# SU(16) grand unification: Breaking scales, proton decay, and neutrino magnetic moment

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We give a detailed renormalization group analysis for the SU(16) grand unified group with general breaking chains in which quarks and leptons transform separately at intermediate energies. Our analysis includes the effects of Higgs bosons. We show that the grand unification scale could be as low as $\sim 10^{8.5}$ GeV and give examples where new physics could exist at relatively low energy ($\sim$ 250 GeV). We consider proton decay in this model and show that it is consistent with a low grand unification scale. We also discuss the possible generation of a neutrino magnetic moment in the range of $10^{-11}$ to $10^{-10}\mu_B$ with a very small mass by the breaking of the embedded $SU(2)_\nu$ symmetry at a low energy.

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## I. INTRODUCTION

Recent treatments of the SU(15) grand unification group [1–5] have shown that in the versions of that model which produce "ununified models" [6] at intermediate energies, grand unification may be reached at a relatively low energy and that the lowest intermediate scale may be within the reach of the Superconducting Super Collider (SSC). It has also been shown recently that the effects of Higgs bosons in the renormalization group equations of SU(15) models are quite large [4]. In the literature, there is also some discussion of the gauge group SU(16) [7], which has some desirable features that are not found in SU(15) models. Among these are separate gauging of baryon and lepton number [7] and the embedding of Voloshin symmetry [8, 9] $SU(2)_\nu$ which might play an important role in the solar neutrino puzzle [10, 11]. In this paper, we will analyze the renormalization group equations (RGE's) of the SU(16) grand unification model with breaking chains in which quarks and leptons transform separately at intermediate energies. We will include the effects of Higgs bosons, which are significant, and use the newest data from the CERN $e^+e^-$ collider LEP for couplings at low energy. We give this analysis in Sec. II.

In the SU(16) model, all known left-handed fermions of a single generation together with a left-handed antineutrino transform like the fundamental representation of the gauge group: i.e.,

$$\Psi_{eL} \equiv \left(\hat{\nu}_e\, \nu_e\, e^- e^+\, u_1\, u_2\, u_3\, d_1\, d_2\, d_3\, \hat{u}_1\, \hat{u}_2\, \hat{u}_3\, \hat{d}_1\, \hat{d}_2\, \hat{d}_3\right)_L . \tag{1.1}$$

Here, the caret over a particle's symbol denotes its antiparticle. Of course, mirror fermions must be introduced to make the model free of anomalies, but we do not need to discuss them explicitly here. Our interest is to look for chains with low unification scale. The existence of such chains is known [1–4] in SU(15) models, for which it has been shown that a low unification scale makes the model free from the monopole problem [12] while being perfectly consistent with the proton lifetime [5, 13]. We discuss proton decay for our SU(16) model in Sec. III.

Another reason for examining the SU(16) model is to implement the suggestion of a previous paper [9] which points out that this group contains the subgroup $SU(4)_l$, in which the left-handed leptons including an antineutrino transform as the fundamental representation. Further, $SU(4)_l$ contains Voloshin symmetry which allows for the magnetic moment of the neutrino to be in the range of $10^{-11}\mu_B$ to $10^{-10}\mu_B$ and at the same time allowing only a small neutrino mass [9, 14, 15]. A magnetic moment in this range might be required [16] if the anticorrelation [11] of solar neutrino flux with sunspot number is confirmed. We shall discuss the implementation in detail in Sec. IV.

## II. SU(16)-BREAKING SCHEME AND RG ANALYSIS

In Fig. 1, we show the symmetry-breaking scheme of our model. The purpose of our scheme is to have the leptonic sector transform separately from the quark sector at intermediate energies and to have the leptonic sector $SU(4)_l$ break in the same manner as in the previous $SU(4)_l$ based model [9]. In the figure, we show the representations of the Higgs fields used to break symmetries at their indicated mass scales. We use the hypothesis of minimal fine-tuning [17], which allows us to choose the mass scales of submultiplets of these fields at different scales. We are interested in scenarios where an intermediate gauge group exists at energies of the order of 1 TeV so that we shall have new physics at observable energies. We will show in this section that such scenarios are con-

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sistent with renormalization group analysis.

In the one-loop approximation, couplings $\alpha_i = g_i^2/4\pi$ evolve as

$$\frac{\partial \alpha_i}{\partial \ln M} = -\frac{B_i}{2\pi}\alpha_i^2 , \tag{2.1}$$

which implies

$$\alpha_i^{-1}(M_2) = \alpha_i^{-1}(M_1) - \frac{B_i}{2\pi} \ln \frac{M_1}{M_2} . \tag{2.2}$$

For the standard model couplings $\alpha_{1Y}$, $\alpha_{2L}$, and $\alpha_{3c}$, we use the conventional normalization which determines the relations

$$\mathrm{Tr}\,(T_i T_j) = 2\delta_{ij} , \quad B_i = \frac{11}{3}N - \frac{4}{3}(2n_g) - \frac{1}{6}T(S_i) , \tag{2.3}$$

where the $T_i$'s are the generators of the SU(16) fundamental representation. To simplify the boundary conditions, we normalize the couplings $\alpha_i$ of the intermediate gauge groups and the SU(16) gauge group such that

$$\mathrm{Tr}\,(T_i T_j) = \frac{1}{2}\delta_{ij} ,$$
$$B_i = \frac{1}{f}\left(\frac{11}{3}N - \frac{f}{3}(2n_g) - \frac{1}{6}T(S_i)\right) . \tag{2.4}$$

