Weak electromagnetic decays of charm baryons

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In view of recent experimental trends we investigate the weak photonic decays of charmed baryons within the framework of the constituent quark model. Decay widths and asymmetry parameters for all $\frac{1}{2}^+ \to \frac{1}{2}^+ + \gamma$ charm-changing modes are calculated with appropriate QCD corrections.

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I. INTRODUCTION

The discovery of the charm particle began a new era in particle physics. Ever since, charm hadrons have been under an active probe, but data constraints directed most of the theoretical efforts to the understanding of the weak decays of charm mesons. The advent of B factories and a change in experimental trends have now brought these charmed baryons under active investigation [1–3], with results encouraging enough to warrant a detailed theoretical analysis. Moreover, the large event samples of B-meson decays will allow for an accurate and extensive study of all charm baryonic decays in the near future. A detailed knowledge of these decay properties is essential not only for understanding the charm sector, but also because it will form the core to the quality of information that can be extracted from $b \to c$ physics.

Charmed hadrons can decay into numerous channels, yet the data on the exclusive modes is very limited. Generally, the spectator diagram is considered to be the significant decay mechanism for these modes. However, considerable speculation exists over other contributing processes such as the W-exchange mechanism [4,5]. Unlike the meson weak decays, this W-exchange mechanism in baryons is neither helicity nor color suppressed since there may exist a spin-0 two-quark system inside the baryon. In fact, the contribution from this process has been found to be proportional to $|\psi(0)|^2$, thereby making it more significant for heavy-baryon weak decays [6]. Experimentally, the lifetime differences among D^0, D^+, Λ_c^+, Ξ_c^+, and Ξ_c^0 are also indicative of the presence of the W-exchange mechanism. The signal [7] for $\Lambda_c^+ \to \Delta^{++} K^-$ and recent measurements by CLEO [1,2] on exclusive modes such as $\Xi_c^0 \to \Omega^- K^+$ and $\Lambda_c^+ \to \Xi^0 K^+$, which can occur most likely via a W-exchange diagram, lend credence to this interpretation.

Previous theoretical attempts to study charm baryons are mostly limited to the weak mesonic modes [8]. Strong-interaction interference effects between different processes, which are prevalent among these modes, cast a shadow on the exact contribution of each process. Final-state interactions (FSI's) among hadrons further complicate the situation. The $B \to B'\gamma$ weak radiative modes can, however, provide a direct estimate for the W-exchange process. First, the charm-changing $\Delta C = \pm \Delta S$ weak radiative modes decay only through a two-quark W-exchange process (2QP). Besides, because of photon emission in the final state, these decays are free from such strong-interaction FSI effects.

In this paper we analyze the weak radiative decays ($\frac{1}{2}^+ \to \frac{1}{2}^+ + \gamma$) of $C=1$ and 2 charm baryons in the Cabibbo-enhanced, -suppressed and -doubly-suppressed modes. Section II deals with the construction of the weak W-exchange Hamiltonian and the calculation of decay amplitudes. In Sec. III we estimate the decay widths and asymmetry parameters for all these charm-changing weak electromagnetic decays including the effect of flavor dependence on the scale. Finally, in Sec. IV we discuss the effects of quantum chromodynamical (QCD) modifications to these decays.

II. W-EXCHANGE WEAK RADIATIVE HAMILTONIAN

The components of the weak Hamiltonian for the charm-changing decays can be classified as follows: Cabibbo enhanced,

$$\Delta C = \Delta S = -1;\ H_{24}^{13} \propto \cos^2\theta_c \ , \tag{1a}$$

Cabibbo suppressed,

$$\Delta C = -1 \ , \ \Delta S = 0;\ H_{34}^{13} - H_{24}^{12} \propto \cos\theta_c \sin\theta_c \ , \tag{1b}$$

and Cabibbo doubly suppressed,

$$ELTAC = -\Delta S = -1;\ H_{34}^{12} \propto -\sin^2\theta_c \ . \tag{1c}$$

In contrast with the strangeness-changing hyperon radiative decays, where, though the W-exchange mechanism is found to be dominant [9–12], single-quark processes (1QP's) can also contribute, for the charm-changing $\Delta C = \pm \Delta S$ modes, the decay occurs only through a 2Q W-exchange process. The mode $\Delta C = -1$, $\Delta S = 0$ may, in addition, have some contribution from 1QP's, but it is expected to be highly suppressed.

We employ the constituent quark model to construct the Hamiltonian for the weak bremsstrahlung process

$$q_1(p_1) + q_2(p_2) \to q_3(p_3) + q_4(p_4) + \gamma(k) \ . \tag{2}$$

Following the procedure as discussed in [9–11] for quark momentum integration (up to first order in $\mathbf{p}_i$) and summing over the four photon-emission permutations, we get the following effective weak Hamiltonian for the 2Q parity-conserving (pc) and parity-violating (pv) process:

$$H_{\text{eff}}^{\text{pc}}=\frac{eG_F(\text{Cabibbo factors})}{\sqrt{2}}|\mathbf{k}|[Aq_3^\dagger i\sigma\cdot\varepsilon\times\hat{\mathbf{k}}q_1\, q_4^\dagger q_2+Bq_3^\dagger q_1\, q_4^\dagger i\sigma\cdot\varepsilon\times\hat{\mathbf{k}}q_2$$
$$+Cq_4^\dagger\sigma\cdot\varepsilon q_1\, q_4^\dagger\sigma\cdot\hat{\mathbf{k}}q_2+Dq_3^\dagger\sigma\cdot\hat{\mathbf{k}}q_1\, q_4^\dagger\sigma\cdot\varepsilon q_2]\ , \tag{3a}$$

