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Hypothetical long-range interactions and restrictions on their parameters from force measurements

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Different experiments on measuring van der Waals (Casimir) and gravitational forces for obtaining restrictions on the constants of hypothetical interactions which decrease with distance on Yukawa or power-law potentials are considered. New restrictions which are based on experiments on direct force measurements (such as Eötvös and Galileo ones, measurements of Casimir and van der Waals forces) are obtained and reviewed. Restrictions on the parameters of light elementary particles which follow from the results of force measurements are discussed. New experiments in which an optimized configuration of test bodies is used for obtaining the strongest restrictions on long-range force constants are suggested. It is possible to strengthen all contemporary restrictions by a factor of 10 and even by a factor of ten million depending on the region of parameter values in such experiments.

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I. INTRODUCTION

Recent developments in quantum field theory have revealed the possibility of the existence of a large number of light and massless elementary particles of different types. Among them are particles such as the axion [1,2], scalar neutrino [3] (which arises in supersymmetric theories), spin-1 antigraviton [4,5] (which arises in all extended supergravity schemes), dilaton (which appears in the theories with broken scale invariance [6,7]), etc. Exchange of such particles may show up not only in the scattering of elementary particles and in their reactions and decays, but also leads to some new effects in atomic spectroscopy. Moreover, it can reveal itself as an additional interaction between macrobodies. The last possibility is under investigation now in more than 40 laboratories all over the world. Additional long-range interactions may result in a deviation of the gravitational force from Newtonian law (experiments of Cavendish type), in the difference between inertial and gravitational masses (experiments of Eötvös and Galileo types), in deviations of the Casimir force from the theoretical predictions under their experimental verification, in atomic force microscopy, etc. (for a review of gravitational experiments and best limits of the inverse-square law see [8,9]).

There are statements in the literature that a hypothetical (so-called "fifth") force was observed in the experiments of Eötvös and Cavendish types [10–12]. But at the same time other research groups do not find this force [13–18].

In the present paper, an analysis of different terrestrial experiments on measuring the van der Waals and gravitational forces is carried out in connection with the possible existence of new long-range interactions. The implications of such experiments on the search for hypothetical interactions are investigated too. The best limits on the parameters of hypothetical forces which are obtained from the experimental data on force measurements are given. Moreover, some new experiments are suggested, the realization of which will cause quick progress in the search for new long-range forces.

It should be noted that when speaking about long-range interactions we mean here the forces with an action range λ from one angstrom to hundreds of meters with an effective potential of Yukawa type, $\sim(1/r)\exp(-r/\lambda)$, and power-law forces $\sim r^{-n}$, where $n=2,3,4,\ldots$. The choice of such potentials is discussed in Sec. II. Also, the restrictions on hypothetical forces resulting from modern experiments on force measurements are given there. As a consequence, restrictions on the parameters of elementary particles which generate these interactions are given also.

It should be emphasized that although the restrictions from the force measurement experiments sometimes are not so strong as the ones obtained from astrophysics and elementary particle physics, they are most reliable and model independent.

In Sec. III new experiments on force measuring are suggested in order to search for hypothetical long-range interactions and stronger restrictions on their parameters. In particular, the optimal shape of test bodies is found. It provides the best sensitivity of experiment to the presence of an additional long-range force.

In Sec. IV all the best restrictions on the parameters of new hypothetical interactions are presented in Fig. 5 and in Table V together with the prospective limits which would be achieved if the experiments suggested in Sec. III were performed.

The paper has two main purposes. One of them is to obtain and review all best restrictions which follow from experiments on force measurements as was done in Ref. [19] 12 years ago. The second goal is to awaken interest in the optimum experiments of Casimir and Cavendish types which were discussed by us in [43,44] and promise great progress in the problem of new long-range interactions and light elementary particles.

Throughout the paper the units in which $h=c=1$ are used.

II. LONG-RANGE FORCES IN MODERN PHYSICS AND RESTRICTIONS ON THEIR PARAMETERS FROM FORCE MEASUREMENTS

A. Types of potentials

As was noted in the Introduction, long-range forces with an interaction radius from 10^{-10} to 100 m (and more) are predicted by almost every modern unified theory of fundamental interactions. Let us consider the types of potentials which correspond to such forces.

The effective interaction potential between two particles is calculated in quantum field theory in the form

$$V(r)=\frac{1}{(2\pi)^3 M_1 M_2}\int d^3\mathbf{k}\, e^{-i\mathbf{k}\cdot\mathbf{r}}T(s_0,t)\ , \qquad (1)$$

where $T(s,t)$ is an invariant amplitude of elastic scattering of particles (in Mandelstam's variables), $r=|\mathbf{r}|$ is the distance between them. $\mathbf{k}$ is the transferred momentum, and $M_{1,2}$ are the masses of particles. Moreover, it is supposed that $s=s_0=(M_1+M_2)^2$ for extracting the velocity-independent part of the potential.

The potential of a force acting between two atoms can be calculated by means of Feynman rules. The result is the well-known potential of Yukawa type under exchange by scalar or vector particles with mass $m=\lambda^{-1}$:

$$V(r)=\alpha(2z)^2\frac{1}{r}e^{-r/\lambda}\ . \qquad (2)$$

Here the factor z, which stands for the number of protons in the atom, is introduced for taking off the dependence of α on the sort of atom, i.e., on z (number of neutrons is also supposed to be equal to z). The exchange of hypothetical particles such as a scalar axion [1,2], scalar neutrino [3], spin-1 antigraviton [4,5], etc., leads to an effective potential [Eq. (2)].

Another light particle which is interesting for us is the dilaton. It is a boson which appears due to the breaking of scale invariance [6,7]. In contrast with the case of usual Goldstone bosons, the effective potential (2) arises even by exchanging one such particle because of the mixing of a dilaton with a graviton. Different examples of hypothetical particles and non-Newtonian forces can be found in [20].

The value of the constant α in the effective Yukawa-type potential for a spin-1 antigraviton is predicted by theory: $\alpha=8\pi G m_0^2\simeq 10^{-40}$, where m_0 is the sum of the current quark masses of a nucleon and G is the gravitational constant. The value of α predicted for the dilaton is equal to $\frac{1}{3}GM_N^2\approx 2\times 10^{-39}$, where m_N is the nucleon mass.

For the case of $m=0$ in Eq. (1), one has the usual Coulomb potential. If the particles exchange pseudoscalar particles, then the resulting potential depends on the spins of the interacting particles. Such spins are added up under the interaction of macrobodies. Hence additional interaction arises only if the total spin of each macrobody is not equal to zero. But in this case the macrobodies have magnetic moments which are enormous in comparison with additional long-range interactions. Therefore, hereafter, it will be supposed that macrobodies have no magnetic moments, and consequently we will not consider the exchange of one pseudoscalar particle.

However, if the exchange of an even number of such particles is considered, then the long-range interaction appears again. Thus, in Ref. [21], it was shown that the exchange of two massless arions [22] leads to the interaction potential between electrons, $V\sim r^{-3}$.

There is a whole class of unified theories based on supersymmetry in which the massless Fermi particle arises, the so-called Goldstino [3]. In [23] the exchange by two such particles was studied, and it was found that the interaction potential can be written independently of the type of theoretical model as $V\sim r^{-7}$.

