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Tests of the factorization hypothesis and the determination of meson decay constants

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We discuss various tests of the factorization hypothesis making use of the close relationship between semileptonic and factorized nonleptonic decay amplitudes. It is pointed out that factorization leads to truly model-independent predictions for the ratio of nonleptonic to semileptonic decay rates, if in the nonleptonic decay a spin-1 meson of arbitrary mass or a pion takes the place of the lepton pair. Where the decay constants of those mesons are known, these predictions represent ideal tests of the factorization hypothesis. In other cases they may be used to extract the decay constants. Currently available data on the decays $\bar{B}^0 \to D^+\pi^-$, $D^{*+}\pi^-$, $D^+\rho^-$, $D^{*+}\rho^-$ are shown to be in excellent agreement with the factorization results. A weighted average of the four independent values for the QCD coefficient a_1 extracted from the data gives $a_1 = 1.15 \pm 0.06$, suggesting that it may be equal to the Wilson coefficient $c_1(\mu)$ evaluated at the scale $\mu = m_b$.

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The dynamics of nonleptonic weak decays is strongly influenced by the confining color forces among the quarks. In contrast with semileptonic transitions, where the lepton current naturally factorizes and one is left with the hadronic matrix element of a color-singlet quark current, nonleptonic processes are complicated by the phenomenon of quark rearrangement due to the exchange of soft and hard gluons. The theoretical description involves matrix elements of local four-quark operators, which are much harder to deal with than current operators.

A great simplification can be accomplished if one is willing to adopt the factorization hypothesis, which relates the complicated nonleptonic decay amplitudes to products of meson decay constants and hadronic matrix elements of current operators similar to the ones encountered in semileptonic decays. Despite its remarkable success in the description of two-body decays of B and D mesons, precise tests of the factorization hypothesis are of utmost importance in order to find out its realm of applicability as well as its limitations. While many tests have been suggested or already carried out [1–8], most of them do not simply test the factorization hypothesis, but rather factorization together with some phenomenological model or, alternatively, together with heavy-quark symmetry for dealing with the hadronic current matrix elements. It is the main objective of this Brief Report to concentrate on such tests that do not suffer from additional uncertainties due to our unsatisfactory ways of dealing with nonperturbative QCD.

As there exist several versions of factorization in the literature, let us begin by giving an unambiguous prescription of how to calculate the rate of some exclusive nonleptonic B decay in the factorization approximation. We will concentrate on $b \to c$ transitions, which are induced by the effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\, V_{cb}\, \left[c_1(\mu)\, Q_1^{cb} + c_2(\mu)\, Q_2^{cb}\right] \\ +\text{penguin operators.} \tag{1}$$

It consists of products of local four-quark operators with scale-dependent Wilson coefficients $c_i(\mu)$. The operators Q_1 and Q_2, written as products of color-singlet currents, are given by

$$Q_1^{cb} = \left[(\bar{d}'u)_{V-A} + (\bar{s}'c)_{V-A}\right](\bar{c}b)_{V-A}\,, \\ Q_2^{cb} = (\bar{c}u)_{V-A}\,(\bar{d}'b)_{V-A} + (\bar{c}c)_{V-A}\,(\bar{s}'b)_{V-A}\,, \tag{2}$$

where d' and s' denote weak eigenstates of the down and strange quarks, respectively, and $(\bar{c}b)_{V-A} = \bar{c}\gamma_\mu(1-\gamma_5)\,b$, etc. The Wilson coefficients of so-called penguin operators [9] in Eq. (1) are very small. Their contribution to the dominant decay amplitudes may be neglected.

If QCD was turned off, the Wilson coefficients of the operators Q_1^{cb} and Q_2^{cb} would be $c_1 = 1$ and $c_2 = 0$. These values are modified by hard gluon exchange. Evaluated at the scale $\mu = m_b \simeq 5.0$ GeV one finds, in the leading logarithmic approximation [10], $c_1(m_b) = 1.12$ and $c_2(m_b) = -0.26$.