$$H_{\text{eff}}^{\text{pv}}=\frac{eG_F(\text{Cabibbo factors})}{\sqrt{2}}|\mathbf{k}|[Cq_3^\dagger\sigma\cdot\varepsilon q_1\, q_4^\dagger q_2+Dq_3^\dagger q_1\, q_4^\dagger\sigma\cdot\varepsilon q_2+(A+H(k))q_3^\dagger i\sigma\cdot\varepsilon\times\hat{\mathbf{k}}q_1\, q_4^\dagger\sigma\cdot\hat{\mathbf{k}}q_2$$
$$+(B-H(k))q_3^\dagger\sigma\cdot\hat{\mathbf{k}}q_1\, q_4^\dagger i\sigma\cdot\varepsilon\times\hat{\mathbf{k}}q_2+i\varepsilon\cdot q_3^\dagger\sigma q_1\times q_4^\dagger\sigma q_2 H(k)]\ , \tag{3b}$$

where q_i indicate the Pauli spinors of the relevant flavors, with external quarks assumed to be on the mass shell.

Coefficients of the quark operators are

$$\begin{aligned}A&=-H(k)+(Q_-G(k)+P_+H(k))\ ,\\ B&=+H(k)-(P_-G(k)+Q_+H(k))\ ,\\ C&=+G(k)-(P_+G(k)+Q_-H(k))\ ,\\ D&=-G(k)+(Q_+G(k)+P_-H(k))\ ,\end{aligned} \tag{4}$$

where $G(k)$ and $H(k)$ are propagator factors defined as

$$\begin{aligned}G(k)&=\frac{1}{2}\left[\frac{e_3}{p_3\cdot k}+\frac{e_1}{p_1\cdot k}-\frac{e_4}{p_4\cdot k}-\frac{e_2}{p_2\cdot k}\right]\ ,\\ H(k)&=\frac{1}{2}\left[\frac{e_3}{p_3\cdot k}-\frac{e_1}{p_1\cdot k}-\frac{e_4}{p_4\cdot k}+\frac{e_2}{p_2\cdot k}\right]\ .\end{aligned} \tag{5}$$

$p_i\cdot k=p_\mu k^\mu$, and e_i are quark charges in electron units.

The terms with coefficients

$$P_\pm=\frac{k^0}{24}\left[\frac{-5}{m_4}\pm\frac{1}{m_2}\right]\ \text{and}\ Q_\pm=\frac{k^0}{24}\left[\frac{-5}{m_3}\pm\frac{1}{m_1}\right]\ , \tag{6}$$

where m_i are the constituent quark masses with $m_u=m_d=0.336$ GeV, $m_s=0.54$ GeV, and $m_c=1.5$ GeV, arise through the quark momenta (p_i) integration. If these terms are neglected, the Hamiltonian reduces to the form obtained by Kamal [13] for the Cabibbo-enhanced mode.

The propagator factors in Eq. (5) are approximated by using

$$\frac{1}{p_i\cdot k}=\frac{1}{E_ik-\mathbf{p}_i\cdot\mathbf{k}}\approx\frac{1}{\bar{E}_ik}\ . \tag{7}$$

Since the photon momentum is large in these decays, the use of the naive nonrelativistic reduction of $G(k)$ and $H(k)$, through a p/m expansion, may be incorrect. We therefore follow the improvised quark model [13,14] and replace the quark energy E_i by its average value, i.e.,

$$\bar{E}_i=(m_i^2+\langle\mathbf{p}_i^2\rangle)^{1/2}\ . \tag{8}$$

This then allows an expansion in the parameter p/E. Physically, one would expect this to be a better approximation than a p/m expansion, as the average energy $\bar{E}_i$ of the quark has a dependence on the momentum of the final baryon, as well as on the cutoff α provided by the wave functions. Using the harmonic-oscillator (HO) wave functions for baryons, we evaluate $\langle\mathbf{p}_i^2\rangle$ as shown in Refs. [13,14] and obtain

$$\begin{aligned}\bar{E}_{1,2}&=\left[m_{1,2}^2+\frac{7}{4}\alpha^2+\frac{k^2}{144}\right]^{1/2}\ ,\\ \bar{E}_{3,4}&=\left[m_{3,4}^2+\frac{7}{4}\alpha^2+\frac{25k^2}{144}\right]^{1/2}\ ,\end{aligned} \tag{9}$$

where α^2 is the HO parameter.

In addition, one may expect a contribution from the internal radiative process where the photon is emitted by the W boson. This, however, has been found to be suppressed by a factor $m_uk/m_W^2\approx10^{-5}$ as compared to the bremsstrahlung process [15].

A. Decay amplitudes

Among the $J^P=\frac{1}{2}^+$ charmed baryons comprising the **20** multiplet of SU(4), only members of the SU(3) submultiplets **3***, **3**, and Ω_c^0 of **6** decay weakly. The remaining decay strongly or radiatively to **3***. Masses of most of the $C=1$ baryons have been experimentally measured [16], and the remaining are taken from a theoretical estimate based on a central two-body potential supplemented by a spin-spin interaction resulting from the Briet-Fermi reduction of a one-gluon-exchange contribution (Table I). We illustrate the evaluation of the decay amplitudes at the baryonic level for $B(\mathbf{3}^*)\to B'(\mathbf{8})+\gamma$ in the Cabibbo-enhanced mode. Using Eq. (3) and the quark-model wave functions [17], we derive the pc and pv amplitudes for the decays of $\Lambda_c^+\to\Sigma^++\gamma$ and $\Xi_c^0\to\Xi^0+\gamma$. Up to an overall scale factor $(eG_F/\sqrt{2})\cos^2\theta_c$, these are

TABLE I. Masses of charmed baryons [**20** of SU(4)].