Here, $f$ is the number of fundamentals of the subgroup per generation in the 16 of Eq. (1.1). To be explicit, for the $\mathrm{SU}(2)_{qL}$ group $f = 3$, for the $\mathrm{SU}(3)_{qL}$ and $\mathrm{SU}(3)_{qR}$ groups $f = 2$, and for all other intermediate groups and the SU(16) group $f = 1$. The above equations for $B_i$ hold for SU($N$) gauge groups and with $N = 0$ hold for U(1) gauge groups. In the second terms, $n_g$, the number of fermion generations, is multiplied with a factor of 2 to account for mirror fermions. This term does not affect the scales of symmetry breaking. In the third terms, $T(S_i)$ is the quadradic Casimir invariant for all Higgs submultiplets with masses less than the scale of interest. For a complex field, the value of $T(S_i)$ should be doubled.

With the above normalizations, the U(1) generators that enter our symmetry breaking pattern (see Fig. 1) are

$$Q_{qB} = \frac{1}{2\sqrt{6}}\,\mathrm{diag}\,(0_{(4)}, 1_{(6)}, -1_{(6)}) , \tag{2.5}$$

$$Q_{q\Lambda} = \frac{1}{2\sqrt{3}}\,\mathrm{diag}\,(0_{(4)}, 0_{(6)}, 1_{(3)}, -1_{(3)}) , \tag{2.6}$$

$$Q_{qY} = \frac{1}{2\sqrt{33}}\,\mathrm{diag}\,(0_{(4)}, 1_{(6)}, -4_{(3)}, 2_{(3)}) , \tag{2.7}$$

$$Q_{lX} = \frac{1}{2\sqrt{6}}\,\mathrm{diag}\,(1,1,1,-3,0_{(12)}) , \tag{2.8}$$

$$Q_{lY} = \frac{1}{2\sqrt{3}}\,\mathrm{diag}\,(0,-1,-1,2,0_{(12)}) , \tag{2.9}$$

$$Q_{Y} = \frac{1}{2\sqrt{15}}\,\mathrm{diag}\,(0,-3,-3,6,1_{(6)},-4_{(3)},2_{(3)}) . \tag{2.10}$$

In these equations, the notation of the form $a_{(b)}$ stands for $b$ successive entries of $a$. For example, "$1_{(6)}$" stands for "1,1,1,1,1,1." The ordering is the same as in Eq. (1.1).

The breaking scheme shown in Fig. 1 has the following boundary conditions at the breaking scales $M_i$:

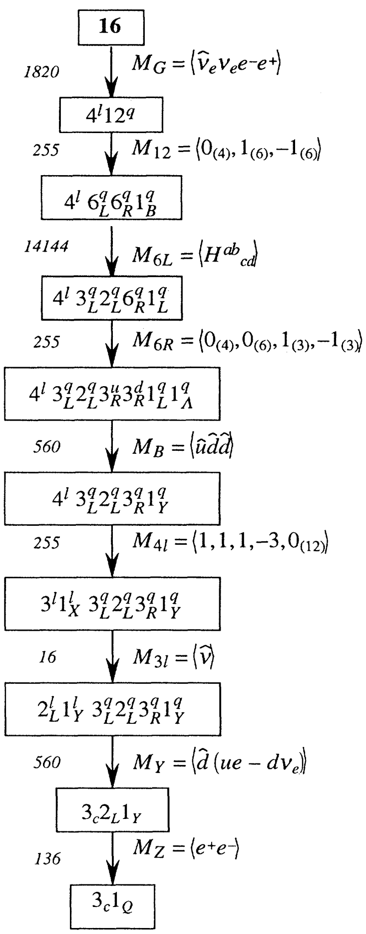


FIG. 1. A possible chain of symmetry breaking. The numbers $n$ denote a factor SU($n$) in the gauge group if $n > 1$, and a U(1) factor if $n = 1$. The superscripts $q$ or $l$ indicate whether only quarks (and antiquarks) or only leptons (and antileptons) are non-singlets under that part of the gauge group. If one considers the **255** as a traceless matrix, its VEV's are diagonal and the notation $1_{(6)}$, e.g., stands for six consecutive entries of unity. In the **560**, the symbol $\langle \hat{d}ue \rangle$, e.g., stands for the VEV of the color singlet combination of the components with one index having the quantum numbers of $\hat{d}$, another of $u$ and another of $e$. One can contemplate chains with fewer steps by equating two or more energy scales. Note the following transformation properties: $A^{[ijkl]}$ : **1820**, $T^k_l$ : **255**, $H^{[kl]}_{[pq]}$ : **14144**, $B^{[ijk]}$ : **560**, $V^i$ : **16**, $\Phi^{\{ij\}}$ : **136**.

