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Implications of results from the CERN e^+e^- collider LEP for SO(10) grand unification with two intermediate stages

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We consider the breaking of the grand unification group SO(10) to the standard model gauge group through several chains containing two intermediate stages. Using the values of the gauge coupling constants at a scale M_Z derived from recent data from the CERN e^+e^- collider LEP, we determine the range of their intermediate and unification scales. In particular, we identify those chains that permit new gauge structure at relatively low energy ($\sim$ 1 TeV).

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Recently, SO(10) [1] breaking chains with one intermediate stage [2] have been examined in light of the latest LEP data [3] from the CERN e^+e^- collider LEP. These data give

$$\alpha_1(M_Z) = 0.016887 \pm 0.000040,$$
$$\alpha_2(M_Z) = 0.03322 \pm 0.00025, \tag{1}$$
$$\alpha_3(M_Z) = 0.120 \pm 0.007,$$

where the α_is are normalized such that they would be equal when SO(10) is a good symmetry and refer to $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ respectively. Our conclusion was that if SO(10) breaks through a single intermediate scale to the standard model, then this scale is in the range of 10^9 to 10^{11} GeV. In this report, we extend our analysis to two intermediate-stage breaking schemes. Such analysis has been done previously [4]. Our analysis differs from these in the use of the most recent data given above. We are primarily interested in identifying those chains that permit low-energy gauge groups containing the standard model as a subgroup. We find that it is possible to have extra neutral gauge bosons in the low-energy regime, but definitely no extra charged ones below about 10^7 GeV.

We start by noting that all possibilities with grand unified SU(5) in the intermediate stage are already ruled out by the data. We have checked that the intermediate breaking to flipped [5] SU(5) $\times$ U(1) does not work as well without supersymmetry. So we look at symmetry breaking chains where the intermediate level gauge groups are either $\{2_L 2_R 4_C P\}$ or any of its subgroups, where 2_L, for example, stands for the group $SU(2)_L$ and P denotes an unbroken $L \leftrightarrow R$ parity symmetry. We denote these chains by the following notation:

$$\mathrm{SO}(10) \xrightarrow[\langle H_2\rangle]{M_U} G_2 \xrightarrow[\langle H_1\rangle]{M_2} G_1 \xrightarrow[\langle h\rangle]{M_1} \{2_L 1_Y 3_c\} \xrightarrow[\langle 10\rangle]{M_Z} \{3_c 1_Q\}\,. \tag{2}$$

Here SO(10) breaks to G_2 at the unification scale M_U due to the vacuum expectation value (VEV) of the Higgs multiplet H_2, and subsequently G_2 breaks to the subgroup G_1 with the Higgs multiplet H_1. All possible choices of G_2 and G_1, along with the lowest-dimensional Higgs representations which can do the breaking, have been listed in Table I. The notation for Higgs representations used in Table I have been defined in Table II. In the tables, $X = (B-L)/2$. In all cases, $\{2_L 1_Y 3_c\}$ breaks to $\{3_c 1_Q\}$ with a complex **10**. The breaking to the standard model is done with h, where h can be a 16- or 126-dimensional representation of SO(10). In either case, we can achieve a see-saw mechanism to generate small neutrino masses, at the tree level [6] with **126**, or through loops [7] using a **16**.

An important point to note in the tables is that, for some chains of symmetry breaking, the parity symmetry P remains unbroken at some intermediate stages [4]. This is achieved by a judicious choice of Higgs VEV's which have even parity. For example, under the subgroup $\{2_L 2_R 4_C\}$, the 16-dimensional multiplet of SO(10) decomposes as

$$\mathbf{16} \to (2,1,4) + (1,2,\overline{4})\,. \tag{3}$$

Under the P symmetry, these two submultiplets transform into each other. Thus, the product $\mathbf{16} \times \overline{\mathbf{16}}$ contains two singlets of $\{2_L 2_R 4_C\}$, which transform into each other under P. Making linear combinations, we can form a singlet with even P and one with odd P. Since