According to the factorization hypothesis one may now write the hadronic matrix elements of Q_1^{cb} and Q_2^{cb} as products of two current matrix elements [11]. As an example, we consider the decay amplitude of the transition $\bar{B}^0 \to D^+\pi^-$, which in the factorization approximation is given by

$$A^{\text{fact}} = \frac{G_F}{\sqrt{2}}\, V_{cb}\, V_{ud}^*\, a_1\, \langle\, \pi^- \,|\, (\bar{d}\,u)_A \,|\, 0\,\rangle \langle\, D^+ \,|\, (\bar{c}\,b)_V \,|\, \bar{B}^0\,\rangle\,. \tag{3}$$

Class I transitions such as the one considered above, in which only a charged meson can be generated directly from a current, are proportional to the QCD coefficient a_1. Its relation to the Wilson coefficients will be discussed

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below. Correspondingly, those decays in which the meson generated directly from the current is neutral, such as the J/ψ particle in the decay $\bar{B} \to \bar{K}\, J/\psi$, are called class II, and their decay amplitudes are proportional to the QCD coefficient a_2:

$$A^{\rm fact} = \frac{G_F}{\sqrt{2}}\, V_{cb}\, V_{cs}^*\, a_2 \,\langle\, J/\psi \,|\, (\bar{c}c)_V \,|\, 0 \,\rangle\, \langle\, \bar{K} \,|\, (\bar{s}b)_V \,|\, \bar{B} \,\rangle \; . \tag{4}$$

Factorized amplitudes in which there is interference between a_1 and a_2 terms are categorized as class III.

Usually, form factor suppressed weak annihilation topologies (W-exchange and quark-annihilation diagrams) are neglected in the calculation of factorization amplitudes. This is not an inherent property of the factorization approximation. Rather it is necessary from a practical point of view, since little more is known about form factors at such large timelike momentum transfer than that they should be strongly suppressed. What really is an inherent property of the factorization approximation is the neglect of final-state interactions (FSI's). However, unlike D decays, the decays of B mesons do not take place in a resonance region. Thus one has good reason to believe that ignoring the effects of FSI is a good approximation in B decays.

Let us now turn to the relation between the Wilson coefficients and the QCD coefficients a_1 and a_2. Naively, one would expect $a_1 = c_1(\mu_f) + \xi c_2(\mu_f)$ and $a_2 = c_2(\mu_f) + \xi c_1(\mu_f)$, with $\xi = 1/N_c$, and μ_f denoting the factorization point, in B decays usually identified with m_b. However, experience in D decays has shown that setting $\xi = 0$ allows for a better description of the data, and it has been suggested to treat ξ or even a_1 and a_2 independently as a free parameter [12]. Thus one can test the factorization hypothesis by checking whether or not the values for the QCD coefficient a_1 (a_2) as extracted from different class I (class II) transitions agree with each other. For a_1 also an absolute prediction becomes possible, by observing that varying the parameter ξ in the range $0 < \xi < 1/3$ induces no more than a 10% change in a_1. One would therefore expect $a_1 = 1.1 \pm 0.1$ which has been confirmed in a recent extraction of a_1 from all available nonleptonic two-body decays of B mesons [6]. However, it should be stressed that the fit for a_1 has been strongly dominated by the two decay modes[1] $\bar{B}^0 \to D^{(*)+}\,\pi^-$. It remains to be seen whether the same value of a_1 will be found from other decay modes as more precise data become available.

As first pointed out by Bjorken, the close relationship between factorized amplitudes and semileptonic decay amplitudes provides the most direct test of the factorization assumption [1–3]. To this end, a nonleptonic decay width is related to the corresponding differential semileptonic decay width evaluated at the same q^2. Let us consider the ratios

$$\begin{aligned} R_P^{(*)} &= \frac{\Gamma(\bar{B}^0 \to D^{(*)+}\,P^-)}{\mathrm{d}\left\{\Gamma(\bar{B}^0 \to D^{(*)+}\,\ell^-\,\bar{\nu}_\ell)\right\}/\mathrm{d}q^2\,\big|_{q^2=m_P^2}} \\ &= 6\pi^2\, f_P^2\, |a_1|^2\, |V_{ij}|^2\, X_P^{(*)} \; , \end{aligned} \tag{5}$$