Additionally, the exchange of the usual neutrinos also leads to the appearance of a long-range interaction. The potential of such an interatomic interaction was found in [24] and has the form $V\sim r^{-5}$.

The list of such examples can be continued. However, it is essential here to find out what types of potentials and light elementary particles exist really in nature. So it is interesting to investigate the restrictions on the parameters of a Yukawa-type potential (2) and power-law ones following from all up to date experiments.

We shall use the power-law potential of the interaction between the atoms such as in [17]:

$$V_n(r)=\lambda_n(2z)^2\frac{1}{r}\left[\frac{r_0}{r}\right]^{n-1}\ , \qquad (3)$$

where λ_n is dimensionless constant and $r_0=1$ F$=10^{-15}$ m. In such a form, the constant λ_n does not depend on the type of atom [such as α in Eq. (2)].

Subsequently, we shall suppose that the hypothetical long-range interaction field of a macrobody is the additive sum of the fields of its separate atoms. Such a hypothesis is justified because the additional interaction is rather small. The atomic field in its turn is the additive sum of the fields of nucleons and electrons.

Let us consider experiments in which the new hypothetical forces can be revealed.

B. Eötvös- and Galileo-type experiments

In the experiments of this type, the so-called weak principle of equivalence was verified; i.e., the difference between inertial and gravitational masses of a body was measured. In Eötvös experiments two bodies of equal gravitational mass were hung at the torque balance. If the inertial masses of the bodies were different, then the torque moment would appear due to the interaction with the Earth [17,18,25,26].

In Galileo-type experiments, the bodies such as those in Eötvös experiments fall down and the time of their fall is registered [27–30]. The existence of an additional hypothetical force which is not proportional to the masses of interacting bodies can lead to the appearance of the effective difference between inertial and gravitational masses. Therefore some restrictions on hypothetical interaction emerge from the experiments of Eötvös and Galileo types.

Following the Ref. [19], let us show what restrictions on λ_n from Eq. (3) and on λ from Eq. (2) can be obtained on the basis of Eötvös experiments. The typical result of such experiments is that the relative difference between the accelerations imparted by the Earth to various substances of the same mass must be less than 10^{-9}. Recently, a new experiment of Eötvös type was achieved [25], in which this quantity was shown to be less than 10^{-11}.

The restrictions of Ref. [25] are shown in Fig. 1 where the region of α,λ permitted by the experiment lies below curve 1. Curve 2 was obtained in [17,18] where the Eötvös-type experiment was carried out with massive laboratory bodies instead of the Earth.

If we admit that the hypothetical interaction arises due to a spin-1 antigraviton or dilaton, then one can obtain restrictions on their masses. So taking into account the values of the coupling constants predicted by theory, $\alpha \approx 10^{-40}$ for a spin-1 antigraviton and $\alpha \approx 2\times 10^{-39}$ for a dilaton, one obtains from Fig. 1 for a spin-1 antigraviton $\lambda = m^{-1} < 5$ m, and $\lambda < 0.3$ m for a dilaton.

Another sort of Eötvös experiment was carried out in 1971 [26]. In that experiment the difference in accelerations imparted to test bodies by the Sun was measured. The restrictions which follow from [26] are $\lambda_1 < 10^{-47}$ $\lambda_2 < 10^{-20}$, $\lambda_3 < 10^{7}$ [7].

In conclusion, it is useful to note that the best restrictions on λ_n from other Eötvös experiments made before 1979 were collected in the Ref. [19]: $\lambda_1 < 10^{-45}$, $\lambda_2 < 10^{-23}$, $\lambda_3 < 10^{-2}$.

C. Cavendish-type experiments

Originally, the gravitational force between two test bodies (usually between two balls) of known mass has been measured. As a result, the value of the gravitational constant $G \approx 6.66\times 10^{-11}$ Nm²/kg² was calculated [32,33].

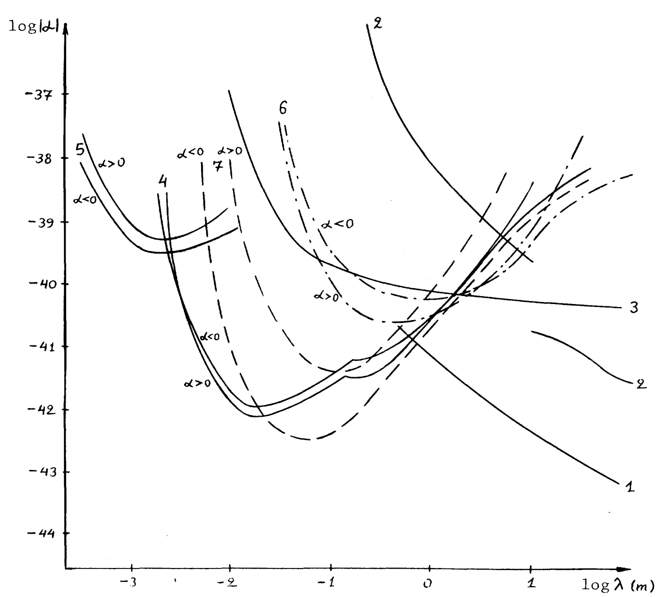

FIG. 1. Permitted regions of Yukawa interaction constants lie under the curves: 1 [25], 2 [17,18] (from Eötvös-type experiments), 3 [32,33], 4 [13], 5 [15], 6 [16], and 7 [14] (from Cavendish-type experiments).

The value of G obtained from the experiments of other types (see, e.g., the geophysical data [10,11]) is equal to the Cavendish laboratory value of G with an accuracy up to 1%. It should be emphasized that the difference between the values of G from [10,11,32,33] are understood here not as the real effect, but as the accuracy limit. Consequently, starting from the fact that the hypothetical force must not lead to a shift in the value of G by more than 1%, one finds restrictions on the hypothetical force parameters. These restrictions were obtained for α,λ in [31]. Permitted values of α,λ belong to the region which is below curve 3 in Fig. 1.

In the last decade, experiments have been carried out on measuring the deviations of the gravity force from Newtonian law [10,11,14,15,18]. It is not necessary to know the value of G (which is known in fact with an accuracy up to 1%) to calculate such deviations because the characteristic value of the deviation includes only the ratios of gravitational forces. Therefore these experiments are more sensitive to the possible existence of an additional hypothetical force in comparison with classical Cavendish experiments. Moreover, interest in such experiments was stimulated by Long's announcement [34] about the experimental discovery of deviations from Newtonian law. Later, experiments with contradictory results were carried out, part of them confirming and part rejecting Long's result. For the time being, however, there are no sufficiently strong reasons to believe that such deviations really exist.

The characteristic value of the deviation may be written as

$$\varepsilon = \frac{1}{rF}\frac{d}{dr}(r^2F) , \tag{4}$$

where r is a distance between point bodies and F is a force acting between them. At present, the value of ε is of the order of $\pm 10^{-4}$ with $r \sim 10^{-2}$–1 m [14,15].

For the parameters of the Yukawa interaction, α,λ from Eq. (2), one then has

$$\varepsilon = \frac{\alpha}{Gm_N^2}\frac{r^2}{\lambda^2}e^{-r/\lambda} . \tag{5}$$

This equation results in restrictions on the values of α,λ. The corresponding permitted regions for a number of experiments [13–16] are shown in Fig. 1 (curves 4–7). The difference between the cases of $\alpha > 0$ and $\alpha < 0$ are explained by the nonzero mean value of ε in the experiments under investigation.