SU(3)	Particle	Mass	Lifetime (10^{-12} sec) [5,8]
3*	Λ_c^+	2285.0±0.6	0.196±0.016
	Ξ_c^+	2466.2±2.2	0.57±0.14
	Ξ_c^0	2472.8±1.7	0.082±0.06
6	Σ_c	2453	
	Ξ_c'	2561	
	Ω_c^0	2740	0.79±0.34
3	Ξ_{cc}	3616	
	Ω_{cc}	3706	

$$\langle \Sigma^+\gamma | H_w^{pc} | \Lambda_c^+ \rangle = \frac{2}{\sqrt{6}}[H(k) - 2G(k) - X\{(2+3\zeta_c - 30\zeta_s)G(k) + (8-6\zeta_c)H(k)\}] , \tag{10a}$$

$$\langle \Sigma^+\gamma | H_w^{pv} | \Lambda_c^+ \rangle = \frac{2}{\sqrt{6}}[2H(k) - G(k) + X\{(2+3\zeta_c - 30\zeta_s)H(k) + (8-6\zeta_c)G(k)\}] , \tag{10b}$$

$$\langle \Xi^0\gamma | H_w^{pc} | \Xi_c^0 \rangle = -\frac{2}{\sqrt{6}}[H(k) - 2G(k) - X\{(2+3\zeta_c - 30\zeta_s)G(k) + (8-6\zeta_c)H(k)\}] , \tag{11a}$$

$$\langle \Xi^0\gamma | H_w^{pv} | \Xi_c^0 \rangle = -\frac{2}{\sqrt{6}}[2H(k) - G(k) + X\{(2+3\zeta_c - 30\zeta_s)H(k) + (8-6\zeta_c)G(k)\}] , \tag{11b}$$

where $X = k^0/24m_u$ and the flavor symmetry-breaking parameters are denoted by $6\zeta_s = 1 - m_u/m_s$ and $3\zeta_c = 1 - m_u/m_c$. In the same manner, we calculate the decay amplitudes for the other weak decays, which are shown in Table II.

III. DETERMINATION OF DECAY ASYMMETRIES AND BRANCHING RATIOS

A. Decay rate and asymmetry methodology

The gauge-invariant form of the radiative weak decay amplitude $B \to B' + \gamma$ is written as

$$M = \frac{eG_F}{\sqrt{2}} \bar{B}'(F_1 + F_2\gamma_5) k\!\!\!/ \epsilon\!\!/ B , \tag{12}$$

where k^μ and ε^μ are the momentum and polarization vectors of the emitted photon, B and B' are the Dirac spinors for the initial- and final-state baryons, and F_1 and F_2 are the pc and pv decay amplitudes. The decay rates are then given by

$$\Gamma(B \to B'\gamma) = \frac{e^2 G_F^2}{2\pi}[|F_1|^2 + |F_2|^2]k^3 , \tag{13}$$

and the asymmetry is

$$\alpha(B \to B'\gamma) = \frac{2\,\mathrm{Re}(F_1 F_2^*)}{|F_1|^2 + |F_2|^2} . \tag{14}$$

The baryon decay amplitudes, expressed in terms of F_1 and F_2, are extracted in the spirit of our earlier works [9–11]. Using nonrelativistic reduction, Eq. (12) takes the form

$$M = \frac{eG_F}{\sqrt{2}} B_f^\dagger (iF_1 \boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\times\mathbf{k} + F_2 k \boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}) B_i , \tag{15}$$

where B_f and B_i are now Pauli spinors. Considering the two helicity states of the photon, one gets

$$M(\lambda_\gamma = \pm 1) = \frac{eG_F}{\sqrt{2}} k(\pm F_1 - F_2) B_f^\dagger \sigma_\mp B_i . \tag{16}$$

Thus

$$A^{pc(pv)} = \langle \gamma(\lambda_\gamma = +1), B_f \downarrow | H_w^{pc(pv)} | B_i \uparrow \rangle \propto kF_1(-kF_2) . \tag{17}$$

The decay rate then can be expressed using the harmonic-oscillator wave functions for baryons [9,13] as

$$\Gamma(B \to B'\gamma) = \frac{e^2 G_F^2 (\text{Cabibbo factors})}{2\pi} \times k[|A^{pc}|^2 + |A^{pv}|^2](I_{2q})^2 \exp\left[\frac{-k^2}{12\alpha^2}\right] . \tag{18}$$

Ensuring necessary kinetic matching between the constituents and baryons introduces a scale factor I_{2q} (dimension GeV^3) for the 2QP's and is given by [9,13]

$$I_{2q} = \delta\left[\sum \mathbf{p}_i - \sum \mathbf{p}_f - \mathbf{k}\right](2\pi^{1/2}\alpha)^3 . \tag{19}$$

This scale factor I_{2q}, which corresponds to the spatial matrix element $|\psi(0)|^2 = \langle \psi | \delta(r_1 - r_2) | \psi \rangle$, is as yet uncertain for baryons. Its evaluation is complicated because, unlike the mesons, these are three-body systems. Furthermore, the harmonic-oscillator potential gives good results only for peripheral processes, but is not realistic for a central quantity such as $|\psi(0)|^2$. A relative scale can be estimated using hyperon radiative decays.

