Minijets and the behavior of inelasticity at high energies

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(Received 30 November 1992)

We present an extension of the interacting gluon model, used previously to calculate inelasticities and leading particle spectra in hadronic and nuclear collisions, which incorporate also the production of minijets. As a result we get inelasticity slowly increasing towards some limited value.

PACS number(s): 13.85.Hd, 12.38.Mh, 12.40.−y, 13.87.Ce

The concept of inelasticity plays an important role in cosmic ray and accelerator physics. Whereas in the former it is crucial for the understanding and interpretation of the development of cosmic ray cascades, in the latter it is an indispensable ingredient of any statistical model of multiparticle production processes. Inelasticity is usually defined as the fraction K of the available energy $\sqrt{s}$, in a given interaction, effectively employed for multiparticle production. The energy dependence of inelasticity is a problem of great interest both for the interpretation of cosmic ray data and also for quark-gluon plasma (QGP) physics since inelasticity decreasing with energy would make the formation of QGP more difficult. Experimentally the situation is not clear and many authors have proposed different behavior of the average inelasticity $\langle K \rangle$ as a function of $\sqrt{s}$ [1].

One of the models which in a natural way leads to $\langle K \rangle$ decreasing with energy is the interacting gluon model (IGM) [2]. It included originally only soft gluonic interactions and used the phenomenological soft gluon-gluon cross section as an input. However, it was claimed recently that semi-hard QCD interactions (which produce the so-called minijets) represent an important fraction ($\sim 25\%$) of the total cross section already at the CERN collider energies and are expected to be even more important at higher energies [3]. In this paper we discuss therefore the effect of the inclusion of such semihard component to the original IGM.

In the framework of the IGM, in a first approximation, valence quarks do not interact at all but instead form leading particles. The interaction is supposed to come entirely from the gluonic contents of the colliding hadrons via the formation of gluonic fireballs (clusters). The originally predicted *decrease* of $\langle K \rangle$ with energy can be traced to the assumption that the phenomenological behavior of gluon-gluon cross section $\sigma_{gg}(\hat{s})$ is limited to $1/\hat{s} < \sigma_{gg} <$ const behavior, to the $1/x$ form of the gluonic structure functions for small x (see below for details) and to the assumed constancy with energy of the percentage p of the energy-momentum of the projectile allocated to gluons. Here we shall relax the first condition by allowing the QCD semi-hard interaction mechanism which leads to σ_{gg} increasing with energy. The probability of depositing fractions x and y of the energy momenta of the incoming hadrons in the central region of reaction, by means of the gluon-gluon interactions, is given by the formula [2]

$$\chi(x,y)=\frac{\chi_0}{2\pi\sqrt{D_{xy}}}\exp\left\{-\frac{1}{2D_{xy}}[\langle y^2\rangle(x-\langle x\rangle)^2+\langle x^2\rangle(y-\langle y\rangle)^2-2\langle xy\rangle(x-\langle x\rangle)(y-\langle y\rangle)]\right\}, \tag{1}$$

where

$$D_{xy}=\langle x^2\rangle\langle y^2\rangle-\langle xy\rangle^2 ,$$
$$\langle x^n y^m\rangle=\int_0^1 dx\, x^n\int_0^1 dy\, y^m w(x,y) , \tag{2}$$

and χ_0 is a normalization constant defined by the condition that

$$\int_0^1 dx\int_0^1 dy\, \chi(x,y)\theta(xy-K_{\min}^2)=1 ,$$

with $K_{\min}$ being the minimal inelasticity,

$$K_{\min}=\frac{m_0}{\sqrt{s}} , \tag{3}$$

which is defined by the mass m_0 of the lightest possible produced state.