It is easy to obtain from Fig. 1 restrictions on the spin-1 antigraviton $m > 6\times 10^{-5}$ eV ($\lambda < 3\times 10^{-3}$ m) and on the dilaton $m > 5\times 10^{-4}$ eV ($\lambda < 4\times 10^{-4}$ m).

In the same way restrictions on λ_n in Eq. (3) can be obtained from the experimental values of ε defined in Eq. (4).

It gives stronger restrictions than from Eötvös data (see Sec. II B) for $n = 2,3$: $\lambda_2 < 10^{-26}$ and $\lambda_3 < 10^{-12}$.

Today there are new, more exact experiments of Cavendish type. In addition, the measurements were carried out [13] in a more beneficial way to search for restrictions on the power-law interaction region of distances between the test bodies. The restrictions resulting from [13] were found in [35] and proved to be $\lambda_2 < 7\times 10^{-30}$, $\lambda_3 < 7\times 10^{-17}$, $\lambda_4 < 1\times 10^{-3}$.

D. Casimir force measurements

One of the types of experiments for the search for hypothetical long-range interactions is the Casimir force measurement (i.e., the retarded van der Waals force) [36–38]. In these experiments the experimental force value $E_{\rm expt}$ was compared with the theoretical one $F_{\rm theor}$. Within the accuracy of the experimental relative error δ, the difference between $F_{\rm theor}$ and $F_{\rm expt}$ was not found. Therefore a hypothetical force $F_{\rm add}$ must obey the inequality

$$F_{\rm add} < F_{\rm expt}\delta \; . \tag{6}$$

All the restrictions under consideration can be obtained from Eq. (6).

In [36–38] the Casimir force between the plane plate and the spherical lens of R radius ($R \gg l$, where l is a distance between the plate and lens) for $l \in 0.05–2\ \mu$m was measured. An approximative theoretical value of the Casimir force for such a configuration was given in [36,39].

Hypothetical forces for such a configuration due to potentials (2) and (3) were obtained in [21,31,40]. Taking into account that $\delta \approx 10–20\,\%$ [37] for $l \sim 0.8–1\ \mu$m, one obtains restrictions on α,λ, shown in Fig. 2 (as usual, the permitted region lies below the curve).

The restrictions on the parameters of the power-law interactions from (7) can be obtained in the same way [21].

E. Measurements of van der Waals forces between the crossed cylinders and in atomic force microscopy

The majority of experiments on gravitational force measurements were performed, as a rule, for distances between test bodies greater than 10^{-2} m. For smaller distances ($<10^{-6}$ m), the subject of experimental research was the Casimir and van der Waals forces. In this section the results of experiments on force measurements between the crossed cylinders [37] and between a tip of the atomic force microscope (AFM) and a sample are expounded with the aim of searching the Yukawa-type long-range forces on supersmall distances [41].

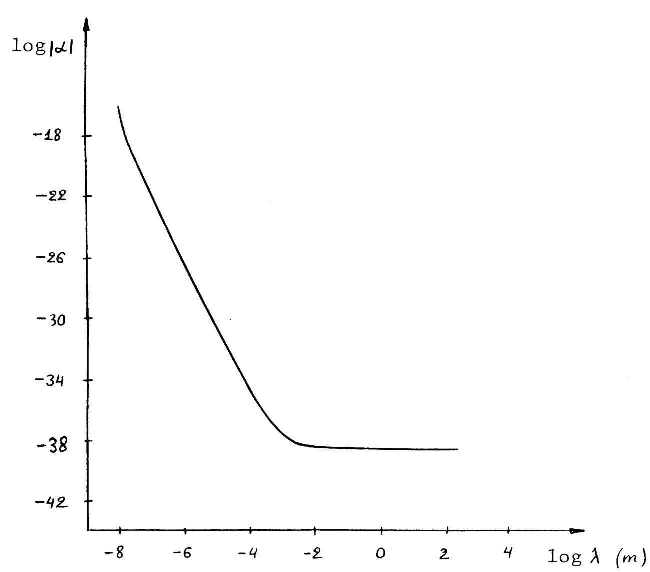

FIG. 2. Restrictions on Yukawa-type interaction constants α,λ from the Casimir effect.

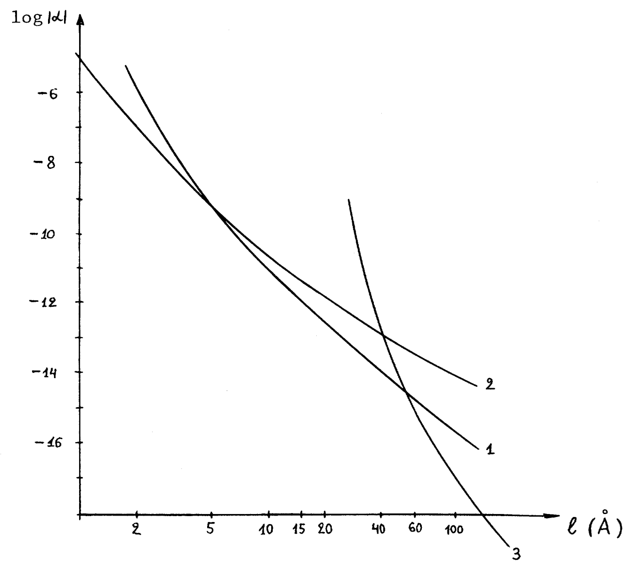

FIG. 3. Restrictions on Yukawa-type interaction constants α,λ from van der Waals force measurements between the crossed cylinders (curve 1 [38]), from AFM (curve 2 [41]), and from the Casimir effect (curve 3 [37]).

In experiments [38] the force between the crossed cylinders with mica plates stuck to them was measured in a distance region from 15 to 1300 Å. The resulting restrictions are shown in the usual way in Fig. 3 (curve 1).

Another type of experiment for the search of Yukawa-type interactions with such small λ is atomic force microscopy [41]. Experimental research on the force dependence $F(l)$ between a plane sample and a tip of the AFM (l is the distance between them) was performed in [42]. The force was measured with a relative error of 70% and the distance with the absolute error ≈ 1 Å.

The corresponding restrictions on parameters α,λ are shown in Fig. 3 (curve 2). In the same figure, the restrictions from the Casimir effect are represented in accordance with Sec. II D for the region of λ under consideration (curve 3).

As is seen from Fig. 3, the best restrictions on α with $\lambda > 60$ Å follow from the Casimir force measurements, with 10 Å $\lesssim \lambda \lesssim$ 60 Å from the van der Waals force measurements [38] and with 1 Å $\lesssim \lambda \lesssim$ 10 Å from the force measurements in the AFM [41].

III. OPTIMIZATION OF EXPERIMENTS WITH TWO MACROBODIES

A. Formulation of the variational problem

As was mentioned in Sec. II, the theoretical values of Casimir or van der Waals forces are confirmed by experiment with an accuracy of experimental error δ. Therefore restrictions on the hypothetical additional force $F_{\rm add}$ were obtained from the inequality (6). Hence, to obtain stronger restrictions on the parameters of the hypothetical force, it is necessary to make the ratio $\gamma \equiv F_{\rm add}/F$ (where F is the known experimentally measured force) as large as possible. Here two test body configurations only will be examined. In principle, a similar problem could be solved for three- and more-body configurations.