$$\alpha_G^{-1}(M_G) = \alpha_{12q}^{-1}(M_G) = \alpha_{4l}^{-1}(M_G)\,, \tag{2.11}$$

$$\alpha_{12q}^{-1}(M_{12}) = \alpha_{6qL}^{-1}(M_{12}) = \alpha_{6qR}^{-1}(M_{12}) = \alpha_{1qB}^{-1}(M_{12})\,, \tag{2.12}$$

$$\alpha_{6qL}^{-1}(M_{6L}) = \alpha_{3qL}^{-1}(M_{6L}) = \alpha_{2qL}^{-1}(M_{6L})\,, \tag{2.13}$$

$$\alpha_{6qR}^{-1}(M_{6R}) = \alpha_{3uR}^{-1}(M_{6R}) = \alpha_{3dR}^{-1}(M_{6R}) = \alpha_{1\Lambda q}^{-1}(M_{6R})\,, \tag{2.14}$$

$$\begin{gathered} \frac{1}{2}\alpha_{3uR}^{-1}(M_B) + \frac{1}{2}\alpha_{3dR}^{-1}(M_B) = \alpha_{3qR}^{-1}(M_B)\,, \\ \frac{2}{11}\alpha_{1qB}^{-1}(M_B) + \frac{9}{11}\alpha_{1\Lambda q}^{-1}(M_B) = \alpha_{1qY}^{-1}(M_B)\,, \end{gathered} \tag{2.15}$$

$$\alpha_{4l}^{-1}(M_{4l}) = \alpha_{3l}^{-1}(M_{4l}) = \alpha_{1lX}^{-1}(M_{4l})\,, \tag{2.16}$$

$$\begin{gathered} \alpha_{3l}^{-1}(M_{3l}) = \alpha_{2lL}^{-1}(M_{3l})\,, \\ \frac{1}{9}\alpha_{3l}^{-1}(M_{3l}) + \frac{8}{9}\alpha_{1lX}^{-1}(M_{3l}) = \alpha_{1lY}^{-1}(M_{3l})\,, \end{gathered} \tag{2.17}$$

$$\begin{gathered} 2\alpha_{3Lq}^{-1}(M_Y) + 2\alpha_{3qR}^{-1}(M_Y) = \alpha_{3c}^{-1}(M_Y)\,, \\ 3\alpha_{2qL}^{-1}(M_Y) + \alpha_{2lL}^{-1}(M_Y) = \alpha_{2L}^{-1}(M_Y)\,, \\ \frac{11}{5}\alpha_{1qY}^{-1}(M_Y) + \frac{9}{5}\alpha_{1lY}^{-1}(M_Y) = \alpha_{1Y}^{-1}(M_Y)\,. \end{gathered} \tag{2.18}$$

In writing Eq. (2.17), we have used the fact that in the group $SU(3)_l$ there is a generator

$$\lambda_{3l} = \frac{1}{2\sqrt{3}}\,\mathrm{diag}\,(2,-1,-1,0,0_{(12)}) \tag{2.19}$$

which is broken at the scale $M_{3l}$. At the same scale $U(1)_{lX}$ also breaks, leaving unbroken the combination

$$Q_{lY} = \frac{1}{3}\lambda_{3l} - \frac{2\sqrt{2}}{3}Q_{lX}\,. \tag{2.20}$$

In deriving Eq. (2.18) for the breakings at the scale $M_Y$, we have used similar considerations. In particular, $SU(3)_{qL}$, which has the two diagonal generators

$$\lambda_{3qL} = \frac{1}{2\sqrt{2}}\,\mathrm{diag}\,(0_{(4)},1,-1,0,1,-1,0,0_{(6)})\,, \tag{2.21}$$

$$\Lambda_{3qL} = \frac{1}{2\sqrt{6}}\,\mathrm{diag}\,(0_{(4)},-1,-1,2,-1,-1,2,0_{(6)})\,, \tag{2.22}$$

breaks, as does $SU(3)_{qR}$, which has the two diagonal generators

$$\lambda_{3qR} = \frac{1}{2\sqrt{2}}\,\mathrm{diag}\,(0_{(4)},0_{(6)},-1,1,0,-1,1,0)\,, \tag{2.23}$$

$$\Lambda_{3qR} = \frac{1}{2\sqrt{6}}\,\mathrm{diag}\,(0_{(4)},0_{(6)},1,1,-2,1,1,-2)\,, \tag{2.24}$$

leaving unbroken $SU(3)_c$ which has the two diagonal generators

$$\lambda_{3c} = \sqrt{2}\lambda_{3qL} + \sqrt{2}\lambda_{3qR}\,, \tag{2.25}$$

$$\Lambda_{3c} = \sqrt{2}\Lambda_{3qL} + \sqrt{2}\Lambda_{3qR}\,. \tag{2.26}$$

Similarly, $SU(2)_{qL}$ with the diagonal generator

$$\lambda_{2qL} = \frac{1}{2\sqrt{3}}\,\mathrm{diag}\,(0_{(4)},1_{(3)},-1_{(3)},0_6), \tag{2.27}$$

and $SU(2)_{lL}$ with the diagonal generator

$$\lambda_{2lL} = \frac{1}{2}\,\mathrm{diag}\,(0,1,-1,0,0_{(12)}) \tag{2.28}$$

are broken, leaving $SU(2)_L$ unbroken with the diagonal generator

$$\lambda_{2L} = \sqrt{3}\lambda_{2qL} + \lambda_{2lL}. \tag{2.29}$$

Now, using

$$n_i \equiv \log_{10}\left(\frac{M_i}{1\,\mathrm{GeV}}\right)\,, \tag{2.30}$$

the one-loop equations and boundary conditions give us the following equations for the standard model couplings:

$$\begin{aligned} \alpha_{3c}^{-1}(M_Z) = 4\alpha_G^{-1} - \frac{\ln 10}{2\pi}[\, & (n_Y - n_Z)B_{3c} + (n_{3l} - n_Y)(2B_{3qL} + 2B_{3qR}) \\ & +(n_{4l} - n_{3l})(2B_{3qL} + 2B_{3qR}) + (n_B - n_{4l})(2B_{3qL} + 2B_{3qR}) \\ & +(n_{6R} - n_B)(2B_{3qL} + B_{3uR} + B_{3dR}) + (n_{6L} - n_{6R})(2B_{3qL} + 2B_{6qR}) \\ & +(n_{12} - n_{6L})(2B_{6qL} + 2B_{6qR}) + (n_G - n_{12})4B_{12q}]\,, \end{aligned} \tag{2.31}$$