The function $w(x,y)$ (called "spectral function") contains all the dynamical input of the model and is proportional to the mean number of gluon-gluon interactions with given x and y. It reads

$$w(x,y)=w_S(x,y)+w_H(x,y) , \tag{4}$$

where

$$w_S(x,y)=A\frac{\sigma_{gg}^S(\hat{s})}{\sigma_{hN}^{in}(s)}G_h(x)G_N(y)\theta(xy-K_{\min}^2)\theta(\xi-xy) , \tag{5}$$

$$w_H(x,y)=A\frac{\sigma_{gg}^H(\hat{s})}{\sigma_{hN}^{in}(s)}G_h(x)G_N(y)\theta(xy-\xi) . \tag{6}$$

The cross sections σ_{gg}^S and σ_{gg}^H are the gluon-gluon cross sections in the nonperturbative (soft) and in the perturbative (hard) regime respectively. For the former we take the previously used phenomenological ansatz [2] and for the latter the lowest-order perturbative QCD results; $\sigma_{hN}^{\rm in}$ is the inelastic hadron-hadron cross section, A is a constant parameter, and the $G_{h,N}$ are the effective numbers of gluons, which we approximate by the gluonic structure functions of corresponding hadrons normalized to the percentage p of hadronic momentum allocated to the glue:

$$\int_0^1 dx\, xG_{h,N}(x)=p_{h,N} . \tag{7}$$

The soft gluon-gluon cross section is chosen to be

$$\sigma_{gg}^S=\frac{\alpha}{\hat{s}} , \tag{8}$$

where α is a parameter. The hard gluon-gluon cross section is given by [4]

$$\sigma_{gg}^H=\frac{\pi}{18p_{T\,\min}^2}[\alpha_s(Q^2)]^2H , \tag{9}$$

with

$$H=16T+\frac{4\xi}{xy}\ln\left[\frac{1-T}{1+T}\right]+\frac{2\xi}{xy}T ,$$

$$T=\left[1-\frac{\xi}{xy}\right]^{1/2} ,$$

$$\xi=\frac{4p_{T\,\min}^2}{s} ,$$

$$\alpha_s(Q^2)=\frac{12\pi}{25\ln(Q^2/\Lambda^2)} ,$$

where $p_{T\,\min}$ is a cutoff parameter and $\Lambda=0.2$ GeV. The gluon distribution is taken to be the same as before [2], i.e.,

$$G(x)=p\frac{1+n}{x}(1-x)^n . \tag{10}$$

The new element introduced in this work with respect to Ref. [2] is the inclusion of w_H in the spectral function. It was introduced here in the same way as the semi-hard component of the eikonal function was introduced by Durand and Pi [5] in their diffraction-scattering formalism for total cross sections, cf. Eq. (4).

The QCD parameters are fixed to their most accepted values namely $\Lambda=0.2$ GeV and $p_{T\,\min}=2$ GeV. The scale is chosen to be $Q^2=p_{T\,\min}^2$. Since we want to compare our results with those obtained previously in Ref. [2] we keep $m_0=0.35$ GeV, $n=5$, and $p=0.5$, the only modification being the introduction of the semi-hard spectral function, w_H. We have then only two parameters to adjust, A and α, which will be fixed by two experimental constraints. The first one is that for p-$\bar{p}$ reactions at $\sqrt{s}=540$ GeV the following relation holds [3]:

$$\frac{\sigma^{\rm minijets}}{\sigma_{pp}^{\rm in}}\cong\frac{\sigma_{gg}^H}{\sigma_{gg}^S+\sigma_{gg}^H}\cong\frac{1}{4} ; \tag{11}$$

this fixes the value of α. The second constraint is given by the requirement [1] that for proton-proton collisions at $\sqrt{s}=16.5$ GeV the mean inelasticity $\langle K\rangle\cong0.50$. This condition fixes the value of A. We have checked that at 16.5 GeV the product $A\alpha$ is equal to the old value of α found in Ref. [2] as it should be since at such low energies minijets have no importance.

