (16) imply

$$B_+\equiv B(D^{*+}\rightarrow D^+\gamma)=1.7\% \ ,$$

to be compared with the experimental value [20] of $(1.1\pm1.4\pm1.6)\%$. For this value, in order to obtain a given fractional error on m_u/m_c, we need to measure B_0/B_+ to about 1.5 times this fractional error. Thus, improvement of our estimate of the present error on m_u (about 20%) would be demanding, but not inconceivable.

We have ignored $1/m_Q$ corrections to the relation (1). While the value of f_B in (12) is compatible with analysis of B-$\bar{B}$ mixing in the context of information about elements of the Cabibbo-Kobayashi-Maskawa matrix [21], it lies below predictions of lattice QCD [3]. These, as well as the work of Ref. [4], suggest that such corrections could be substantial.

While $1/m_Q$ corrections to (1) have been discussed in Ref. [4], a corresponding discussion for the electromagnetic hyperfine term (8) has not yet been given [22]. It does not make sense to incorporate only partial information on such corrections. Corrections to the *ratio* of the two terms are what we need, and probably make more sense. Nonetheless, to estimate the likely size of such corrections, we can attempt to apply our result (13) for $|\Psi(0)|^2$ to the kaon decay constant as well, multiplying the right-hand side of (1) by a correction factor $(1-A/\overline{m}_M)$. For the meson mass $\overline{m}_M$ we use the spin-averaged masses, $\overline{m}_K=794$ MeV, as well as the values for D and B mesons mentioned above. We obtain the observed kaon decay constant $f_K=160$ MeV with $A=357$ MeV, giving rise to $f_D=227$ MeV and $f_B=164$ MeV. If we apply instead a similar correction to the relation for the *square* of the decay constant, we obtain $f_D=244$ MeV, $f_B=168$ MeV. We summarize these values in Table I.

TABLE I. Values of pseudoscalar meson decay constants f_D and f_B (in MeV) obtained under various assumptions about $1/m_Q$ corrections. Errors of about 10% are associated with uncertainties in light-quark masses.

Corrections	f_D	f_B
None	290	177
Applied to f_M^2	227	164
Applied to f_M	244	168

The above values of f_D are compatible with the range of recent theoretical estimates (see, e.g., [3,5]). The values of f_B, as mentioned, lie below lattice estimates. The use of (13) to estimate f_K is much more dubious than the use of kaons in the relation (11), where we do not have to assume that wave functions of kaons are the same as those of D or B mesons.

A further correction which has been neglected is that the ratio $f_B/f_D=(M_D/M_B)^{1/2}$ implied by the use of identical B and D wave functions in (1) should be multiplied by a QCD correction of 1.11 [23]. Without a discussion of QCD corrections to (8), however, such a factor is only part of the present approach.

To summarize, we have found that isospin splittings in the D, D^*, B, and B^* systems permit one to estimate meson decay constants. We find a range of values depending on how $1/m_Q$ corrections are taken into account, but our most naïve treatment (ignoring these corrections) predicts a value of f_D at its experimental limit of 290 MeV, and a value $f_B=177$ MeV.

Our predictions can be checked in the near future not only by measurement of f_D (via detection of the decay $D\rightarrow\mu^+\nu$), but also by verification of the very small predicted isospin splitting in the B^* system. The eventual goal of measuring f_B represents a considerably greater experimental challenge. A measurement of f_D would allow us to estimate the magnitude of $1/m_Q$ corrections, and thus could lead to a more reliable estimate for f_B.

We are grateful to Glenn Boyd, Harry Lipkin, and Mark Wise for helpful discussions. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-90ER-40560, and by the National Science Foundation.

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