$$\mathbf{16} \times \overline{\mathbf{16}} = \mathbf{1} + \mathbf{45} + \mathbf{210}\,, \tag{4}$$

the P-even singlet must be the singlet of SO(10). Noting that the **45** multiplet does not have any singlet [8] of $\{2_L 2_R 4_C\}$, we conclude that the P-odd singlet must be in **210**. Similarly, from the product of $\mathbf{10} \times \mathbf{10}$, we can show that **54** has a P-even singlet under the same subgroup. Similar considerations can be applied to the subgroup $\{2_L 2_R 1_X 3_c\}$, for which we find a P-odd singlet in **45**. In **210**, apart from the singlet of the larger group $\{2_L 2_R 4_C\}$ described above, we find an extra P-even singlet.

To examine the different chains of symmetry breaking, we use the one-loop renormalization group equations (RGE's)

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TABLE I. Different chains of symmetry breaking considered in this paper. The highest stage of symmetry breaking is performed by the VEV of the G_2 singlet of the H_2 multiplet of SO(10), and plays no role in the evolution of the gauge couplings below M_U. In the next stage, the field obtaining VEV is the G_1 singlet contained in the H_1, whose transformation under G_2 is displayed. The notations for H_1 submultiplets are explained in Table II.

Chain	Higher intermediate stage H_2	G_2	Lower intermediate stage H_1 submultiplet	G_1
I	210	$\{2_L2_R4_C\}$	Λ^{45}	$\{2_L2_R1_X3_c\}$
II	54	$\{2_L2_R4_CP\}$	Λ^{210}	$\{2_L2_R1_X3_cP\}$
III	54	$\{2_L2_R4_CP\}$	Λ^{45}	$\{2_L2_R1_X3_c\}$
IV	210	$\{2_L2_R1_X3_cP\}$	Λ^{45}	$\{2_L2_R1_X3_c\}$
V	210	$\{2_L2_R4_C\}$	Σ_R^{45}	$\{2_L1_R4_C\}$
VI	54	$\{2_L2_R4_CP\}$	Σ_R^{45}	$\{2_L1_R4_C\}$
VII	54	$\{2_L2_R4_CP\}$	λ^{210}	$\{2_L2_R4_C\}$
VIII	45	$\{2_L2_R1_X3_c\}$	Σ_R^{45}	$\{2_L1_R1_X3_c\}$
IX	210	$\{2_L2_R1_X3_cP\}$	Σ_R^{45}	$\{2_L1_R1_X3_c\}$
X	210	$\{2_L2_R4_C\}$	σ_R^{210}	$\{2_L1_R1_X3_c\}$
XI	54	$\{2_L2_R4_CP\}$	σ_R^{210}	$\{2_L1_R1_X3_c\}$
XII	45	$\{2_L1_R4_C\}$	Λ^{45}	$\{2_L1_R1_X3_c\}$

TABLE II. The submultiplet, whose representation under the subgroup is shown, contributes to the evolution of the gauge coupling of that subgroup.

SO(10) multiplet	Relevant submultiplet in SO(10) subgroup $\{2_L1_R4_C\}$	$\{2_L2_R4_C\}$	$\{2_L2_R1_X3_c\}$	$\{2_L1_R1_X3_c\}$	Notation
10	$(2,\frac{1}{2},1)$	$(2,2,1)$	$(2,2,0,1)$	$(2,\frac{1}{2},0,1)$	ϕ^{10}
16	$(1,-\frac{1}{2},4)$	$(1,2,4)$	$(1,2,\frac{1}{2},1)$	$(1,-\frac{1}{2},\frac{1}{2},1)$	δ_R^{16}
16		$(2,1,4)$	$(2,1,-\frac{1}{2},1)$		δ_L^{16}
126	$(1,1,10)$	$(1,3,10)$	$(1,3,-1,1)$	$(1,1,-1,1)$	Δ_R^{126}
126		$(3,1,10)$	$(3,1,1,1)$		Δ_L^{126}
45	$(1,0,15)$	$(1,1,15)$	$(1,1,0,1)$		Λ^{45}
210		$(1,1,15)$			Λ^{210}
45		$(1,3,1)$	$(1,3,0,1)$		Σ_R^{45}
45		$(3,1,1)$	$(3,1,0,1)$		Σ_L^{45}
210		$(1,3,15)$			σ_R^{210}
210		$(3,1,15)$			σ_L^{210}
210		$(1,1,1)$			λ^{210}