For the pure 2QP's, the ratios

$$\frac{\Gamma(\Lambda_c^+ \to \Sigma^+\gamma)}{\Gamma(\Sigma^+ \to p\gamma)} = \frac{\cos^2\theta_c}{\sin^2\theta_c} \frac{k_{\Lambda_c}}{k_{\Sigma^+}} \frac{[|A_{\Lambda_c}^{pc}|^2 + |A_{\Lambda_c}^{pv}|^2]}{[|A_{\Sigma^+}^{pc}|^2 + |A_{\Sigma^+}^{pv}|^2]} \exp\left[-\frac{k_{\Lambda_c}^2 - k_{\Sigma^+}^2}{12\alpha^2}\right] \approx 14.9 ,$$

$$\frac{\Gamma(\Xi_c^0 \to \Xi^0\gamma)}{\Gamma(\Sigma^+ \to p\gamma)} = \frac{\cos^2\theta_c}{\sin^2\theta_c} \frac{k_{\Xi_c}}{k_{\Sigma^+}} \frac{[|A_{\Xi_c}^{pc}|^2 + |A_{\Xi_c}^{pv}|^2]}{[|A_{\Sigma^+}^{pc}|^2 + |A_{\Sigma^+}^{pv}|^2]} \exp\left[-\frac{k_{\Xi_c}^2 - k_{\Sigma^+}^2}{12\alpha^2}\right] \approx 15.1 , \tag{20}$$

TABLE II. Two-quark decay amplitude with scale $[eG_F(\text{Cabibbo factors})/\sqrt{2}]I_{2q}(k)$. Here $\zeta_s=\frac{1}{6}(1-m_u/m_s)$, $\zeta_c=\frac{1}{3}(1-m_u/m_c)$, and $X=k^0/24m_u$.

