L^2 part of the heavy-quark potential from dual QCD and heavy-quark spectroscopy

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We use the classical approximation to the dual QCD field equations to calculate the term in the heavy-quark potential that is proportional to angular momentum squared. This potential combined with the potentials obtained in our earlier work gives a result which is essentially the dual of the potential acting between a monopole-antimonopole pair carrying Dirac electric dipole moments and rotating in a relativistic superconductor. These potentials are used to fit the masses of the low-lying states of the $c\bar{c}$ and $b\bar{b}$ systems. The agreement, achieved with only four parameters, two of which are roughly determined in advance, is better than 1%. We also predict the masses of the lightest $c\bar{b}$ states.

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I. INTRODUCTION

In a series of recent papers we have calculated the heavy-quark–antiquark central potential [1, 2], the spin-spin potential and the tensor force [3], and finally the spin-orbit potential [4]. In this paper we wish to complete our study of the static heavy-quark potential through order $(\text{mass})^{-2}$ by including the term proportional to the orbital angular momentum squared. This potential is then used to fit the known energy levels of the $c\bar{c}$ and $b\bar{b}$ quark-antiquark systems. We also predict the as yet unobserved levels of these systems as well as those of the, as yet unseen, $c\bar{b}$ system.

II. V_{L^2} FROM THE DUAL QCD LAGRANGIAN

In dual QCD the dynamical field is C_μ, the vector potential dual to the ordinary vector potential A_μ. While the dual QCD Lagrangian has not been derived from first principles, we have motivated its construction on the basis of long-range QCD [4].

In the absence of quarks, the Lagrangian for C_μ is given by

$$\mathcal{L} = 2\text{tr}\left[\tfrac{1}{2}(\mathbf{H}^2 - \mathbf{D}^2) + \tfrac{1}{2}(\mathcal{D}_\mu \mathbf{B})^2\right] - W(B) \ , \qquad (1)$$

where

$$\mathcal{D}_\mu \mathbf{B} = \partial_\mu \mathbf{B} - ig[\mathbf{C}, \mathbf{B}] \ , \qquad (2)$$

and $\mathbf{D}$ and $\mathbf{H}$ are the non-Abelian generalizations of the color-electric displacement and magnetic field:

$$\mathbf{D} = -\boldsymbol{\nabla} \times \mathbf{C} - \tfrac{1}{2} ig[\mathbf{C}, \times \mathbf{C}] \ , \qquad (3)$$

and

$$\mathbf{H} = -\boldsymbol{\nabla} C_0 - \partial_0 \mathbf{C} - ig[\mathbf{C}, C_0], \qquad (4)$$

where $C_\mu = \sum C_\mu^a \frac{1}{2}\lambda_a$ and $\frac{1}{2}\lambda_a$ are the generators of SU(3). The dual coupling constant $g = \frac{2\pi}{e}$ where e is the ordinary Yang-Mills coupling constant. The quantity $\mathbf{B}$ represents the three scalar octets necessary to give mass to all color components of C_μ, $\mathbf{B} = (B_1, B_2, B_3)$. The function $W(\mathbf{B})$ is the counterterm needed for renormalization and plays the role of a Higgs potential. Since they couple to the dual potentials the scalar fields $\mathbf{B}$ carry color-magnetic charge.

For the calculation of flux tubes, we make the simplest color ansatz that produces a closed set of nontrivial field equations [5]. The fields $\mathbf{D}$, $\mathbf{H}$, and $\mathbf{C}$ are all proportional to the color matrix $Y = \lambda_8/\sqrt{3}$. Two of the three B fields can be chosen to be equal: $B_1 = B_2 \equiv B$. B_1, B_2, and B_3 are chosen to be in the color directions $(\lambda_7, -\lambda_5, \lambda_2)$ respectively. Finally, the counterterm is given explicitly elsewhere [6].