TABLE III. Expressions for $T(S_i)$ for different intermediate gauge groups.

Intermediate gauge group	Higgs contribution $T(S_i)$
$\{2_L1_R4_C\}$	$T_{2L} = 1\phi^{10}$
	$T_{1R} = 1\phi^{10} + 20\Delta_R^{126} + 2\delta_R^{16}$
	$T_{4C} = 6\Delta_R^{126} + 1\delta_R^{16} + 4\Lambda^{45} + 4\Lambda^{210}$
$\{2_L2_R4_C\}$	$T_{2L} = 2\phi^{10} + 40\Delta_L^{126} + 4\delta_L^{16} + 2\Sigma_L^{45} + 30\sigma_L^{210}$
	$T_{2R} = 2\phi^{10} + 40\Delta_R^{126} + 4\delta_R^{16} + 2\Sigma_R^{45} + 30\sigma_R^{210}$
	$T_{4C} = 18\Delta_R^{126} + 18\Delta_L^{126} + 2\delta_R^{16} + 2\delta_L^{16} + 12\sigma_R^{210} + 12\sigma_L^{210}$ $+4\Lambda^{45} + 4\Lambda^{210}$
$\{2_L2_R1_X3_c\}$	$T_{2L} = 2\phi^{10} + 4\Delta_L^{126} + 1\delta_L^{16} + 2\Sigma_L^{45}$
	$T_{2R} = 2\phi^{10} + 4\Delta_R^{126} + 1\delta_R^{16} + 2\Sigma_R^{45}$
	$T_{1X} = 9\Delta_L^{126} + 9\Delta_R^{126} + \frac{3}{2}\delta_L^{16} + \frac{3}{2}\delta_R^{16}$
	$T_{3c} = 0$
$\{2_L1_R1_X3_c\}$	$T_{2L} = 1\phi^{10}$
	$T_{1R} = 1\phi^{10} + 2\Delta_R^{126} + \frac{1}{2}\delta_R^{16}$
	$T_{1X} = 3\Delta_R^{126} + \frac{3}{4}\delta_R^{16}$
	$T_{3c} = 0$

$$\mu \frac{\partial \alpha_i}{\partial \mu} = \frac{a_i}{2\pi} \alpha_i^2 \,, \tag{5}$$

which gives

$$\alpha_i^{-1}(M_2) = \alpha_i^{-1}(M_1) - \frac{a_i}{2\pi} \ln \frac{M_2}{M_1} \,. \tag{6}$$

Here,

$$a_i = \frac{4}{3} n_g - \frac{11}{3} N + \frac{T(S_i)}{6} \tag{7}$$

where n_g is the number of fermion generations which we take as 3 and N is the value of N in SU(N) with $N = 0$ for U(1). The necessary expressions for $T(S_i)$ are given in Table III. If the Higgs fields are complex then the value of $T(S_i)$ has been multiplied by a factor of 2. We use the hypothesis of minimal fine-tuning [9], which fixes the masses of Higgs bosons according to their transformation under the unbroken subgroups at any scale. The Higgs fields that enter Table III are those submultiplets that have masses below the energy level of interest and contribute to evolution of the couplings. These submultiplets are defined in Table II.