It was shown in [21] that an increase in γ may be achieved by modification of the test body configuration, of the distance between them, and of their density. With an increase of the distance between test bodies, it is necessary to take into account both Casimir (van der Waals) and gravitational forces. Therefore, the force F in the general case turns out to be equal to $F_{\rm Cas}+F_{\rm grav}$ and hence the quantity γ must now be defined as

$$\gamma = \frac{F_{\rm add}}{F_{\rm Cas}+F_{\rm grav}} \ . \tag{7}$$

To obtain the strongest restrictions on the constants of hypothetical forces, it is necessary to have in experiment a maximal value of γ from (7); i.e., the variation of γ with respect to the variation of the test body configuration for optimum configuration vanishes: $\delta\gamma = 0$ [43].

For the case of measuring deviations from the known force law, the variational problem is more complicated. As was mentioned in Sec. II C, such deviations may be parametrized by the dimensionless quantity ε from (4), which describes deviations from the inverse square law for point masses. However, in real experiments, test bodies are not pointlike. Hence the law for the gravitational force acting between them is more complicated than the inverse square law. Therefore, in Refs. [13–16,34], an additional value $\Delta\beta$ was introduced which characterizes deviations of the experimental force $F_{\rm expt}$ from the theoretical value $F_{\rm theor}$:

$$\Delta\beta = \frac{(F_2/F_1)_{\rm expt}}{(F_2/F_1)_{\rm theor}} - 1 \ , \tag{8}$$

where $F_{1,2} \equiv F(l_{1,2})$ and $l_{1,2}$ are the distances between the test bodies. The quantity (8) is independent of the value of G, which is known with low accuracy.

The value of $\Delta\beta$ can be connected with the value of some effective ε, which already does not depend directly on the configuration of test bodies.

Before starting with the optimization of experiments on measuring the deviations from the known force law, we must take into account that in the general case both gravitational and Casimir (van der Waals) forces act upon the test bodies. So it is necessary to generalize the definitions of ε in Eq. (4) and of $\Delta\beta$ in Eq. (8).

As shown in [44], such generalizations may be written in the form

$$\frac{d\beta}{dl} = \lim_{\Delta l \to 0} \frac{\Delta\beta}{\Delta l} \ ,$$
$$\varepsilon = \frac{1}{m-2}\,\frac{1}{r^{m-1}F_p(r)}\,\frac{d}{dr}\left[r^{m-1}\frac{d}{dr}[F_p(r)r^2] \right] \ , \tag{9}$$

$$l\frac{d\beta}{dl} = -l\frac{\mathcal{F}_{\rm Cas}(l)}{F(l)}\,\frac{d}{dl} \times \left\{ \left[\frac{d}{dl}\left[\frac{\mathcal{F}_{\rm Cas}(l)}{\mathcal{F}_{\rm grav}(l)} \right] \right]^{-1} \frac{d}{dl}\,\frac{F(l)}{\mathcal{F}_{\rm grav}(l)} \right\} \ , \tag{10}$$

for the force between the test bodies, which consists of three parts:

$$F(l) = G\mathcal{F}_{\rm grav}(l) + C_{\rm Cas}\mathcal{F}_{\rm Cas}(l) + F_{\rm add}(l) \tag{11}$$

(gravitational, molecular, hypothetical). Here also the following notation is used:

$$F_p(r) = G\frac{m_1 m_2}{r^2} + \frac{C_m}{r^m} + F_{p,\rm add}(r) \ , \tag{12}$$

which is the force between two pointlike bodies of the same material such as the test bodies. C_m is a constant of Casimir (van der Waals) force between material points. The value of the integer number m depends on the distances r under consideration. Thus $m=7$ for $r \gtrsim 3$ μm (the so-called temperature Casimir forces) and for $r \lesssim 0.05$ μm (unretarded van der Waals forces), $m=8$ for $0.05\ \mu{\rm m} \lesssim r \lesssim 3\ \mu$m (retarded Casimir forces). The coefficient in (10) is chosen in such a way that the quantity $l\,d\beta/dl$ turns into ε when the test bodies collapse into material points.

Let us show how the restrictions on parameters of hypothetical interactions can be obtained with the help of the quantities in Eqs. (9) and (10). Taking, for example, the quantity (10) and substituting into the force (11) the concrete form of hypothetical interaction, one calculates the expression for $l\,d\beta/dl$ which depends on the long-range constants λ_n or α,λ. On the other hand, assuming that in the experiment there were no deviations of $l\,d\beta/dl$ from zero within the accuracy of experimental error $\delta(l\,d\beta/dl)_{\rm expt}$, one can conclude

$$l\frac{d\beta(\lambda_n \text{ or } \alpha,\lambda)}{dl} < \delta\left[l\frac{d\beta}{dl} \right]_{\rm expt} \ . \tag{13}$$

From this inequality, the restrictions under consideration can be obtained in the same way as the restrictions found from the value of ε [Eq. (9)] (see e.g., Sec. II C for the case of purely gravitational forces). It is obvious that the restrictions obtained in these two ways are equivalent because the values of ε and $l\,d\beta/dl$ are unambiguously connected [44].

Let us now return to the problem of optimization of experiment on measuring deviations from the known force law. To obtain the best restrictions, it is necessary to choose the configuration of test bodies in such a way that the quantity

$$\bar{\varepsilon} = \frac{1}{V_1 V_2}\int_{V_1}\int_{V_2} d^3\mathbf{r}_1 d^3\mathbf{r}_2\,\varepsilon(|\mathbf{r}_1-\mathbf{r}_2|) \tag{14}$$

(or the quantity $l\,d\beta/dl$) would have a maximum value (here $V_{1,2}$ are the volumes of the test bodies). So we have again the variational problem $\delta\bar{\varepsilon}=0$, or $\delta(l\,d\beta/dl)=0$ with respect to variations of the test body configuration, similar to the case of direct force measurements discussed above.

B. Approximative method for the calculations of van der Waals (Casimir) forces

Before going to the solution of the variational problem, we must calculate the forces in Eqs. (7) and (11) for any given configuration. The gravitational force may be found by a simple addition of the forces acting between the atoms of the test bodies. Let us suppose that the same procedure may be used for the determination of hypothetical forces. As was mentioned in Sec. II, such an approach is reasonable because of the smallness of the forces (and also because of conservation of the corresponding charge). An exact calculation of Casimir (or van der Waals) forces for arbitrary test body configurations has been impossible up to now because of the great difficulties connected with the impossibility of separating variables in the wave equation and constructing photon Green's functions in the medium. Exact results are known for the force between two plane-parallel plates [45] and for the energy of a perfectly conducting spherical shell [46] (some other cases are, e.g., a small ball over the plane or two small balls considered in the same way).

In this connection, in Ref. [39], a simple approximative method of calculation was suggested. It is based on the following assumptions.

(1) The potential of, generally speaking, the nonadditive interaction of macrobodies is derived by simply adding the interactions of their atoms.

