Universal seesaw mechanisms for quark-lepton mass spectrum

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Problems of fermion mass hierarchies and generation mixings are investigated through universal seesaw mechanisms (USM's) in an extension of the standard model with a left-right-symmetric gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_y$. Electroweak Higgs doublets and singlets induce USM's between ordinary fermion multiplets and exotic electroweak singlets of fermions. The USM's work singly in the charged-fermion sectors to suppress their masses below the electroweak mass scale, and doubly in the neutral-fermion sector to make neutrinos superlight. The wide gap between vanishingly small neutrino masses and the 100 GeV scale of the top-quark mass is explained by multiple USM suppressions without presuming a huge Majorana mass. A global chiral $U(1)_A$ symmetry is introduced so as to circumvent the strong *CP* violation, to distinguish generations, and to restrict the pattern of the Yukawa interactions. Three kinds of electroweak Higgs singlets bring about USM's and cause the generation mixing leading to a realistic variety in each charge sector of the fermion mass spectrum. A fourth Higgs singlet with the largest vacuum expectation value is introduced to make the neutrino masses tiny and to make the axion invisible. By assigning chiral charges to make effective mass matrices of all fermion sectors of the extended Fritzsch type, characteristics of the mass spectra of charged fermions and the quark mixing matrix are described without introducing unnatural hierarchies in the Yukawa coupling constants. Neutrinos have a spectrum comprising doubly degenerate states with a smaller mass and a singlet state with a larger mass. The vacuum mixing angle takes a small value which is favorable for explaining both the new results of the GALLEX Collaboration and the data of the Homestake and Kamiokande experiments.

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I. INTRODUCTION

If the solar-neutrino puzzle owes its resolution to matter-induced flavor conversion, viz., the Mikheyev-Smirnov-Wolfenstein mechanism [1,2], neutrinos changing their flavors should have tiny masses around and less than 10^{-3} eV [3]. On the other hand, high-precision measurements of Z decay at the CERN e^+e^- collider LEP [4] have shown that the top-quark mass takes a value of 124 ± 34 GeV [5]. Therefore it is an important challenge to theorists to account for why quarks and leptons exist in a surprisingly broad mass spectrum ranging from 10^{-3} to 10^{11} eV. In this paper we investigate such a characteristic feature of the quark-lepton mass spectrum, in which neutrinos hold a special position through universal seesaw mechanisms in an extension of the standard model with the left-right-symmetric gauge group [6]

$$G = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_y ,$$

where a group $U(1)_y$ is generated by a new charge y.

The universal seesaw mechanism (USM) [7–9] is designed to explain the smallness of the masses of quarks and charged leptons relative to the electroweak scale by postulating the existence of exotic fermions belonging to electroweak singlets. Davidson and Wali [9] have combined a global chiral $U(1)_A$ symmetry with the underlying gauge group G by introducing two left-right-symmetric sets of electroweak Higgs doublets and derived effective mass matrices of extended Fritzsch type for up- and down-quark sectors at the first seesaw approximation [9]. In their approach, neutral fermions are presumed to have Dirac and Majorana masses, so as to make neutrinos superlight through the original seesaw mechanism [10]. To describe the masses of charged and neutral fermions in a unified way, we have developed a new scheme of USM [11,12] within a framework of a gauge field theory with $SU(4)_L \times SU(4)_R$ color symmetry in which leptons are described as the fourth color fermions in the manner of Pati and Salam [13]. The distinguishing aspect of the new scheme from the ordinary USM is that exotic electroweak-singlet fermions with and without color are assumed to exist for each generation so as to effectuate the USM singly in the charged-fermion sectors and doubly in the neutral-fermion sector. As a result, the broad width of the quark-lepton mass spectrum is explained naturally without resorting to a huge Majorana mass. Three Higgs biquartets trigger the USM's and work to bring diversity in the mass spectrum. Colored Higgs quartets make the color interaction of leptons superweak and lead to tiny neutrino masses through multiple seesaw suppressions.

The model in this paper has the same number of fundamental fermions as the previous scheme [11,12]. Namely, in addition to the fermions presumed to exist in the forms of electroweak doublets and singlets in the Davidson-Wali approach, we postulate the existence of additional electroweak-singlet neutral fermions per generation. Higgs fields of the model consist of a left-right-symmetric pair of electroweak doublets and four kinds of electroweak singlets. The Higgs doublets determine scales of the standard left-handed electroweak interaction and of a high-energy right-handed electroweak interaction. The Higgs singlets carrying different chiral charges work to resolve the strong *CP* problem and to bring about multiple USM suppressions. Three of them create variety in the mass spectrum of each charge sector of fermions (horizontal hierarchy). The remaining one, with the largest vacuum expectation value (VEV), plays the roles of explaining the wide mass gap in each generation of observed fermions (vertical hierarchy) and making the so-called axion invisible through the double universal seesaw mixings. Chiral charges are assigned to the fermion and Higgs fields so as to distinguish generations, to get effective mass matrices of extended Fritzsch type with an extra (2,2) entry for the charged-fermion sectors, and to realize the maximum suppression of neutrino masses at the first seesaw approximation.

In contrast with the conventional seesaw approach where both the original and universal seesaw mechanisms are implemented separately, the present scheme enables us to describe the main characteristics of the fermion mass spectrum in a unified manner. All fermions acquire masses through the ordinary Higgs mechanism and the fermion number is conserved. The vertical and horizontal hierarchies in the spectrum of the charged fermions and the magnitude of the quark mixing matrix are explained without adjustment of the Yukawa coupling constants over an unnatural range of values. Neutrinos turn out to have a distinctive mass spectrum consisting of doubly degenerate states with smaller mass and a singlet state with larger mass. As a result of such a unique spectrum, the matter-induced flavor conversion occurs among three neutrino species, and the vacuum mixing angle takes a small value which is favorable for explaining the data of the Homestake [14] and Kamiokande [15] experiments and the new gallium counting rate results of the GALLEX Collaboration [3].

In the next section, fundamental fermions and Higgs bosons in the model are introduced and their classification is made with respect to the underlying gauge group G and the chiral symmetry $U(1)_A$. For the ordinary USM to take place, every conventional fermion must be accompanied by an exotic fermion belonging to an electroweak singlet, and an extra neutral fermion is assumed to exist per generation so as to induce the USM doubly in the neutral-fermion sector. The Higgs structure is simple in the sense that only electroweak doublets and singlets are introduced. The electroweak bidoublet of Higgs fields [6] is forbidden to exist, since it obstructs implementation of the USM. In Sec. III the Higgs potential is given and the condition for the existence of a left-right-asymmetric vacuum is derived. To analyze the pattern of the Yukawa interaction, we introduce a subsidiary matrix $\mathcal{K}$ [9] whose (i,j) component consists of the sum of chiral charges of the fermion fields of the ith and jth generations in Sec. IV. There are several different choices of chiral charge assignments which distinguish generations and lead to effective mass matrices of extended Fritzsch type. We adopt an assignment of chiral charges that produces the horizontal hierarchies and induces the strongest suppression in the neutrino effective mass matrix. In Sec. V a generic form of effective mass matrices is derived for all fermion sectors at the first seesaw approximation. In Sec. VI an analysis is made of the effective mass matrices for the charged-fermion sectors and neutrino sector. The magnitude of the elements of the quark mixing matrix is found to be consistent with the observed values. Strong USM suppressions give unique features to the neutrino mass spectrum and the lepton mixing matrix, i.e., degenerate masses and three-flavor oscillation with a small vacuum mixing angle. Section VII is allotted for discussion. Order estimation of VEV's of the Higgs fields in the model shows that the superlightness of neutrinos is deeply related to the invisibleness of the axion. In Appendix A different assignments of chiral charges and properties of the resulting mass matrix are shown. Appendix B describes briefly a formalism for the matter-induced neutrino oscillation among three flavors with a specific mass spectrum.