To illustrate the use of our tables, let us consider one example. Take chain I, for which $G_2 = \{2_L 2_R 4_C\}$, $G_1 = \{2_L 2_R 1_X 3_c\}$, and we shall take $h = 126$. For the range M_Z to M_1, the only contribution to $T(S_i)$ is from a doublet of SU$(2)_L$ contained in the 10-dimensional multiplet. Between M_1 and M_2, the symmetry group is $\{2_L 2_R 1_X 3_c\}$. Looking into Table III for the row of $\{2_L 2_R 1_X 3_c\}$, one can now determine the various $T(S_i)$ by putting $\phi^{10} = 1$ and $\Delta_R^{126} = 1$, and all other greek symbols equal to zero. (If the P-symmetry were unbroken at this stage, as for example in chain II, then we should set $\Delta_L^{126} = 1$ as well.) For evolution from M_2 to M_U, we note from Table I that Λ^{45} contributes to the symmetry breaking at the scale M_2. Thus, in Table III, we now use the row of $\{2_L 2_R 4_C\}$, with $\Lambda^{45} = 1$, and the other symbols set to the same value as in the previous stage. The coefficients of various terms given in Table III have been derived from the transformation of the corresponding submultiplets given in Table II.

In our analysis, we match couplings at each stage of symmetry breaking. We assume that all fermions have