We next wish to extend this to a system consisting of a heavy quark of charge e and an antiquark of charge $-e$ having masses m_1 and m_2 (we can ignore spins for the purpose of this paper) and rotating around their center of mass with an angular velocity $\boldsymbol{\omega}$. The quark change

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density must also lie in the Y direction in order to absorb the flux of $\mathbf{D}$. Because quarks in our dual theory are like magnetic monopoles in ordinary electrodynamics, we must modify (3) so that Gauss's law is satisfied. This is achieved by adding a string field $\mathbf{D}_s$ to (3), which behaves like a polarization, and adding to (4) the magnetization produced by the motion of the polarization. The resulting relation between the electric displacement $\mathbf{D}$ and the magnetic field $\mathbf{H}$ and the dual potentials is

$$\mathbf{D} = -\nabla \times \mathbf{C} + \mathbf{D}_s \ , \tag{5}$$

and

$$\mathbf{H} = -\nabla C_0 - \dot{\mathbf{C}} + \mathbf{v} \times \mathbf{D}_s \ , \tag{6}$$

where $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{x}$ and $\mathbf{D}_s$ is the string field [1] joining the quarks:

$$\nabla \cdot \mathbf{D}_s = e[\delta^3(\mathbf{x} - \mathbf{x}_1(t)) - \delta^3(\mathbf{x} - \mathbf{x}_2(t))]Y \ , \tag{7}$$

where $\mathbf{x}_{1,2}(t)$ are the positions of the quarks at time t.

The resulting Lagrangian is

$$\mathcal{L} = \tfrac{2}{3}\mathbf{H}^2 - \tfrac{2}{3}\mathbf{D}^2 - 4g^2B^2(\mathbf{C}^2 - C_0^2) + 4B\nabla^2 B - 4B\ddot{B} + 2B_3\nabla^2 B_3 - 2B_3\ddot{B}_3 - W(B) \ , \tag{8}$$

where the double dot denotes the second derivative with respect to time.

The field equations following from $\mathcal{L}$ are readily written down. They are

$$\nabla^2 C_0 - \nabla \cdot \mathbf{v} \times \mathbf{D}_s + \nabla \cdot \dot{\mathbf{C}} - 6g^2B^2C_0 = 0 \ , \tag{9a}$$

$$-\nabla \times (\nabla \times \mathbf{C}) - \ddot{\mathbf{C}} + \nabla \times \mathbf{D}_s + \partial_0(-\nabla C_0 + \mathbf{v} \times \mathbf{D}_s) - 6g^2B^2\mathbf{C} = 0 \ , \tag{9b}$$

$$\nabla^2 B - \ddot{B} - g^2B(\mathbf{C}^2 - C_0^2) = \frac{1}{2}\frac{\partial W}{\partial B} \ , \tag{9c}$$

and

$$\nabla^2 B_3 - \ddot{B}_3 = \frac{\partial W}{\partial B_3} \ . \tag{9d}$$

To obtain the term proportional to L^2 in the heavy-quark potential we need only expand the above Lagrangian to order $\mathbf{v}^2$. This can be done by noting that when the quark sources rotate the static fields $\mathbf{C}, B$, and B_3 move rigidly with the sources through first order in $\mathbf{v}$. Let us denote these rotating static configurations by $\bar{\mathbf{C}}, \bar{B}$, and $\bar{B}_3$. These barred quantities satisfy the equations of Ref. [1] determining the central potential. Their time dependence is due solely to the rotation of the quark sources:

$$\dot{\bar{\mathbf{C}}} = \boldsymbol{\omega} \times \bar{\mathbf{C}} - (\mathbf{v} \cdot \nabla)\bar{\mathbf{C}} \ , \tag{10}$$

$$\dot{\bar{B}} = -(\mathbf{v} \cdot \nabla)\bar{B} \ , \tag{11}$$

and similarly for $\bar{B}_3$.

Next, we can replace $\mathbf{C}$ by $\bar{\mathbf{C}}, B$ by $\bar{B}$, and B_3 by $\bar{B}_3$ in order to calculate the Lagrangian to order v^2. This is because the Lagrangian is stationary with respect to variations around $\bar{\mathbf{C}}, \bar{B}$, and $\bar{B}_3$, since these are solutions to the equations of motion. Hence there are no terms in $\mathcal{L}$ linear in the differences $\mathbf{C} - \bar{\mathbf{C}}$, etc., which themselves are of order v^2. Thus the part of the Lagrangian quadratic in the velocity is

$$\mathcal{L}_2 = \tfrac{2}{3}\mathbf{H}^2 + 4g^2\bar{B}^2C_0^2 - 4B\ddot{\bar{B}} - 2B_3\ddot{\bar{B}}_3 \ , \tag{12}$$

where the magnetic field (which is first order in $\mathbf{v}$) is

$$\mathbf{H} = -\nabla C_0 - \boldsymbol{\omega} \times \bar{\mathbf{C}} + (\mathbf{v} \cdot \nabla)\bar{\mathbf{C}} + \mathbf{v} \times \mathbf{D}_s \ . \tag{13}$$