Decay	A^{pc}	A^{pv}
	$\Delta C=\Delta S=-1\ \ (c+d\to s+u+\gamma)$	
$\Lambda_c^+\to\Sigma^++\gamma$	$\frac{2}{\sqrt{6}}[H(k)-2G(k)-X\{(2+3\zeta_c-30\zeta_s)G(k)+(8-6\zeta_c)H(k)\}]$	$\frac{2}{\sqrt{6}}[2H(k)-G(k)+X\{(2+3\zeta_c-30\zeta_s)H(k)+(8-6\zeta_c)G(k)\}]$
$\Xi_c^0\to\Xi^0+\gamma$	$-\frac{2}{\sqrt{6}}[H(k)-2G(k)-X\{(2+3\zeta_c-30\zeta_s)G(k)+(8-6\zeta_c)H(k)\}]$	$-\frac{2}{\sqrt{6}}[2H(k)-G(k)+X\{(2+3\zeta_c-30\zeta_s)H(k)+(8-6\zeta_c)G(k)\}]$
$\Xi_{cc}^+\to\Xi_c^++\gamma$	$\frac{2}{\sqrt{6}}[(G(k)-H(k))\{1-X(2-3\zeta_c)\}]$	$\frac{2}{\sqrt{6}}[(G(k)-H(k))\{1-X(2-3\zeta_c)\}]$
$\Xi_{cc}^+\to\Xi_c^++\gamma$	$-\sqrt{2}[(H(k)+G(k))\{1+X(10-\zeta_c-20\zeta_s)\}-20X\zeta_sG(k)]$	$\sqrt{2}[(H(k)+G(k))\{1+X(10-\zeta_c-20\zeta_s)\}-20X\zeta_sH(k)]$
	$\Delta C=-1,\ \ \Delta S=0\ \ (c+d\to d+u+\gamma)$	
$\Lambda_c^+\to p+\gamma$	$\frac{2}{\sqrt{6}}[H(k)-2G(k)-X\{(2+3\zeta_c)G(k)+8H(k)\}]$	$\frac{2}{\sqrt{6}}[2H(k)-G(k)+X\{(2+3\zeta_c)H(k)+8G(k)\}]$
$\Xi_c^0\to\Sigma^0+\gamma$	$-\frac{1}{\sqrt{3}}[(H(k)+G(k))(1+10X)+6\zeta_cXG(k)]$	$\frac{1}{\sqrt{3}}[(H(k)+G(k))(1+10X)+6\zeta_cXH(k)]$
$\Xi_c^0\to\Lambda+\gamma$	$[(G(k)-H(k))(1-2X)]$	$[(G(k)-H(k))(1-2X)]$
$\Xi_{cc}^+\to\Lambda_c^++\gamma$	$\frac{2}{\sqrt{6}}[(G(k)-H(k))(1-2X)]$	$\frac{2}{\sqrt{6}}[(G(k)-H(k))(1-2X)]$
$\Xi_{cc}^+\to\Sigma_c^++\gamma$	$-\sqrt{2}[(G(k)+H(k))\{1+X(10+2\zeta_c)\}+2X\zeta_cG(k)]$	$\sqrt{2}[(G(k)+H(k))\{1+X(10+2\zeta_c)\}-2X\zeta_cH(k)]$
$\Xi_{cc}^{++}\to\Sigma_c^{++}+\gamma$	0	0
	$\Delta C=-1,\ \ \Delta S=0\ \ (c+s\to s+u+\gamma)$	
$\Xi_c^+-\Sigma^++\gamma$	$\frac{2}{\sqrt{6}}[H(k)-2G(k)-X\{(2+3\zeta_c-18\zeta_s)G(k)+(8-6\zeta_c-6\zeta_s)H(k)\}]$	$\frac{2}{\sqrt{6}}[2H(k)-G(k)+X\{(2+3\zeta_c-18\zeta_s)H(k)+(8-6\zeta_c-6\zeta_s)G(k)\}$
$\Xi_c^0\to\Sigma^0+\gamma$	$\frac{1}{\sqrt{3}}[H(k)-2G(k)-X\{(2+3\zeta_c-18\zeta_s)G(k)+(8-6\zeta_c-6\zeta_s)H(k)\}]$	$\frac{1}{\sqrt{3}}[2H(k)-G(k)+X\{(2+3\zeta_c-18\zeta_s)H(k)+(8-6\zeta_c-6\zeta_s)G(k)\}$
$\Xi_c^0\to\Lambda+\gamma$	$[H(k)+X\{(6-3\zeta_c-30\zeta_s)G(k)+(4+6\zeta_s)H(k)\}]$	$-[G(k)+X\{(6-3\zeta_c-30\zeta_s)H(k)+(4+6\zeta_s)G(k)\}]$
$\Omega_c^0\to\Xi^0+\gamma$	$-2[G(k)+X\{(4+2\zeta_c-8\zeta_s)G(k)+(6-\zeta_c-24\zeta_s)H(k)\}]$	$2[H(k)+X\{(4+2\zeta_c-8\zeta_s)H(k)+(6-\zeta_c-24\zeta_s)G(k)\}]$
$\Omega_{cc}^+\to\Xi_c^++\gamma$	$\frac{2}{\sqrt{6}}[G(k)-H(k)][1-X(2-3\zeta_c-6\zeta_s)]$	$\frac{2}{\sqrt{6}}[G(k)-H(k)][1-X(2-3\zeta_c-6\zeta_s)]$
$\Omega_{cc}^+\to\Xi_c'^++\gamma$	$-\sqrt{2}[G(k)+H(k)+X\{(10-\zeta_c-38\zeta_s)G(k)+(10-\zeta_c-18\zeta_s)H(k)\}$	$\sqrt{2}[G(k)+H(k)+X\{(10-\zeta_c-38\zeta_s)H(k)+(10-\zeta_c-18\zeta_s)G(k)\}]$
	$\Delta C=-\Delta S=-1\ \ (c+s\to d+u+\gamma)$	
$\Xi_c^+\to p+\gamma$	$\frac{2}{\sqrt{6}}[H(k)-2G(k)-X\{(2+3\zeta_c+12\zeta_s)G(k)+(8-6\zeta_s)H(k)\}]$	$\frac{2}{\sqrt{6}}[2H(k)-G(k)+X\{(2+3\zeta_c+12\zeta_s)G(k)+(8-6\zeta_s)G(k)\}$
$\Xi_c^0\to n+\gamma$	$\frac{2}{\sqrt{6}}[2H(k)-G(k)+X\{(8+3\zeta_c-6\zeta_s)G(k)+(2+12\zeta_s)H(k)\}]$	$-\frac{2}{\sqrt{6}}[2G(k)-H(k)+X\{(2+12\zeta_s)G(k)+(8+3\zeta_c-6\zeta_s)H(k)\}]$
$\Omega_c^0\to\Lambda+\gamma$	$-\frac{2}{\sqrt{6}}[G(k)-H(k)]\{1-2X(1-3\zeta_s)\}$	$-\frac{2}{\sqrt{6}}[G(k)-H(k)]\{1-2X(1-3\zeta_s)\}$
$\Omega_c^0\to\Sigma^0+\gamma$	$-\sqrt{2}[(G(k)+H(k))\{1+X(10+2\zeta_c-2\zeta_s)\}+2X\zeta_cH(k)]$	$\sqrt{2}[(G(k)+H(k))\{1+X(10+2\zeta_c-2\zeta_s)\}+2X\zeta_cG(k)]$
$\Omega_{cc}^+\to\Lambda_c^++\gamma$	$\frac{2}{\sqrt{6}}[G(k)-H(k)]\{1-2X(1-3\zeta_s)\}$	$\frac{2}{\sqrt{6}}[G(k)-H(k)]\{1-2X(1-3\zeta_s)\}$
$\Omega_{cc}^+\to\Sigma_c^++\gamma$	$-\sqrt{2}[(G(k)+H(k))\{1+X(10+2\zeta_c+2\zeta_s)\}+2X\zeta_cG(k)]$	$\sqrt{2}[(G(k)+H(k))\{1+X(10+2\zeta_c+2\zeta_s)\}-2X\zeta_cH(k)]$

where $\alpha^2 \approx \frac{1}{6}$ GeV^2 is fixed from the excitation energy of hyperons [18].