where f_P is the decay constant of the pseudoscalar meson P, V_{ij} is the appropriate Kobayashi-Maskawa (KM) matrix element (associated with P) and (in the limit of vanishing lepton mass)

$$\begin{aligned} X_P &= \frac{(m_B^2 - m_D^2)^2}{[m_B^2 - (m_D + m_P)^2]\;[m_B^2 - (m_D - m_P)^2]} \\ &\quad\times \left|\frac{F_0(m_P^2)}{F_1(m_P^2)}\right|^2 , \\ X_P^* &= \left[m_B^2 - (m_{D^*} + m_P)^2\right]\;\left[m_B^2 - (m_{D^*} - m_P)^2\right] \\ &\quad\times \frac{|A_0(m_P^2)|^2}{m_P^2 \sum\limits_{i=0,\pm} |H_i(m_P^2)|^2} \; . \end{aligned} \tag{6}$$

The form factors $F_0(q^2)$, $F_1(q^2)$, and $A_0(q^2)$ as well as the helicity amplitudes $H_0(q^2)$ and $H_\pm(q^2)$ are defined in Ref. [13].

Bjorken has suggested this test with $P = \pi$, in which case $X_\pi \simeq X_\pi^* \simeq 1$ to within less than 0.5% as can be shown by expanding those quantities in powers of m_π^2/m_B^2 [6]. For heavier pseudoscalar mesons, X_P and X_P^* become model dependent and may quite substantially deviate from 1. In the infinite quark mass limit, one finds, for example, $X_{D_s} \simeq 1.36$ and $X_{D_s}^* \simeq 0.37$ [6].

We can get rid of this model dependence altogether by replacing the pseudoscalar meson P in Eq. (5) by a vector or pseudovector meson. In the factorization approximation one then finds

$$\begin{aligned} R_V^{(*)} &= \frac{\Gamma(\bar{B}^0 \to D^{(*)+}\,V^-)}{\mathrm{d}\left\{\Gamma(\bar{B}^0 \to D^{(*)+}\,\ell^-\,\bar{\nu}_\ell)\right\}/\mathrm{d}q^2\,\big|_{q^2=m_V^2}} \\ &= 6\pi^2\, f_V^2\, |a_1|^2\, |V_{ij}|^2 \; , \end{aligned} \tag{7}$$

where now, of course, the KM matrix element is associated with the (pseudo)vector meson and f_V denotes its decay constant. The reason for all form factors and additional kinematical factors to cancel in the ratio can be easily understood. For zero lepton masses, the lepton pair that in the semileptonic decay is generated by the $(V-A)$ current carries spin 1 in its c.m. frame. Integrated over the lepton angles keeping $q^\mu = (p_\ell + p_\nu)^\mu$ fixed, the production of the lepton pair is therefore kinematically equivalent to the production of a (pseudo)vector particle with four-momentum q^μ [summed over all polarizations of the (pseudo)vector particle]. Corrections to Eq. (7) due to finite lepton masses are of order m_ℓ^2/m_V^2. With the ρ meson being the lightest spin-1 meson these corrections may safely be neglected for electrons and muons.

Setting $V = \rho$, we can use Eq. (7) to obtain two independent values for a_1, since the decay constant f_ρ is known.[2] These values should be compared with those

[1]Here and in the following, "$D^{(*)}$" stands for "D or D^*."