II. FUNDAMENTAL FERMIONS AND HIGGS BOSONS

Fundamental fermions in the model are classified with respect to the underlying gauge group G and the global chiral group $U(1)_A$. The electroweak doublets of fermions destined to constitute main components of ordinary quarks and leptons of the ith generation are described by the chiral fields with transformation properties

$$
\begin{aligned}
q_{iL} &= \begin{bmatrix} u^r & u^g & u^b \\ d^r & d^g & d^b \end{bmatrix}_{iL} \sim (3,2,1;\tfrac{1}{3};x_i) , \\
q_{iR} &= \begin{bmatrix} u^r & u^g & u^b \\ d^r & d^g & d^b \end{bmatrix}_{iR} \sim (3,1,2;\tfrac{1}{3};-x_i) ,
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
l_{iL} &= \begin{bmatrix} \nu \\ e \end{bmatrix}_{iL} \sim (1,2,1;-1;x_i) , \\
l_{iR} &= \begin{bmatrix} \nu \\ e \end{bmatrix}_{iR} \sim (1,1,2;-1;-x_i) ,
\end{aligned}
\tag{2}
$$

where the fourth and fifth entries in parentheses are, respectively, y charges and chiral charges. As the seesaw partners, we introduce electroweak singlets of exotic fermions with the chiral fields transforming as

$$
\begin{aligned}
U_{iL} &= (U^r\ U^g\ U^b)_{iL} \sim (3,1,1;\tfrac{4}{3};y_i) , \\
U_{iR} &= (U^r\ U^g\ U^b)_{iR} \sim (3,1,1;\tfrac{4}{3};-y_i) ,
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
D_{iL} &= (D^r\ D^g\ D^b)_{iL} \sim (3,1,1;-\tfrac{2}{3};y_i) , \\
D_{iR} &= (D^r\ D^g\ D^b)_{iR} \sim (3,1,1;-\tfrac{2}{3};-y_i) ,
\end{aligned}
\tag{4}
$$

$$N_{iL} \sim (1,1,1;0;y_i), \quad N_{iR} \sim (1,1,1;0;-y_i) , \tag{5}$$

$$E_{iL} \sim (1,1,1;-2;y_i), \quad E_{iR} \sim (1,1,1;-2;-y_i) \tag{6}$$

for each generation. To bring about the USM doubly in the neutral-fermion sector, it is necessary to assume the existence of electroweak singlets of neutral fermions

$$N'_{iL} \sim (1,1,1;0;z_i), \quad N'_{iR} \sim (1,1,1;0;-z_i) \tag{7}$$

for each generation. Although the symmetry between charged and neutral sectors is broken, these additional singlets make the neutrinos superlight without violating fermion-number conservation.

The model has a rather simple Higgs structure. A left-right-symmetric pair of electroweak Higgs doublets,

$$\chi_L \sim (1,2,1;-1;0), \quad \chi_R \sim (1,1,2;-1;0) , \tag{8}$$

with VEV's

$$\langle \chi_L \rangle = \begin{bmatrix} w_L \\ 0 \end{bmatrix}, \quad \langle \chi_R \rangle = \begin{bmatrix} w_R \\ 0 \end{bmatrix} \tag{9}$$

works to break the left-right symmetry and the Weinberg-Salam symmetry in the electroweak interaction. We assign zero chiral charge to them so that quarks of up and down sectors can acquire masses through VEV's of $\chi_{L,R}$ and their conjugates $\tilde{\chi}_{L,R} = i\sigma_2 \chi^*_{L,R}$. Three kinds of Higgs singlets carrying different chiral charges,

$$\sigma_i \sim (1,1,1;0;h_i) \quad (i=1,2,3) , \tag{10}$$

with VEV's

$$\langle \sigma_i \rangle = v_i , \tag{11}$$

induce the USM's in all fermion sectors and bring in different scales to afford realistic mass spectra for quarks and leptons. An extra Higgs singlet

$$\sigma_0 \sim (1,1,1;0;h_0) \tag{12}$$

with the largest VEV

$$\langle \sigma_0 \rangle = v_0 \tag{13}$$

is introduced to bring about doubly the USM in the neutral-fermion sector. Without loss of generality, the chiral charges of Higgs singlets are restricted to be positive, i.e., $h_i > 0$ $(i=0,1,2,3)$. The complex-conjugate field σ_i^* with negative chiral charge $-h_i$ is the chiral partner of σ_i transforming as $\sigma_i \leftrightarrow \sigma_i^*$ under $L \leftrightarrow R$.

The VEV's are assumed to be real and to satisfy the hierarchy

$$v_0 > v_3 > v_2 > v_1 > w_R > w_L \tag{14}$$

$(v_0^2 \gg v_3^2 \gg v_2^2 \gg v_1^2 \gg w_R^2 \gg w_L^2)$. The G symmetry breaks down to $SU(3)_c \times U(1)_Q$ via the Weinberg-Salam symmetry $G_{WS} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$. The hypercharge Y of the group $U(1)_Y$ at the Weinberg-Salam symmetry is given by

$$Y = 2T_{3R} + y , \tag{15}$$

in terms of the third component of the electroweak isospin T_{3R} and the y charge.

The Lagrangian with the symmetry group G is required to be invariant under the chiral $U(1)_A$ transformation of the fermions and Higgs scalars defined by

$$q_i \to e^{-i\gamma_5 x_i \theta} q_i, \quad l_i \to e^{-i\gamma_5 x_i \theta} l_i , \tag{16}$$

$$\begin{aligned} (U_i, D_i) &\to e^{-i\gamma_5 y_i \theta}(U_i, D_i) , \\ (N_i, E_i) &\to e^{-i\gamma_5 y_i \theta}(N_i, E_i) , \end{aligned} \tag{17}$$

$$N'_i \to e^{-i\gamma_5 z_i \theta} N'_i , \tag{18}$$

$$\chi_L, \chi_R \to \chi_L, \chi_R , \tag{19}$$

$$\sigma_i \to e^{ih_i\theta} \sigma_i \quad (i=0,1,2,3) , \tag{20}$$

for an arbitrary real number θ. Chiral charges are determined so as to place restrictions on the structure of quark-lepton mass matrices and to distinguish the generation of fundamental fermions and the type of Higgs singlet as

$$x_i \neq x_j, \quad y_i \neq y_j, \quad z_i \neq z_j, \quad h_i \neq h_j \quad (i \neq j) . \tag{21}$$

Table I categorizes the fundamental fermions and Higgs bosons in the present theory.

TABLE I. Classification of the fermion and Higgs multiplets.

	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_y$	$U(1)_A$
q_{iL}	3	2	1	$\frac{1}{3}$	x_i
q_{iR}	3	1	2	$\frac{1}{3}$	$-x_i$
l_{iL}	1	2	1	-1	x_i
l_{iR}	1	1	2	-1	$-x_i$
U_{iL}	3	1	1	$\frac{4}{3}$	y_i
U_{iR}	3	1	1	$\frac{4}{3}$	$-y_i$
D_{iL}	3	1	1	$-\frac{2}{3}$	y_i
D_{iR}	3	1	1	$-\frac{2}{3}$	$-y_i$
N_{iL}	1	1	1	0	y_i
N_{iR}	1	1	1	0	$-y_i$
E_{iL}	1	1	1	-2	y_i
E_{iR}	1	1	1	-2	$-y_i$
N'_{iL}	1	1	1	0	z_i
N'_{iR}	1	1	1	0	$-z_i$
σ_i	1	1	1	0	h_i
σ_0	1	1	1	0	h_0
χ_L	1	2	1	-1	0
χ_R	1	1	2	-1	0

III. HIGGS POTENTIAL

The renormalizable Higgs potential which is invariant under the gauge group G, the left-right symmetry $(L \leftrightarrow R, \sigma_i \leftrightarrow \sigma_i^*)$, and the global chiral transformation in Eqs. (16)–(20) is generally given by

$$\begin{aligned} V_H = & -\mu_\chi^2(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) - \sum_{i=0}^{3} \{\mu_\sigma^{(i)}\}^2 |\sigma_i|^2 \\ & + \lambda_\chi(\chi_L^\dagger \chi_L \chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R \chi_R^\dagger \chi_R) + \lambda'_\chi \chi_L^\dagger \chi_L \chi_R^\dagger \chi_R \\ & + \sum_{i,j=0}^{3} \lambda_\sigma^{(ij)} |\sigma_i|^2 |\sigma_j|^2 \\ & + \sum_{i=0}^{3} \gamma_i(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R)|\sigma_i|^2 + V'_H , \end{aligned} \tag{22}$$

where V'_H consists of cubic and quartic terms of Higgs singlets, other than $|\sigma_i|^2|\sigma_j|^2$, allowed by chiral charge conservation.

The gauge symmetry is broken by the VEV's of the Higgs fields which minimize the potential V_H. Substituting the VEV's into V_H and taking derivatives with respect to them, we get conditions for the extremum. Noting that all the VEV's must take nonvanishing values [see Eq. (14)], we obtain the relations for w_L and w_R as follows:

$$-\mu_\chi^2+2\lambda_\chi w_L^2+\lambda'_\chi w_R^2+\sum_{i=0}^{3}\gamma_i v_i^2=0\ , \tag{23}$$

$$-\mu_\chi^2+2\lambda_\chi w_R^2+\lambda'_\chi w_L^2+\sum_{i=0}^{3}\gamma_i v_i^2=0\ . \tag{24}$$

Therefore, in order to ensure the existence of a left-right-asymmetric vacuum ($w_L^2\neq w_R^2$), it is necessary to demand the condition

$$\lambda'_\chi=2\lambda_\chi\ . \tag{25}$$

Namely, the Higgs potential depends on the electroweak doublets only through the combination $\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R$. This fact does not mean the appearance of an additional symmetry, since the gauge symmetry prohibits mixing between χ_L and χ_R. Note that minimization of the Higgs potential determines $w_L^2+w_R^2$ and v_i^2 $(i=0,1,2,3)$, leaving the ratio of w_L^2 and w_R^2 unfixed. To realize the hierarchy of VEV's in Eq. (14), it is necessary to adjust the parameters in the Higgs potential and to specify the w_L/w_R ratio. Owing to a relation between chiral charges h_1 and h_3 [see Eq. (34) below], V'_H may have a term such as $\sigma_3\sigma_1^{*3}+\sigma_3^*\sigma_1^3$. Note that the stability of the potential minimum is guaranteed if the coefficients λ_χ and $\lambda_\sigma^{(ii)}$ of diagonal quartic terms in V_H are sufficiently larger than those of off-diagonal quartic terms such as γ_i and $\lambda_\sigma^{(ij)}$ $(i\neq j)$.