(2) The nonadditivity is taken into account by decreasing (renormalizing) the potential constant obtained by means of assumption (1) in that proportion in which the exact interaction constant for plane plates (of the same materials as the bodies under consideration) differs from that obtained by simply adding.

It was proved in [39] from a consideration of the extreme configurations for which the exact results also exist that the maximum error of this method for the retarded Casimir forces between arbitrary bodies is not greater than 20%.

The retarded forces dominate for distances between bodies which are greater than several hundreds angstroms but less than several micrometers. For greater distances the usual Casimir forces turn into the temperature ones. For such forces the maximum error of suggested method turns out to be [44] less than 64% in the case of two arbitrary metallic bodies. For two dielectric bodies the maximum error of the method was shown to be [44] about 25%. This accuracy is quite sufficient for the solution of the variational problem where we are looking for the qualitative character of the configuration.

C. Optimal experiments on direct force measurements

Let us find now the solution of the variational equation $\delta\gamma=0$, where γ was defined in Eq. (7). We shall suppose for simplicity that the test bodies under consideration are the rotation figures around the z axis (this supposition allows us to exclude unnecessary turning moments). Assuming also that the test bodies are homogeneous, one can write

$$\gamma=\frac{\int_{V_1}\int_{V_2}d^3\mathbf{r}_1 d^3\mathbf{r}_2 f_{\text{add}}(|\mathbf{r}_1-\mathbf{r}_2|)}{\int_{V_1}\int_{V_2}d^3\mathbf{r}_1 d^3\mathbf{r}_2[f_{\text{Cas}}(|\mathbf{r}_1-\mathbf{r}_2|)+f_{\text{grav}}(|\mathbf{r}_1-\mathbf{r}_2|)]}\ , \tag{15}$$

where f is a projection of a volume force density on the z axis.

Using the standard variational technique, we have, from Eq. (7),

$$\delta F_{\text{add}}(F_{\text{Cas}}+F_{\text{grav}})-F_{\text{add}}(\delta F_{\text{Cas}}+\delta F_{\text{grav}})=0\ . \tag{16}$$

In a cylindrical coordinate system, the projection of every force on the z axis can be written as

$$F=\int_0^{2\pi}d\varphi_1\int_l^{\infty}dz_1\int_0^{\phi_1(z_1)}\rho_1 d\rho_1\int_0^{2\pi}d\varphi_2\int_{-\infty}^{0}dz_2\int_0^{\phi_2(z_2)}\rho_2 d\rho_2 f(|\mathbf{r}_1-\mathbf{r}_2|)\ , \tag{17}$$

where $\phi_{1,2}$ $(z_{1,2})$ are the boundary functions of upper and lower bodies, respectively, and l is a distance between them. A pole is on the top of the lower body.

Let us write Eq. (16) taking into account Eq. (17) and with the variation only of the boundary of the upper body:

$$\begin{aligned}(F_{\text{Cas}}+F_{\text{grav}})&\int_{V_2}d\varphi_2 dz_2 d\rho_2\int_0^{2\pi}d\varphi_1\int_l^{\infty}dz_1\rho_1 f_{\text{add}}(|\mathbf{r}_1-\mathbf{r}_2|)|_{\rho_1=\phi_1(z_1)}\delta\phi_1(z_1)\\ &-F_{\text{add}}\int_{V_2}d\varphi_2 dz_2 d\rho_2\int_0^{2\pi}d\varphi_1\int_l^{\infty}dz_1\rho_1[f_{\text{Cas}}(|\mathbf{r}_1-\mathbf{r}_2|)+f_{\text{grav}}(|\mathbf{r}_1-\mathbf{r}_2|)]|_{\rho_1=\phi_1(z_1)}\delta\phi_1(z_1)=0\ .\end{aligned} \tag{18}$$

Since Eq. (18) is valid for any $\delta\phi_1(z_1)$, it can be reduced to

$$\rho_1\{f_{\text{add}}(|\mathbf{r}_1-\mathbf{r}_2|)-\gamma[f_{\text{Cas}}(|\mathbf{r}_1-\mathbf{r}_2|)+f_{\text{grav}}(|\mathbf{r}_1-\mathbf{r}_2|)]\}|_{\rho_1=\phi_1(z_1)}=0\ . \tag{19}$$

A similar equation may be obtained for the second body:

$$\rho_2\{f_{\text{add}}(|\mathbf{r}_1-\mathbf{r}_2|)-\gamma[f_{\text{Cas}}(|\mathbf{r}_1-\mathbf{r}_2|)+f_{\text{grav}}(|\mathbf{r}_1-\mathbf{r}_2|)]\}|_{\rho_2=\phi_2(z_2)}=0\ . \tag{20}$$

Trivial solutions of Eqs. (19) and (20), $\rho_{1,2}=\phi_{1,2}(z_{1,2})=0$, are not interesting. After reducing Eqs. (19) and (20) on $\rho_{1,2}$, one has

$$\left.\frac{f_{\text{add}}}{f_{\text{Cas}}+f_{\text{grav}}}\right|_{\rho_i=\phi_i(z_i)}=\gamma, \quad i=1,2 \; . \qquad (21)$$

The value of γ is constant for the configuration given (it does not depend on $|\mathbf{r}_1-\mathbf{r}_2|$). At the same time, the left-hand side of Eq. (21) contains the quantity $|\mathbf{r}_1-\mathbf{r}_2|$. Hence the unique solution of Eq. (21) is the configuration for which

$$|\mathbf{r}_1-\mathbf{r}_2|\,|_{\rho_i=\phi_i(z_i)}=\text{const} \; . \qquad (22)$$

In other words, for the optimum configuration every boundary point of the first body is at the same distance from each point of the second body and vice versa; i.e., one of the bodies, e.g., the upper body, is concentrated in a point, while the lower body is part of an infinitely thin spherical shell. In the real experiment, the point will be replaced by a small ball with some radius r and a spherical shell will have thickness ΔR and radius R so that r, $\Delta R \ll R$ (see Fig. 4). Equation (16) is valid also in the case of $F_{\text{Cas}}=F_{\text{grav}}=0$, $\delta F_{\text{Cas}}=\delta F_{\text{grav}}=0$, which can be realized for a small test mass off center inside a spherical shell (for the calculation of such a configuration, see, e.g., [47]).

Finally, to fix the optimum configuration, it is necessary to find a value of R for which the quantity γ would be maximum. It can be done by calculation of γ for the configuration of the ball over the spherical shell and by solving the equation $\partial\gamma/\partial R=0$. For example, in the case of the power-law interaction (3), the result is $R_{\max}\approx 200\ \mu$m.

Unfortunately, the forces acting in such a configuration are too small for experimental registration: $F_{\text{grav}}\approx F_{\text{Cas}}\sim 10^{-20}$ N. In connection with this, the question arises of how to change the optimum configuration to increase the force considerably without a large loss in the value of γ. Such a "semioptimal" configuration turns out to be the configuration of two plane-parallel plates with thickness D placed on a distance l one from the other. For this case the force becomes experimentally observable and the optimal values of D and l may be found by solving the equations

$$\frac{\partial\gamma}{\partial D}=\frac{\partial\gamma}{\partial l}=0 \qquad (23)$$

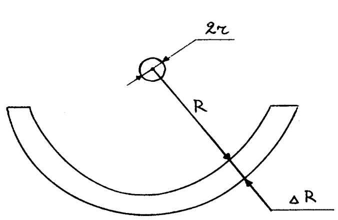

FIG. 4. Optimum configuration of a small ball over a thin spherical shell.