The gluons deposited in the central region are supposed to form of a fireball (gluonic cluster) of mass $M=\sqrt{xys}$. The inelasticity variable K is defined then as

$$K=\frac{M}{\sqrt{s}}=\sqrt{xy} \tag{12}$$

and the inelasticity distribution $\chi\langle K\rangle$ can be obtained from $\chi(x,y)$ by a simple change of variables:

$$\chi(K)=\int_0^1 dx\int_0^1 dy\,\delta(\sqrt{xy}-K)\chi(x,y) . \tag{13}$$

Finally we can calculate the average inelasticity as

$$\langle K\rangle=\int_0^1 dK\,K\chi(K) \tag{14}$$

and leading particle spectra $[x_L\in(0,1-K_{\min}^2)]$:

$$f(x_L)=\int_0^1 dx\int_0^1 dy\,\theta(xy-K_{\min}^2)\delta(1-x-x_L)\chi(x,y) . \tag{15}$$

One can easily see that for symmetrical (e.g., proton-proton) collisions $\langle K\rangle\sim\langle x\rangle$ and the width of the K and x_L distributions is controlled by $\langle x^2\rangle$. In order to investigate qualitatively the energy dependence of $\langle K\rangle$ it is then enlightening to consider what happens to $\langle x\rangle$ and $\langle x^2\rangle$. Approximating $G(x)$ by its most singular term, $G(x)=1/x$, we can calculate $\langle x\rangle$, $\langle x^2\rangle$, and $\langle xy\rangle$ analytically, considering the effect of the soft and hard components separately. In the high-energy limit $(s\to\infty)$ we obtain

$$\begin{gathered}\langle x\rangle_S\sim\frac{1}{m_0^2}\frac{\alpha}{\sigma_{hN}^{\rm in}(s)},\quad \langle x^2\rangle_S\sim\frac{1}{2m_0^2}\frac{\alpha}{\sigma_{hN}^{\rm in}(s)} ,\\ \langle xy\rangle_S\to0,\quad \langle x\rangle_H\sim\frac{1}{\sigma_{hN}^{\rm in}(s)}\ln\left[\frac{s}{p_{T\,\min}^2}\right] ,\\ \langle x^2\rangle_H\sim\frac{1}{\sigma_{hN}^{\rm in}(s)}\ln\left[\frac{s}{4p_{T\,\min}^2}\right],\quad \langle xy\rangle_H\sim\frac{1}{\sigma_{hN}^{\rm in}(s)} ,\end{gathered} \tag{16}$$

where $\langle x^ny^m\rangle_S$ $(\langle x^ny^m\rangle_H)$ were calculated with w_S (w_H). It is then clear that the soft component contribution to energy deposition decreases with the reaction energy and therefore $\langle K\rangle$ will be asymptotically dominated by the semi-hard component. Whether the total average inelasticity will increase or not will depend on the exact form of the hadron-hadron cross section.

As one can see from Eq. (16), both $\langle x\rangle_S$ and $\langle x^2\rangle_S$ decrease with energy whereas $\langle x\rangle_H$ and $\langle x^2\rangle_H$ remain essentially constant. [One can easily check that the $\sigma_{hN}^{\rm in}(s)$ we have used, cf. below, essentially cancels the log

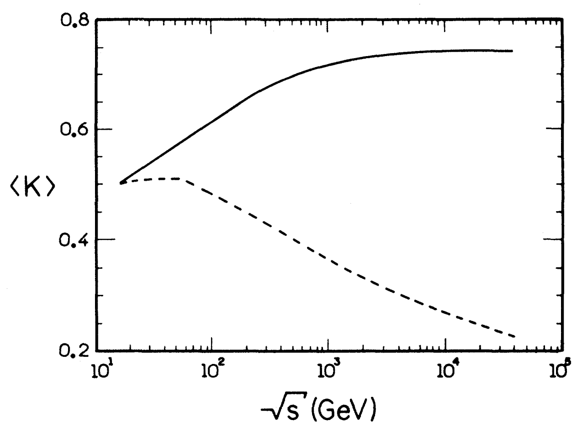

FIG. 1. Average inelasticity as a function of $\sqrt{s}$ in proton-proton collisions. The dashed line represents previous results with w_s alone and the solid curve shows $\langle K \rangle$ calculated with both contributions, i.e., with $w = w_S + w_H$.