IV. CHIRAL CHARGES AND SEESAW MASS MATRICES

The Lagrangian density for fermion–Higgs-boson interactions $\mathcal{L}_Y$, being invariant under the fundamental group G and the left-right symmetry, has the general form

$$\begin{aligned}\mathcal{L}_Y=&\sum_{i,j}\{Y_{ij}^{uU}\overline{q}_{iL}\chi_L U_{jR}+Y_{ij}^{dD}\overline{q}_{iL}\widetilde{\chi}_L D_{jR}+Y_{ij}^{uU}\overline{q}_{iR}\chi_R U_{jL}+Y_{ij}^{dD}\overline{q}_{iR}\widetilde{\chi}_R D_{jL}\}\\&+\sum_{i,j}\{Y_{ij}^{\nu N}\overline{l}_{iL}\chi_L N_{jR}+Y_{ij}^{eE}\overline{l}_{iL}\widetilde{\chi}_L E_{jR}+Y_{ij}^{\nu N}\overline{l}_{iR}\chi_R N_{jL}+Y_{ij}^{eE}\overline{l}_{iR}\widetilde{\chi}_R E_{jL}\}\\&+\sum_{i,j}\{Y_{ij}^{\nu N'}\overline{l}_{iL}\chi_L N'_{jR}+Y_{ij}^{\nu N'}\overline{l}_{iR}\chi_R N'_{jL}\}+\sum_{i,j,k}\{Y_{ij}^{U}\overline{U}_{iL}U_{jR}+Y_{ij}^{D}\overline{D}_{iL}D_{jR}+Y_{ij}^{N}\overline{N}_{iL}N_{jR}+Y_{ij}^{E}\overline{E}_{iL}E_{jR}\\&+Y_{ij}^{N'}\overline{N}'_{iL}N'_{jR}+Y_{ij}^{NN'}\overline{N}_{iL}N'_{jR}+Y_{ij}^{NN'}\overline{N}'_{iL}N_{jR}\}(\sigma_k+\sigma_k^*)+\text{H.c.}\end{aligned} \tag{26}$$

The G symmetry prohibits the fermions belonging to electroweak doublets from interacting directly with each other. This is the key ingredient for implementing USM. The chiral $U(1)_A$ symmetry requiring the conservation of chiral charges governs which Yukawa coupling constants take nonvanishing values [see the $\mathcal{H}$ matrix in Eq. (31)]. Note that it is not necessary to specify the dependence of the Yukawa coupling constant on the Higgs singlets, since the chiral charges are assigned so that any pair of singlet fermions can couple to only one kind of Higgs singlet. The Yukawa coupling constants are assumed here to be Hermitian, viz.,

$$Y_{ij}=Y_{ji}^*\ . \tag{27}$$

Chiral charges must be assigned so that no fermion field carries a bare mass.

Spontaneous breakdowns of the G symmetry create 6×6 seesaw mass matrices for the charged-fermion sectors and a 9×9 seesaw mass matrix for the neutral-fermion sector. It is convenient to represent the mass matrices, with respect to the following bases in the generation space, as

$$f=\begin{bmatrix}f_1\\f_2\\f_3\end{bmatrix},\quad F=\begin{bmatrix}F_1\\F_2\\F_3\end{bmatrix},\quad F'=\begin{bmatrix}F'_1\\F'_2\\F'_3\end{bmatrix}, \tag{28}$$

where f_i, F_i, and F'_i are the components of the multiplets (q_i,l_i), (U_i,D_i,N_i,E_i), and N'_i, respectively. The mass matrices take the block-matrix forms

$$M^f=\begin{bmatrix}0 & M_L^f\\ M_R^f & M_\sigma^F\end{bmatrix} \tag{29}$$

for the charged-fermion sectors in the (f,F) base, and

$$M^f=\begin{bmatrix}0 & M_L^f & 0\\ M_R^f & M_\sigma^F & M_\sigma^{FF'}\\ 0 & M_\sigma^{F'F} & M_\sigma^{F'}\end{bmatrix} \tag{30}$$

for the neutral-fermion sector in the $(f,F,F')=(\nu,N,N')$ base. The 3×3 submatrices M_L^f and M_R^f are composed of the VEV's of the electroweak doublets χ_L and χ_R, while M_σ^F and $M_\sigma^{F'}$ are of the VEV's of the electroweak singlets σ_i $(i=1-3)$. In order to make neutrinos superlight, it is necessary to require that the submatrices $M_\sigma^{FF'}$ and $M_\sigma^{F'F}$ be composed mainly of the largest VEV v_0 of the Higgs singlet σ_0. To achieve the USM doubly in the neutral-fermion sector, the top-right and down-left submatrices in the 9×9 seesaw mass matrix M^f must be zero. Otherwise the neutrinos acquire masses of comparable order with the charged leptons through direct Yukawa couplings between l_i and N'_i.

As basic postulates, we presume here that all the 3×3 submatrices in Eqs. (29) and (30) are regular and that the submatrix M_σ^F takes an extended Fritzsch form. To visualize the pattern of the Yukawa couplings in $\mathcal{L}_Y$, it is convenient to introduce a matrix $\mathcal{H}$ which has the component $\mathcal{H}_{ij}$ comprising the sum of chiral charges of the fermion fields of the ith and jth generations as follows [9,12]:

$$\mathcal{H}_{ij}\equiv u_i+u_j \quad \begin{cases} u_i=x_i & (i=1,2,3)\ , \\ u_i=y_{i-3} & (i=4,5,6)\ , \\ u_i=z_{i-6} & (i=7,8,9)\ . \end{cases} \tag{31}$$

The roles of the submatrices (M_L^f, M_R^f) and M_σ^F are interchanged in the Davidson-Wali scheme and the present scheme. Namely, the variety in the quark mass matrices is brought in through (M_L^f, M_R^f) in the former, while it is through M_σ^F in the latter. For M_σ^F to be an extended Fritzsch form and to explain the diversity in mass spectra, the components of the $\mathcal{H}$ matrix must satisfy the conditions

$$\mathcal{H}_{ij}=\begin{cases} s_2h_2, & (ij)=(45),(54)\ , \\ -s_3h_3, & (ij)=(55)\ , \\ -s_1h_1, & (ij)=(56),(65)\ , \\ s_1h_1, & (ij)=(66) \end{cases}$$
$$\mathcal{H}_{ij}\neq\pm h_1,\pm h_2,\pm h_3,\pm h_0,0 \ \text{ otherwise}\ , \tag{32}$$

provided that

$$s_3h_3=3s_1h_1\ , \tag{33}$$

where the sign factors $s_i=\pm1$ depend on whether σ_i or σ_i^* couples with fermion singlets. Equation (33) is equivalent to

$$h_3=3h_1,\quad s_3=s_1\ . \tag{34}$$

Since the parameter θ in the chiral transformation in Eqs. (16)–(20) is arbitrary, it is possible to specify $s_3=s_1\equiv1$. Henceforth, let us write $s_2\equiv s$. Then, the chiral charges y_i are determined to be

$$y_1=\tfrac{3}{2}h_1+sh_2,\quad y_2=-\tfrac{3}{2}h_1,\quad y_3=\tfrac{1}{2}h_1\ . \tag{35}$$

Note that this is a unique assignment for the chiral charges y_1, y_2, and y_3 which can reduce the common features of the horizontal hierarchies to that of the VEV's, i.e., $v_3^2\gg v_2^2\gg v_1^2$. In order to get effective mass matrices of the extended Fritzsch type at the first seesaw approximation, it is necessary to restrict the components of the $\mathcal{H}$ matrix as

$$\mathcal{H}_{ij}=0,\quad (ij)=(15),(24),(36),(42),(51),(63)\ ,$$
$$\mathcal{H}_{ij}\neq0 \ \text{ otherwise}\ , \tag{36}$$

which fixes the chiral charges x_i as

$$x_1=-y_2=\tfrac{3}{2}h_1,\quad x_2=-y_1=-\tfrac{3}{2}h_1-sh_2\ ,$$
$$x_3=-y_3=-\tfrac{1}{2}h_1\ . \tag{37}$$

With the chiral charges obtained so far, the structure of the submatrices M_L^f, M_R^f, and M_σ^F is determined as

$$M_{L,R}^f=\begin{pmatrix} 0 & Y_{12}^{fF}w_{L,R} & 0 \\ Y_{21}^{fF}w_{L,R} & 0 & 0 \\ 0 & 0 & Y_{33}^{fF}w_{L,R} \end{pmatrix} \tag{38}$$

for $(f,F)=(u,U),(d,D),(\nu,N),(e,E)$, and

$$M_\sigma^F=\begin{pmatrix} 0 & Y_{12}^Fv_2 & 0 \\ Y_{21}^Fv_2 & Y_{22}^Fv_3 & Y_{23}^Fv_1 \\ 0 & Y_{32}^Fv_1 & Y_{33}^Fv_1 \end{pmatrix} \tag{39}$$

for $F=U,D,N,E$.