(a direct calculation shows that the value of γ for the semioptimal configuration is less than for the optimum one only in several times).

The solutions of the system (23) for the hypothetical power-law interactions are given in Table I where also the values of the force between the plates are shown.

Let us discuss what strongest restrictions on λ_n can be obtained with the help of an optimal experiment. They are limited first by the error of experiment, δ. Because the total force F_{expt} consists of more than one-half of F_{grav}, which is known up to 1%, now the error δ cannot be less than 1%. Therefore, dividing the numbers from the last column of Table I over 100, we conclude that the strongest up to date restrictions (from Cavendish-type experiments; see Sec. II C) can be improved 30 times with $n=3$ and 2500 times with $n=4$.

In the same way, it is possible to obtain the strongest restrictions for a Yukawa-type interaction of Eq. (2) from the optimal force measurements between two plane-parallel plates. But in contrast with power-law interactions where the additional force contains only one constant, the value of which must be restricted, here there are two parameters α and λ. So if we are looking for restrictions on the first constant, e.g., on α, the optimal parameters of the test bodies (l,D) will depend on the second one, i.e., on λ. Therefore it must be taken into account that starting with some value of λ it is possible to go out of the region of temperature Casimir forces to the region of retarded ones. Calculating the value of γ with a Yukawa-type additional interaction and solving the system (23), we shall obtain the best restrictions on α which depend on the value of λ. Such results together with the optimal values of D,l and corresponding force F_{opt} are represented in Table II. The best known restrictions $\alpha_{\max}^{\text{expt}}$ to date are shown in the third column of Table II in accordance with the results of Sec. II.

As seen from Table II for $\lambda\sim 10^{-2}$ m, new, stronger restrictions on α can be obtained only if $\delta<0.1\%$. However, for $\lambda>10^{-4}$ m, the total force consists mainly of gravitation, which is known nowadays within an error of 1%. That is why it is not realistic to demand a decrease in the error of the force measurements to a value which is less than 1%.

Another situation takes place for the region $10^{-8}\lesssim\lambda\lesssim 10^{-3}$ m. In this region, as is evident from Table II, presently the best known restrictions $\alpha_{\max}^{\text{expt}}$ may be increased by several orders of magnitude if an optimal experiment measuring Casimir forces and gravitation is realized.

TABLE I. Prospective limits on the parameters $\lambda_n^{\max}$ that could be obtained with a configuration of two plane-parallel plates of thicknesses D_{opt} placed at the distance l_{opt} in the experiments on direct force measurements (F_{opt} is net force between plates).

n	l_{opt} (μm)	D_{opt} (μm)	F_{opt} (N/cm^2)	$\lambda_n^{\max}\frac{100\%}{\delta}$
2	120	100	5×10^{-14}	2×10^{-27}
3	90	80	5×10^{-14}	3×10^{-16}
4	80	80	7×10^{-14}	4×10^{-5}

TABLE II. Prospective limits on the parameters of Yukawa-type interactions α, λ that could be obtained in direct force measurements. In addition, the best experimental restrictions on α for different λ are displayed.

λ (m)	$\alpha_{max}\frac{100\%}{\delta}$	α_{max}^{expt}	l_{opt} (μm)	D_{opt} (m)	F_{opt} (N/cm^2)
10^{-2}	7.1×10^{-39}	4×10^{-42}	50–500	10^{-3}	4.2×10^{-12}
10^{-3}	1.8×10^{-38}	3×10^{-38}	50–100	10^{-3}	4.2×10^{-12}
10^{-4}	6.3×10^{-38}	1×10^{-35}	~100	10^{-4}	5.9×10^{-14}
10^{-5}	4.0×10^{-34}	2×10^{-32}	~50	10^{-4}	1.8×10^{-13}
10^{-6}	4.8×10^{-29}	4×10^{-29}	3	10^{-5}–10^{-2}	1.5×10^{-9}
10^{-7}	4.6×10^{-23}	1×10^{-23}	0.4	10^{-6}–10^{-2}	1.5×10^{-5}
10^{-8}	4.6×10^{-17}	2×10^{-17}	0.05	10^{-6}–10^{-2}	2.0×10^{-2}

D. Optimal experiments measuring deviations from the known force law

Similarly to the previous section, it can be shown that the maximum value of $\bar{\varepsilon}$ from Eq. (4) is realized if all points of the first test body are at the same distance r_0 from each point of the second one and vice versa, where r_0 is the maximum point of $\varepsilon(r)$ from Eq. (9) (see Fig. 4).

Because the forces in such configuration usually are very small, it is worthwhile to take instead the "semioptimal" configuration of two plane plates for which, as a direct calculation shows, the forces are considerably larger, but the quantity $\bar{\varepsilon}$ has almost the same value as for the optimum configuration. The distance between the plates is chosen in such a way that the value of $\bar{\varepsilon}$ would be maximal. Therefore the optimal configuration of the test bodies here is the same as in the case of experiments on direct force measurements (see Sec. III).

The solution of the system of equations (23) (which is written now not for γ but for $\bar{\varepsilon}$) in the case of a power-law interaction leads to the best restrictions $\lambda_2 < 5\times10^{-29}$, $\lambda_3 < 1\times10^{-18}$, $\lambda_4 < 4\times10^{-7}$ (here the sensitivity to force variations of the order of $\Delta F \sim 10^{-12}$ N is taken to be 2.5×10^{-14} N [16,48]; the plate area is put equal to 10 cm^2 and the distance between the plates changes from 50 to 500 μm).

As is seen from these results, the restrictions which can be obtained in such a way practically coincide with those from the direct force measurements (see Table I). However, in this case there is a better prospect because, e.g., increasing the plate area up to several 100 cm^2 would lead to a strengthening of the restrictions on λ_n of 10 times.

In Table III the restrictions are collected on the constants of the Yukawa-type interaction which can be obtained from the optimum experiments measuring deviations from the known force law for different parameters of plates.

Concluding this inquiry, let us note that in the case of measuring deviations from the known force law there is an opportunity to use for the strengthening restrictions on Yukawa constants the optimum configuration of Fig. 4 itself. The fact is that preserving the ratio $R \gg \Delta R$ it is possible to increase R, say, up to several meters. Let $\Delta R = R/10$. Hence the force acting in such a system will be a purely gravitational one and large enough for experimental registration. A detailed consideration of such a configuration was carried out in [49]. Results concerning the possibilities of strengthening the restrictions on the Yukawa-type interaction are given in Table IV. Here δF is the sensitivity to the force variation and the absolute error of distance measurements is suggested to be $\Delta R = 10^{-8}$ m.

IV. CONCLUSION

In this paper modern experiments on force measurements were analyzed from the point of view of searching for new hypothetical interactions. Such experiments, in

TABLE III. Prospective limits on the parameters of Yukawa-type interactions α, λ that could be obtained with plane plates (thickness D, distance l, square S) in the experiments on measuring the deviations from the known force law.