[2]We use $f_\rho = 205$ MeV [6].

obtained from Eq. (5) with $P = \pi$. However, as long as the differential q^2 spectrum of the semileptonic decay $\bar{B}^0 \to D^+ \ell^- \bar{\nu}$ has not yet been measured, we must again resort to some form factor model in decays with a D meson in the final state. In Table I, we have used the predictions of Ref. [6] for those two decays. They are based on an Isgur-Wise function extracted from data on the decay $\bar{B}^0 \to D^{*+} \ell^- \bar{\nu}$ with perturbative QCD corrections and (model-dependent) $1/m_Q$ corrections added on. Nonleptonic decay data used in Table I have been taken from CLEO [14] and ARGUS [15]. The ARGUS data as well as the predictions of Ref. [6] have been rescaled using the new CLEO measurement $B(D^{*+} \to D^0 \pi^+) = (68 \pm 2)\%$ [16]. The experimental number for the ratio R^*_ρ/R^*_π, where some of the systematic uncertainties drop out, has been taken from CLEO alone. We observe that all four values for the QCD coefficient a_1 presented in Table I are in excellent agreement with each other and with the expectation from perturbative QCD, thus providing strong support for the factorization hypothesis in B decays with large recoil. Taking the weighted average of all four values gives $a_1 = 1.15 \pm 0.06$, which suggests that, just like in D decays, we may have $\xi = 0$, i.e., the QCD coefficient a_1 may be equal to the Wilson coefficient $c_1(\mu)$ evaluated at the scale of the decaying quark.

As better statistics becomes available the decays $\bar{B}^0 \to D^{(*)+} K^{(*)-}$ should be included in the above analysis, since the decay constants f_K and f_{K^*} are also known (the latter can be extracted from exclusive τ decay data). On the other hand, Eq. (7) may be used together with the experimentally determined value of the QCD coefficient a_1 to extract yet unknown decay constants of spin-1 mesons like the a_1 meson or the D^*_s meson without resorting to some particular form factor model or to heavy-quark symmetry.

From our kinematical argument about the equivalence of the lepton pair in the semileptonic decay and the spin-1 particle in the nonleptonic decay it is clear that Eq. (7) must be valid, separately, for the different polarizations of the D^* meson in the final state. This amounts to the factorization prediction that the polarization of the D^* meson in the nonleptonic decay $\bar{B}^0 \to D^{*+} V^-$ should be equal to the polarization in the corresponding semileptonic decay at the same q^2. This prediction is currently being tested by the CLEO Collaboration [14]. However, in interpreting the results of such a test, one has to bear in mind that in the semileptonic as well as in the nonleptonic case the D^* polarization at the points $q^2 = 0$ and $q^2 = q^2_{\rm max}$ is unambiguously determined by kinematics alone to be 100% longitudinal and 1/3 longitudinal, respectively. At zero recoil, there is no preferred direction and thus the value 1/3 just expresses the fact that there are two transverse, but only one longitudinal polarization. At $q^2 = 0$, corresponding to maximum recoil in the semileptonic decay, the left-handed electron or muon and the right-handed antineutrino go off parallel to each other, thereby forcing the D^* into longitudinal polarization. In the corresponding nonleptonic decay $\bar{B}^0 \to D^* V^-$ we know (even without the factorization approximation) that the decay amplitude must be proportional to the polarization vector of the (pseudo)vector meson V. Now, for small $q^2 = m_V^2 \ll m_B^2/4$ the (pseudo)vector meson V is highly relativistic (in the B meson rest frame) so that for longitudinal polarization of V (and consequently D^*) the components of the polarization vector acquire very large values, causing longitudinal polarization to dominate.

The above discussion shows that in comparing polarizations in semileptonic and nonleptonic decays, one needs polarization data of quite high precision in order to make a statement about the validity of factorization. Thus, at low q^2, it is the amount of transverse polarization that has to be measured with a small relative uncertainty. Especially for the semileptonic decay such a high precision measurement of the q^2 dependence of the D^* polarization seems hardly possible at present. Fortunately, this seems to be a case where heavy-quark symmetry predictions receive only minor corrections. In the infinite quark mass limit one finds, for the ratio of transverse to longitudinal polarization at some fixed q^2,

$$\frac{d\Gamma_T}{d\Gamma_L} = \frac{4q^2(m_B^2 + m_{D^*}^2 - q^2)}{(m_B - m_{D^*})^2\,[(m_B + m_{D^*})^2 - q^2]}, \tag{8}$$