term there.] This implies that the soft contribution will produce distributions for K and x_L narrowing with energy (as already observed in Ref. [2]) while the semi-hard component will lead to spectra broadening with energy. The numerical evaluation of $\langle K \rangle$ [as given by Eq. (14)] as a function of $\sqrt{s}$ is shown in Fig. 1 (for the proton-proton cross section we have used the following form for $\sigma_{hN}^{\text{in}}(s)$ [6]: $\sigma_{hN}^{\text{in}}(s) = 39.5 s^{-0.38} + 21.7 s^{0.08}$(mb)). As can be seen in Fig. 1 the inclusion of minijets reverses the trend of decreasing inelasticities found in the previous calculations with the IGM. It seems that the value of $\langle K \rangle$ tends to a saturation point (as suggested by the full line in Fig. 1), its precise value depending on the asymptotic behavior of σ_{hN}^{in}. This is the main result of this paper. The idea that minijets are responsible for increasing $\langle K \rangle$ was already advanced by some authors [7] and here it was brought to the IGM. One can therefore argue that here we provide *a model* for the parameter κ appearing in

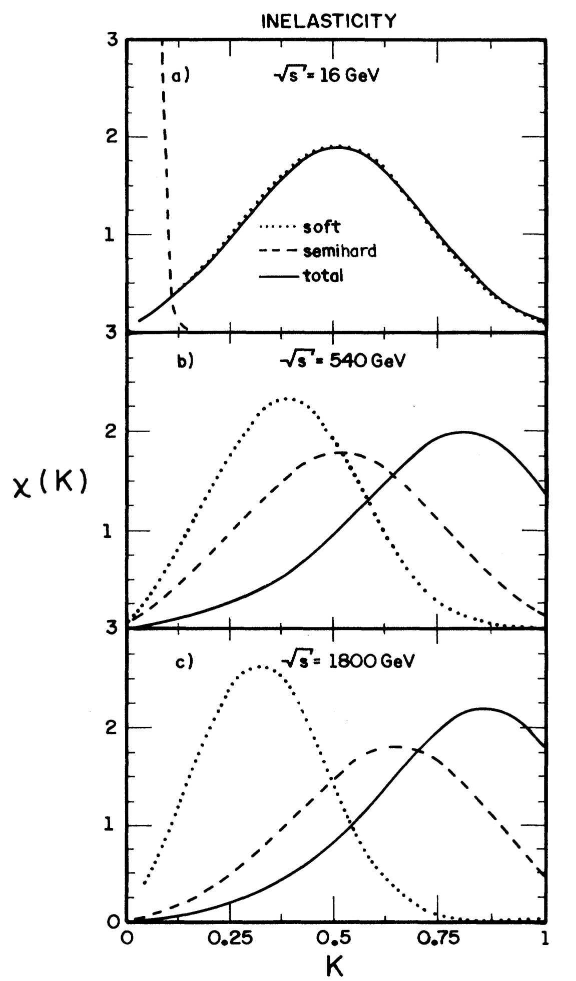

FIG. 2. (a) Inelasticity distribution for proton-proton collisions at $\sqrt{s} = 16$ GeV. The dotted line represents Eq. (13) with $w = w_S$, the dashed line is the same with $w = w_H$ and the solid curve includes both soft and semi-hard contributions $w = w_S + w_H$. (b) The same as (a) for $\sqrt{s} = 540$ GeV. (c) The same as (a) for $\sqrt{s} = 1800$ GeV.