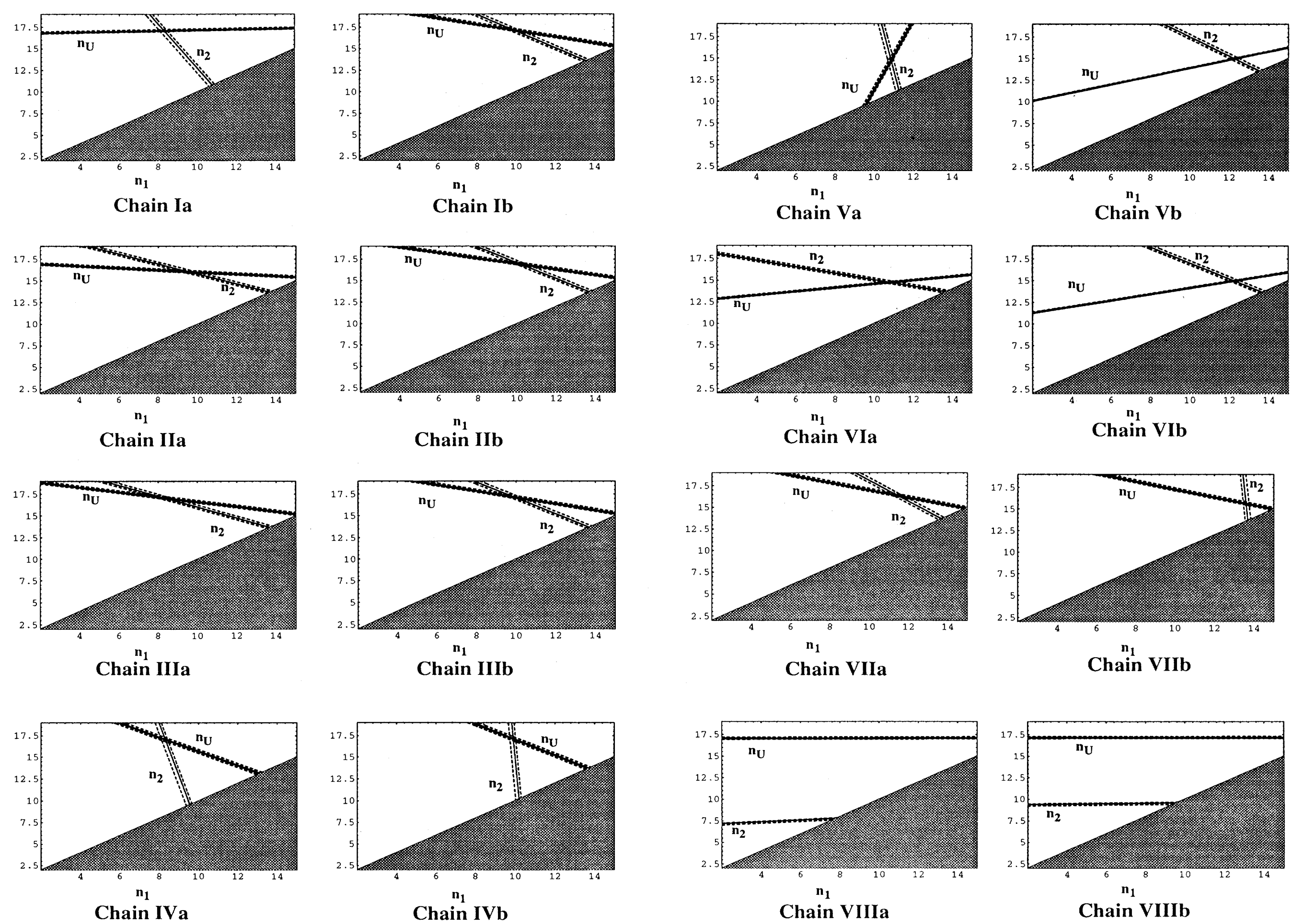

FIG. 1. $n_U = \log_{10}\left(\frac{M_U}{\text{GeV}}\right)$ and $n_2 = \log_{10}\left(\frac{M_2}{\text{GeV}}\right)$ plotted vs $n_1 = \log_{10}\left(\frac{M_1}{\text{GeV}}\right)$ for chains I through XII. For each chain we refer to case (a) where $h = 126$ and case (b) where $h = 16$. The constraint $n_U, n_2 \geq n_1$ is violated by n_U or n_2 being in the shaded portion of the graphs. The acceptable domain for n_1 in each case is given in Table IV.

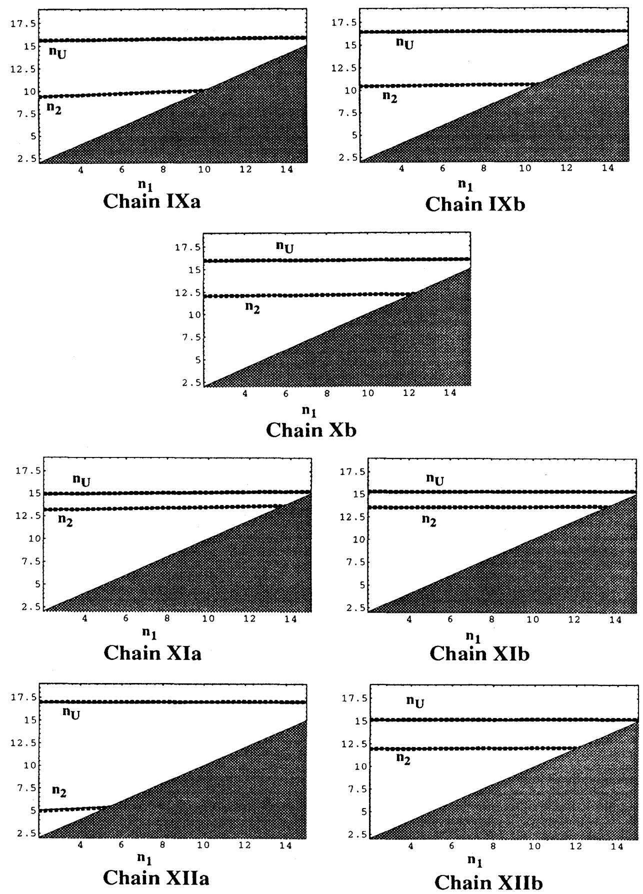

FIG. 1. (*Continued*).

masses less than M_Z. Although the mass of the t-quark is expected to be slightly larger than M_Z, and the mass of ν_R could be much larger than M_Z, the corrections due to these are negligible for the purposes of our calculations.