$$\begin{aligned} \alpha_{2L}^{-1}(M_Z) = 4\alpha_G^{-1} - \frac{\ln 10}{2\pi}[\, & (n_Y - n_Z)B_{2L} + (n_{3l} - n_Y)(3B_{2qL} + B_{2lL}) \\ & +(n_{4l} - n_{3l})(3B_{2qL} + B_{3l}) + (n_B - n_{4l})(3B_{2qL} + B_{4l}) \\ & +(n_{6R} - n_B)(3B_{2qL} + B_{4l}) + (n_{6L} - n_{6R})(3B_{2qL} + B_{4l}) \\ & +(n_{12} - n_{6L})(3B_{6qL} + B_{4l}) + (n_G - n_{12})(3B_{12qL} + B_{4l})]\,, \end{aligned} \tag{2.32}$$

$$\alpha_{1Y}^{-1}(M_Z) = 4\alpha_G^{-1} - \frac{\ln 10}{2\pi}\Bigg[ (n_Y - n_Z)B_{1Y} + (n_{3l} - n_Y)\left(\frac{11}{5}B_{1qY} + \frac{9}{5}B_{1lY}\right) + (n_{4l} - n_{3l})\left(\frac{11}{5}B_{1qY} + \frac{1}{5}B_{3l} + \frac{8}{5}B_{1lX}\right) + (n_B - n_{4l})\left(\frac{11}{5}B_{1qY} + \frac{9}{5}B_{4l}\right) + (n_{6R} - n_B)\left(\frac{2}{5}B_{1qB} + \frac{9}{5}B_{1\Lambda q} + \frac{9}{5}B_{4l}\right) + (n_{6L} - n_{6R})\left(\frac{2}{5}B_{1qB} + \frac{9}{5}B_{6qR} + \frac{9}{5}B_{4l}\right) + (n_{12} - n_{6L})\left(\frac{2}{5}B_{1qB} + \frac{9}{5}B_{6qR} + \frac{9}{5}B_{4l}\right) + (n_G - n_{12})\left(\frac{11}{5}B_{12q} + \frac{9}{5}B_{4l}\right)\Bigg]. \quad (2.33)$$

For the couplings at $M_Z$ we use the experimental values [18]

$$\begin{aligned} \alpha_{3c}^{-1}(M_Z) &= 8.197\,, \\ \alpha_{2L}^{-1}(M_Z) &= 30.102\,, \\ \alpha_{1Y}^{-1}(M_Z) &= 59.217\,, \\ M_Z &= 91.176\,\text{GeV}\,. \end{aligned} \quad (2.34)$$

The $B_i$'s are determined by Eq. (2.3) and Eq. (2.4) with the $T(S_i)$'s being determined by the Higgs structure given in Fig. Ref. 1 along with the principal of minimal fine-tuning. We note that to impliment the suggestion of ref. [9] an additional rank-2 antisymmetric tensor is to be included. However, this Higgs field has little effect on the RGE's and is not included in the analysis of this section.

Now, the above equations for the standard model couplings can be solved silmutaneously in terms of $n_G$ and $n_{12}$ to obtain

$$n_G = -3.28 - 0.09n_Y - 0.21n_{3l} + 0.90n_{4l} + 0.32n_B + 2.99n_{6R} - 2.60n_{6L}\,, \quad (2.35)$$

$$n_{12} = -2.08 - 0.02n_Y - 0.22n_{3l} + 0.73n_{4l} + 0.37n_B + 2.70n_{6R} - 2.34n_{6L}\,. \quad (2.36)$$

We note that Higgs fields make significant contributions to the above equations. From the structure of the above equations, we see that $n_{4l}$ and $n_{6R}$ being relatively high helps to meet the requirement $n_G \geq n_{12}$. In fact, no solution with low $n_{4l}$ is found to be acceptable.

We are further interested in seeing that it is possible to have new physics, including breaking the Voloshin symmetry, at less than 1 TeV. Since our lowest intermediate stage, which contains the Voloshin symmetry, is broken at $M_{3l}$, we are interested in $n_{3l} \lesssim 3$. So as to investigate this possibility, we make the simplification

$$\begin{aligned} M_{6L} &= M_{12} = M_G\,, \\ M_{4l} &= M_B = M_{6R}\,. \end{aligned} \quad (2.37)$$

This yields

$$n_G = 9.35 + 0.66n_Y - 0.28n_{3l}\,, \quad (2.38)$$

$$n_{6R} = 8.77 + 0.59n_Y - 0.19n_{3l}\,. \quad (2.39)$$

In this example both $M_Y$ and $M_{3l}$ may be low. The particular case of this solution which has $M_Y = M_{3l}$ is graphed in Fig. 2 for the allowed region where $M_G \geq M_{12} \geq M_{3l} \gtrsim 250$ GeV. This gives the range of unification scale to be $10^{10}\,\text{GeV} \leq M_G \leq 10^{15}\,\text{GeV}$.