Using $B(\Sigma^+ \to p\gamma) = 1.25 \times 10^{-3}$ as an input [16], we obtain

$$B(\Lambda_c^+ \to \Sigma^+\gamma) = (4.54 \times 10^{-3})\% , \quad \alpha(\Lambda_c^+ \to \Sigma^+\gamma) = -0.013 , \tag{21a}$$

$$B(\Xi_c^0 \to \Xi^0\gamma) = (1.93 \times 10^{-3})\% , \quad \alpha(\Xi_c^0 \to \Xi^0\gamma) = -0.042 . \tag{21b}$$

Naively, the branching ratio may have been expected to be larger because of the $\cos^2\theta_c$ factor as well as the large photon momentum. These effects are counteracted by the Gaussian form factor appearing through the spatial integral, which amounts to having an extra fine-interaction radius term.

In the above estimate, we have considered the spatial overlap to be the same for the strangeness- and charm-changing modes, i.e., $|\psi(0)|^2 \approx 6.3 \times 10^{-3}$ GeV^3. However, $|\psi(0)|^2$ being a dimensionful quantity, it may be incorrect to ignore its variation with flavor. Evidence to corroborate this is found in quark-model [18,19] as well as in lattice calculations [20]. In fact, the charm baryons may provide a good and perhaps even dramatic way of testing the flavor dependence of the confinement forces. The absence of a dynamical theory of interactions between quarks limits our evaluation of $|\psi(0)|^2$ from first principles. Hence we make a naive estimate for the scale parameter by using a hyperfine splitting (HFS), i.e.,

$$\Delta E_{\rm HFS} = \frac{4\pi\alpha_s}{9m_1 m_2} |\psi(0)|^2 \langle \sigma_1 \cdot \sigma_2 \rangle , \tag{22}$$

which leads to

$$\frac{\Sigma_c - \Lambda_c}{\Sigma - \Lambda} = \frac{|\psi(0)|_c^2}{|\psi(0)|_s^2} \frac{\alpha_s(m_c)}{\alpha_s(m_s)} \frac{m_c - m_u}{m_s - m_u} \frac{m_s}{m_c} . \tag{23}$$

For a choice of $\alpha_s(m_c)/\alpha_s(m_s) \approx 2.7$, we get

$$\frac{|\psi(0)|_c^2}{|\psi(0)|_s^2} \approx 2.83 , \tag{24}$$

which is consistent with the estimates of (2)–(3) given by lattice calculations [20].

In addition, the HO parameter α is also expected to have flavor dependence, increasing for heavier quark systems. Following the analysis of Copley, Isgur, and Karl [19] for evaluating the excitation energies, we find $\alpha^2 \approx \frac{1}{3}$ for charm baryons. The net effect of the above scaling is to enhance the ratios in Eq. (20) to yield

$$B(\Lambda_c^+ \to \Sigma^+\gamma) = (2.91 \times 10^{-2})\% , \quad \alpha(\Lambda_c^+ \to \Sigma^+\gamma) = 0.023 , \tag{25a}$$

$$B(\Xi_c^0 \to \Xi^0\gamma) = (1.26 \times 10^{-2})\% , \quad \alpha(\Xi_c^0 \to \Xi^0\gamma) = -0.010 . \tag{25b}$$

The asymmetries remain almost unaffected by variation in the scale. The amplitude calculations for these decays indicate the major contribution to be via the parity-violating mode. It may be remarked here that if the propagators $G(k)$ and $H(k)$ in Eq. (5) are approximated using a naive nonrelativistic p/m expansion, as done for the strange baryons [9–11], the results are interestingly significant, with the asymmetry parameters showing a dramatic change. The asymmetry parameters for both $\alpha(\Lambda_c^+ \to \Sigma^+\gamma)$ and $\alpha(\Xi_c^0 \to \Xi^0\gamma)$ are enhanced by an order of magnitude to -0.27 and -0.30, respectively, with an enhanced parity-conserving contribution. An experimental observation of these results will provide a clearer insight into the validity of the improvised quark-model scheme as compared to the naive nonrelativistic approach.

TABLE III. Decay rates and asymmetry parameters for charm baryon weak radiative decays.

Process	Decay width $\times 10^9$(sec^{-1})	Asymmetry
$\Delta C = \Delta S = -1$:		
$\Lambda_c^+ \to \Sigma^+ + \gamma$	1.48	0.02
$\Xi_c^0 \to \Xi^0 + \gamma$	1.54	−0.01
$\Xi_{cc}^+ \to \Xi_c^+ + \gamma$	0.41	1.00
$\Xi_{cc}^+ \to \Xi_c'^+ + \gamma$	10.15	−0.99
$\Delta C = -1, \Delta S = 0$:		
$\Lambda_c^+ \to p + \gamma$	0.11	−0.25
$\Xi_c^+ \to \Sigma^+ + \gamma$	0.09	−0.07
$\Xi_c^0 \to \Sigma^0 + \gamma$	0.06	−0.48
$\Xi_c^0 \to \Lambda + \gamma$	0.14	−0.01
$\Xi_{cc}^+ \to \Lambda_c^+ + \gamma$	0.02	1.00
$\Xi_{cc}^+ \to \Sigma_c^+ + \gamma$	0.88	−0.99
$\Xi_{cc}^{++} \to \Sigma_c^{++} + \gamma$	0	
$\Omega_c^0 \to \Xi^0 + \gamma$	0.38	−0.79
$\Omega_{cc}^+ \to \Xi_c^+ + \gamma$	0.02	1.00
$\Omega_{cc}^+ \to \Xi_c'^+ + \gamma$	0.59	−0.99
$\Delta C = -\Delta S = -1$:		
$\Xi_c^+ \to p + \gamma$	2.38 $\tan^4\theta_c$	−0.33
$\Xi_c^0 \to n + \gamma$	2.26 $\tan^4\theta_c$	0.41
$\Omega_c^0 \to \Lambda + \gamma$	0.32 $\tan^4\theta_c$	1.00
$\Omega_c^0 \to \Sigma^0 + \gamma$	19.01 $\tan^4\theta_c$	−0.99
$\Omega_{cc}^+ \to \Lambda_c^+ + \gamma$	0.32 $\tan^4\theta_c$	1
$\Omega_{cc}^+ \to \Sigma_c^+ + \gamma$	18.16 $\tan^4\theta_c$	−0.99

Branching ratios and asymmetry parameters are similarly calculated for all Cabibbo-enhanced, -suppressed, and -doubly-suppressed decay modes and are given in Table III.