As in [1], it is convenient to decompose $\bar{\mathbf{C}}$ into

$$\bar{\mathbf{C}} = \mathbf{c} + \mathbf{C}_D \ , \tag{14}$$

where $\mathbf{C}_D$ is the Dirac monopole field associated with the two sources [1]:

$$\mathbf{C}_D = \frac{e}{4\pi\rho}\left\{ \frac{z - R/2}{\sqrt{\rho^2 + (z - R/2)^2}} - \frac{z + R/2}{\sqrt{\rho^2 + (z + R/2)^2}} \right\}\hat{e}_\phi \ , \tag{15}$$

and to make an analogous decomposition of C_0 into

$$C_0 = c_0 + C_{0D} \ , \tag{16}$$

where

$$C_{0D} = \mathbf{v} \cdot \mathbf{C}_D + \frac{e}{4\pi}\left\{ -\frac{\boldsymbol{\omega} \cdot (\mathbf{x} - \mathbf{x}_1)}{|\mathbf{x} - \mathbf{x}_1|} + \frac{\boldsymbol{\omega} \cdot (\mathbf{x} - \mathbf{x}_2)}{|\mathbf{x} - \mathbf{x}_2|} \right\} \tag{17}$$

is the scalar potential associated with the Biot-Savart magnetic field of the two moving sources. It is readily verified that replacing $\bar{\mathbf{C}}$ by $\mathbf{C}_D$ and C_0 by C_{OD} in Eq. (13) yields a magnetic field

$$\mathbf{H} = \mathbf{H}_{BS} \equiv \mathbf{v}_1 \times \mathbf{D}_{c_1} + \mathbf{v}_2 \times \mathbf{D}_{c_2} \ , \tag{18}$$

where $\mathbf{v}_{1,2} = \boldsymbol{\omega} \times \mathbf{x}_{1,2}$ and $\mathbf{D}_{c_{1,2}}$ are the Coulomb fields of sources 1 and 2.

Inserting the decompositions (16) and (17) into $\mathbf{H}$ (13) and using (18), we obtain for the Lagrangian $\mathcal{L}_2$ the expression

$$\mathcal{L}_2 = \tfrac{2}{3}[-\nabla c_0 + \mathbf{v}_1 \times \mathbf{D}_{c_1} + \mathbf{v}_2 \times \mathbf{D}_{c_2} - \nabla \times (\mathbf{v} \times \mathbf{c})]^2 + 4g^2\bar{B}^2(c_0 + C_{0D})^2 + 4[(\mathbf{v} \cdot \nabla)\bar{B}]^2 + 2[(\mathbf{v} \cdot \nabla)\bar{B}_3]^2 \ . \tag{19}$$

Varying c_0 in $\mathcal{L}_2$ (note that c_0 is the only unknown field in $\mathcal{L}_2$) produces the field equation

$$\nabla^2 c_0 - 6g^2\bar{B}^2(c_0 + C_{0D}) = 0 \ . \tag{20}$$

Since we now have a static problem, the L^2 part of the heavy-quark potential is

$$V_{L^2} = -\int d^3\mathbf{x}\mathcal{L}_2 \ . \tag{21}$$

Using Eq. (20), we can now express V_{L^2} in the final form

$$V_{L^2} = -\int d^3\mathbf{x}\Big\{ \tfrac{2}{3}(\mathbf{v}_1 \times \mathbf{D}_{c_1} + \mathbf{v}_2 \times \mathbf{D}_{c_2})^2 + \tfrac{2}{3}[\boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{c})]^2 + \tfrac{4}{3}\mathbf{v} \times \mathbf{c} \cdot [(\mathbf{v}_1 \cdot \boldsymbol{\nabla})\mathbf{D}_{c_1} + (\mathbf{v}_2 \cdot \boldsymbol{\nabla})\mathbf{D}_{c_2}]$$
$$+\tfrac{2}{3}c_0 \boldsymbol{\nabla} \cdot \mathbf{v} \times \mathbf{D}_s + 4[(\mathbf{v} \cdot \boldsymbol{\nabla})\bar{B}]^2 + 2[(\mathbf{v} \cdot \boldsymbol{\nabla})\bar{B}_3]^2 \Big\} \ . \tag{22}$$