We also investigate another possible solution where

$$\begin{aligned} &M_{4l} = M_B = M_{6R} = M_{6L} = M_{12}\,, \\ &M_Y = 10^2\,\text{GeV}\,. \end{aligned} \quad (2.40)$$

This yields

$$n_G = 3.99 + 0.56n_{3l}\,, \quad (2.41)$$

$$n_{12} = 4.62 + 0.48n_{3l}\,. \quad (2.42)$$

We graph this case in Fig. 3. Note that $M_G$ can be as low as $10^{8.5}$ GeV. In fact, from the constraint $M_{12} \geq M_{3l}$, we note from Fig. 3 that the unification scale has to be smaller than $10^{8.9}$ GeV. One characteristic of this solution is that all intermediate scales other than $M_Y$ are larger than $10^8$ GeV, so that the only observable new physics comes from the scale $M_Y$. However, we shall see in the next section that this solution is inconsistent with the constraints arising from nonobservation of proton decay, unless the discrete symmetry $V^\alpha \to -V^\alpha$ is imposed on the Lagrangian.

Each of the solutions discussed above has the feature that one can achieve grand unification at scales much lower than what is expected in standard unification models based on gauge groups SU(5) or SO(10). Such low grand unification scale has many interesting features not

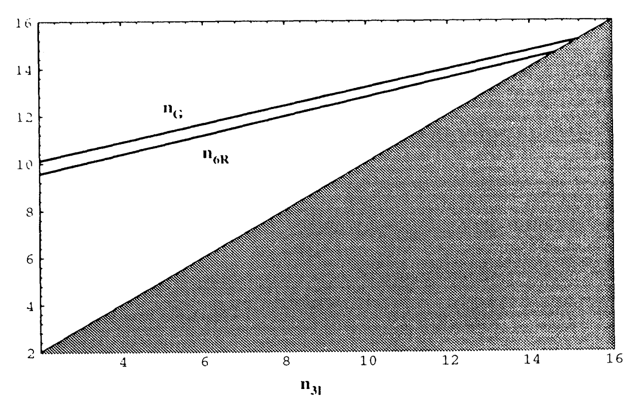


FIG. 2. $n_G \equiv \log_{10}\frac{M_G}{\text{GeV}}$ and $n_{6R} \equiv \log_{10}\frac{M_{6R}}{\text{GeV}}$ as a function of $n_{3l} \equiv \log_{10}\frac{M_{3l}}{\text{GeV}}$ with $M_Y = M_{3l}$, $M_{4l} = M_B = M_{6R}$ and $M_{6L} = M_{12} = M_G$ in Fig.1. The shaded area is not acceptable since by definition $M_G \geq M_{6R} \geq M_{3l}$.

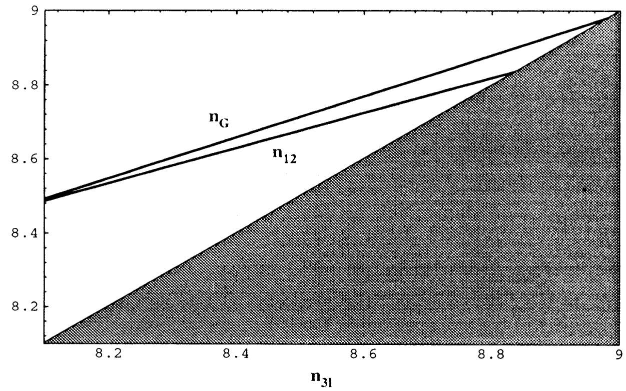


FIG. 3. $n_G \equiv \log_{10} \frac{M_G}{\text{GeV}}$ and $n_{12} \equiv \log_{10} \frac{M_{12}}{\text{GeV}}$ as a function of $n_{3l} \equiv \log_{10} \frac{M_{3l}}{\text{GeV}}$ with $M_{4l} = M_B = M_{6R} = M_{6L} = M_{12}$ and $M_Y = 10^2$ GeV in Fig. 1. The shaded area is not acceptable since by definition $M_G \geq M_{12} \geq M_{3l}$.

present in standard unification models. For example, it has been shown [12] that, unlike the SU(5) model, one can circumvent the cosmological monopole problem easily. Also, the intermediate scales are low, and some of them can be in reach of the next generation of experiments. If new gauge structure exists at TeV energies, as occurs for example in the second solution with low $M_Y$, there will be new gauge bosons to mediate a lot of processes [3]. Direct production of these gauge bosons at the SSC will also provide exciting physics, some of which has been discussed in the context of SU(15) models [2, 19].

## III. PROTON DECAY

Since baryon and lepton numbers are part of the gauge symmetry of the model, proton decay diagrams must involve baryon number and lepton number violating vacuum expectation values (VEV's) in the Higgs sector. The only VEV in our symmetry breaking scheme which violates baryon number is the VEV of the 560-dimensional representation $B^{klm}$ having the quantum numbers of $\left\langle \hat{u}\hat{d}\hat{d} \right\rangle$, which has $B = -1$. Lepton-number violation arises from the VEV of $B^{klm}$ having the quantum numbers of $\left\langle \hat{d}\,(ue - d\nu_e) \right\rangle$, which has $L = 1$, and the VEV of 16-dimensional representation $V^\alpha$ having the quantum numbers of $\langle \hat{\nu}_e \rangle$, which has $L = -1$. Note that lowering indices changes the signs of the quantum numbers.

To examine proton decay, we use the method of effective operators. In this method, one writes down all effective operators involving fermions and scalars which are invariant under the full SU(16) group. When the scalars develop VEV's, one obtains an effective operator involving fermions only. These VEV's are responsible for baryon or lepton violation.