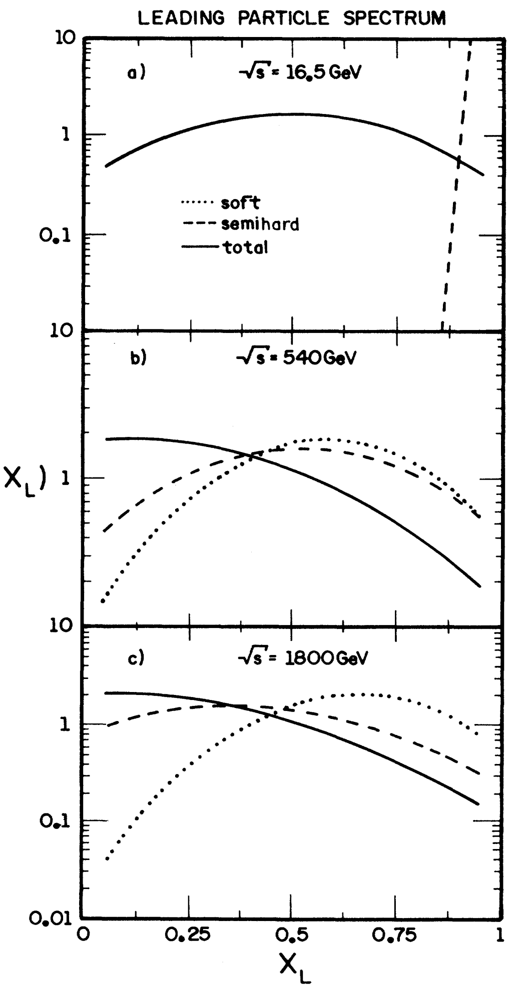

FIG. 3. (a) Leading particle distribution for proton-proton collisions at $\sqrt{s} = 16$ GeV. The dotted line represents Eq. (15) with $w = w_S$, the dotted line is the same with $w = w_H$, and the solid curve includes both soft and semihard contributions $w = w_S + w_H$. (b) The same as (a) for $\sqrt{s} = 540$ GeV. (c) The same as (a) for $\sqrt{s} = 1800$ GeV.

the formula for inelasticity presented in Ref. [7]. In this sense the remarks made in Ref. [1] about the expected limiting asymptotic behavior of inelasticity K as being caused by the assumed energy independence of the amounts of the energy-momenta p of the projectiles allocated to gluons are also valid here. Although we did not attempt to make a detailed analysis of existent data our values of $\langle K \rangle$ are very close to those found in cosmic ray studies [6,8].

Figure 2 shows inelasticity distributions for three different energies $\sqrt{s} = 16$ GeV [Fig. 2(a)], 540 GeV [Fig. 2(b)], and 1800 GeV [Fig. 2(c)]. The total distribution (solid line) is at lower energies strongly dominated by the soft component (dotted line) but at higher energies the semi-hard component (dashed line) becomes increasingly important.

Figure 3 shows leading particle spectra for the same CERN Intersecting Storage Rings (ISR), CERN Super Proton Synchrotron (S*pp*S), and Tevatron collider energies. As it can be seen, the distributions move to the left implying a softening of leading particles. This is consistent with increasing inelasticities. Apart from showing the effect of minijet dynamics these results are interesting because leading baryon spectra at such energies will soon be available [9]. We would like to remind the reader that the results in both Figs. 2(a) and 3(a) are the same as those already presented in Ref. [2] where they were shown to be in agreement with ISR data.

Some comments are in order here. We have shown that contrary to some claims [10] the IGM model can incorporate, in a quite natural way, also, the inelasticity $\langle K \rangle$ growing towards some limited value. However, it is quite clear from the present work (and was also discussed at length in Ref. [1]) that to get $\langle K \rangle$ increasing so fast as demanded by some other models (cf. Ref. [1] again) one would either have to use $\sigma^{\text{in}}_{hN}(s)$ increasing very slowly with $\sqrt{s}$ (not faster than $\ln s$) or to allow for the increase with the energy of the parameter p, i.e., the amount of energy-momenta allocated to gluons. In view of the above results and of the results presented in Refs. [1,6] we do not see a need for such a scenario for the time being.

One should be also aware of the fact that $\langle K \rangle$ as calculated above (i.e., containing both *soft* and *hard* components) can be used as initial fractional energy in statistical models only in the cases where one can expect thermalization of the produced fireball [11] (i.e., practically only in very-high-energy nuclear collisions). However, our inelasticity is perfectly usable for any cosmic ray applications [12].

This work was supported in part (G.W.) by the Polish Committee for Scientific Research Grant No. 2 0423 91 01. We would like to thank the Brazilian agencies FAPESP and CNPq for financial support.

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