λ (m)	$D=10$ cm	$S=1$ m^2	$D=1$ cm	$S=10^{-2}$ m^2
	Interval of l (μm)	α_{max}	Interval of l (μm)	α_{max}
10^{2}	10^5–10^6	4×10^{-47}	10^4–10^5	8×10^{-43}
10^{1}	10^5–10^6	5×10^{-48}	10^4–10^5	1×10^{-43}
10^{0}	10^5–10^6	1×10^{-48}	10^4–10^5	1×10^{-44}
10^{-1}	10^4–10^5	5×10^{-48}	10^4–10^5	4×10^{-45}
10^{-2}	10^3–10^4	2×10^{-46}	10^3–10^4	1×10^{-44}
10^{-3}	10^3–2×10^3	5×10^{-44}	10^3–2×10^3	1×10^{-42}
10^{-4}	10^2–3×10^2	4×10^{-43}	10^2–3×10^2	8×10^{-41}
10^{-5}	50–100	9.4×10^{-38}	50–100	3×10^{-37}
10^{-6}			10–20	2×10^{-30}

TABLE IV. Prospective limits on the parameters of Yukawa-type interactions α, λ that could be obtained with the configuration of a small ball in the center of a thin spherical shell of radius R.

λ (m)	R (m)	F (N)	δF (N)	$\alpha_{\max}$
10^2	5	5.5×10^{-3}	2×10^{-11}	2×10^{-44}
	1	8.8×10^{-6}	2×10^{-13}	4×10^{-42}
10^1	5	5.5×10^{-3}	2×10^{-11}	4×10^{-46}
	1	8.8×10^{-6}	2×10^{-13}	4×10^{-44}
10^0	2	1.4×10^{-4}	1×10^{-12}	4×10^{-46}
10^{-1}	0.2	1.4×10^{-8}	2×10^{-15}	6×10^{-45}
10^{-2}	0.02	1.4×10^{-12}	10^{-15}	3×10^{-41}

contrast with astrophysical observations and the consideration of new decay channels of elementary particles, allow one to obtain more model-independent restrictions on the constants of long-range interactions from Eqs. (2) and (3).

In Sec. II experiments of Eötvös, Galileo, and Cavendish types [10–18,27,28] were considered, and the verification of the Casimir effect [36,37], and the measurements of van der Waals forces between crossed cylinders [38] and the force measurements in atomic force microscopy [41] were discussed.

The best restrictions on the constants λ_n of power-law interactions [Eq. (3)] from different experiments on force measurements are assembled in Table V.

The prospects of strengthening the restrictions on λ_n are shown in the fifth column of Table V. For the cases of $n=3$ and 4, they are practically equal for experiments of Casimir and Cavendish types. The absence of reasonable prospects with $n=1$ and 2 is connected with the fact that all presently available experimental facilities have been already used in experiments in which the actual results of Table V were obtained.

In the present paper restrictions on parameters of Yukawa long-range interactions [Eq. (2)] are also reviewed. The best restrictions to date are shown in Fig. 5. The region of α, λ permitted by all experiments performed up to now lies below curve 1. The best restrictions on α with $\lambda > 1$ m follow from experiments of Eötvös type [18], with 3×10^{-4} m $\lesssim \lambda \lesssim 1$ m from Cavendish-type experiments [13–15], with 10^{-8} m $\lesssim \lambda \lesssim 3\times10^{-4}$ m from the Casimir effect [36–37], and with 10^{-10} m $\lesssim \lambda \lesssim 10^{-8}$ m from measurement of the nonretarded van der Waals forces [38,41].

In Secs. II B–II D the experiments on Casimir force measurements and Cavendish- and Eötvös-type experiments were analyzed for obtaining restrictions on masses of light hypothetical particles—the dilaton and spin-1 antigraviton—which arise in modern gauge theories of gravity [4–7]. It was shown that nowadays the accuracy of Casimir force measurements is insufficient to obtain reliable restrictions on the masses of these particles, although experiments of Eötvös and Cavendish types are able to do that. As a result, the restrictions found are $m > 6\times10^{-5}$ eV for the spin-1 antigraviton and $m > 5\times10^{-4}$ eV for the dilaton.

In Sec. III the variational problem of the optimization of test body configurations for obtaining stronger restrictions on hypothetical long-range interactions was discussed. The restrictions on α, λ, which can be obtained in future experiments described in Sec. III, are shown in Fig. 5. Curve 2 corresponds to prospective restrictions from the experiments on measurements of the Casimir force and gravitation. The restrictions from the Cavendish-type experiments with plane plates as test bodies are represented by curve 3 (the thickness of the plates is $D=1$ cm, and their area $S=10^{-2}$ m^2) and by curve 4 ($D=10$ cm, $S=1$ m^2) drawn in accordance with the results from Table III. Curves 5 and 6 restrict the re-

TABLE V. Modern and prospective limits on power-law long-range interactions.

		$\lambda_n^{\max}$		
n	Casimir	Eötvös	Cavendish	Prospective
1	1×10^{-40}	10^{-47}		no better
2	1×10^{-27}	10^{-23}	7×10^{-30}	no better
3	5×10^{-15}	10^{-2}	7×10^{-17}	1×10^{-18}
4	3×10^{-3}	10^{18}	1×10^{-3}	4×10^{-7}

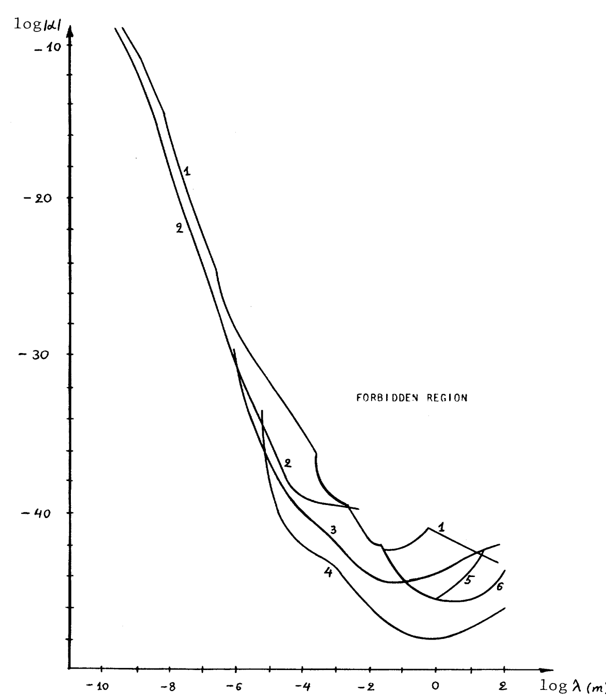

FIG. 5. Permitted regions of Yukawa interaction constants lie under the curves: 1, the best restrictions to date from different experiments; 2, from prospective optimum experiments of Casimir type; 3,4, from prospective Cavendish-type experiments with plane plates [thickness $D=1$ cm, area $S=10^{-2}$ m^2 (curve 3) and $D=10$ cm, $S=1$ m^2 (curve 4)]; 5, 6, from prospective experiments with the configuration of Fig. 4 [$R=1$ m (curve 5) and $R=5$ m (curve 6)].

gion of permitted α, λ from the optimum Cavendish-type experiment with the ball over the spherical shell (Fig. 4).