The lowest order effective operators for proton decay involve four fermionic fields [5]. From the discussion above, we find two SU(16)-invariant effective operators which can induce proton decay [20]. These are

$$\mathcal{O}_1 = \{\Psi^k \Psi^l\}\{\Psi^m \Psi^n\} B_{klr} B^{pqr} \Phi_{mp} \Phi_{nq} \,, \qquad (3.1)$$

$$\mathcal{O}_2 = \{\Psi^k \Psi^l\}\{\Psi^m \Psi^n\} B_{klm} V_n \,. \qquad (3.2)$$

Here, $\{\Psi^k \Psi^l\} \equiv (\Psi^k)^T C \Psi_l$, where $C$ is the conjugation matrix for fermions.

Typical diagrams generating $\mathcal{O}_1$ and $\mathcal{O}_2$ are shown in Fig. 4 and Fig. 5, respectively. In Fig. 4, $\Phi_{u\hat{u}}$ gets a VEV. In determining the RGE's, we only gave a VEV to the $\Phi^{e^- e^+}$ component of **136**. However, in order to give masses to the quarks as well as the charged leptons a VEV must be given to each of the $\Phi^{u\hat{u}}$, $\Phi^{d\hat{d}}$, and $\Phi^{e^- e^+}$ components of **136**, i.e., a linear combination of $\Phi^{u\hat{u}}$, $\Phi^{d\hat{d}}$, and $\Phi^{e^- e^+}$ represents the standard model Higgs boson. At higher scales additional components make contributions to the RGE's. These contributions are small compared to other Higgs contributions. Therefore, we ignore them.

From the figures we obtain an order-of-magnitude estimate of the coeficients $\kappa_1$ and $\kappa_2$ of the 4-fermion operators $\mathcal{O}_1$ and $\mathcal{O}_2$ respectively:

$$\kappa_1 \sim \left(\frac{m_f}{M_W}\right)^2 \frac{\lambda_{B\Phi} \lambda_{BB} M_Y M_B M_W^2}{M_G^6} \,, \qquad (3.3)$$

$$\kappa_2 \sim \left(\frac{m_f}{M_W}\right)^2 \frac{\lambda M_B M_{3l}}{M_G^4} \,. \qquad (3.4)$$

Here, the quantity $m_f$ is the mass of a typical fermion, and comes from the Yukawa couplings. Antisymmetry of $B_{klr}$ and fermion indices require use of second generation fermions [5]. Therefore, we use $m_f \simeq 100$ MeV. Also, $\lambda_{B\Phi}$, $\lambda_{BB}$, and $\lambda$ denote the scalar couplings. We have assumed that all virtual colored scalars have masses of order $M_G$, the largest scale of this model. The mass scales in the numerator are the scales of the VEV's. Making a rough estimate, we have neglected factors of gauge coupling constants.

Known bounds on proton lifetime imply

$$\kappa_1 \,, \kappa_2 \lesssim 10^{-30}\,\text{GeV}^{-2} \,. \qquad (3.5)$$

If we use the above constraint with $M_B \sim M_G$, which can be seen to be true from Sec. II, and assume $\lambda_{B\Phi}$, $\lambda_{BB}$, and $\lambda$ are $\sim 1$, then we find the constraints

$$\frac{M_G^5}{M_Y} \gtrsim 10^{28}\,\text{GeV}^4 \,, \qquad (3.6)$$

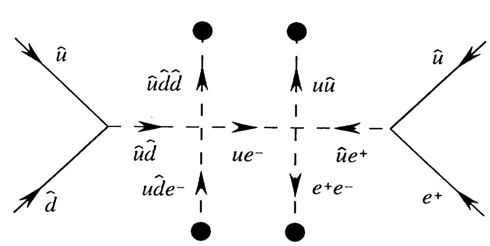


FIG. 4. Tree diagram generating $\mathcal{O}_1$. The labels on the Higgs boson lines represent, via Eq. (1.1), the transformation properties under SU(16). The indices should all be considered as upper indices.

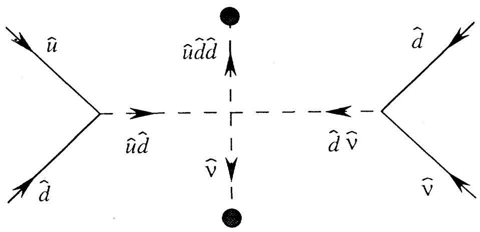


FIG. 5. Tree diagram generating $\mathcal{O}_2$.

$$\frac{M_G^3}{M_{3l}} \gtrsim 10^{24}\,\mathrm{GeV}^2\,. \tag{3.7}$$

from $\mathcal{O}_1$ and $\mathcal{O}_2$ respectively. Equation (3.6) puts no restriction on the solutions to the RGE's, but Eq. (3.7), resulting from $\mathcal{O}_2$, does. For example, Eq. (3.7) rules out the entire special case defined by Eq. (2.40) which gives $M_G$ as low as $\sim 10^{8.5}$ GeV, although it does not rule out any region of the case defined by Eq. (2.37) which allows for a low energy $M_{3l}$. However, the effective operator $\mathcal{O}_2$, which yields this constraint, is no longer allowed if we impose the discrete symmetry $V^\alpha \rightarrow -V^\alpha$ on the Lagrangian. Another feature that would exist if this discrete symmetry is imposed is mentioned in the next section.