In the study of each chain, we solve analytically in one loop order for allowed scales, $n_U = \log_{10}\left(\frac{M_U}{\text{GeV}}\right)$ at grand unification, $n_2 = \log_{10}\left(\frac{M_2}{\text{GeV}}\right)$, the higher of the intermediate scales, and $n_1 = \log_{10}\left(\frac{M_1}{\text{GeV}}\right)$, the lower of the intermediate scales. The graphs for each case considered are drawn with 1σ errors from LEP data. Only those portions of the graphs are meaningful where $n_U \geq n_2 \geq n_1$. Further n_U has to be sufficiently high to escape the constraint from non-observation of proton decay. We take this constraint to be approximately [2]

$$\left(\frac{\alpha_U^{-1}(M_U)}{40}\right)\left(\frac{M_U}{10^{15}\,\text{GeV}}\right)^2 > 2.5 \qquad (8)$$

Therefore, we consider the portion of any chain where $M_U < 10^{15}$ GeV to be unacceptable. We show our findings in the graphs of Fig. 1, and in Table IV we present the acceptable regions of the lower intermediate scale n_1 for these graphs. For each chain we refer to case (a) where $h = 126$ and case (b) where $h = 16$. Chain Xa is not featured because it has no meaningful solution.

We find no chain where extra light charged bosons can occur. In all chains with $SU(2)_R$ or $SU(4)_C$ in the lower intermediate scale, the allowed regions of n_1 tend to be small with both intermediate scales at very high values.

TABLE IV. Acceptable domains of n_1 for all chains. The chains are defined in Table I, and a, b refer to the choice of h being **126** or **16** respectively.

Chain	Allowed values of n_1	
	Lowest	Highest
Ia	8.4±0.2	10.8 ± 0.2
Ib	10.0±0.2	13.5 ± 0.2
IIa	9.6±0.2	13.6 ± 0.2
IIb	10.2±0.2	13.8 ± 0.2
IIIa	8.2±0.2	13.7 ± 0.2
IIIb	9.8±0.2	13.6 ± 0.2
IVa	8.5±0.2	9.6 ± 0.2
IVb	9.9±0.2	10.2 ± 0.2
Va	10.8±0.2	11.2 ± 0.2
Vb	12.2±0.2	13.5 ± 0.2
VIa	12.0±0.1	13.7 ± 0.2
VIb	12.4±0.1	13.6 ± 0.2
VIIa	11.4±0.2	13.6 ± 0.2
VIIb	13.6±0.2	13.8 ± 0.2
VIIIa	2.0	7.7 ± 0.1
VIIIb	2.0	9.5 ± 0.1
IXa	2.0	10.0 ± 0.2
IXb	2.0	10.5 ± 0.2
Xb	2.0	12.2 ± 0.2
XIa	2.0	13.6 ± 0.2
XIb	2.0	13.7 ± 0.2
XIIa	2.0	5.3 ± 0.1
XIIb	2.0	12.1 ± 0.2

We find that additional gauge bosons in the range of TeV's are permitted only in the chains VIII through XII except for chain Xa which has no meaningful solution. All of these chains have $\{2_L 1_R 1_X 3_c\}$ as the lower intermediate gauge group. This means that there is only one extra gauge boson in the TeV range, and this one is neutral. Even in this set, the chains XIa and XIIb are of very marginal acceptability due to the constraint of Eq. (8) set by experiments on proton decay.

In see-saw models of neutrino mass, ordinary neutrinos are light due to a large Majorana mass of the right-handed neutrinos [6]. This large Majorana mass cannot be generated unless the $\{1_R\}$ symmetry is broken. Thus the magnitude of this Majorana mass is expected to be similar to the scale of the $\{1_R\}$ breaking. Our conclusions stated above thus show that it is possible to obtain this Majorana mass in the TeV range. However, the full parity symmetry is not restored until at much higher energy since $\{2_R\}$ breaking always occurs at high scale.

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