From Fig. 5 it is seen that for 10^{-8} m $< \lambda < 10^{-3}-10^{-2}$ m the restrictions on α can be improved with the help of suggested experiments on direct force measurements at $10-10^4$ times for different values of λ. Starting from $\lambda > 10^{-6}-10^{-5}$ m, the restrictions on α can be improved with the help of the Cavendish-type experiments suggested in this paper. Moreover, in this region it is possible to strengthen the restrictions known to date by a factor of the order of tens of millions for some values of λ.

In conclusion, let us mention that the strengthening of restrictions on α, λ with the help of the suggested experiments will lead to an improvement of the limits on the masses of the spin-1 antigraviton, dilaton, and other light elementary particles. As seen, from Fig. 5, the restrictions $m > 10^{-2}$ eV ($\lambda < 2\times10^{-5}$ m) for the spin-1 antigraviton and $m > 2\times10^{-2}$ eV ($\lambda < 1\times10^{-5}$ m) for the dilaton would be found if the suggested experiments were done.

Thus compact and relativity inexpensive laboratory experiments on force measurements between macrobodies can form a new direction alternative to the acceleration technique for obtaining important new information about elementary particles and their interactions.

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[1] H. P. Nilles and S. Raby, Nucl. Phys. **B198**, 102 (1982).
[2] K. Tamvakis and D. Wyler, Phys. Lett. **112B**, 451 (1982).
[3] S. Deser and B. Zumino, Phys. Rev. Lett. **38**, 1433 (1977).
[4] J. Scherk, Phys. Lett. **88B**, 265 (1979).
[5] C. K. Zachos, Phys. Lett. **76B**, 329 (1978).
[6] J. O'Hanlon, Phys. Rev. Lett. **29**, 137 (1972).
[7] Y. Fujii, Ann. Phys. (N.Y.) **69**, 494 (1972).
[8] E. G. Adelberger *et al.*, Annu. Rev. Nucl. Sci. **41**, 269 (1991).
[9] C. Talmage *et al.*, Phys. Rev. Lett. **61**, 1159 (1988).
[10] S. C. Holding, F. D. Stacey, and G. J. Tuck, Phys. Rev. D **33**, 3487 (1986).
[11] F. D. Stacey *et al.*, Rev. Mod. Phys. **59**, 157 (1987).
[12] E. Fishbach *et al.*, Phys. Rev. Lett. **56**, 3 (1986).
[13] J. K. Hoskins *et al.*, Phys. Rev. D **32**, 3084 (1985).
[14] Y. T. Chen, A. H. Cook, and A. J. F. Metherell, Proc. R. Soc. London **A394**, 47 (1984).
[15] V. P. Mitrofanov and O. I. Ponomareva, Zh. Eksp. Teor. Fiz. **94**, 16 (1988) [Sov. Phys. JETP **67**, 1963 (1988)].
[16] V. I. Panov and V. N. Frontov, Zh. Eksp. Teor. Fiz. **77**, 1701 (1979) [Sov. Phys. JETP **50**, 852 (1979)].
[17] C. W. Stubbs *et al.*, Phys. Rev. Lett. **58**, 1070 (1987).
[18] C. W. Stubbs *et al.*, Phys. Rev. Lett. **62**, 609 (1989).
[19] G. Feinberg and J. Sucher, Phys. Rev. D **20**, 1717 (1979).
[20] J. Kim, Phys. Rep. **150**, 1 (1987).
[21] V. M. Mostepanenko and I. Yu. Sokolov, Phys. Lett. A **125**, 405 (1987).
[22] A. A. Anselm, N. G. Uraltzev, and V. A. Hoze, Usp. Fiz. Nauk. **145**, 185 (1985) [Sov. Phys. Usp. **28**, 113 (1985)].
[23] E. E. Radescu, JINR Report No. NP-84-497, 1984 (unpublished).
[24] G. Feinberg, Commun. Mod. Phys. A **19**, 51 (1989).
[25] B. R. Heckel *et al.*, Phys. Rev. Lett. **63**, 2705 (1989).
[26] V. B. Braginskii and V. I. Panov, Zh. Eksp. Teor. Fiz. **61**, 873 (1971) [Sov. Phys. JETP **34**, 463 (1972)].
[27] T. M. Niebauer, M. P. McHugh, and J. B. Faller, Phys. Rev. Lett. **59**, 2705 (1989).
[28] V. Cavasinni *et al.*, Phys. Lett. A **116**, 157 (1986).
[29] K. Kuroda and N. Mio, Phys. Rev. Lett. **62**, 1941 (1989).
[30] K. Kuroda and N. Mio, Phys. Rev. D **42**, 3903 (1990).
[31] V. M. Mostepanenko and I. Yu. Sokolov, Phys. Lett. A **132**, 313 (1988).
[32] G. T. Gillies, Metrologia Suppl. **24**, 1 (1987).
[33] G. Müller, W. Zurn, K. Linder, and N. Rosch, Phys. Rev. Lett. **63**, 2621 (1989).
[34] D. R. Long, Phys. Rev. D **9**, 850 (1974).
[35] V. M. Mostepanenko and I. Yu. Sokolov, Phys. Lett. A **146**, 373 (1990).
[36] B. V. Derjaguin, I. I. Abrikosova, and E. M. Lifshitz, Q. Rev. **10**, 295 (1968).
[37] S. Hunklinger, H. Beisselman, and W. Arnold, Rev. Sci. Instrum. **43**, 584 (1972).
[38] Y. N. Israelachvili and D. Tabor, Proc. R. Soc. London **A331**, 19 (1988).
[39] V. M. Mostepanenko and I. Yu. Sokolov, Dokl. Akad. Nauk SSSR **298**, 1380 (1988) [Sov. Phys. Dokl. **33**, 140 (1988)].
[40] V. A. Kuzmin, I. I. Tkachev, and M. E. Shaposhnikov, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 49 (1982) [JETP Lett. **36**, 59 (1982)].
[41] V. I. Panov, Yu. N. Moiseev, V. M. Mostepanenko, and I. Yu. Sokolov, in *Fifth Marcel Grossmann Meeting,* Proceedings, Perth, Australia, edited by D. G. Blair and M. J. Buckingham (World Scientific, Singapore, 1989), Pt. B.
[42] Yu. N. Moiseev, V. M. Mostepanenko, V. I. Panov, and I. Yu. Sokolov, Phys. Lett. A **132**, 354 (1988).
[43] V. M. Mostepanenko and I. Yu. Sokolov, in *Modern Progress of Theoretical Physics,* edited by P. I. Pronin and Yu. N. Obukhov (World Scientific, Singapore, 1991).
[44] V. M. Mostepanenko and I. Yu. Sokolov, in *Quantum Gravity,* Proceedings of the Fifth Seminar, Moscow, Russia, edited by M. A. Markov, V. A. Berezin, and V. P. Frolov (World Scientific, Singapore, 1991).
[45] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **29**, 94 (1955) [Sov. Phys. JETP **2**, 73 (1956)].
[46] B. Davies, J. Math. Phys. **13**, 1324 (1972).
[47] N. I. Kolosnitzin, Izmer. Tekh. (to be published).
[48] G. Binning, C. F. Quate, and Ch. Gerber, Phys. Rev. Lett. **56**, 930 (1986).
[49] V. M. Mostepanenko and I. Yu. Sokolov, Found. Phys. Lett. (to be published).