which is subject to QCD as well as $1/m_Q$ corrections. The general structure of the corrections can be found in Ref. [17]. While the QCD corrections can be reliably calculated using perturbation theory (see, e.g., Ref. [18]), the $1/m_Q$ corrections are model dependent. We have calculated the corrections to Eq. (8) using the QCD corrections of Ref. [18] and the $1/m_Q$ corrections resulting from (a) an analysis of the wave function model of Bauer, Stech, and Wirbel [19, 20] and (b) a sum rule calculation [21]. Although corrections to individual form factors in both models are as large as 30% at maximum recoil and furthermore vary strongly between both models, the correction factor to Eq. (8) in neither one of the two models deviates from one by more than 5% (though the deviations in the two models go in opposite directions). In Table II, we present the prediction for the D^* polarization in semileptonic B decays as a function of q^2 obtained from Eq. (8). The quoted errors result from the conservative estimate of a 10% relative uncertainty for Γ_T/Γ_L at maximum recoil (i.e., at $q^2 = 0$), decreasing linearly

TABLE I. Determination of the QCD coefficient a_1 from several nonleptonic B decay modes as a test of the factorization assumption. The data are taken from CLEO and ARGUS. The theoretical predictions for the branching ratios in the last two rows (given in %) are those of Ref. [6]. They have only been rescaled according to the lower value of V_{cb} that is a consequence of the new CLEO measurement $B(D^{*+} \to D^0\pi^+) = (68 \pm 2)\%$ [16].

Quantity	Experiment	Theory	a_1
R^*_π [see Eq. (5)]	1.29 ± 0.22	$0.97a_1^2$	1.15 ± 0.10
R^*_ρ [see Eq. (7)]	3.0 ± 0.7	$2.37a_1^2$	1.13 ± 0.13
R^*_ρ/R^*_π	2.5 ± 0.6	$f_\rho^2/f_\pi^2 = 2.4$	$\cdots$
$B(\bar{B}^0 \to D^+\pi^-)$	0.28 ± 0.05	$0.214a_1^2$	1.15 ± 0.10
$B(\bar{B}^0 \to D^+\rho^-)$	0.74 ± 0.22	$0.502a_1^2$	1.21 ± 0.18

TABLE II. Amount of transverse polarization (in %) of the D^* in semileptonic B decay.

q^2	0	m_ρ^2	$m_{a_1}^2$	$m_{D_s^*}^2$	$q_{\max}^2$
$d\Gamma_T/d\Gamma_{\text{tot}}$	0	12± 1	26± 2	48± 1	$\frac{2}{3}$

to the point of zero recoil, where the polarization is fixed model independently.

In nonleptonic B decays, the only polarization measurement presently available is that of the D^* polarization in the decay $\bar{B}^0 \to D^{*+}\,\rho^-$. CLEO finds $\Gamma_T/\Gamma_{\text{tot}} = (10 \pm 9)\%$ [14], which has to be compared with the 12% transverse polarization predicted for the semileptonic decay at $q^2 = m_\rho^2$ (see Table II). In order for this test to be sensitive to deviations from factorization, the experimental uncertainty will have to be reduced.

The situation may be more favorable in the decay $\bar{B}^0 \to D^{*+}\,D_s^*$ with predicted 48% of transverse polarization, hopefully allowing for a measurement with smaller relative uncertainties. Also, it will be particularly interesting to see whether this decay obeys the factorization prediction, as this would indicate that the factorization assumption may be justified even in decays with only medium energy release, where a larger influence of final-state interactions might be expected. However, one should keep in mind that the QCD coefficient a_1 drops out of the ratio Γ_T/Γ_L, so that from polarization tests alone it will not be possible to decide whether the short range QCD corrections represented by the values of a_1 and a_2 are really independent of the energy release. To this end, one would like to test the validity of Eq. (7) with $V = D_s^*$ using a value for the decay constant of the D_s^* as determined independently from a measurement of the rate for the decay $D_s \to \mu\,\bar{\nu}$ (employing $f_{D_s} \simeq f_{D_s^*}$, predicted by heavy-quark symmetry). In the absence of such a measurement $f_{D_s^*}$ may be taken from sum rule or lattice calculations.

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