In obtaining the solution to Eq. (20) it is convenient to employ a coordinate system which reflects the symmetry of the fields. In this system the quark and antiquark lie on the z axis at $z = R/2$ and $z = -R/2$, respectively. This of course means that for unequal mass quarks the center of mass $\mathbf{x}_{\text{c.m.}} = z_{\text{c.m.}}\hat{e}_z$ is not at the origin. Choosing the rotation in the x direction for quarks of mass m_1 and m_2 we obtain the following expressions for the various velocities:

$$\mathbf{v} = \boldsymbol{\omega} \times (\mathbf{x} - \mathbf{x}_{\text{c.m.}}) = \omega[y\hat{e}_z - (z - z_{\text{c.m.}})\hat{e}_y], \tag{23a}$$

$$\mathbf{v}_1 = \boldsymbol{\omega} \times (\mathbf{x}_1 - \mathbf{x}_{\text{c.m.}}) = -\omega\left(\frac{R}{2} - z_{\text{c.m.}}\right)\hat{e}_y, \tag{23b}$$

and

$$\mathbf{v}_2 = \boldsymbol{\omega} \times (\mathbf{x}_2 - \mathbf{x}_{\text{c.m.}}) = \omega\left(\frac{R}{2} + z_{\text{c.m.}}\right)\hat{e}_y, \tag{23c}$$

where

$$z_{\text{c.m.}} = \frac{m_1 - m_2}{m_1 + m_2}\frac{R}{2}. \tag{24}$$

Substituting these expressions into C_{0D}, we find that it separates naturally into a part even in z and one odd in z.

$$C_{0D} = \omega \cos\phi[C^o_{0D} + C^e_{0D}], \tag{25}$$

where

$$C^o_{0D} = \frac{e}{4\pi\rho}\Big\{ \frac{\rho^2 + z(z + R/2)}{\sqrt{\rho^2 + (z + R/2)^2}} - \frac{\rho^2 + z(z - R/2)}{\sqrt{\rho^2 + (z - R/2)^2}} \Big\}, \tag{26}$$

and

$$C^e_{0D} = z_{\text{c.m.}}\frac{e}{4\pi\rho}\Big\{ \frac{z - R/2}{\sqrt{\rho^2 + (z - R/2)^2}} - \frac{z + R/2}{\sqrt{\rho^2 + (z + R/2)^2}} \Big\}. \tag{27}$$

A similar decomposition of c_0 and the fact that $\bar{B}$ is even in z allows Eq. (20) to be separated into an equation for the even part and one for the odd. Removing the ϕ dependence changes the ∇^2 to $\nabla^2 - 1/\rho^2$. One further simplification results from a comparison of C^e_{0D} and C_D. The coefficient of $\hat{e}_\phi$ in the latter is proportional to the former. This means that the field equation satisfied by c^e_0 and its boundary value are simply proportional to the c (where $\mathbf{c} = c\hat{e}_\phi$) of our unperturbed solution. Therefore $c^e_0 = z_{\text{c.m.}}c$ and Eq. (20) need only be solved for c^o_0.

Using

$$\mathbf{L} = \frac{m_1 m_2}{m_1 + m_2}R^2\boldsymbol{\omega}, \tag{28}$$

and evaluating all explicit functions we obtain

$$V_{L^2} = \frac{L^2}{4R^2}\left[\left(\frac{1}{m_1} + \frac{1}{m_2}\right)^2 V_+ + \left(\frac{1}{m_1} - \frac{1}{m_2}\right)^2 V_-\right], \tag{29}$$