It is important to note that the decay modes of the proton obtained from the operators $\mathcal{O}_1$ and $\mathcal{O}_2$ are different from the predictions of standard SU(5) or SO(10) grand unification models. In $\mathcal{O}_1$, the indices $k, l$ are antisymmetrized and so are $m, n$. Thus, the quark level operator for proton decay [5] arising from it is $\hat{u}\hat{s}\hat{u}\mu^+$. This gives rise to the decay mode $p \rightarrow K^0\mu^+$. On the other hand, in $\mathcal{O}_2$, the quark level operator is $\hat{u}\hat{s}\hat{d}\hat{\nu}$, where the neutrino can belong to any generation since the indices $m, n$ are not necessarily antisymmetric for this operator. Thus, we expect a decay mode $p \rightarrow K^+\hat{\nu}$. Note that both operators give rise to $(B-L)$-conserving decays.

## IV. NEUTRINO MAGNETIC MOMENT

We now discuss generation of a sizeable magnetic moment for the neutrino. The most general Yukawa couplings of the model are given by

$$-\mathcal{L}_Y = \sum_a h_a \Psi^\alpha_{aL}\Psi^\beta_{aL}\Phi_{\alpha\beta} + \sum_{a\neq b} f_{ab}\Psi^\alpha_{aL}\Psi^\beta_{bL}\varphi_{\alpha\beta} + \mathrm{H.c.} \tag{4.1}$$

Here, latin indices refer to the generation. $\Phi$ is the symmetric 136-dimensional rank-2 SU(16) tensor representation, whose couplings in the generation space are chosen diagonal without loss of generality. The additional multiplet $\varphi$ is a 120-dimensional antisymmetric rank-2 tensor, so its coupling $f_{ab}$ is antisymmetric in its generation indices. This field is put into the model to generate a magnetic moment for the neutrino. Since quarks play no role in the magnetic moment of the neutrino, we focus our attention on the leptonic part of the interactions.

$\Phi$ is the Higgs field which breaks $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ down to $\mathrm{SU}(3)_c \times \mathrm{U}(1)_Q$. In the leptonic sector we assume only $\Phi_{34}$ gets a VEV [9]. This VEV gives masses to charged leptons, but not to neutrinos. The multiplet $\varphi$, on the other hand, is assumed to have no VEV [9]. The dominant contributions to the mass of $\nu_e$ then come from the one-loop graphs of Fig. 6. The diagrams involve a Higgs potential term of the form $\gamma\varphi^{\alpha\beta}\varphi^{\gamma\delta}A_{\alpha\beta\gamma\delta}$, where $\gamma$ is a coupling with the dimension of mass and $A_{\alpha\beta\gamma\delta}$ is the antisymmetric rank-4 SU(16) tensor whose VEV $\langle A_{1234}\rangle$ breaks SU(16) to $\mathrm{SU}(12)_q \times \mathrm{SU}(4)_l$ at $M_G$. Because $A_{\alpha\beta\gamma\delta}$ is antisymmetric, $\langle A_{2314}\rangle = -\langle A_{2413}\rangle$, and therefore the mass contributions of the two diagrams of Fig. 6 have opposite sign.

From Fig. 6, we estimate

$$m_{\nu_e} \approx \frac{f_{13}^2\gamma\langle A_{2413}\rangle m_\tau}{16\pi^2 M_{24}^2}\ln\left(\frac{M_{24}^2 M_{13}^2}{M_{14}^2 M_{23}^2}\right), \tag{4.2}$$

where $M_{\alpha\beta} \equiv (\text{mass of } \varphi_{\alpha\beta})$. The important point to realize here is that, in the limit of unbroken $\mathrm{SU}(3)_l$, $M_{23} = M_{13}$ and $M_{24} = M_{14}$, and so the mass contributions from the two diagrams cancel each other.

The diagrams of Fig. 6 with one photon line attached give the dominant diagrams for the magnetic moment of $\nu_e$. These diagrams add because of an extra negative sign between the two diagrams arising from the photon vertex. We estimate

$$\mu_{\nu_e} \approx \frac{ef_{13}^2\gamma\langle A_{2413}\rangle m_\tau}{16\pi^2}\left[\frac{1}{M_{24}^2 M_{13}^2} + \frac{1}{M_{14}^2 M_{23}^2}\right]. \tag{4.3}$$

In the limit of unbroken $\mathrm{SU}(3)_l$, we can write $M_{13}^2 = M_{23}^2 \equiv M^2 - \frac{1}{2}\Delta M_{4l}^2$ and $M_{14}^2 = M_{24}^2 \equiv M^2 + \frac{1}{2}\Delta M_{4l}^2$, where $\Delta M_{4l}^2$ is the mass difference due to $\mathrm{SU}(4)_l$ breaking. The breaking of $\mathrm{SU}(3)_l$ then changes the masses according to

$$M_{13}^2 = M^2 - \frac{1}{2}\Delta M_{4l}^2 + \Delta M_{3l}^2\,,\; M_{23}^2 = M^2 - \frac{1}{2}\Delta M_{4l}^2\,, \tag{4.4}$$

$$M_{14}^2 = M^2 + \frac{1}{2}\Delta M_{4l}^2 + \Delta M_{3l}^2\,,\; M_{24}^2 = M^2 + \frac{1}{2}\Delta M_{4l}^2\,, \tag{4.5}$$

where $\Delta M_{3l}^2$ to lowest order is due to the term $\lambda\langle V_1\rangle\langle V^1\rangle\varphi^{1\alpha}\varphi_{1\alpha}$. Here, $V_\alpha$ is the vector representation Higgs field and $\lambda$ is a dimensionless coupling. Putting Eq. (4.4) and Eq. (4.5) into the expressions for $m_{\nu_e}$ and $\mu_{\nu_e}$ gives us