To make the submatrices $M_\sigma^{FF'}$ and $M_\sigma^{F'F}$ regular, it is necessary to impose the restrictions

$$z_{j_i}=-y_i+s_{i+3}h_0 \quad (j_1\neq j_2\neq j_3) \tag{40}$$

on the chiral charges z_{j_i} $(i=1-3)$. Here the sign factors s_{i+3} distinguish whether σ_0 or σ_0^* enters in the Yukawa interactions. Without loss of generality, we are able to rename the exotic neutral-fermion fields as

$$F'_{j_i}\to F'_i\ . \tag{41}$$

After this relabeling, the component matrix $(\mathcal{H}_{ij})$ takes the form

$$(\mathcal{H}_{ij})=\begin{pmatrix} 2s_4h_0-3h_1-2sh_2 & (s_4+s_5)h_0-sh_2 & (s_4+s_6)h_0-2h_1-sh_2 \\ (s_5+s_4)h_0-sh_2 & 2s_5h_0+3h_1 & (s_5+s_6)h_0+h_1 \\ (s_6+s_4)h_0-2h_1-sh_2 & (s_6+s_5)h_0+h_1 & 2s_6h_0-h_1 \end{pmatrix} \tag{42}$$

for $i,j=7-9$. To get such an effective mass matrix for the neutrino sector that realizes the strongest USM suppression, we impose the restrictions

$$s_4=-s_5=-s_6,\quad 2s_6h_0=h_1+s'h_2\ , \tag{43}$$

leading to $\mathcal{H}_{78}=\mathcal{H}_{87}=-sh_2$ and $\mathcal{H}_{99}=s'h_2$. Then the chiral charges for all fermions are determined as in Table II, and the remaining submatrices for the neutral-fermion

TABLE II. Chiral charges of the fermion multiplets.

	1	2	3
x_i	$\frac{3}{2}h_1$	$-\frac{3}{2}h_1-sh_2$	$-\frac{1}{2}h_1$
y_i	$\frac{3}{2}h_1+sh_2$	$-\frac{3}{2}h_1$	$\frac{1}{2}h_1$
z_i	$-2h_1-(s+\frac{1}{2}s')h_2$	$2h_1+\frac{1}{2}s'h_2$	$\frac{1}{2}s'h_2$

sector are derived as

$$M_\sigma^{FF'}=M_\sigma^{F'F}=\begin{bmatrix} Y_{11}^{FF'}v_0 & 0 & 0 \\ 0 & Y_{22}^{FF'}v_0 & 0 \\ 0 & 0 & Y_{33}^{FF'}v_0 \end{bmatrix} \quad (44)$$

and

$$M_\sigma^{F'}=\begin{bmatrix} 0 & Y_{12}^{F'}v_2 & 0 \\ Y_{21}^{F'}v_2 & 0 & 0 \\ 0 & 0 & Y_{33}^{F'}v_2 \end{bmatrix} . \quad (45)$$

In Appendix A different assignments of chiral charges are given which lead to an effective mass matrix of extended Fritzsch type also for the neutrino sector.

All the components of the $\mathcal{H}$ submatrix for $(i,j)=(1\text{–}3,7\text{–}9)$ are required not to vanish in order to forbid the direct coupling of the electroweak Higgs doublets $\chi_{L,R}$ with l_i and N_j' fields. Namely, $x_i+z_j\neq 0$ so as to make $Y_{ij}^{\nu N'}=0$ in Eq. (26). Otherwise, the seesaw mass matrix M^ν acquires the top-right and down-left submatrices, canceling out the effect of multiple seesaw suppressions. Here it is relevant to show concretely an example of the possible assignment of chiral charges satisfying these conditions. Probably the most simple example is to put $(h_1,h_2,h_3)=(1,2,3)$, $h_0=\frac{3}{2}$, $s=s'=1$, and $s_4=-s_5=-s_6=-1$. For this choice of chiral charges, V_H' in Eq. (22) can include a cubic term $\sigma_1\sigma_2\sigma_3^*+\sigma_1^*\sigma_2^*\sigma_3$.

V. EFFECTIVE MASS MATRICES

On the assumption that $v_i^2 \gg w_L^2, w_R^2$ $(i=0,1,2,3)$, the seesaw mass matrices in Eqs. (29) and (30) are block diagonalized approximately by the bi-unitary transformations as [16,17]

$$U_L M^f U_R^\dagger \simeq \begin{bmatrix} M_{\text{eff}}^f & 0 \\ 0 & M^F \end{bmatrix} , \quad (46)$$

with

$$U_{L,R}=\begin{bmatrix} 1-\frac{1}{2}\rho_{L,R}\rho_{L,R}^\dagger & -\rho_{L,R} \\ \rho_{L,R}^\dagger & 1-\frac{1}{2}\rho_{L,R}^\dagger\rho_{L,R} \end{bmatrix} . \quad (47)$$

Here M_{eff}^f is an effective 3×3 mass matrix for each sector of ordinary quarks and leptons, and M^F is a 3×3 (6×6) mass matrix for heavy charged-fermion sectors (for a heavy neutral-fermion sector). We get the effective mass matrices

$$M_{\text{eff}}^f \equiv -M_L^f (M_\sigma^F)^{-1} M_R^f \quad (48)$$

and the component ρ_L and ρ_R matrices of the bi-unitary transformations

$$\rho_L = M_L^f (M_\sigma^F)^{-1}, \quad \rho_R^\dagger = (M_\sigma^F)^{-1} M_R^f \quad (49)$$

for the charged-fermion sectors $[(f,F)=(u,U),(d,D),(e,E)]$. Similarly, we find the effective mass matrix

$$\begin{aligned} M_{\text{eff}}^\nu &= -(M_L^\nu \;\; 0)\begin{bmatrix} M_\sigma^N & M_\sigma^{NN'} \\ M_\sigma^{N'N} & M_\sigma^{N'} \end{bmatrix}^{-1} \begin{bmatrix} M_R^\nu \\ 0 \end{bmatrix} \\ &= -M_L^\nu [M_\sigma^N - M_\sigma^{NN'}(M_\sigma^{N'})^{-1} M_\sigma^{N'N}]^{-1} M_R^\nu \end{aligned} \quad (50)$$

and the component matrices

$$\begin{aligned} \rho_L &= (M_L^\nu \;\; 0)\begin{bmatrix} M_\sigma^N & M_\sigma^{NN'} \\ M_\sigma^{N'N} & M_\sigma^{N'} \end{bmatrix}^{-1} \\ &= (\, M_L^\nu [M_\sigma^N - M_\sigma^{NN'}(M_\sigma^{N'})^{-1} M_\sigma^{N'N}]^{-1} \;\; M_L^\nu [M_\sigma^{N'N} - M_\sigma^{N'}(M_\sigma^{NN'})^{-1} M_\sigma^N]^{-1}) \end{aligned} \quad (51)$$

$$\begin{aligned} \rho_R^\dagger &= \begin{bmatrix} M_\sigma^N & M_\sigma^{NN'} \\ M_\sigma^{N'N} & M_\sigma^{N'} \end{bmatrix}^{-1} \begin{bmatrix} M_R^\nu \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} (M_\sigma^N - M_\sigma^{NN'}(M_\sigma^{N'})^{-1} M_\sigma^{N'N})^{-1} M_R^\nu \\ (M_\sigma^{NN'} - M_\sigma^N (M_\sigma^{N'N})^{-1} M_\sigma^{N'})^{-1} M_R^\nu \end{bmatrix} \end{aligned} \quad (52)$$

for the neutral-fermion sector [12].