B. Inclusion of the single-quark transition

In the above estimate [Eqs. (20) and (21)], we have neglected the possible single-quark contribution to $\Sigma^+ \to p\gamma$. The presence of the single-quark transitions, though weaker in strength than the W-exchange process, has been well established in hyperon decays [16] by the nonzero $B(\Xi^- \to \Sigma^- + \gamma) = 0.227 \times 10^{-3}$. Moreover, on inclusion of the single-quark transition, the asymmetry $\alpha(\Sigma^+ \to p\gamma)$ is enhanced from -0.35, for the pure 2QP, to -0.62, which is in better agreement with the recently determined experimental value [21] of (-0.72 ± 0.10).

The 1QP is essentially a transition involving the W loop and is expected to be suppressed by the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This transition is allowed in the standard model and is not removed with any renormalization as the photon is emitted at short distances $\sim 1/m_W$ by internal quarks or the W boson [22]. It arises out of the matrix element

$$H_W^{1q} = \frac{eG_F \sin\theta_c \cos\theta_c}{\sqrt{2}} \bar{q}'(a+b\gamma_5)\not{k}\not{\epsilon} q \ , \tag{26}$$

where the parameters a and b govern the pc and pv contributions, respectively. These parameters have been predicted in various models, but are still uncertain. In the Glashow-Wienberg-Salam electroweak gauge theory [22], the ratio $b/a \approx \frac{1}{4}$ and a is of the order of 10^{-5}. In the presence of short-distance QCD corrections [23] due to gluonic exchange, a is enhanced by about two orders of magnitude to -0.96×10^{-2} and the ratio b/a becomes $+1$. Long-distance strong-interaction effects [24] are expected to effectively lower its value to $b/a = -\frac{1}{3}$. We studied the effect of varying the ratio b/a from -1 to $+1$ on the scale parameter I_{2q}. Using $B(\Xi^- \to \Sigma^- + \gamma)$ and $B(\Sigma^+ \to p\gamma)$ as inputs, we find $a \approx (-3.25$ to $-4.60)\times10^{-2}$ GeV and that I_{2q} lies in the range $(5.5-6.8)\times10^{-3}$ GeV^3, shown in Fig. 1. This then yields Cabibbo-enhanced modes in the range

$$\begin{aligned} B(\Lambda_c^+ \to \Sigma^+\gamma) &= [(2.19-3.40)\times10^{-2}]\% \ , \\ B(\Xi_c^0 \to \Xi^0\gamma) &= [(0.96-1.48)\times10^{-2}]\% \ . \end{aligned} \tag{27}$$

The single-quark contribution in the charm-changing decays proceeds via the $c \to u + \gamma$ transition. This process, which occurs through SU(3) breaking, is expected to be suppressed by at least an order of magnitude relative to the corresponding $s \to d + \gamma$ transition in the strange baryons which occurs due to SU(4) breaking. This single-quark transition, which is present only in suppressed charm-changing decays, is neglected for the present analysis. It may be remarked here that the decay $\Xi_{cc}^{++} \to \Sigma_c^{++}\gamma$ is forbidden by the W-exchange process and can proceed only through a 1QP. The signal for this decay will be able to provide some estimate for the strength of the charm-changing single-quark transitions.

IV. QCD MODIFICATIONS

As a consequence of introducing QCD short-distance gluon exchange, the four-Fermi interaction gets modified to

$$H_w^{\rm QCD} = \frac{eG_F(\text{Cabibbo factors})}{\sqrt{2}} [c_1 \bar{q}_3 \Gamma_\mu q_1 \, \bar{q}_4 \Gamma^\mu q_2 + c_2 \bar{q}_4 \Gamma_\mu q_1 \, \bar{q}_3 \Gamma^\mu q_2] \ , \tag{28}$$

where $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$ and $c_1 = (c_+ + c_-)/2$, $c_2 = (c_+ - c_-)/2$ represent combinations of the QCD coefficients c_- and c_+. In the leading-logarithmic approximation, these are given by

$$c_\pm(\mu) = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(m_W^2)} \right]^{d_\pm/2b} \ , \tag{29}$$

with $d_- = -2d_+ = 8$ and $b = 11 - \frac{2}{3}N_f$, N_f being the number of flavors, μ the mass scale, and α_s is the strong fine-structure constant. The precise value of these QCD coefficients is difficult to assign, depending as they do on the mass scale and $\Lambda_{\rm QCD}$. In the free-field limit, $c_+ = c_- = 1$, but at the charm mass scale, these quantities are estimated to give substantial enhancements for $c_- \approx 1.3-2.1$ and suppressions for $c_+ \approx 0.6-0.9$ [8].