where

$$V_+ = \frac{4e}{3R}\int_0^\infty \rho^2 d\rho \int_{-\infty}^\infty dz \frac{c}{[\rho^2 + (z - R/2)^2]^{\frac{5}{2}}}\left(2\rho^2 + 2z^2 - \frac{Rz}{2} - \frac{R^2}{4}\right) - \frac{2}{3}\frac{\alpha_s}{R} - \frac{8e}{3R^2}\int_0^{R/2} z dz \frac{1}{\rho}\frac{\partial \rho c_0}{\partial \rho}\bigg|_{\rho=0}$$
$$-\frac{2\pi}{R^2}\int_0^\infty \rho\, d\rho \int_{-\infty}^\infty dz \left[\frac{4}{3}\left(\rho\frac{\partial c}{\partial z} - z\frac{\partial c}{\partial \rho}\right)^2 + \frac{4}{3}\frac{r^2c^2}{\rho^2} + 8\left(\rho\frac{\partial \bar{B}}{\partial z} - z\frac{\partial \bar{B}}{\partial \rho}\right)^2 + 4\left(\rho\frac{\partial \bar{B}_3}{\partial z} - z\frac{\partial \bar{B}_3}{\partial \rho}\right)^2\right] \tag{30}$$

and

$$V_- = 2e\int_0^\infty \rho^2 d\rho \int_{-\infty}^\infty dz \frac{(z - R/2)c}{[\rho^2 + (z - R/2)^2]^{\frac{5}{2}}} + \frac{2e}{3}\int_0^{R/2} dz \frac{1}{\rho}\frac{\partial \rho c}{\partial \rho}\bigg|_{\rho=0}$$
$$+\frac{2}{3}\frac{\alpha_s}{R} - 2\pi\int_0^\infty \rho\, d\rho \int_{-\infty}^\infty dz \left\{\frac{1}{3}\left[\frac{c^2}{\rho^2} + \left(\frac{\partial c}{\partial \rho}\right)^2\right] + 2\left(\frac{\partial \bar{B}}{\partial \rho}\right)^2 + \left(\frac{\partial \bar{B}_3}{\partial \rho}\right)^2\right\} \ . \tag{31}$$

It is this final form which we will employ for our numerical computations.

III. NUMERICAL EVALUATION OF V_{L^2}

The procedure which is followed here is exactly the same as that of Ref. [3], as are the numerical methods. The azimuthal dependence in the field equation (20) for c_0 can be factored out leaving a second-order partial differential equation in the variables ρ and z. The fact that c_0 goes exponentially to $-C_{OD}$ means that we can use a finite cutoff in both variables and solve the equation for positive z. We used a two-dimensional lattice, typically 64 by 64, on which the values of $\bar{B}$ are known from our previous calculations. For each value of R we used the Gauss-Seidel method with the successive overrelaxation (SOR) technique to produce the solutions to (20). The rate of convergence required 400 to 600 iterations to produce accurate results. For each value of R, we use c_0 and the previously determined solutions to the field equations for $\bar{c}$, $\bar{B}$, and $\bar{B}_3$ to evaluate (22). The results of this calculation are shown in Fig. 1. An analytic fit to our numerical results, useful in the calculation of the energy levels of $c\bar{c}$ and $b\bar{b}$ systems, is the following:

$$V_{L^2} = \frac{L^2\alpha_s}{m_q^2 R^3}(-0.344 + 1.553x - 0.288x^2 + 0.0169x^3) \tag{32}$$

where R is the separation of the quark and antiquark, α_s is the conventional QCD coupling constant, and x is the dimensionless length variable $x = \sqrt{-\lambda \tilde{F}_0^2} R$. Here λ and $-\tilde{F}_0^2$ are the two standard parameters of dual QCD [5, 6]. To avoid solving the field equations in the fitting process, we have fixed the third parameter of dual QCD, $g'^2 = 5$. As a result, α_s is given in terms of λ by $\alpha_s = \frac{\pi}{5\lambda}$.

As was the case for all other $(\text{mass})^{-2}$ potentials, the R behavior is too singular for use in the Schrödinger equation and therefore must be treated as a perturbation. All of our previous empirical potentials contained Yukawa forms as well as powers of $1/R$. This potential, at least over the range necessary for our fit (from a few tenths of a fm to a few fm), has no obvious Yukawa dependence. We note that in our numerical calculations the boundary conditions for $\bar{B}$ along the z axis change at the charge position. For this reason it was necessary to have the charge on a mesh point. This means that the smallest value of R for which we could do the calculation would be the z cutoff (fixed by the QCD length scale) divided by the number of mesh points. In reality, more points are needed between the origin and the charge for an accurate solution. The integral of the $\mathbf{D}_s$ term in (22) runs over z from the origin to the charge location. Clearly this cannot be evaluated accurately with only one or two mesh points.