Thanks to the generic form of M_σ^F and the simple structure of $M_\sigma^{N'N}=M_\sigma^{NN'}$ and $M_\sigma^{N'}$, the effective mass matrices in Eqs. (48) and (50) are cast into the common formula as

$$M_{\text{eff}}^f = \begin{bmatrix} 0 & A & 0 \\ A^* & D & B \\ 0 & B^* & C \end{bmatrix} \quad (53)$$

for all fermion sectors $(f=u,d,\nu,e)$. The coefficients of the mass matrices for all fermion sectors are calculated to be

$$\begin{aligned} A &= -\frac{(Y_{12}^{fF})^2}{Y_{12}^F}\frac{w_L w_R}{\hat{v}_2} , \\ B &= \frac{Y_{21}^{fF} Y_{33}^{fF} Y_{23}^F}{Y_{21}^F Y_{33}^F}\frac{w_L w_R \hat{v}_{1a}}{\hat{v}_2^* \hat{v}_{1b}} , \\ C &= -\frac{(Y_{33}^{fF})^2}{Y_{33}^F}\frac{w_L w_R}{\hat{v}_{1b}} , \\ D &= \frac{|Y_{12}^{fF}|^2}{|Y_{12}^F|^2}\left[Y_{22}^F \hat{v}_3 - \frac{|Y_{23}^F|^2}{Y_{33}^F}\frac{|\hat{v}_{1a}|^2}{\hat{v}_{1b}} \right]\frac{w_L w_R}{|\hat{v}_2|^2} , \end{aligned} \quad (54)$$

where

$$\hat{v}_{1a}=\hat{v}_{1b}=v_1, \quad \hat{v}_2=v_2, \quad \hat{v}_3=v_3 \quad (55)$$

for the charged-fermion sectors, and

$$\hat{v}_{1a}=v_1,\quad \hat{v}_{1b}=v_1-\frac{(Y_{33}^{NN'})^2}{Y_{33}^{N}Y_{33}^{N'}}\frac{v_0^2}{v_2},$$
$$\hat{v}_2=v_2-\frac{Y_{11}^{NN'}Y_{22}^{NN'}}{Y_{12}^{N}Y_{21}^{N'}}\frac{v_0^2}{v_2},\quad \hat{v}_3=v_3 \tag{56}$$

for the neutrino sector.

VI. FERMION MASSES AND FLAVOR MIXINGS

Evidently the USM lowers the masses of charged fermions from the electroweak scale w_L by the factors w_R/v_i. As for the neutral-fermion sector, the neutrino masses receive further strong USM suppressions with the factors $(v_2/v_0)^2$, $(v_2/v_0)^4$, and $(v_2/v_0)^6$.

For the sake of definite arguments, the signs of the diagonal Yukawa coupling constants are fixed to be

$$Y_{22}^F>0,\quad Y_{33}^F<0,\quad Y_{33}^N Y_{33}^{N'}<0\ , \tag{57}$$

so as to make $C>0, CD-|B|^2>0$. Let us express the effective mass matrices for all sectors in the form [18]

$$M_{\text{eff}}^f=\begin{bmatrix} 0 & |A|e^{ia} & 0\\ |A|e^{-ia} & D & |B|e^{ib}\\ 0 & |B|e^{-ib} & C\end{bmatrix}, \tag{58}$$

which is transformed, by adjustment of phases, into the symmetric real matrix as

$$\overline{M_{\text{eff}}^f}\equiv \mathcal{P}M_{\text{eff}}^f\mathcal{P}^\dagger=\begin{bmatrix} 0 & |A| & 0\\ |A| & D & |B|\\ 0 & |B| & C\end{bmatrix}, \tag{59}$$

where

$$\mathcal{P}=\begin{bmatrix} 1 & 0 & 0\\ 0 & e^{ia} & 0\\ 0 & 0 & e^{i(a+b)}\end{bmatrix}. \tag{60}$$

Eigenvalues of $\overline{M_{\text{eff}}^f}$ designated generically by $-m_1$, m_2, and m_3 satisfy the relations

$$-m_1+m_2+m_3=C+D\ ,$$
$$m_1m_2-m_2m_3+m_3m_1=|A|^2+|B|^2-CD\ , \tag{61}$$
$$m_1m_2m_3=|A|^2C\ .$$

The symmetric matrix $\overline{M_{\text{eff}}^f}$ is diagonalized as

$$\mathcal{O}\overline{M_{\text{eff}}^f}\mathcal{O}^T=\text{diag}(-m_1,m_2,m_3)\ , \tag{62}$$

by the orthogonal matrix [9,12]

$$\mathcal{O}^T=\begin{bmatrix} \left[\frac{m_2m_3(C+m_1)}{C(m_2+m_1)(m_3+m_1)}\right]^{1/2} & \left[\frac{m_1m_3(C-m_2)}{C(m_2+m_1)(m_3-m_2)}\right]^{1/2} & \left[\frac{m_1m_2(m_3-C)}{C(m_3-m_2)(m_3+m_1)}\right]^{1/2}\\ -\left[\frac{m_1(C+m_1)}{(m_2+m_1)(m_3+m_1)}\right]^{1/2} & \left[\frac{m_2(C-m_2)}{(m_2+m_1)(m_3-m_2)}\right]^{1/2} & \left[\frac{m_3(m_3-C)}{(m_3-m_2)(m_3+m_1)}\right]^{1/2}\\ \left[\frac{m_1(C-m_2)(m_3-C)}{C(m_2+m_1)(m_3+m_1)}\right]^{1/2} & -\left[\frac{m_2(C+m_1)(m_3-C)}{C(m_2+m_1)(m_3-m_2)}\right]^{1/2} & \left[\frac{m_3(C+m_1)(C-m_2)}{C(m_3-m_2)(m_3+m_1)}\right]^{1/2}\end{bmatrix}. \tag{63}$$

In the first seesaw approximation, the mass eigenstate of ordinary fermions $f_{L,R}^{(M)}$ in the generation space is given by

$$f_{L,R}^{(M)}=\mathcal{O}\mathcal{P}(f_{L,R}-\rho_{L,R}F_{L,R}) \tag{64}$$

for the charged-fermion sectors and

$$f_{L,R}^{(M)}=\mathcal{O}\mathcal{P}\left[f_{L,R}-\rho_{L,R}\begin{bmatrix}F_{L,R}\\ F'_{L,R}\end{bmatrix}\right] \tag{65}$$

for the neutral-fermion sector.

A. Charged fermions

Observed masses [19,20] of the three generations of quarks and charged leptons are subject to the horizontal hierarchy in each sector

$$m_3^2\gg m_2^2\gg m_1^2\ , \tag{66}$$

and our current knowledge on the weak mixing matrix [20] suggests also the order relation

$$|V_{ud}|^2\gg|V_{us}|^2\gg|V_{ub}|^2\ . \tag{67}$$

The present scheme can reduce the main features of these properties essentially to the VEV's hierarchy $v_3^2\gg v_2^2\gg v_1^2$. In fact, from Eq. (54), the coefficients of the effective mass matrix for charged-fermion sectors are proved to satisfy

$$|C|^2\gg CD-|B|^2\gg|A|^2,|B|^2\ . \tag{68}$$

With these relations we are able to solve Eq. (61) approximately, getting the expressions

$$m_1 \simeq \frac{|A|^2 C}{CD - |B|^2}, \quad m_2 \simeq D - \frac{|B|^2}{C}, \quad m_3 \simeq C + \frac{|B|^2}{C} \quad (\approx C) , \tag{69}$$

which obey the order relation in Eq. (66). At a similar approximation the orthogonal matrix in Eq. (63) is reduced to

$$\mathcal{O}^T \simeq \begin{bmatrix} 1 - \dfrac{m_1}{2m_2} & \left[\dfrac{m_1}{m_2}\right]^{1/2} & \left[\dfrac{m_1 m_2}{C m_3}\left[1-\dfrac{C}{m_3}\right]\right]^{1/2} \\ -\left[\dfrac{m_1}{m_2}\right]^{1/2} & 1 - \dfrac{m_1}{2m_2} - \dfrac{1}{2}\left[1-\dfrac{C}{m_3}\right] & \left[1-\dfrac{C}{m_3}\right]^{1/2} \\ \left[\dfrac{m_1}{m_2}\left[1-\dfrac{C}{m_3}\right]\right]^{1/2} & -\left[1-\dfrac{C}{m_3}\right]^{1/2} & 1-\dfrac{1}{2}\left[1-\dfrac{C}{m_3}\right] \end{bmatrix} , \tag{70}$$

where the leading terms and the first-order corrections (the leading terms) with respect to the factors m_1/m_2, m_1/m_3, m_2/m_3, and $1-C/m_3$ are retained in the diagonal components (the off-diagonal components).