This then alters the Hamiltonian in Eq. (3), which, written in terms of spin operators and for a fixed photon helicity $\lambda_\gamma = +1$, is given below, up to an overall scale $[eG_F(\text{Cabibbo factors})/\sqrt{2}]|\mathbf{k}|$,

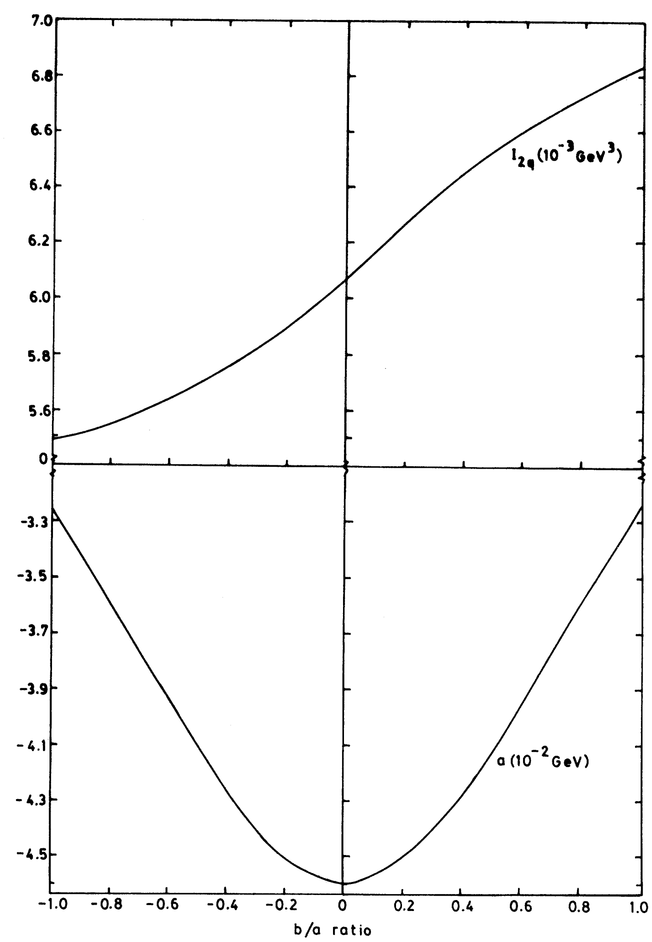

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FIG. 1. Variation of spatial integral I_{2q} with b/a ratio.

$$H^{\rm pc}_{w_{\rm QCD}}=c_1[Aq_3^\dagger\sigma_-q_1\,q_4^\dagger q_2+Bq_3^\dagger q_1\,q_4^\dagger\sigma_-q_2+Cq_3^\dagger\sigma_-q_1\,q_4^\dagger\sigma_3q_2+Dq_3^\dagger\sigma_3q_1\,q_4^\dagger\sigma_-q_2]$$
$$+c_2[A'q_4^\dagger\sigma_-q_1\,q_3^\dagger q_2+B'q_4^\dagger q_1\,q_3^\dagger\sigma_-q_2+C'q_4^\dagger\sigma_-q_1\,q_3^\dagger\sigma_3q_2+D'q_4^\dagger\sigma_3q_1\,q_3^\dagger\sigma_-q_2]\,, \qquad (30a)$$

$$H^{\rm pv}_{w_{\rm QCD}}=c_1[Cq_3^\dagger\sigma_-q_1\,q_4^\dagger q_2+Dq_3^\dagger q_1\,q_4^\dagger\sigma_-q_2+Aq_3^\dagger\sigma_-q_1\,q_4^\dagger\sigma_3q_2+Bq_3^\dagger\sigma_3q_1\,q_4^\dagger\sigma_-q_2]$$
$$+c_2[C'q_4^\dagger\sigma_-q_1\,q_3^\dagger q_2+D'q_4^\dagger q_1\,q_3^\dagger\sigma_-q_2+A'q_4^\dagger\sigma_-q_1\,q_3^\dagger\sigma_3q_2+B'q_4^\dagger\sigma_3q_1\,q_3^\dagger\sigma_-q_2]\,, \qquad (30b)$$

where A', B', C', and D' are

$$A'=+G(k)-(Q'_-H(k)+P'_+G(k))\,,\quad B'=-G(k)+(P'_-H(k)+Q'_+G(k))\,,$$
$$C'=-H(k)+(P'_+H(k)+Q'_-G(k)),\quad D'=+H(k)-(Q'_+H(k)+P'_-G(k))\,, \qquad (31)$$

and

$$P'_\pm=\frac{k^0}{24}\left[\frac{-5}{m_3}\pm\frac{1}{m_2}\right]\quad\text{and}\quad Q'_\pm=\frac{k^0}{24}\left[\frac{-5}{m_4}\pm\frac{1}{m_1}\right]\,.$$

For the radiative weak decays, the effect of these QCD modifications is to alter the decay amplitudes in Eqs. (10) and (11) by an overall scale of $c_-(\mu)$. The presence of c_- in the overall scale may be understood by noting that the portion of the Hamiltonian corresponding to c_+ is symmetric in color indices and hence does not contribute. For a choice of $c_-(m_s)=2.80$ and $c_-(m_c)=1.84$ at $\Lambda^2_{\rm QCD}=0.1$ GeV determined using Eq. (22), the decay rates for the charm-changing modes are scaled down by a factor $[c_-(m_c)/c_-(m_s)]^2\approx0.43$.

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