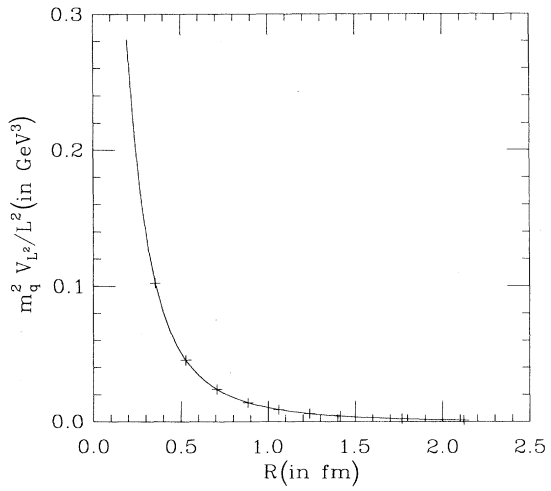

FIG. 1. The potential $m_q^2 V_{L^2}(R)/L^2$ plotted as a function of quark separation R. The ×'s are our calculated points and the solid curve is our empirical form.

Finally, we note that there are also problems in the calculation of this potential for very large values of R and that the fit given in Eq. (32) only describes the potential in the region that we have calculated it. It then cannot be used for either very small or large values of R. We do not view this limitation as a serious problem since the perturbation theory calculation of the energy shifts of p-wave state (this potential vanishes for s waves) is strongly suppressed near the origin due to the vanishing of the wave function and the exponential fall of the wave function at large R makes the contribution from this region unimportant.

IV. FIT TO THE ENERGY LEVELS OF THE $C\bar{C}$ AND $B\bar{B}$ SYSTEMS

The procedure for obtaining a best fit to the energy levels of the known $c\bar{c}$ and $b\bar{b}$ states is the same as that used in Ref. [7]. We define an effective χ^2 as

$$\chi^2 = \sum \left(\frac{\text{experiment} - \text{theory}}{0.01 \times \text{experiment}} \right)^2 . \tag{33}$$

This would be the actual χ^2 if the experimental statistical error was in fact 1% or equivalently what might be expected to be equal to the number of degrees of freedom if the theory was good to 1%. Our four free parameters, λ, $\sqrt{-\tilde{F}_0^2}$, and the two quark masses m_c and m_b, are then varied to minimize the effective χ^2. Our procedure is the following. Using our spin independent potential we solve the Schrödinger equation to determine the eigenvalues and the wave functions for the necessary orbital angular momentum states. The spin and angular momentum dependent potential are then used perturbatively to calculate the energies of the individual states and the χ^2 is evaluated. The four parameters are then varied to minimize χ^2. It should be emphasized that these are our only parameters and the dependence of potentials on these parameters is completely determined by dual QCD.

Once the best-fit parameters are determined we can predict the unobserved energy levels. Our best fit to the sixteen observed states is given in Table I. The resulting

TABLE I. Predicted and experimental masses [8] of all observed $c\bar{c}$ and $b\bar{b}$ states below threshold.

State	Predicted mass (GeV)	Experimental mass (GeV)
$\eta_c(1S)$	2.968	2.980
$\psi(1S)$	3.110	3.097
$\psi(2S)$	3.693	3.686
$\chi_{c_0}(1P)$	3.440	3.415
$\chi_{c_1}(1P)$	3.498	3.511
$\chi_{c_2}(1P)$	3.537	3.556
$\Upsilon(1S)$	9.489	9.460
$\Upsilon(2S)$	10.010	10.023
$\Upsilon(3S)$	10.341	10.355
$\Upsilon(4S)$	10.613	10.580
$\chi_{b_0}(1P)$	9.866	9.860
$\chi_{b_1}(1P)$	9.900	9.892
$\chi_{b_2}(1P)$	9.918	9.913
$\chi_{b_0}(2P)$	10.212	10.232
$\chi_{b_1}(2P)$	10.236	10.255
$\chi_{b_2}(2P)$	10.255	10.268

χ^2 is 1.64, corresponding to an average least-squares error of 0.3%. This represents a substantial improvement over our preliminary results [7], where we fit the 13 states that were independent of V_{LS} obtaining $\chi^2 = 2.24$. The largest percentage error occurs for the $\chi_{c_0}(1P)$, where our result is too heavy by 25 MeV.