B. Quark flavor mixing

The quark mixing matrix for weak current V is given by

$$V = \mathcal{O}_u \mathcal{P}_{ud} \mathcal{O}_d^T , \tag{71}$$

where

$$\mathcal{P}_{ud} = \mathcal{P}_u \mathcal{P}_d^\dagger = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_1} & 0 \\ 0 & 0 & e^{i\phi_2} \end{bmatrix} , \tag{72}$$

with

$$\phi_1 = a^u - a^d, \quad \phi_2 = \phi_1 + b^u - b^d . \tag{73}$$

The components of this mixing matrix have approximately the following magnitudes:

$$\begin{aligned} |V_{us}| \simeq |V_{cd}| &\simeq \left[\frac{m_u}{m_c} + \frac{m_d}{m_s} - 2\left[\frac{m_u m_d}{m_c m_s}\right]^{1/2} \cos\phi_1\right]^{1/2} , \\ |V_{ub}| &\simeq \left[\frac{m_u}{m_c}\right]^{1/2} \left\{2 - \frac{C_u}{m_t} - \frac{C_d}{m_b} - 2\left[\left[1-\frac{C_u}{m_t}\right]\left[1-\frac{C_d}{m_b}\right]\right]^{1/2} \cos(\phi_2-\phi_1)\right\}^{1/2} , \\ |V_{cb}| \simeq |V_{ts}| &\simeq \left\{2 - \frac{C_u}{m_t} - \frac{C_d}{m_b} - 2\left[\left[1-\frac{C_u}{m_t}\right]\left[1-\frac{C_d}{m_b}\right]\right]^{1/2} \cos(\phi_2-\phi_1)\right\}^{1/2} , \\ |V_{td}| &\simeq \left[\frac{m_d}{m_s}\right]^{1/2} \left\{2 - \frac{C_u}{m_t} - \frac{C_d}{m_b} - 2\left[\left[1-\frac{C_u}{m_t}\right]\left[1-\frac{C_d}{m_b}\right]\right]^{1/2} \cos(\phi_2-\phi_1)\right\}^{1/2} . \end{aligned} \tag{74}$$

Eliminating the C parameters, we get the relations [21]

$$\left|\frac{V_{ub}}{V_{cb}}\right| \simeq \left[\frac{m_u}{m_c}\right]^{1/2} , \qquad \left|\frac{V_{td}}{V_{cb}}\right| \simeq \left[\frac{m_d}{m_s}\right]^{1/2} , \tag{75}$$

which are consistent with the observed values of the quark masses [19,20] and the components of the weak mixing matrix [20] within remaining uncertainties.

Unless further assumptions are imposed on the Yukawa coupling constants, it is impossible to fix the value of the coefficient C. To estimate the elements of the weak mixing matrix, let us regard C as an adjustable parameter which takes a value close to the third-generation quark mass m_3 from below as implied by the last relation in Eq. (69). For instance, when $D_u = m_c$, $D_d = m_s$, and $\phi_1 = \phi_2 = 85°$, the magnitude of the elements of the weak mixing matrix are calculated to be [22]

$$\begin{bmatrix} 0.975\,32 & 0.220\,79 & 2.3958\times10^{-3} \\ 0.220\,67 & 0.974\,64 & 3.7187\times10^{-2} \\ 7.7520\times10^{-3} & 3.6449\times10^{-2} & 0.999\,31 \end{bmatrix} , \tag{76}$$

which are consistent with the available experimental data [20]. Analysis on the weak CP phase, which appears in this theory as a free parameter, will be made elsewhere.

C. Specific features of neutrinos

The effective mass matrix for the neutrino sector has quite different characteristics than those for the charged-fermion sectors. The relation $|B_\nu|,|D_\nu| \ll |A_\nu|,|C_\nu|$ among the coefficients of the mass matrix holds since B_ν and D_ν are suppressed by the huge factors $(v_2/v_0)^4$ and $(v_2/v_0)^6$, while A_ν and C_ν are suppressed by the factor $(v_2/v_0)^2$. Therefore it is a good approximation to express the effective mass matrix for the neutrino sector by

$$M^\nu_{\rm eff} \approx \begin{bmatrix} 0 & A_\nu & 0 \\ A^*_\nu & 0 & 0 \\ 0 & 0 & C_\nu \end{bmatrix} = \begin{bmatrix} 0 & |A_\nu|e^{ia_\nu} & 0 \\ |A_\nu|e^{-ia_\nu} & 0 & 0 \\ 0 & 0 & C_\nu \end{bmatrix} . \quad (77)$$

Consequently the neutrinos appear in a distinctive spectrum consisting of doubly degenerate states with smaller mass m_S and a singlet state with larger mass m_L such that

$$m_{\nu_1} \simeq m_{\nu_2} \simeq |A_\nu| \equiv m_S, \quad m_{\nu_3} \simeq C_\nu \equiv m_L , \quad (78)$$

and the orthogonal transformation matrix has the simple form

$$\mathcal{O}_\nu \simeq \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad (79)$$

which has no dependence on parameters in the mass matrix.

The weak mixing matrix for leptons is obtained by

$$U = \mathcal{O}_e \mathcal{P}_{e\nu} \mathcal{O}_\nu^T \equiv (\mathbf{u}^{(1)}, \mathbf{u}^{(2)}, \mathbf{u}^{(3)}) , \quad (80)$$

where

$$\mathcal{P}_{e\nu} = \mathcal{P}_e \mathcal{P}_\nu^\dagger = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_3} & 0 \\ 0 & 0 & e^{i\phi_4} \end{bmatrix} , \quad (81)$$

with

$$\phi_3 = a^e - a^\nu, \quad \phi_4 = \phi_3 + b^e . \quad (82)$$

It is convenient to introduce the column vectors $\mathbf{u}^{(i)}$ to analyze the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism among three neutrinos with the mass spectrum in Eq. (78). In particular, an important role is played by the "eth" component of the third column vector $\mathbf{u}^{(3)}$, whose magnitude is given by

$$|(\mathbf{u}^{(3)})_e| = |u_e^{(3)}| = \left[\frac{m_e(C_e - m_\mu)(m_\tau - C_e)}{C_e(m_\mu + m_e)(m_\tau + m_e)} \right]^{1/2} \equiv |\sin\theta| . \quad (83)$$

Note that unfixed phases a^e, a^ν, and b^e do not enter into this expression since the eth component of the vector $\mathbf{u}^{(3)}$ consists of a single term owing to the simple structure of the orthogonal matrix $\mathcal{O}_\nu$ in Eq. (79).

In Appendix B it is explicitly shown that the matter-induced flavor conversion among three neutrino species with the mass spectrum in Eq. (78) is reduced to the well-known problem for two-neutrino oscillation. Namely, the MSW oscillation is proved to occur solely between two flavor states of the ν_e and one linear combination of ν_μ and ν_τ. It is essential to observe that the angle θ defined in Eq. (83) appears as the vacuum mixing angle between two such states, and its value becomes small as a natural consequence of three-flavor oscillation. In fact, $|\sin\theta|$ takes a small value since C_e is very close to m_τ, as implied by the last relation in Eq. (69).

The GALLEX Collaboration has reported the results of the solar-neutrino measurement with a gallium detector covering a 295-day exposure as [3]

$$^{71}\text{Ga count rate} = 83 \pm 19(\text{stat}) \pm 8(\text{syst}) \text{ solar neutrino units (SNU)} . \quad (84)$$

Compared with the preliminary data by the SAGE group [23], which had reported a counting rate of 20^{+15}_{-20}(stat)$\pm$32(syst) SNU, the new results are rather close to the value 124–132 SNU predicted from the standard solar model calculations [24–26]. On the other hand, Bahcall and Bethe [27] have found that the Homestake [14] and Kamiokande II [15] data are in excellent agreement with a nonadiabatic solution of the matter-induced MSW oscillation satisfying the condition

$$(m_L^2 - m_S^2)\sin^2\theta \simeq 1 \times 10^{-8} \text{ eV}^2 . \quad (85)$$

Their analysis is applicable, as it stands, to our theory. The set of the parameters which explain both results in Eqs. (84) and (85) is estimated as follows [3]:

$$m_L^2 - m_S^2 \simeq 6 \times 10^{-6} \text{ eV}^2, \quad |\sin\theta| \simeq 0.042 . \quad (86)$$

From Eq. (83), this value of the vacuum mixing angle is shown to correspond to the parameter $C_e \simeq 1.06$ GeV. These estimates are very close to the results $|\sin\theta| \simeq 0.043 \pm 0.004$ and $C_e \simeq (1.10 \pm 0.12)$ GeV obtained in the previous model with the color SU(4) symmetry [28], where the parameter C_e is calculable due to the large gauge group.

VII. SUMMARY AND DISCUSSION

With motivation to describe the mass spectrum of all the quarks, charged leptons, and neutrinos in a unified way, we have developed a new scheme of USM's in the left-right-symmetric extension of the standard model. All masses of fundamental fermions and gauge particles are created through the ordinary Higgs mechanisms and the hierarchical variety in the fermion mass spectrum is produced through multiple universal seesaw mixings. This scheme is simple in the sense that no Majorana mass appears and fermion-number conservation is not violated.

For all four sectors of quarks and leptons the present scheme provides effective mass matrices of the same Hermitian form reduced from the 6×6 and 9×9 seesaw mass matrices. Coefficients of the effective mass matrices for the charged-fermion sectors are suppressed below the electroweak scale w_L by the seesaw factors w_R/v_i, and those for the neutrino sector receive further strong suppression by the multiple seesaw factor $(v_2/v_0)^2$. As a result the fermion mass spectrum acquires a huge width ranging from $(v_2/v_0)^2(w_R/v_2)w_L$ to $(w_R/v_1)w_L$. Upon such a broad spectrum all charge sectors have their own intrasectorial mass hierarchies which are essentially ascribable to differences among VEV's of the three Higgs singlets σ_i $(i=1,2,3)$. For the charged-fermion sectors the effective mass matrices have a structure of extended Fritzsch type with a diagonal (2,2) component. This is particularly important for the up-quark sector, because the top-quark mass is now known to be well above the upper bound of the original Fritzsch ansatz [21]. Adjusting the Yukawa coupling constants over a not so unnatural range of values, we are able to explain the hierarchies in mass spectra of charged fermions and the weak mixing matrix. For the neutrino sector, the multiple USM suppression results in an effective mass matrix of very simple form which predicts doubly degenerate states with smaller masses and a singlet state with larger mass. Consequently, the matter-induced neutrino oscillation takes place among three flavors, leading naturally to a small vacuum mixing angle. This result seems to be favorable for explaining all the available data on the solar-neutrino experiments.