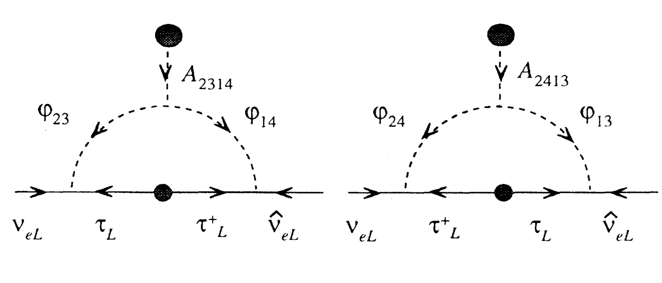


FIG. 6. The diagrams that contribute to the mass of the neutrinos at the 1-loop level. Magnetic moment arises by attaching a photon line to any internal line.

$$m_{\nu_e} \approx \frac{f_{13}^2 \gamma M_G m_\tau}{16\pi^2 M^6} \Delta M_{3l}^2 \Delta M_{4l}^2 \left[1 - \left(\frac{\Delta M_{4l}^2}{2M^2}\right)^2\right]^{-1}, \tag{4.6}$$

$$\mu_{\nu_e} \approx \frac{e f_{13}^2 \gamma M_G m_\tau}{8\pi^2 M^4} \left[1 - \left(\frac{\Delta M_{4l}^2}{2M^2}\right)^2\right]^{-1}, \tag{4.7}$$

where we have used $\langle A_{2413} \rangle \sim M_G$ and assumed $\frac{\Delta M_{3l}^2}{M^2} \ll 1$. Now, requiring $m_{\nu_e} \lesssim 10eV$ and $\mu_{\nu_e} \gtrsim 10^{-11}\mu_B$, gives from Eq. (4.6) and Eq. (4.7) the constraints

$$\frac{f_{13}^2 \gamma M_G}{M^6} \Delta M_{3l}^2 \Delta M_{4l}^2 \left[1 - \left(\frac{\Delta M_{4l}^2}{2M^2}\right)^2\right]^{-1} \lesssim 10^{-6}, \tag{4.8}$$

$$\frac{f_{13}^2 \gamma M_G}{M^4} \left[1 - \left(\frac{\Delta M_{4l}^2}{2M^2}\right)^2\right]^{-1} \gtrsim 10^{-6}\,\mathrm{GeV}^{-2}. \tag{4.9}$$

Also, demanding the mass-mixing matrix for $\varphi$ to have a positive determinant gives the constraint

$$\gamma < \frac{M^2}{M_G} \left[1 - \left(\frac{\Delta M_{4l}^2}{2M^2}\right)^2\right]^{\frac{1}{2}}. \tag{4.10}$$

We remark that if we impose the discrete symmetry $V_\alpha \to -V_\alpha$ on the Lagrangian, then we should expect $\gamma$ to be small because then $\gamma \varphi^{\alpha\beta} \varphi^{\gamma\delta} A_{\alpha\beta\gamma\delta}$ is the only term of the Lagrangian not invariant under $A_{\alpha\beta\gamma\delta} \to -A_{\alpha\beta\gamma\delta}$. An example of this is when in each of Eq. (4.8), Eq. (4.9) and Eq. (4.10) the left and right sides are approximately equal. In this case, $\Delta M_{3l}^2 \Delta M_{4l}^2 \sim 10^6 f_{13}^2\,\mathrm{GeV}^4$ and if also $\left(\frac{\Delta M_{4l}^2}{2M^2}\right)^2 \ll 1$ then $M \sim 10^3\,\mathrm{GeV}$. Typically, M is of the order of 1 TeV with $\mathrm{SU}(3)_l$ and $\mathrm{SU}(4)_l$ breaking inducing small mass changes in the $\varphi_{\alpha\beta}$'s with $\Delta M_{3l}^2 \sim \Delta M_{4l}^2 \sim 10^3\,\mathrm{GeV}^2$.

## V. SUMMARY

We have shown in this paper breaking chains for the SU(16) grand unification group which lead to the standard model with the quark and lepton sectors transforming separately at intermediate energies. The grand unification scale for this model can be as low as $\sim 10^{10}$ GeV. We have shown that this does not produce any conflict with know bounds of proton lifetime. In fact, if a discrete symmetry is imposed on the model, one can obtain chains with the unification scale as low as $\sim 10^{8.5}$ GeV. Also, low intermediate breaking scales can exist in the $\lesssim 1$ TeV range. This has many observable consequences, including guage bosons at the TeV range which can give rise to a rich phenomenology.

Further, the model embeds the Voloshin symmetry $SU(2)_\nu$ into its subgroup $\mathrm{SU}(4)_l$. By using a rank-2 antisymmetric Higgs field, it is possible to get a significant magnetic moment for a neutrino with small mass. For this to work, this Higgs field typically should have masses $\sim 1$ TeV with relatively small mass differences induced by $\mathrm{SU}(3)_l$ and $\mathrm{SU}(4)_l$ breaking.


## ACKNOWLEDGMENTS

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[20] One can easily construct other operators which violate baryon and lepton numbers, but they do not contribute to proton decay. For example, consider $\{\Psi^k\Psi^l\}\{\Psi^m\Psi^n\}B_{klr}B^{pqr}A_{mnpq}$. If only VEV's are those shown in Fig. 1, it is easy to see that no four-fermion operator results.