Our best-fit parameters are

(i) $\lambda = 1.705$,

(ii) $\sqrt{-\tilde{F}_0^2} = 428$ MeV,

(iii) $m_c = 1.359$ GeV,

(iv) $m_b = 4.778$ GeV.

These parameters, which are only slightly changed from our previous fits [7], result in $\alpha_s = 0.3685$, a string tension $\sigma = 0.1885$ GeV^2, a flux tube radius of 0.5 fm, and a scaling parameter $\sqrt{-\lambda\tilde{F}_0^2} = 558$ MeV.

The predicted energy levels for the as yet unobserved (nearly) stable states of these systems are shown in Table II. Our potentials should be applicable to any state composed of sufficiently heavy quarks. The only required changes for S states are the use of the correct reduced mass in the Schrödinger equation and the replacement of m_q^2 by the product of the quark masses. (The quark mass dependence of V_{LS} and V_{L^2} is more complicated.) The results for the lowest-mass $c\bar{b}$ states are $\eta_{c\bar{b}} = 6.276$ GeV and $^3S_1 = 6.365$ GeV. The higher states could also be calculated, but in the absence of any experimental candidates this seems premature.

TABLE II. Predicted masses of the unobserved $c\bar{c}$ and $b\bar{b}$ states.

State	Predicted mass (GeV)
$\eta_c(2S)$	3.592
$(^1P_1)c\bar{c}$ $(n=1)$	3.510
$(^1D_2)c\bar{c}$ $(n=1)$	3.817
$(^3D_1)c\bar{c}$ $(n=1)$	3.806
$\eta_b(1S)$	9.344
$\eta_b(2S)$	9.946
$\eta_b(3S)$	10.291
$\eta_b(4S)$	10.569
$(^1P_1)b\bar{b}$ $(n=1)$	9.904
$(^1P_1)b\bar{b}$ $(n=2)$	10.244
$(^1D_2)b\bar{b}$ $(n=1)$	10.144
$(^1D_2)b\bar{b}$ $(n=2)$	10.429
$\chi_{b_0}(3P)$	10.492
$\chi_{b_1}(3P)$	10.515
$\chi_{b_2}(3P)$	10.533
$(^3D_1)b\bar{b}$ $(n=1)$	10.135
$(^3D_2)b\bar{b}$ $(n=1)$	10.142
$(^3D_3)b\bar{b}$ $(n=1)$	10.149
$(^3D_1)b\bar{b}$ $(n=2)$	10.421
$(^3D_2)b\bar{b}$ $(n=2)$	10.428
$(^3D_3)b\bar{b}$ $(n=2)$	10.435

V. CONCLUSIONS

We have found that the classical approximation to dual QCD gives an excellent fit to the (nearly) stable states of the heavy-quark–antiquark system, in terms of only four parameters. It should be emphasized that this is a calculation from first principles in dual QCD and as such should correctly predict long-range phenomena. We have deliberately not included effects associated with the short-range behavior of QCD such as letting the coupling constant run, or allowing the use of different α_s values for the ψ and Υ and still a third value for the leptonic widths. Our purpose is to show what predictions follow directly from the theory, and what phenomena require a more complete description of QCD.

It appears that all of the potentials have the correct sign and reasonable behavior as a function of R. Those terms that differ in sign from one gluon exchange seem to agree with the experimental data.

Finally, we note that for all of these fits we fixed $g'^2 = 5$. Because varying this parameter requires a complete solution to the dual QCD field equations, which are nonlinear partial differential equations, it was not practical to include this parameter in our minimization search. However, we have done the complete calculation for $g'^2 = 2$ and $g'^2 = 10$. Over this range the final results showed very little dependence on g'^2 with the χ^2 for $g'^2 = 5$ being slightly smaller than for the other values. The dual QCD parameters λ and $\sqrt{-\tilde{F}_0^2}$ change in such a way as to keep the physical quantities α_s and the string tension essentially constant. As result, the exact value of g'^2 does not affect the fit and we can safely fix it to be 5, which was determined from other considerations [5].

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