This model has six kinds of VEV's which are assumed to satisfy the hierarchy in Eq. (14). Against the lowest-energy scale of left-handed electroweak interaction $w_L\simeq250$ GeV, the scale of the right-handed weak current is estimated as $w_R\geq2.0\times10^3$ GeV [29]. Therefore, the VEV v_1 must be at least larger than 10^4 GeV in order that the USM works well and so that the flavor-changing neutral current [30] is safely suppressed by the seesaw factors w_R/v_i. To make the arguments definite, let us assume here that v_1 is of the order of 10^4-10^5 GeV. Then the larger VEV's v_2 and v_3 should be of the order of 10^5-10^6 and 10^6-10^7 GeV, respectively, to ascribe the common features of horizontal hierarchies of charged-fermion sectors to the VEV's v_i $(i=1,2,3)$. Further, the largest VEV's v_0 must be of the order of $10^{12}-10^{13}$ GeV in order to explain the huge gap in the quark-lepton mass spectrum.

In the present scheme, the strong *CP* problem is presumed to be circumvented by the global $U(1)_A$ symmetry. Breakdown of this symmetry accompanies the Nambu-Goldstone boson, viz., the axion. The phases of the Higgs singlets σ_i carrying chiral charges accommodate the axion's degree of freedom in proportion to the magnitude of their VEV's. The mass m_a and decay constant f_a of the axion are related by the relation $m_a f_a\simeq6\times10^{-3}$ GeV^2, and f_a is given by $f_a\simeq|h_0|v_0$ in good approximation, since $v_0^2\gg v_i^2$ $(i=1,2,3)$. Astrophysics and cosmology [31] claim that the axion can exist only with tiny mass m_a in a narrow window of $10^{-6}\leq m_a\leq10^{-3}$ eV [32,33], leading to an approximate restriction $10^{10}\leq v_0\leq10^{13}$ GeV on the largest VEV v_0. Accordingly, if our interpretation on the axion is correct, the largest VEV v_0 fits in the range of values which can reasonably explain the broad width of the quark-lepton mass spectrum. In this model the superlightness of the neutrinos is deeply related to the invisibleness of the axion.

Although the common features of the horizontal mass hierarchies in Eq. (66) are reducible to the difference among VEV's v_i $(i=1,2,3)$, adjustment of the Yukawa coupling constants is still unavoidable in this model. Approximate equations $m_1m_2\simeq|A|^2$ and $m_3\approx C$ derived from Eq. (69) lead readily to the following intersectorial relations depending solely on the Yukawa coupling constants:

$$\begin{aligned} m_e m_\mu : m_d m_s : m_u m_c &\simeq \frac{(Y_{12}^{eE})^4}{(Y_{12}^{E})^2} : \frac{(Y_{12}^{dD})^4}{(Y_{12}^{D})^2} : \frac{(Y_{12}^{uU})^4}{(Y_{12}^{U})^2} , \\ m_\tau : m_b : m_t &\approx \frac{(Y_{33}^{eE})^2}{Y_{33}^{E}} : \frac{(Y_{33}^{dD})^2}{Y_{33}^{D}} : \frac{(Y_{33}^{uU})^2}{Y_{33}^{U}} . \end{aligned} \tag{87}$$

These examples imply that the intersectorial relations are ruled essentially by the Yukawa coupling constants. Our model with the small underlying symmetry is not able to explain these relations even if the effect of the renormalization group is taken into account. The symmetry group must be enlarged in order to make our model more predictive. In fact, the previous model with $SU(4)_L\times SU(4)_R$ symmetry [12,28] can generate relations among fermion masses and predict the counting rate of the gallium experiment for the solar *pp* neutrinos. In this connection it should be remembered that rich contents of exotic fermions and Higgs scalars are presumed to exist to implement multiple USM's. Therefore, it is necessary to develop a model with a large symmetry group that can accommodate more naturally those exotic fields as well as the ordinary fermions and Higgs fields.

Our model predicts inevitably the existence of electroweak singlets of heavy charged fermions U_i, D_i, and E_i in the mass range of 10^4-10^7 GeV and very massive neutral fermions N_i and N_i' with masses of order $10^{10}-10^{13}$ GeV per generation. The so-called generation puzzle must be looked at from a new angle because the existence of heavy fermions seems to make the puzzle more sharp and profound. Absorbing six degrees of freedom of the electroweak scalar doublets χ_L and χ_R, gauge particles $(W_L^\pm,Z_L)$ and $(W_R^\pm,Z_R)$ acquire masses of order w_L and w_R, leaving two Higgs particles. The scalar singlets σ_i $(i=0,1,2,3)$ are introduced *a posteriori* to implement multiple USM's and to induce diversity in the quark-lepton mass spectrum. Except for the axion, the role of their seven remaining degrees of freedom is not clarified at this stage of the model. Including a model in which the chiral $U(1)_A$ symmetry is gauged [34], we will investigate the latent significance of such scalar modes in a future publication.

Finally, let us note that there still remains the possibility of massless neutrinos. The GALLEX Collaboration [3] has pointed out that their results together with the Homestake and Kamiokande experiments require severe stretching of solar models but do not rule out the possibility of massless neutrinos. In this connection, it is interesting to observe that $m_\nu=0$ is realized at the tree level as the limit of the multiple universal seesaw mechanism. In fact, as pointed out at the end of Appendix A, it is possible to assign chiral charges so that the Higgs singlets σ_i $(i=1,2,3)$ do not couple with the exotic fermion singlets N_i' $(i=1,2,3)$. Then the rank of the seesaw mass matrix M^ν degrades down to 6, leading to zero neutrino masses.

APPENDIX A: DIFFERENT ASSIGNMENTS OF CHIRAL CHARGES

The conditions in Eq. (43) have been imposed on chiral charges in order to realize the maximum USM suppression in the effective neutrino mass matrix. As a result, we get the unique mass matrix in Eq. (77), leading to the degenerate masses and the simple transformation matrix for the neutrinos. However, it should be noted that there are different choices of the chiral charges leading to the effective mass matrix of extended Fritzsch type for all fermion sectors at the first seesaw approximation. Namely, in place of the restrictions in Eq. (43), it is possible to set the conditions

$$s_4=s_5=s_6,\quad 2s_6h_0=h_1+sh_2\ , \tag{A1}$$

which result in $\mathcal{H}_{78}=\mathcal{H}_{87}=h_1, \mathcal{H}_{79}=\mathcal{H}_{97}=-h_1$, and $\mathcal{H}_{99}=sh_2$. For these choices of chiral charges, the submatrix $M_\sigma^{N'}$ turns out to be the inverse form of the Fritzsch matrix,

$$M_\sigma^{N'}=\begin{bmatrix} 0 & Y_{12}^{N'}v_1 & Y_{13}^{N'}v_1 \\ Y_{21}^{N'}v_1 & 0 & 0 \\ Y_{31}^{N'}v_1 & 0 & Y_{33}^{N'}v_2 \end{bmatrix}, \tag{A2}$$

and $M_\sigma^{NN'}$ and $M_\sigma^{N'N}$ take the forms

$$\begin{aligned} M_\sigma^{NN'}&=\begin{bmatrix} Y_{11}^{NN'}v_0 & 0 & 0 \\ 0 & Y_{22}^{NN'}v_0 & 0 \\ Y_{31}^{NN'}v_0 & 0 & Y_{33}^{NN'}v_0 \end{bmatrix}, \\ M_\sigma^{N'N}&=\begin{bmatrix} Y_{11}^{NN'}v_0 & 0 & Y_{13}^{NN'}v_0 \\ 0 & Y_{22}^{NN'}v_0 & 0 \\ 0 & 0 & Y_{33}^{NN'}v_0 \end{bmatrix}, \end{aligned} \tag{A3}$$

with all the other submatrices being unchanged.

Since the submatrices M_σ^N and $M_\sigma^{NN'}(M_\sigma^{N'})^{-1}M_\sigma^{N'N}$ have the same structure, the effective mass matrices are again cast into the common form in Eqs. (53) and (54). While the relations in Eq. (55) hold for the charged-fermion sectors, Eq. (56) must be replaced by

$$\begin{aligned} \hat{v}_{1a}&=v_1-\frac{Y_{22}^{NN'}}{Y_{23}^{N}}\left[\frac{Y_{13}^{NN'}}{Y_{12}^{N'}}\frac{v_0^2}{v_1}-\frac{Y_{13}^{N'}Y_{33}^{NN'}}{Y_{12}^{N'}Y_{33}^{N'}}\frac{v_0^2}{v_2}\right], \\ \hat{v}_{1b}&=v_1-\frac{(Y_{33}^{NN'})^2}{Y_{33}^{N}Y_{33}^{N'}}\frac{v_0^2}{v_2}, \\ \hat{v}_2&=v_2-\frac{Y_{11}^{NN'}Y_{22}^{NN'}}{Y_{12}^{N}Y_{21}^{N'}}\frac{v_0^2}{v_1}, \\ \hat{v}_3&=v_3-\frac{|Y_{13}^{N'}|^2(Y_{22}^{NN'})^2}{Y_{22}^{N}|Y_{12}^{N'}|^2Y_{33}^{N'}}\frac{v_0^2}{v_2} \end{aligned} \tag{A4}$$

for the neutrino sector. Therefore, the neutrino sector also turns out to take an effective mass matrix of extended Fritzsch type. Accordingly, these assignments of chiral charges predict the mass spectra with hierarchical order of magnitude as in Eq. (66) for all sectors of fermions. In this case it is natural to assume that two-flavor modes are mixtures of two lower mass eigenmodes

$$\begin{aligned} \nu_e&\simeq\nu_1\cos\theta+\nu_2\sin\theta\ , \\ \nu_\mu&\simeq-\nu_1\sin\theta+\nu_2\cos\theta\ , \end{aligned} \tag{A5}$$

where the vacuum mixing angle θ is given by

$$|\sin\theta|\simeq\left[\frac{m_1}{m_2}+\frac{m_e}{m_\mu}-2\left[\frac{m_1m_e}{m_2m_\mu}\right]^{1/2}\cos\phi_3\right]^{1/2}. \tag{A6}$$

In contrast to the case of chiral charge assignments in Eq. (43), the vacuum mixing angle depends on unfixed neutrino masses m_1 and m_2 and an unknown phase $\phi_3=a^e-a^\nu$. Therefore, it is impossible to make a direct comparison between theoretical predictions and experimental results at the present stage. Nevertheless, these assignments of chiral charges also should not be abandoned. Neutrinos are still too unknown to do so.

It is worthwhile to notice that the chiral charges can also be assigned so as to forbid the Yukawa interactions between the exotic N_i' fields and the Higgs singlets σ_i $(i=1,2,3)$, leading to the submatrix $M_\sigma^{N'}=0$. In such a case, the rank of the 9×9 seesaw mass matrix reduces to 6, and all neutrinos turn out to be massless at the tree level. Therefore, it is possible to interpret the massless neutrinos as a limiting case of our multiple universal seesaw mechanisms.

APPENDIX B: MATTER-INDUCED OSCILLATION AMONG THREE NEUTRINO SPECIES

In the previous paper [28] a stationary-state formalism has been developed to describe the matter-induced oscillation among three neutrino species. We summarize here a necessary part of the formalism.

Flavor modes $|\nu_\alpha\rangle$ $(\alpha=e,\mu,\tau)$ and mass eigenmodes in a vacuum $|\nu_j\rangle$ $(j=1,2,3)$ are related by the weak mixing matrix U in Eq. (80) as follows:

$$|\nu_\alpha(t)\rangle=\sum_j U_{\alpha j}|\nu_j(t)\rangle=\sum_j(\mathbf{u}^{(j)})_\alpha|\nu_j(t)\rangle\ . \tag{B1}$$

Observable solar neutrinos carry highly relativistic energy. Therefore the momentum of the neutrino with energy E in the jth eigenmode with mass m_j ($E \gg m_j$) can be approximated by $k_j=\sqrt{E^2-m_j^2}\simeq E-m_j^2/2E$. Let us define the component wave functions $c_\alpha(x)$ and $c_j(x)$ of the flavor modes and mass eigenmodes, respectively, by

$$\langle x|\nu_\alpha(t)\rangle=c_\alpha(x)e^{-iEt},\quad \langle x|\nu_j(t)\rangle=c_j(x)e^{-iEt}\ . \tag{B2}$$

Then the component wave function $c_j(x)$ is subject to the equation

$$\frac{1}{i}\frac{d}{dx}c_j(x)=-\frac{m_j^2}{2E}c_j(x) \tag{B3}$$

up to an overall constant. Transforming this equation into the flavor modes and combining the Wolfenstein term, which represents the effect of neutrino interaction with solar electrons, we find the Schrödinger-like equation

$$\frac{1}{i}\frac{d}{dx}c_\alpha(x)=\sum_\beta \mathcal{P}_{\alpha\beta}c_\beta(x)\ , \tag{B4}$$

where the operator $\mathcal{P}$ generating the spatial evolution of the neutrino system is given by

$$\mathcal{P}_{\alpha\beta}=-\frac{1}{2E}[\Delta m^2(\mathbf{u}^{(3)})_\alpha(\mathbf{u}^{(3)*})_\beta+A\delta_{\alpha e}\delta_{\beta e}]\ , \tag{B5}$$

with

$$\Delta m^2=m_L^2-m_S^2,\quad A=2\sqrt{2}G_F E N_e\ . \tag{B6}$$

In the factor A, G_F is the Fermi constant and N_e is the number density of electrons. Note that the operator $\mathcal{P}$ is real since the overall phase factor $e^{i\phi_4}$ of the column vector $\mathbf{u}^{(3)}$ cancels in Eq. (B5).

The basic equation (B4) shows that the electron number density N_e induces local mixings among three neutrino flavors. This superficial complexity is removed if we use the following globally rotated flavor state vectors:

$$\begin{aligned} |\nu_e'\rangle&=|\nu_e\rangle\ ,\\ |\nu_\mu'\rangle&=\frac{1}{\cos\theta}(|u_\mu^{(3)}||\nu_\mu\rangle+|u_\tau^{(3)}||\nu_\tau\rangle)\ ,\\ |\nu_\tau'\rangle&=\frac{1}{\cos\theta}(-|u_\tau^{(3)}||\nu_\mu\rangle+|u_\mu^{(3)}||\nu_\tau\rangle)\ . \end{aligned} \tag{B7}$$

For the new flavor amplitudes $c_\alpha'(x)$ defined by

$$\langle x|\nu_\alpha'(t)\rangle=c_\alpha'(x)e^{-iEt}\ , \tag{B8}$$

the Schrödinger-like equation (B4) is transformed into the decoupled forms

$$i\frac{d}{dx}\begin{bmatrix}c_e'(x)\\ c_\mu'(x)\end{bmatrix}=\frac{1}{2E}\mathcal{M}^2\begin{bmatrix}c_e'(x)\\ c_\mu'(x)\end{bmatrix}\ , \tag{B9}$$

with

$$\mathcal{M}^2=\begin{bmatrix}\Delta m^2\sin^2\theta+A & \Delta m^2\sin\theta\cos\theta\\ \Delta m^2\sin\theta\cos\theta & \Delta m^2\cos^2\theta\end{bmatrix} \tag{B10}$$

and

$$i\frac{d}{dx}c_\tau'(x)=0\ . \tag{B11}$$

Equation (B9) is equivalent to the well-known evolution equation for the matter-induced oscillation of two neutrino species. The amplitude $c_\tau'(x)$, being unaffected by the vacuum oscillation and the neutrino-electron interaction, remains unchanged through the spatial evolution of the system. Solving Eq. (B9) with the boundary conditions $c_\mu(0)=c_\tau(0)=0$, we find the solution $c_\tau'(x)=0$, which results in

$$c_\mu(x)=\frac{|u_\mu^{(3)}|}{\cos\theta}c_\mu'(x),\quad c_\tau(x)=\frac{|u_\tau^{(3)}|}{\cos\theta}c_\mu'(x)\ . \tag{B12}$$

Thus, the oscillation problem of three neutrino flavors with mass spectrum in Eq. (78) is effectively reduced to that of two flavors $c_e(x)=c_e'(x)$ and $c_\mu'(x)$ with the vacuum mixing angle θ in Eq. (83).

Matter-induced neutrino oscillation between two flavor modes has been analyzed in many papers [27,35,36]. The probability for the detection of electron neutrinos on the earth's surface is given by Parke's formula [36]

$$\langle P_{\nu_e}\rangle\simeq\sin^2\theta+P\cos2\theta\ , \tag{B13}$$

with the "jumping" probability P between the mass eigenstates at the resonance region. Since the vacuum mixing angle θ takes a very small value in the present model, the detection probability $\langle P_{\nu_e}\rangle$ is essentially given by the nonadiabatic jumping probability. If the electron density in the sun is approximated by an exponential distribution

$$N_e(x)=N_0\exp\left[-\frac{x}{R_s}\right]\ , \tag{B14}$$

with scale height $R_s=6.6\times10^9$ cm, the jumping probability is calculated by an analytic continuation of adiabatic solutions of Eq. (B9) in the compact form [37]

$$P=\exp\left[-\frac{\pi R_s\Delta m^2\sin^2\theta}{E}\right]\ . \tag{B15}$$

This formula has been confirmed to be very accurate in the entire $\sin^2 2\theta-\Delta m^2/E$ plane by computer calculation [38].

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